Origins of the method to determine the CKM angle γ using $B^{\pm} \to DK^{\pm}$, $D \to K^0_S\pi^+\pi^-$ decays

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Abstract

The angle γ of the Cabibbo–Kobayashi–Maskawa unitarity triangle is a benchmark parameter of the Standard Model of particle physics. A method to determine γ from $B^+ \to DK^+$ with subsequent $D \to K^0_S\pi^+\pi^-$ or similar multibody decays has been proven to provide good sensitivity. We review the first discussions on the use of this technique, and its impact subsequently. We propose that this approach should be referred to as the BPGGSZ method.

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The angle $\gamma = \arg\left(-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*}\right)$ of the unitarity triangle formed from elements of the Cabibbo–Kobayashi–Maskawa quark mixing matrix [1,2] is a benchmark parameter of the Standard Model of particle physics.[1] The value of γ is a measure of the extent to which the *CP* symmetry between particles and antiparticles is violated in the weak interactions of quarks. It can be determined with negligible theoretical uncertainty from measurements of decay rates and *CP*-violating asymmetries in processes where interference between $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions can occur [3]. The archetypal example, which is also the most experimentally accessible, is that of $B^+ \to DK^+$ decays, where D indicates a neutral D meson that is an admixture of D^0 and $\overline{D}^0$ states, but the same concepts are valid also for related channels such as $B^+ \to D^*K^+$, $B^+ \to DK^{*+}$ and $B^0 \to DK^{*0}$. The observables (relative decay rates and *CP*-violating asymmetries) can be expressed in terms of γ, r_B and δ_B, where r_B is the relative magnitude of the $b \to u\bar{c}s$ and $b \to c\bar{u}s$ transition amplitudes and δ_B is their relative strong (*i.e.* *CP*-conserving) phase. Detailed reviews on the determination of γ from such processes can be found, for example, in Refs. [4–8].

Initial discussions of the use of this approach to obtain experimental sensitivity to γ focussed on the case where the neutral D meson decays to a *CP* eigenstate, such as (*CP*-even) K^+K^-, $\pi^+\pi^-$ or (*CP*-odd) $K^0_S\pi^0$ [9,10]. In this approach, now widely known as the GLW method, the amplitudes for D^0 and $\overline{D}^0$ decays to the final states of interest are related trivially, under the assumption of negligible effects of *CP* violation in the D system. The B decay observables can then be expressed in terms of the unknown parameters $(\gamma,\ r_B,\ \delta_B)$ only. To determine γ from this method alone, however, requires sensitivity to observe the relatively small *CP* asymmetries, expected to be comparable in magnitude to r_B, which is of order 0.1 for $B^+ \to DK^+$ decays. Moreover, such a determination of γ suffers from trigonometric ambiguities since there are only three linearly independent observables in the GLW method, even in the case that both *CP*-even and *CP*-odd D decay final states are used.

To overcome these issues, it is essential to include D decays to non-*CP* eigenstates. The use of $B^+ \to DK^+$ with subsequent $D \to K^\mp\pi^\pm$ decays was noted as being particularly valuable [11], since both doubly Cabibbo-suppressed (*e.g.* $\overline{D}^0 \to K^-\pi^+$) and Cabibbo-favoured (*e.g.* $D^0 \to K^-\pi^+$) amplitudes contribute. The smallness of the relative magnitude of these amplitudes, denoted r_D, complements the size of r_B, so that larger *CP* asymmetries are possible in $B^+ \to DK^+$ decays. Both r_D and the relative strong phase between the D decay amplitudes, δ_D, can be determined independently of the B decay data, so that the observables depend on the same set of unknown parameters $(\gamma,\ r_B,\ \delta_B)$ as in the GLW case. This approach is now widely known as the ADS method. While the two-body $D \to K^\mp\pi^\pm$ decays are particularly attractive experimentally, the ADS method can also be applied for multibody decays such as $D \to K^\mp\pi^\pm\pi^0$ and $D \to K^\mp\pi^\pm\pi^+\pi^-$ [11,12].

Before the start of the BaBar and Belle experiments, γ was expected to be constrained only weakly by the anticipated data [13]. As the first results with the GLW and ADS methods were published [14–17], their precision confirmed that much larger data samples would be necessary to constrain γ to the 10° level or better.

[1] An alternative notation, $\phi_3 \equiv \gamma$, is also in widespread use in the literature.

This situation motivated the investigation of further decay modes that could be usefully employed to determine γ. Several authors noted that the available statistics could be increased by using multibody final states, including those from doubly Cabibbo-suppressed [12] and singly Cabibbo-suppressed [18] D decays. However, these focussed either on inclusive approaches, in which the whole phase-space was integrated over, or on contributions from particular resonances. The key point of how interference between different resonances in the Dalitz plot of a multibody D decay could be exploited to measure γ was not realised until it was proposed to use decays into self-conjugate multibody final states, such as $K^0_S\pi^+\pi^-$. This method has proven over time to have very good sensitivity. The original work was performed separately and independently by two groups. Bondar and Poluektov developed their ideas within the Belle collaboration, while Giri, Grossman, Soffer and Zupan developed theirs for a theoretical paper.

The first presentation on such a method was made by Alex Bondar at an internal Belle collaboration workshop in September 2002 [19]. The slides of this presentation, which have not previously been available publicly, are included in Appendix A. The idea was inspired by the Dalitz plot analysis of $D^0 \to K^0_S\pi^+\pi^-$ decays by the CLEO collaboration that had been published earlier in 2002 [20], in which it was demonstrated that the relative magnitude and phase of the doubly Cabibbo-suppressed $D^0 \to K^{*+}\pi^-$ and Cabibbo-favoured $D^0 \to K^{*-}\pi^+$ decay amplitudes could be determined from a sample of flavour-tagged D mesons. If the same relative phase could be measured separately in samples of neutral D mesons from $B^+ \to DK^+$ and charge-conjugate decays, the difference between these two quantities could be used to obtain 2γ. The solution thus obtained would be unique in the range $[0, \pi]$. The concept was developed further, subsequently to the initial presentation, and implemented with the Belle data by Anton Poluektov. Due to the strong competition between Belle and BaBar at that time, it was decided not to publish the method as a standalone paper, but rather to describe the approach together with the experimental results in a Belle collaboration publication. The first results were presented in preliminary form at the Lepton Photon conference in August 2003 [21], and published not long thereafter [22].

The first publication of the idea was the paper by Anjan Giri, Yuval Grossman, Abner Soffer, and Jure Zupan [23]. This was made available on the arXiv preprint server in March 2003, and presented at the CKM 2003 Workshop in early April, prior to publication. In addition to discussing that γ could be determined from a model-dependent fit to the Dalitz plot distribution of $D \to K^0_S\pi^+\pi^-$ decays produced in $B^+ \to DK^+$ and charge conjugate processes, Ref. [23] proposed a model-independent approach based on binning the Dalitz plot. This model-independent approach removes a potentially large source of uncertainty originating from the description of the *a priori* unknown strong phase variation across the Dalitz plot, at the cost of some statistical precision due to the finite size of the bins. Each bin has associated with it a set of hadronic parameters, corresponding to the r_D and δ_D parameters of the ADS method, that can in principle be determined independently of the B decay data, so that again the only unknowns to be found are $(\gamma,\ r_B,\ \delta_B)$. In particular, Ref. [23] introduced the c_i and s_i parameters, which are the amplitude-weighted averages of the cosine and sine of the strong phase difference between D^0 and $\overline{D}^0$ decay amplitudes within Dalitz plot bin i. These parameters can be determined from quantum-correlated $\psi(3770) \to D\overline{D}$ decays.

Both model-dependent and model-independent variants of the method to determine γ from $B^{\pm} \to DK^{\pm}$ with multibody D decays, which we propose to refer to as the BPGGSZ method (see Appendix B for a recommendation on appropriate citations), have been intensively pursued by experiments.[2] The most recent results from the BaBar [24], Belle [25] and LHCb [26] collaborations with the model-dependent approach, using $D \to K^0_S\pi^+\pi^-$ decay models obtained in Refs. [27,28],[3] have statistical uncertainties on γ as low as 12–15° and model uncertainties that vary in the range 3–9° depending on how conservative a range of model variations is considered. The latest results from the Belle [30] and LHCb [31,32] collaborations with the model-independent approach have uncertainties on γ of 10–15°. A small contribution to this, around 4°, is due to the limited precision with which the c_i and s_i parameters have been measured using data from the CLEOc experiment [33,34] following the scheme for binning of the Dalitz plot proposed in Refs. [35,36].[4] Further improvement in precision can be anticipated as the existing LHCb data sample is analysed, and as much larger data samples are collected in the future with upgrades of the LHCb detector [39–42] and with the Belle II experiment [43,44].

In addition to its use in the $B^+ \to DK^+$, $D \to K^0_S\pi^+\pi^-$ decay chain, the BPGGSZ method has also been used with $B^+ \to D^*K^+$ [24,25], $B^+ \to DK^{*+}$ [24,28] and $B^0 \to DK^{*0}$ [45–47] decays. In addition, D decays to the $K^0_S K^+K^-$ [24,31,32] and $K^0_S\pi^+\pi^-\pi^0$ [48, 49] final states have been exploited and application of the method with other self-conjugate multibody decays is likely in the future. Moreover, the BPGGSZ method has inspired similar methods to determine additional important quantities in heavy flavour physics. Analysis of $B \to D\pi^0$ and similar decays with $D \to K^0_S\pi^+\pi^-$ and other self-conjugate multibody final states can be used to determine the angle β of the unitarity triangle [50]; this method has been implemented in both model-dependent [29,51] and model-independent [52] variants. A "double Dalitz plot" analysis for $B \to D\pi^+\pi^-$ with $D \to K^0_S\pi^+\pi^-$ decays has also been proposed [53], building on a similar concept for γ determination [54,55]. Binning of the $D \to K^0_S\pi^+\pi^-$ Dalitz plot has also been noted to have highly advantageous feature for the experimental determination of D meson mixing parameters from these decays [56–58]. Results obtained with these methods [59,60] currently provide the best sensitivity out of all charm mixing measurements to the mass difference between the neutral D meson eigenstates.

In conclusion, the development of the BPGGSZ method enabled significant improvement in the measurement of γ, removing ambiguities in its determination. This permitted global fits to the parameters of the CKM matrix, allowing for, and deriving stringent bounds on, contributions from physics beyond the SM in loop-dominated processes [61,62]. Such analyses proved that the dominant contributions to flavour-changing processes and to *CP*

[2] This approach has until now been referred to in a variety of ways, sometimes using the GGSZ acronym.

[3] An updated study of the $D \to K^0_S\pi^+\pi^-$ decay amplitude has been published in Ref. [29], but not yet used in any γ determination.

[4] More precise measurements of the c_i and s_i parameters have recently been reported by the BESIII collaboration [37,38], but not yet used in any γ determination.

violation in meson decays are those of the Standard Model, as recognised by the award of the 2008 Nobel Prize in Physics to Kobayashi and Maskawa. Results obtained with the BPGGSZ method continue to provide some of the most precise constraints on γ that enter today's global fits [4, 5], and are expected to continue to do so in the future.

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A First presentation of method to determine γ from $B^+ \to DK^+$, $D \to K_S^0\pi^+\pi^-$ decays

Figures 1– 5 contain the slides of the presentation at the Belle collaboration workshop in September 2002 [19], in which the concept of measuring γ (denoted ϕ_s in the slides) from $B^+ \to DK^+$, $D \to K_S^0\pi^+\pi^-$ decays was first set out.

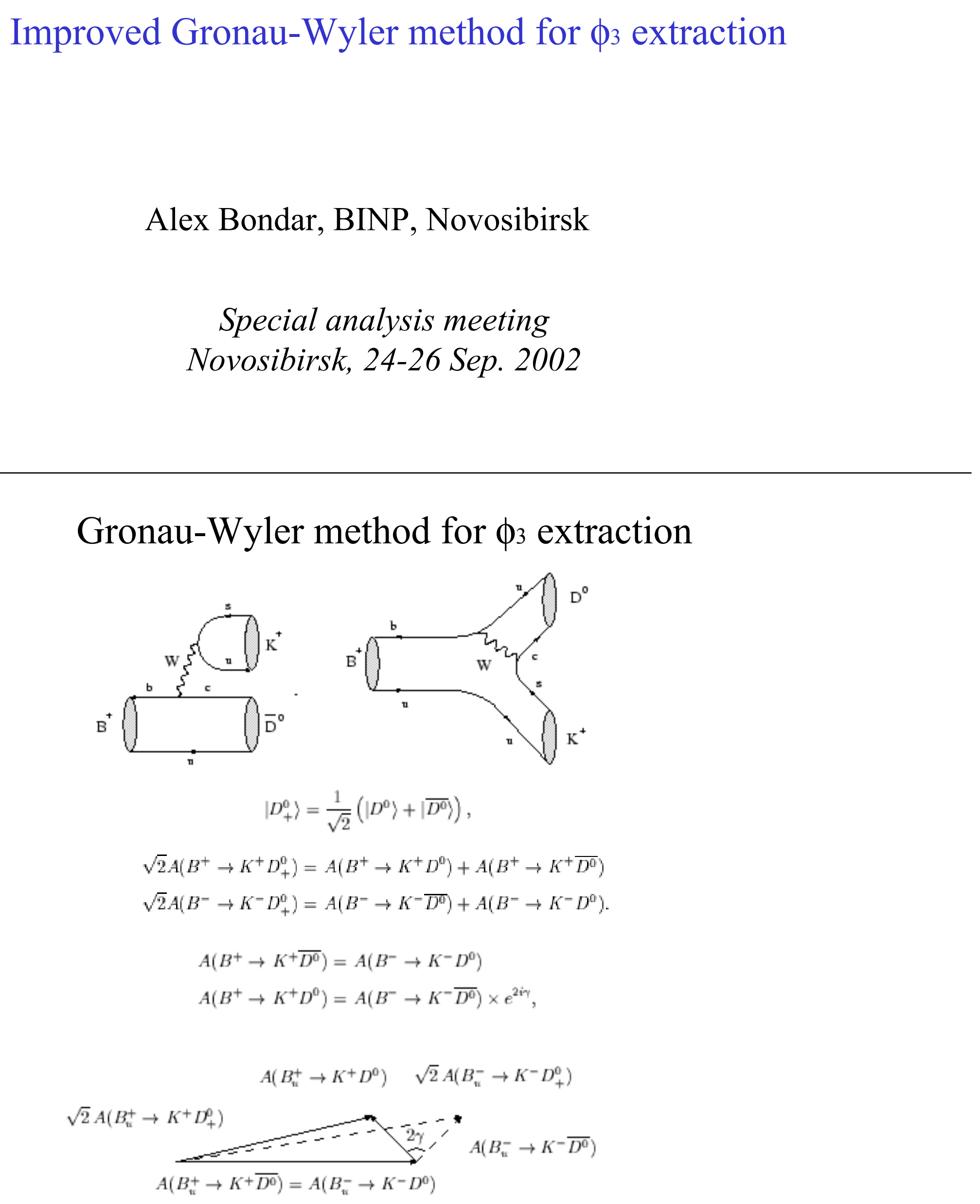
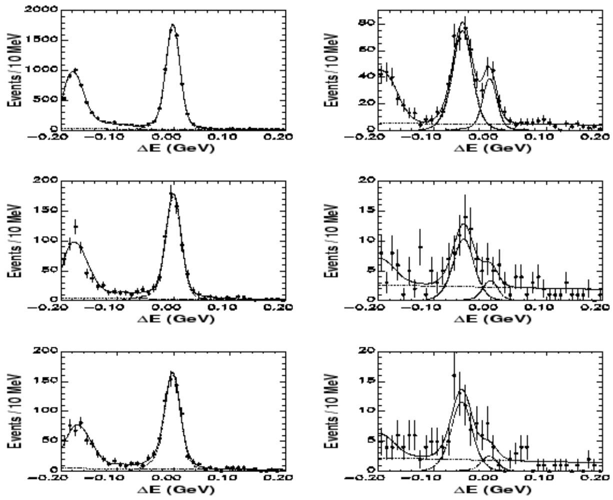


Figure 1: Slides 1–2 of the presentation of Ref. [19].



decay mode	Dπ	DK	Feed-down	Stat. Sig
$B^{\pm} \to D^0 h^{\pm}$	6052±87.7	347.5±21	134.4±14.7	26.1
$B^{\pm} \to D_1 h^{\pm}$	666.9±32.9	51±9.3	12.9±6.3	7.6
$B^{\pm} \to D_2 h^{\pm}$	639.3±29.7	57.3±9.6	9.9±5.6	8.2

decay mode	B^+	B^-
$D^0\pi$	3085.9 ± 68.7	2981.7 ± 62
D^0K	165.4 ± 14.5	179.6 ± 15.0
Feed-across	66.4 ± 10.0	71.8 ± 10.6
$D_1\pi$	302 ± 19.4	339.3 ± 21.6
D_1K	24.5 ± 6.5	26.5 ± 7.9
Feed-across	5.2 ± 4.2	7.9 ± 4.6
$D_2\pi$	339 ± 22.5	291.5 ± 22.7
D_2K	31.8 ± 6.8	23.9 ± 6.3
Feed-across	4.6±3.6	0.7±3.2

Figure 2: Slides 3–4 of the presentation of Ref. [19].

$\mathcal{B}(B^+ \to K^+\overline{D^0})$ can be determined using conventional methods.

$\mathcal{B}(B^+ \to K^+D^0)$ suffers from considerable experimental problems:

- If this colour-suppressed branching ratio is measured through Cabibbo-favoured hadronic decays of the D^0, e.g. through $B^+ \to K^+D^0[\to K^-\pi^+]$, we obtain large interference effects with the colour-allowed decay chain $B^+ \to K^+\overline{D^0}[\to K^-\pi^+]$, where the $\overline{D^0}$ decay is doubly Cabibbo-suppressed.

- All possible hadronic tags of the D^0-meson in $B^+ \to K^+D^0$ will be affected by such interference effects.

- Decays of neutral D-mesons into CP eigenstates, such as $D^0_+ \to \pi^+\pi^-$ or $D^0_+ \to K^+K^-$, involve small efficiencies and are experimentally challenging.

Possible way to overcome these difficulties is reconstruction $D^0(\bar{D}^0) \to K^0_S\pi^+\pi^-$. In this case the interference between these two amplitudes will results in the three body Dalitz plot distribution. The weak phase ϕ_3 can be extracted from the Dalitz plot analysis in the similar way as in CLEOs analysis for double Cabibbo suppressed amplitude in the D^0 decays.

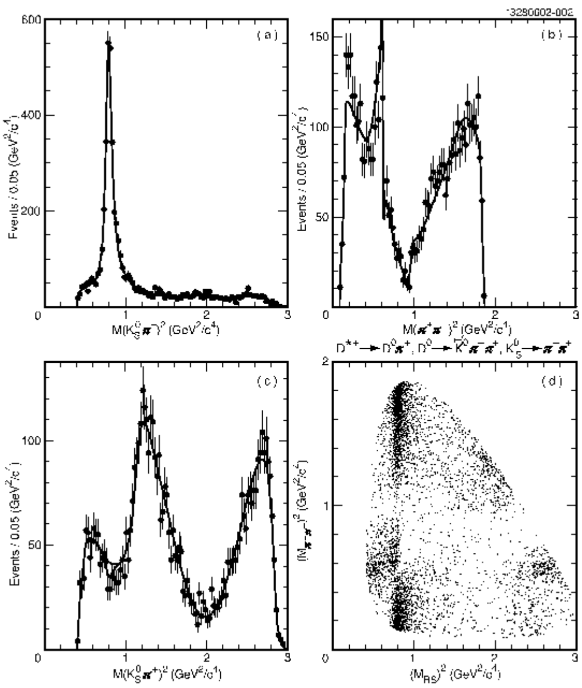


Figure 3: Slides 5–6 of the presentation of Ref. [19].

TABLE I. Standard fit results. The errors shown are statistical, experimental systematic, and modeling systematic respectively. See the text for further discussion.

Component	Amplitude	Phase	Fit Fraction (%)
$K^*(892)^+\pi^- \times B(K^*(892)^+ \to K^0\pi^+)$	$(11 \pm 2\ ^{+4}_{-1}\ ^{+4}_{-1}) \times 10^{-2}$	$321 \pm 10 \pm 3\ ^{+15}_{-5}$	$0.34 \pm 0.13\ ^{+0.31}_{-0.03}\ ^{+0.26}_{-0.02}$
$\overline{K}^0\rho^0$	1.0 (fixed)	0 (fixed)	$26.4 \pm 0.9\ ^{+0.9}_{-0.7}\ ^{+0.4}_{-2.5}$
$\overline{K}^0\omega \times B(\omega \to \pi^+\pi^-)$	$(37 \pm 5 \pm 1\ ^{+3}_{-8}) \times 10^{-3}$	$114 \pm 7\ ^{+6}_{-4}\ ^{+2}_{-5}$	$0.72 \pm 0.18\ ^{+0.04}_{-0.06}\ ^{+0.10}_{-0.07}$
$K^*(892)^-\pi^+ \times B(K^*(892)^- \to \overline{K}^0\pi^-)$	$1.56 \pm 0.03 \pm 0.02\ ^{+0.15}_{-0.03}$	$150 \pm 2 \pm 2\ ^{+2}_{-5}$	$65.7 \pm 1.3\ ^{+1.1}_{-2.6}\ ^{+1.4}_{-3.0}$
$\overline{K}^0 f_0(980) \times B(f_0(980) \to \pi^+\pi^-)$	$0.34 \pm 0.02\ ^{+0.04}_{-0.03}\ ^{+0.04}_{-0.02}$	$188 \pm 4\ ^{+5}_{-3}\ ^{+8}_{-6}$	$4.3 \pm 0.5\ ^{+1.1}_{-0.4} \pm 0.5$
$\overline{K}^0 f_2(1270) \times B(f_2(1270) \to \pi^+\pi^-)$	$0.7 \pm 0.2\ ^{+0.3}_{-0.1} \pm 0.4$	$308 \pm 12\ ^{+15}_{-25}\ ^{+66}_{-6}$	$0.27 \pm 0.15\ ^{+0.24}_{-0.09}\ ^{+0.28}_{-0.14}$
$\overline{K}^0 f_0(1370) \times B(f_0(1370) \to \pi^+\pi^-)$	$1.8 \pm 0.1\ ^{+0.2}_{-0.1}\ ^{+0.2}_{-0.6}$	$85 \pm 4\ ^{+4}_{-1}\ ^{+34}_{-13}$	$9.9 \pm 1.1\ ^{+2.4}_{-1.1}\ ^{+1.4}_{-4.3}$
$K_0^*(1430)^-\pi^+ \times B(K_0^*(1430)^- \to \overline{K}^0\pi^-)$	$2.0 \pm 0.1\ ^{+0.1}_{-0.2}\ ^{+0.5}_{-0.1}$	$3 \pm 4 \pm 4^{+7}_{-15}$	$7.3 \pm 0.7\ ^{+0.4}_{-0.9}\ ^{+3.1}_{-0.7}$
$K_2^*(1430)^-\pi^+ \times B(K_2^*(1430)^- \to \overline{K}^0\pi^-)$	$1.0 \pm 0.1 \pm 0.1\ ^{+0.3}_{-0.1}$	$155 \pm 7\ ^{+1}_{-4}\ ^{+7}_{-24}$	$1.1 \pm 0.2\ ^{+0.3}_{-0.1}\ ^{+0.6}_{-0.3}$
$K^*(1680)^-\pi^+ \times B(K^*(1680)^- \to \overline{K}^0\pi^-)$	$5.6 \pm 0.6\ ^{+0.7}_{-0.4} \pm 4.0$	$174 \pm 6\ ^{+10}_{-3}\ ^{+13}_{-19}$	$2.2 \pm 0.4\ ^{+0.5}_{-0.3}\ ^{+1.7}_{-1.5}$
$\overline{K}^0\pi^+\pi^-$ non-resonant	$1.1 \pm 0.3\ ^{+0.5}_{-0.2}\ ^{+0.9}_{-0.7}$	$160 \pm 11\ ^{+30}_{-18}\ ^{+55}_{-52}$	$0.9 \pm 0.4\ ^{+1.0}_{-0.3}\ ^{+1.7}_{-0.2}$

Base on 5299 events CLEO got the relative phase of the double Cabibbo suppressed amplitude with accuracy of 10^0. In case $B^+ \to D^0(\bar{D}^0)K^+$ decay the ratio of the color suppressed and color allowed amplitudes is in order of $\frac{a_2 |V_{ub}||V_{cs}^*|}{a_1 |V_{cb}||V_{us}^*|} \approx \frac{|V_{ub}|}{3|V_{cb}|\lambda} \approx \frac{1}{8}$, which is greater than in case of double Cabibbo suppressed amplitude by $1/8\lambda^2$ times. This results in the lower number of events (~ 800) necessary for the similar accuracy in ϕ_3 measurement.

The similar method applicable to the $\bar{B}^0 \to D^0(\bar{D}^0)K^{*0}$, $D^0(\bar{D}^0) \to K_S^0\pi^+\pi^-$ decay. In this case the ratio of the amplitudes even more suitable ($\frac{|V_{ub}|}{|V_{cb}|\lambda} \approx 0.4$).

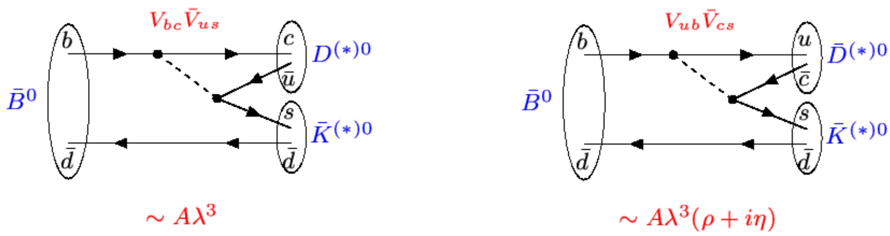
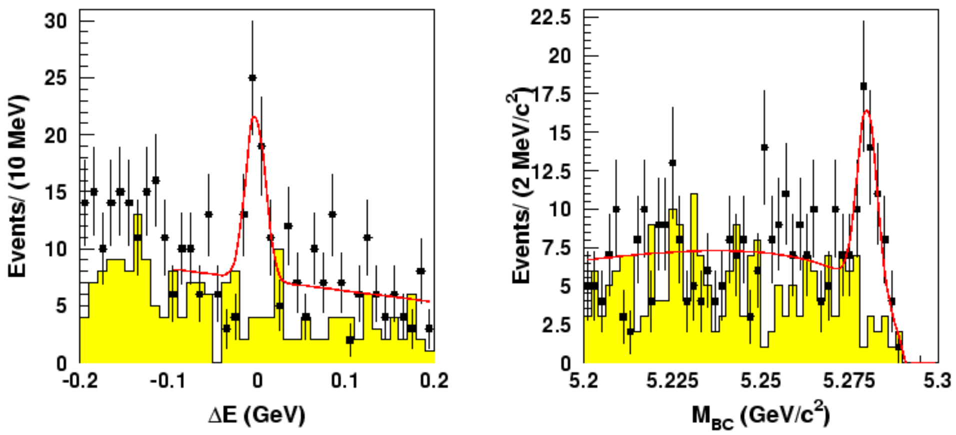


Figure 4: Slides 7–8 of the presentation of Ref. [19].

$\bar{B}^0 \to D^0\bar{K}^{*0}$ decay (sum of three modes)



$40.2^{+9.0}_{-8.5}$ events

$41.0^{+8.7}_{-8.1}$ events

$\mathcal{B} = (4.8^{+1.1}_{-1.0} \pm 0.5) \times 10^{-5}$

5.6σ significance

Conclusion

Suggested method of ϕ_3 extraction has the following advantages:

- Only one final state$(K^{(*)}K^0_S\pi^+\pi^-)$ is used in the analysis.
- Therefore, the experimental systematic errors should cancel.
- The extraction of ϕ_3 is possible with any strong phase difference.
- Double Cabibbo suppressed amplitudes do not effect on ϕ_3 extraction.
- This method helps resolve discrete ambiguities that are usually present in measurements of the weak phase ϕ_3.

Need to do:

- MC study of necessary statistics for different ϕ_3 and strong phase difference.
- Study of the possible model error due to parameterization of the $D^0 \to K^0_S\pi^+\pi^-$ decay amplitude.

Figure 5: Slides 9–10 of the presentation of Ref. [19].

B Recommendation for citation of BPGGSZ method

We recommend to use the following citations for the BPGGSZ method:

[1] A. Bondar. Proceedings of BINP special analysis meeting on Dalitz analysis, 24–26 Sep. 2002, unpublished.

[2] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, Determining γ using $B^+ \to DK^\pm$ with multibody D decays, Phys. Rev. **D68** (2003) 054018, arXiv:hep-ph/0303187.

[3] Belle collaboration, A. Poluektov et al., Measurement of ϕ_3 with Dalitz plot analysis of $B^\pm \to D^{(*)}K^\pm$ decay, Phys. Rev. **D70** (2004) 072003, arXiv:hep-ex/0406067.