COMENIUS UNIVERSITY

Faculty of Mathematics and Physics

Physics Preprint No. **Ph3-94**

November 1994

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Abstract
Dimension of region emitting particles in nuclear collisions can be determined by Hanbury-Brown and Twiss (HBT) correlations of identical particles. In this note we demonstrate that this dimension as determined by HBT correlation of Bremsstrahlung photons is larger than the actual one due to the photon formation length. Qualitatively $r^2 \rightarrow r^2 + a/\omega^2$ where the last term is due to the formation length being proportional to $1/\omega$, where ω is the energy of the photon. The phenomenon permits to "see" experimentally the photon formation length.

The conception of the photon formation length has been known since the classical papers by Landau, Pomeranchuk and Migdal [1]. An elementary explanation of the concept can be found e.g. in Ref.[2]. Formation time $\mathbf{t_f}$ and formation length $\mathbf{l_f}$ for the production of the photon with momentum $\vec{k}$ and energy ω. (in units $\hbar = c = 1$) by a charged particle moving with velocity $\vec{v}$ are given by the expressions [2]

$$t_f = \frac{1}{\omega(1-\vec{n}.\vec{v})} \qquad l_f = \frac{v}{\omega(1-\vec{n}.\vec{v})} \tag{1}$$

where $\vec{n} = \vec{k}/\omega$ is the direction of photon.
Since the emission of a soft Bremsstrahlung photon is essentially a classical phenomenon, the expression in Eq.(1) follows directly from classical electrodynamics [3] without any further assumptions. If a charged particle undergoes multiple scattering in a medium with the mean free path between successive collisions being smaller or of the same order as $\mathbf{l_f}$, the resulting production of soft photons is smaller than what one would expect from the incoherent sum of Bremsstrahlung in individual collisions. The formation length and formation time concept is consistent with the Low theorem [4].
According to the Landau-Pomeranchuk effect and Low theorem very soft photons observed in hadronic or heavy-ion collisions are dominated by the Bremsstrahlung emission off initial or final state particles [2], contributions from intermediate stages being suppressed.
The standard formula for a **single** soft photon emission in hadronic collision leading to multiparticle production reads as follows [2,5]

$$\omega \frac{d\sigma^{\gamma}}{d^3k} = \int \frac{d^3p_1}{E_1} \cdots \frac{d^3p_n}{E_n} E_1 \ldots E_n \frac{d^{3n}\sigma^{h}}{d^3p_1 \ldots d^3p_n} \frac{\alpha}{4\pi^2} \left| J_\mu \varepsilon^\mu \right|^2 \tag{2}$$

where

$$Y_1 \equiv \varepsilon_\mu J^\mu = \sum_{i=1}^{n} Q_i \frac{(\varepsilon . p_i)}{(p_i . k)} - Q_a \frac{(\varepsilon . p_a)}{(p_a . k)} - Q_b \frac{(\varepsilon . p_b)}{(p_b . k)} \tag{3}$$

Superscripts γ, h refer to photon and hadron production respectively, p_i, i = 1,2,...n are momenta of final state hadrons, $(\omega,\vec{k})$ is the four-momentum of the photon, ε denotes its polarisation, indices a,b refer to incoming particles, Q_i (Q_a,Q_b) are charges of outgoing (incoming) particles.
The recipe for calculating Bremsstrahlung photon emission for a given hadronic initial and final state is thus simple. For each outgoing hadron there is the contribution $Q_i(\varepsilon.p_i)/(k.p_i)$ to the amplitude (for the ingoing the sign is changed), all the contributions are added coherently and the sum is squared.
One can ask where has the photon been produced? According to Eq.(1) the photon formation length for $\omega = 20$ MeV is 10 fm and for $\omega = 10$ MeV it is even 20 fm what is much larger than the standard dimensions of the system. Dimension of the system from which soft photons has been produced can be seen from the Hanbury-Brown and Twiss interferometry of two soft photons.
In analogy to the expression $\varepsilon_\mu J^\mu$ the amplitude for the emission of two soft photons with the same polarisation is obtained by the same procedure as above with the result

$$Y_2 = \sum_{i \neq j} Q_i \frac{(\varepsilon \cdot p_i)}{(k_1 \cdot p_i)} Q_j \frac{(\varepsilon \cdot p_j)}{(k_2 \cdot p_j)} + \sum_i Q_i \frac{(\varepsilon \cdot p_i)}{(k_1 \cdot p_i)} Q_i \frac{(\varepsilon \cdot p_i)}{(k_2 \cdot p_i)} \tag{4}$$

In this expression we have left out the incoming particles, but they can be easily included. The first term in the r.h.s. corresponds to the emission of photons by two different external legs, the second term to the emission of two photons by the same leg [5].
The expression for the two photon emission is then obtained from Eq.(2) by replacing $Y_1 = \varepsilon_\mu J^\mu$ by Y_2 and $\omega\, d\sigma^\gamma/d^3k$ by $\omega_1\omega_2 d\sigma^{\gamma\gamma}/d^3k_1 d^3k_2$.
Both cross-sections for one- and two-photon emission correspond to the situation when all charged particles in the final state are emitted from the origin of the coordinate system and at the same time. In order to include the possible emission of charged particles from different positions we have to make the replacement

$$Q_i \frac{(\varepsilon \cdot p_i)}{(k_1 \cdot p_i)} \to e^{ik_1 x}\; Q_i \frac{(\varepsilon \cdot p_i)}{(k_1 \cdot p_i)} \tag{5}$$

This follows either from the classical electrodynamics as discussed in Balek et al.[2] or from the fact that when a state $|\Psi>$ with four-momentum is transferred by a

distance x, the state is changed according to $|\Psi> \rightarrow \exp(ik.x)\,|\Psi>$ where $k.x = \omega t - \vec{k}.\vec{r}$.
To get the expression corresponding to the situation to which HBT interference applies [6,7] we have to introduces phase factors $\exp(i\Phi_i)$ which permit express the phase randomness of the emission of charged particles which bremsstrahl the photons. Each factor in Eq.(5) is thus multiplied by $\exp(i\Phi_i)$. As a net result we have (assuming $t_i = 0$ and the same polarisation of both photons)

$$Y_2(\vec{k}_1,\vec{k}_2) = \sum_{i,j} e^{i(\Phi_i+\Phi_j)}\, e^{-i\vec{k}_1\vec{r}_i}\, e^{-i\vec{k}_2\vec{r}_j}\, Q_i Q_j \frac{(\varepsilon.p_i)(\varepsilon.p_j)}{(k_1.p_i)(k_2.p_j)} \qquad (6)$$

with the single photon amplitude being given as

$$Y_1(\vec{k}) = \sum_i e^{i\Phi_i}\, e^{-i\vec{k}.\vec{r}_i}\, Q_i \frac{(\varepsilon.p_i)}{(k.p_i)} \qquad (7)$$

To be quite complete we should still add to each point i from which a charged particle is emitted the square-root of the corresponding intensity but this can be simulated also by the density of emitters so we have not inserted that explicitly.
Under the assumption of complete incoherence [6,7]

$$\langle\, e^{i(\Phi_i-\Phi_j)} \,\rangle = \delta_{ij} \qquad (8)$$

$$\langle\, e^{i(\Phi_i+\Phi_j-\Phi_k-\Phi_l)} \,\rangle = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \qquad (9)$$

the correlation function

$$R(k_1,k_2)= \frac{\langle\, |Y_2(k_1,k_2)\,|^2\rangle}{\langle\, |Y_1(k_1)\,|^2\rangle\langle\, |Y_1(k_2)\,|^2\rangle} \tag{10}$$

can be rewritten as

$$R(k_1,k_2)= \frac{P(k_1,k_2)}{P(k_1)\ P(k_2)} = 1+ \frac{G(k_1,k_2)}{P(k_1,k_2)} \tag{11}$$

where

$$P(k) = \langle\, |Y_1(k)\,|^2\rangle = \sum_i \left[Q_i \frac{\varepsilon . p_i}{k . p_i} \right]^2 \tag{12}$$

$$P(k_1,k_2)= \langle\, |Y_2(k_1,k_2)\,|^2\rangle = P(k_1)\ P(k_2)+ G(k_1,k_2)$$

$$G(k_1,k_2)= \sum_{i,j} e^{-i\left[\vec{k}_1-\vec{k}_2\right]\left[\vec{r}_i-\vec{r}_j\right]}\ Q_i^2 Q_j^2 \frac{(\varepsilon . p_i)}{(k_1 . p_i)} \frac{(\varepsilon . p_j)}{(k_2 . p_j)} \frac{(\varepsilon . p_i)}{(k_2 . p_i)} \frac{(\varepsilon . p_j)}{(k_1 . p_j)} \tag{13}$$

The terms $(\varepsilon.p_i)/(k.p_i)$ can be simplified. Taking $\varepsilon_0 = 0$, $k = (\omega,\vec{k})$, $p_i = (E_i,\vec{p}_i)$ we obtain

$$\frac{(\varepsilon . p_i)}{(k . p_i)} = - \frac{\vec{\varepsilon} . \vec{v}_i}{\omega(1 - \vec{n} . \vec{v}_i)} \tag{14}$$

where $\vec{n} = \vec{k} / \omega$ is the direction in which the photon moves and $\vec{v}_i = \vec{p}_i / E_i$ is the velocity of the charged particle emitting the photon.

The instructive case of HBT interference of Bremsstrahlung photons emitted by two charged particles will be discussed in a separate paper. Here we shall describe only a simple situation, corresponding to the emission of two photons from a spherical "fireball". Charged particles are produced at the same time $t_i = 0$ from the surface of a sphere with radius R. The magnitude of their velocities is fixed at the same value but the direction of the velocity is chosen randomly within the semi-sphere above the plane tangential to the sphere emitting charged particles at the point from which the charged particle is emitted.
The correlation function is calculated numerically by Eq.(11). The calculation is done numerically for both photons having the same energy $\omega_1 = \omega_2 = \omega$. Using the notation

$$\vec{k}_1 - \vec{k}_2 = \Delta\vec{k}$$
$$\vec{k}_1 + \vec{k}_2 = \vec{K} \qquad (15)$$

we compute the correlation function $R(\vec{k}_1, \vec{k}_2)$ and approximate it for small values of $\Delta\vec{k}$ as

$$R(\vec{k}_1, \vec{k}_2) \approx 1 + \cos(\Delta\vec{k}.\vec{r}_{eff}) \approx 2 - \frac{1}{2}(\Delta\vec{k}.\vec{r}_{eff})^2 \qquad (16)$$

After having computed $R(\vec{k}_1, \vec{k}_2)$ as described above we fit the results at small values of $\Delta\vec{k}$ by Eq.(16) and obtain the value of the adjustable parameter r_{eff}.

In Fig.1 we present the correlation function $R(\vec{k}_1, \vec{k}_2)$ as a function of $|\Delta\vec{k}|$ for two photons bremsstrahled by charged particles emitted from the surface of the sphere with radius R = 6 fm for a set of values of $\omega = \omega_1 = \omega_2$. Polarisation of both photons are the same and perpendicular to the plane given by $\vec{k}_1$ and $\vec{k}_2$. As expected peaks are getting narrower for smaller values of ω.
In Fig.2 we give the dependence of r_{eff} on ω, with r_{eff} being calculated by using Eq.(16). The result is as expected. For small ω the photon formation length represents the dominant contribution to r_{eff}, whereas for large values of ω r_{eff} approaches the radius of the region from which the charged particles are emitted. It should be stressed again that the formation length of the photon has not been inserted by hand into the formalism. It entered into the r_{eff} only on the basis of standard expressions for Bremsstrahlung emission by charged particles. Further aspects of this phenomenon will be given in a more detailed paper.

Before concluding let us present some comments. HBT interferometry of soft photons produced in heavy-ion collisions made possible by the TAPS spectrometer [8] may be an ideal place for studying this phenomenon. It seems that in heavy ion collisions at these energies Bremsstrahlung dominates [8] below and above the photon energy range containing contributions from exited nuclei and fragments.
Recent experiments [9] with heavy ion collisions of the NA-44 collaboration at the CERN-SPS have shown that the dimensions of the volume - as determined by HBT interferometry of positive pions - decrease with increasing transverse mass. The effect goes in the same direction as the one described above. The formation length of pions has been frequently discussed in the literature, see e.g. Refs.[10]. Still, the concept of the formation length of the pion is much less understood than that of the photon and at present we do not see any convincing reasons that the phenomenon discussed above is connected with the most interesting results [9] obtained by the NA-44 collaboration.
It has been recently pointed out that the intermittent behaviour observed in multiparticle production is to some extent due to the HBT interference of identical particles - for recent review see Ref.[11]. Possible analogies between photon and pion formation length could make the efficient region of the emission of pions rather large for soft pions (see Fig.2) in the sense indicated by Bialas and Ziaja [12]. Although being perhaps attractive this points remains at present only at a level of speculation.
In the present note we have started with well known dynamics of soft photon production and we have shown that the photon formation length is experimentally observable via HBT interfometry. For the case of soft pion production one could turn the situation the other way round. A high statistics study of HBT correlations of soft pions in a relatively clean situation, like in pp collisions at low energy could bring a very valuable information on the pion formation length and thus on the mechanism of the pion production in hadronic collisions. The $\bar{p}p$ collisions at low energy or at rest procedes probably via different mechanism [13] and pion formation length effects may be absent.
Note finally that two-photon interferometry in hadronic collisions has already been studied theoretically [17], but to our knowledge the emphasis has not been put onto the interferometry of the soft Bremsstrahlung photons.
In our calculations we have neglected the contribution due to Bremsstrahlung off incoming particles. This issue deserves further attention in particular in case of strong coherent effects [15].

In concluding we can state that the soft photon formation length may be experimentally studied by HBT interference of soft photons.

Acknowledgements On of the authors (J.P.) is indebted to Guy Roche and J.C.Montret for the hospitality extended to him at the University of Blaise Pascal at Clermont-Ferrand and at the Laboratoire de Physique Corpusculaire at this University where J.P. and N.P. started to work on this topic. The authors are grateful to V.Černý, C.Fabjan, P.Filip, J.Ftáčnik, R.Hwa, P.Lichard, R.Lietava, A.Nogová and G.Roche for valuable comments and correspondence.

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Figure Captions

Fig.1 Correlation functions computed according to Eqs.(11-13) for a few values of ω ($= \omega_1 = \omega_2$) as a function of $|\Delta\vec{k}| = |\vec{k}_1 - \vec{k}_2|$. Solid line $\omega = 80$ MeV, $r_{eff} \cong 6{,}00$ fm, long dash: $\omega = 30$ MeV, $r_{eff} = 11$ fm, shorter dash: $\omega = 15$ MeV, $r_{eff} = 20$ fm, very short dash: $\omega = 7$ MeV, $r_{eff} = 39$ fm. The r_{eff} has been calculated by Eq.(16).

Polarisation of both photons is linear and parallel to $\vec{k}_1 x \vec{k}_2$. The radius of the sphere emitting charged particles R = 6 fm, velocity of outgoing particles corresponds to pions with momentum 400 MeV/c.

Fig.2 Dependence of r_{eff} as a function of both photons energy ω calculated by Eq.(16) from data given in Fig.1 completed by some other values of ω . The dashed line is given by $r_{eff} = (R^2 + a/\omega^2)^{1/2}$ with $a = 1.86\ (\hbar c)^2$.

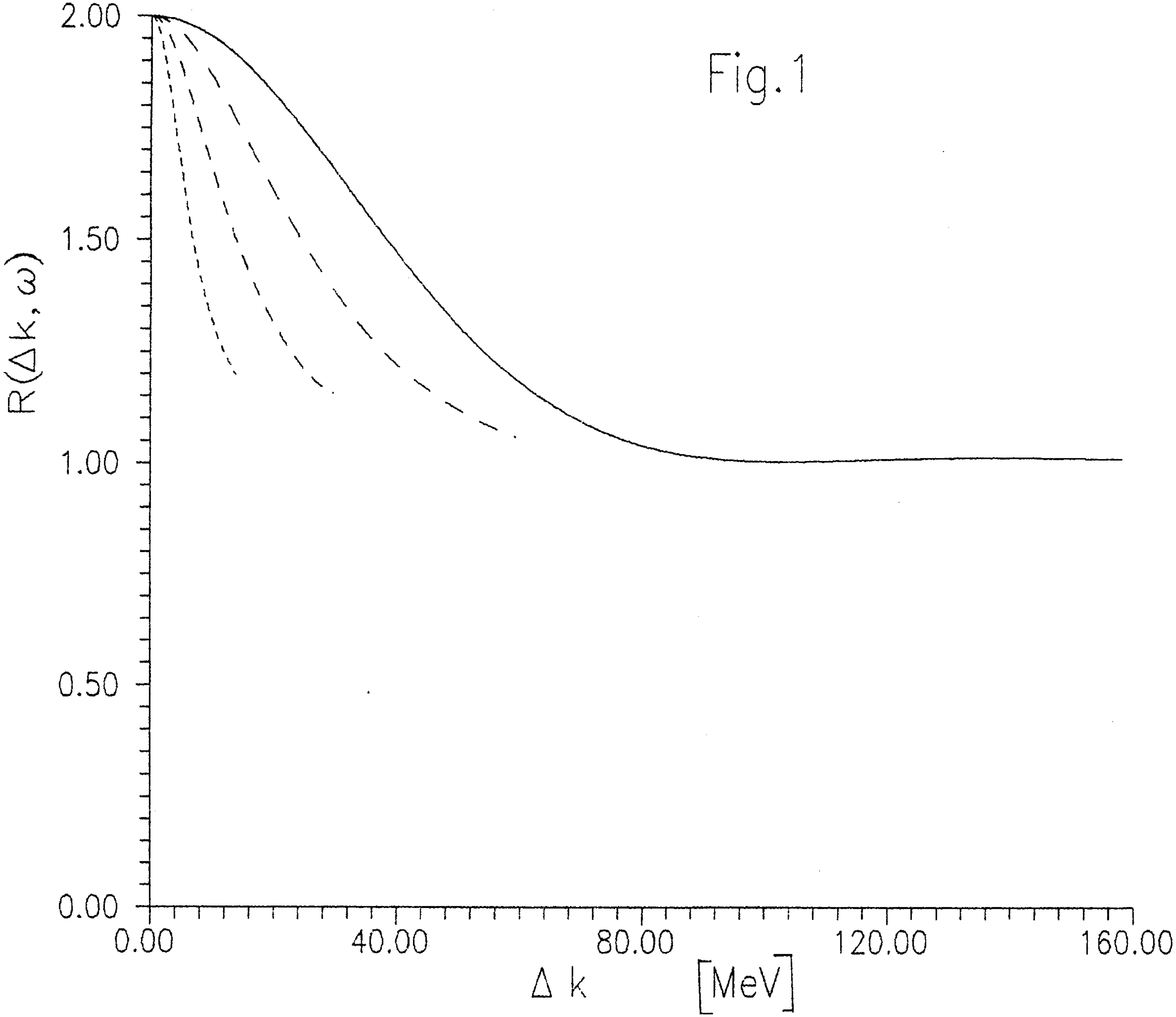
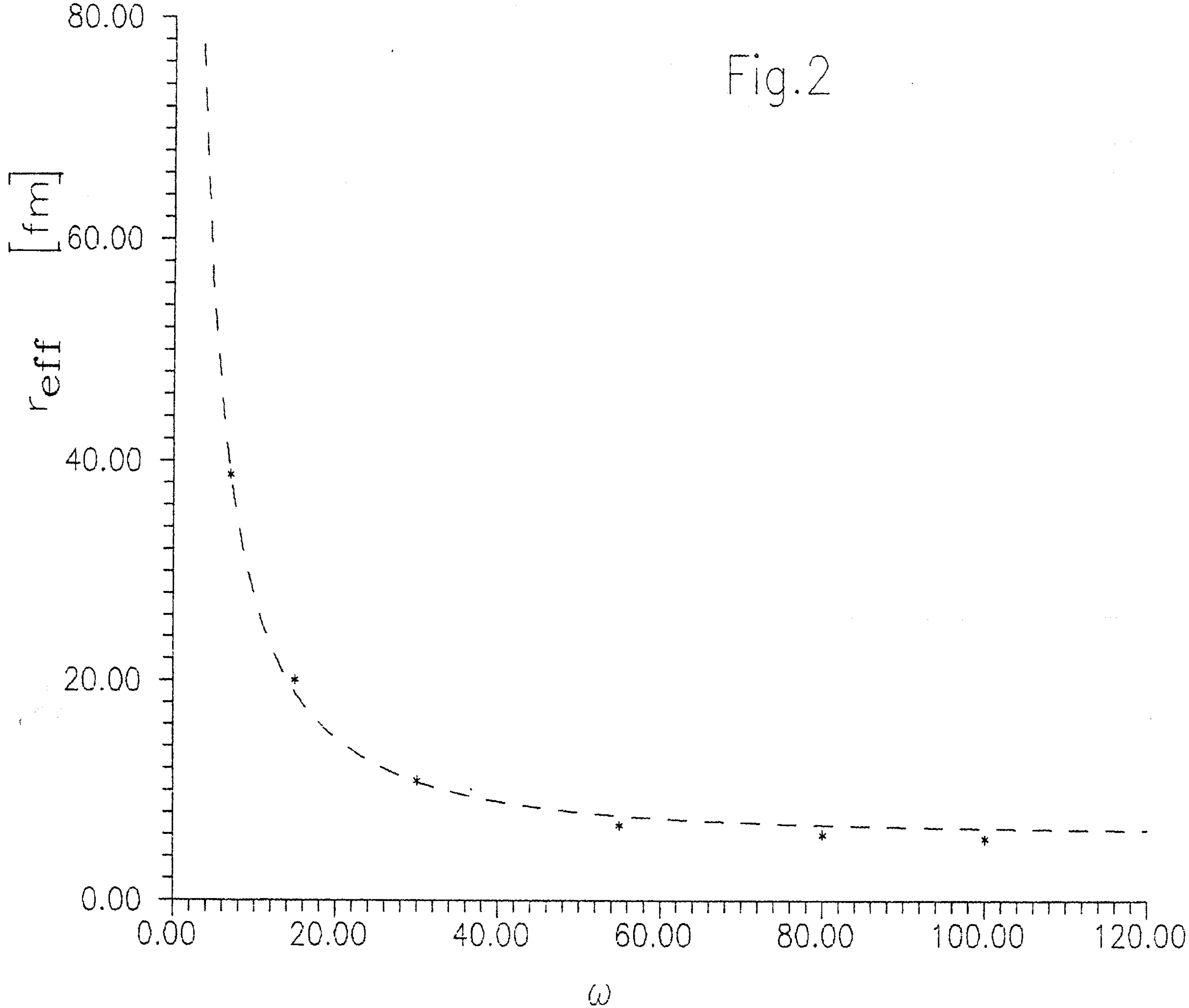


Fig. 1

Fig.2



80.00
60.00
40.00
20.00
0.00
r_{eff} [fm]
0.00
20.00
40.00
60.00
80.00
100.00
120.00
ω