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Compensation of Synchrotron Radiation at Top Energy in the FCC-hh

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Summary

To counteract the significant synchrotron radiation damping in the FCC-hh, a continuous longitudinal emittance blow-up is foreseen at top energy. In this paper, we study the compensation of synchrotron radiation emittance damping via RF phase noise injection. Adapting the noise bandwidth to the targeted bunch length, a good regulation of the bunch length can be achieved in the long run, while on short timescales the fluctuations remain relatively large. The steady-state bunch profile is shown and characterised with a binomial distribution. Finally, possible future developments are being discussed.

Contents

1 Introduction

At flat top, the FCC-hh machine has a design synchronous energy of 50 TeV, combined with a total RF voltage of 42 MV [1]. The energy loss due to synchrotron radiation for protons is significant: 4.67 MeV/turn. The experiments require to level the bunch length to 1.07 ns for several hours for good vertex resolution [2]. Therefore, measures have to be taken in order to compensate the radiation loss in a continuous manner. In this Note, we study the counteraction of synchrotron radiation via continuous injection of RF phase noise, and we determine the steady-state bunch profile resulting from it.

2 Simulation setup

Particle simulations have been performed using the beam dynamics simulation code BLonD [3] and the FCC-hh parameters according to Table 1. Intensity effects have been neglected for these studies. If not otherwise specified, all simulations have been performed with 2.5×10^5 macro-particles and an initially Gaussian bunch profile with a bunch length[1] of $\tau = 1.07$ ns.

Parameter	**Units**	**FCC-hh**
Circumference, C	km	97.75
Bending radius, ρ	km	10.42
Harmonic number, h		130680
RF frequency, f_{rf}	MHz	400.79
Revolution frequency, f_{rev}	Hz	3067
Central synchrotron frequency, f_{s0}	Hz	4.08
Central synchrotron tune, Q_{s0}	1	1.33×10^{-3}
Beam energy (at flat top), E_S	TeV	50
RF voltage, V	MV	42
Momentum compaction factor, α_p	1	101.35×10^{-6}
Energy loss per turn, ΔE_{SR}	MeV	4.67

Table 1: FCC-hh parameters (at top energy)

2.1 Synchrotron radiation damping

First, a benchmark of pure synchrotron radiation damping over 20 million turns has been performed between simulation and theory. The expected bunch length decrease with time is [4, p. 412f.]

$$\tau(t) = \tau_0 e^{-\alpha_E t}, \tag{1}$$

[1]Here and in the following, the bunch length refers to the FWHM bunch length scaled by a factor $4/(2\sqrt{2\ln 2})$, corresponding to the 4-sigma-equivalent bunch length for a Gaussian distribution.

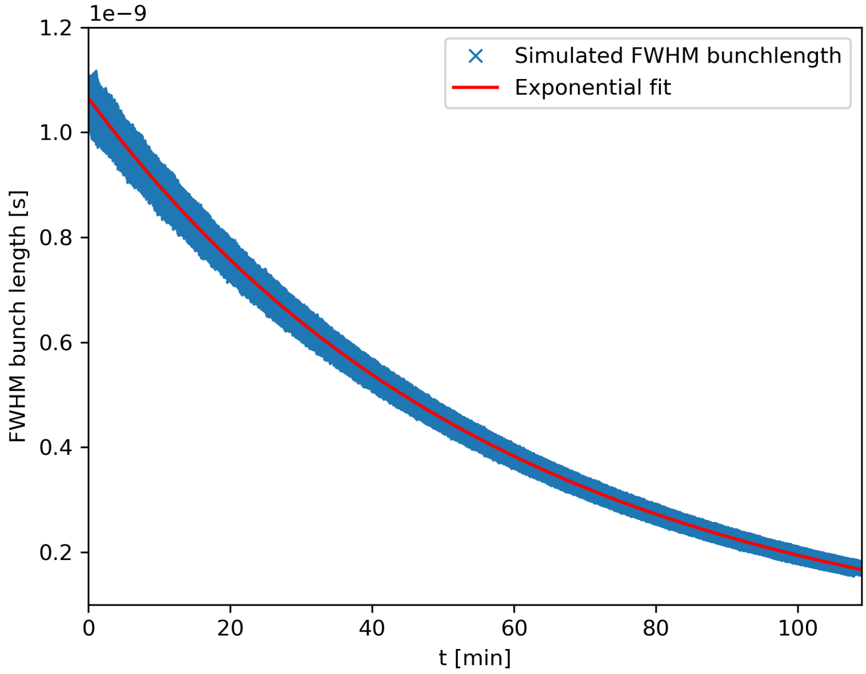

Figure 1: 4σ-equivalent FWHM bunch length evolution over 20 million turns. The simulated depletion with time due to synchrotron radiation is in good agreement with theory.

where τ_0 is the initial bunch length and α_E the damping coefficient. For the FCC-hh, the latter can be approximated assuming separate function magnets as

$$\alpha_E \approx \frac{\Delta E_{SR}}{E_S T_0}, \tag{2}$$

where T_0 denotes the revolution period.

At top energy, the theoretical damping constant is $\alpha_{E,\mathsf{calc}} = 2.86 \times 10^{-4}\,\mathrm{Hz}$, corresponding to a damping time of 0.97 h. An exponential fit applied to the simulated bunch length (see Fig. 1) yields a simulated damping constant of $\alpha_{E,\mathsf{sim}} = 2.85 \times 10^{-4}\,\mathrm{Hz}$, and is thus in good agreement with theory. Taking into account quantum excitation leads to $\alpha_{E,\mathsf{sim}} = 3.11 \times 10^{-4}\,\mathrm{Hz}$, i.e. a roughly 8 % decrease in the damping time. However, due to runtime limitations, quantum excitation has been neglected for the simulations of phase noise injection.

3 Counteraction via phase noise

One option to counteract synchrotron radiation damping is to inject band-limited phase noise. This method is currently used also during the LHC magnetic ramp for controlled emittance blow-up, to counteract loss of Landau damping. Figure 2 shows the synchrotron frequency distribution f_s as a function of maximum particle trajectory phase ϕ_{max} in a single-harmonic RF system, relative to the central synchrotron frequency f_{s0}.

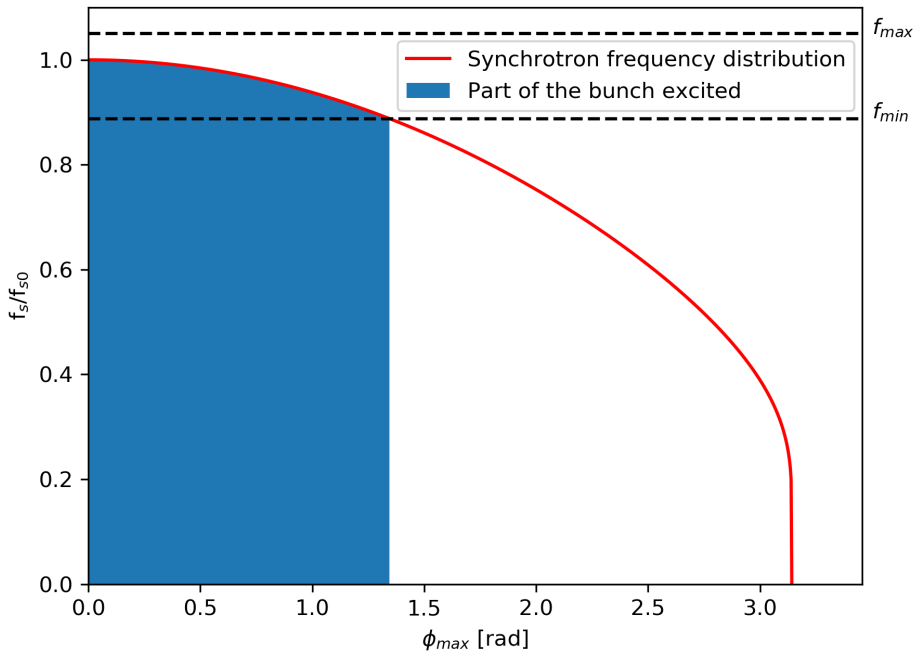

Figure 2: Relative synchrotron frequency distribution as a function of the oscillation phase amplitude for a stationary bucket, in a single-harmonic RF system. The dashed lines represent the RF phase noise band, the blue area the core of the bunch to be excited.

The frequency bandwidth of the phase noise spectrum is chosen in order to selectively excite the particles in the core of the bunch (shaded region in Fig. 2). The lower limit of the frequency band is given by the oscillation amplitude corresponding to the targeted bunch length. As there are no particles with a synchrotron frequency larger than f_{s0}, theoretically, there is no need to extend the noise bandwidth beyond f_{s0}. In practise, however, it is convenient to somewhat increase the upper frequency to avoid missing the bunch core due to discretisation errors.

3.1 Benchmarking phase noise

For a target bunch length of 1.07 ns, a flat noise spectrum with a bandwidth of

$$0.88 f_{\mathsf{s0}} \leq f \leq 1.05 f_{\mathsf{s0}} \tag{3}$$

is thus chosen. In short-bunch approximation, the evolution of the average r.m.s. phase spread $\sigma_\phi(t)$ due to diffusion can be estimated as [5, p. 19]

$$\sigma_\phi(t) = \sigma_{\phi 0} \sqrt{1 + \frac{\omega_{s0}^2 P_{\Delta\phi}^{\mathsf{DS}} t}{2\sigma_{\phi 0}^2}} \approx \sigma_{\phi 0} \left(1 + \frac{\omega_{s0}^2 P_{\Delta\phi}^{\mathsf{DS}} t}{4\sigma_{\phi 0}^2} \right), \tag{4}$$

where $P_{\Delta\phi}^{\mathsf{DS}}$ is the double-sided power spectral density of the phase noise, and $\sigma_{\phi 0} \equiv \sigma_\phi(0) = 2\pi\tau_0 f_{\mathsf{rf}}/4$. To compensate synchrotron radiation damping by noise, the emittance growth

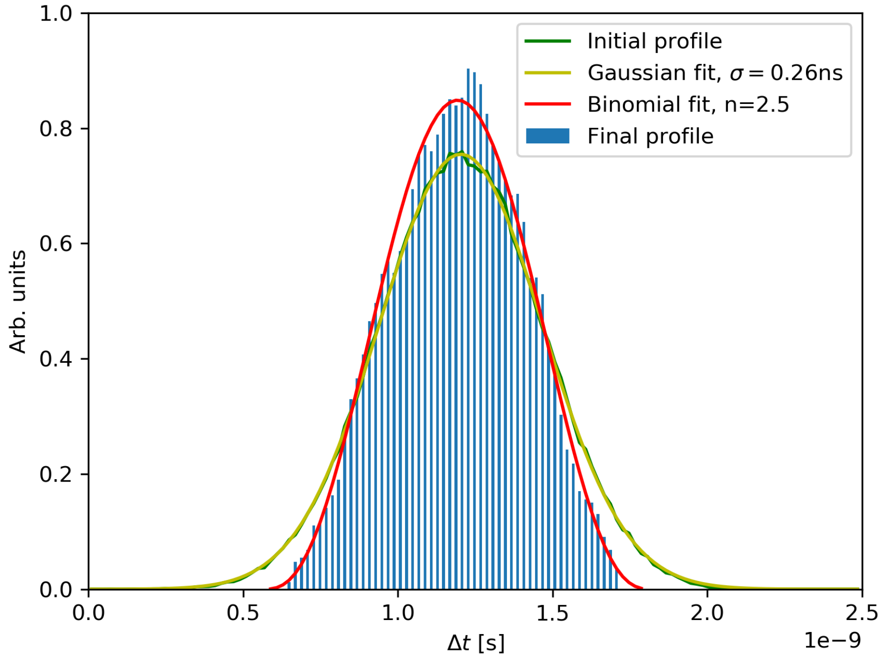

Figure 3: Bunch profile at injection and after 20 million turns, with synchrotron radiation at top energy and continuous phase noise injection of $7.33 \times 10^{-7}\,\frac{\text{rad}^2}{\text{Hz}}$. A Gaussian distribution with $\tau_{4\sigma} = 1.07$ ns is a good fit for the initial profile. For the final distribution, a good fit was found using the binomial distribution (Eq. 7) with exponent $n = 2.5$ and $\tau_{\mathsf{full}} = 1.26\,$ns.

has to match radiation damping,

$$\alpha_E = \frac{\omega_{s0}^2 P_{\Delta\phi}^{\mathsf{DS}}}{4\sigma_{\phi 0}^2}, \tag{5}$$

from where an approximation for the spectral density can be obtained,

$$P_{\Delta\phi}^{\mathsf{DS}} = \frac{1}{4}\frac{\Delta E_{\mathsf{SR}}}{E_S} f_{\mathsf{rev}} \left(\frac{h}{Q_{s0}}\tau_0\right)^2. \tag{6}$$

Hence, for keeping a constant bunch length of 1.07 ns in the FCC-hh at top energy, a power spectral density of $7.33 \times 10^{-7}\,\frac{\text{rad}^2}{\text{Hz}}$ is expected to be necessary. Note, however, that this number remains approximate; Eq. 1 assumes a bunch profile whose width is changing, but not its shape, while in Eq. 4 the bunch profile changes shape and the bunch length is defined from its r.m.s. spread.

Using phase noise with this power spectral density, and starting from a Gaussian bunch profile, the emittance damping due to synchrotron radiation can indeed be roughly compensated. The initial and final bunch profiles are shown in Fig. 3. A Gaussian fit of the initially generated profile confirms that the generated bunch profile has the desired bunch length of 1.07 ns. For the final profile, a good fit was found using the binomial distribution

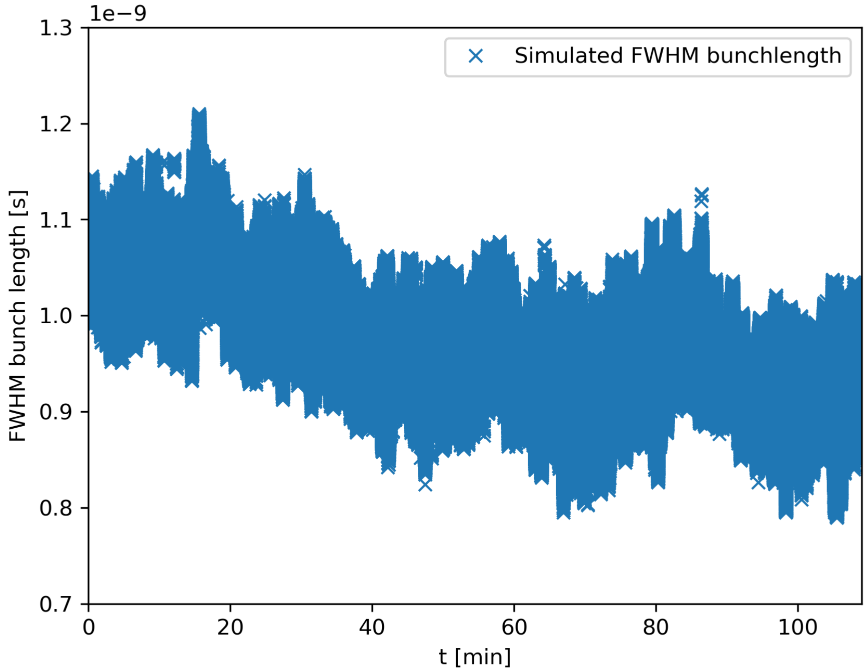

Figure 4: Evolution of the 4σ-equivalent FWHM bunch length over 20 million turns, with synchrotron radiation at top energy and continuous phase noise injection of $7.33 \times 10^{-7} \frac{\text{rad}^2}{\text{Hz}}$. The bunch length is somewhat shrinking while the particle distribution changes, but remains afterwards constant.

as defined in [6, p. 72],

$$\lambda(t) = \lambda_0 \left[1 - \left(\frac{2t}{\tau_{\text{full}}} \right)^2 \right]^{n+1/2} , \tag{7}$$

where λ_0 is a normalisation constant, τ_{full} the full bunch length, and n the exponent of the distribution.

The binomial fit of the final profile is not equally good on the left- and right-hand sides, as we are fitting a symmetric function on an asymmetric bunch profile (the asymmetry of the bunch profile being a result of the accelerating bucket needed to compensate synchrotron radiation energy losses). As expected, the steady-state profile with synchrotron radiation and noise injection has less tails than the Gaussian profile, it is therefore more dense around the core. This is because the noise spectrum is limited by f_{min}, and the diffusion takes place only within the targeted core region, while synchrotron radiation moves tail particles towards the core.

The evolution of the 4σ-equivalent FWHM bunch length along the simulation is shown in Fig. 4. In the first half of the simulation, the phase-space distribution of the bunch is changing and the bunch length is somewhat shrinking. In the second half of the simulation, the bunch length remains constant on average and the steady-state distribution is reached. The average bunch length in the second half of the simulation is (0.94 ± 0.04) ns; the large spread is due to the statistical fluctuations in the diffusion process.

The levelled bunch length is therefore not far from the requested 1.07 ns. For a more exact estimate of the power spectral density needed to maintain exactly 1.07 ns, quantum excitation should be included in future studies.

4 Future plans

Various studies remain to be performed concerning controlled emittance blow-up in the FCC-hh.

4.1 Effect of beam feedback loops

In reality, a beam phase loop is required to reduce the phase noise of the RF system itself, and thus guarantee the desired beam lifetimes.

In the LHC [7], the phase noise for controlled emittance blow-up is injected via the beam phase loop, which corrects its own action with one turn delay. The noise spectrum has to be therefore modified to compensate for the response of the beam feedback.

The effect of beam feedbacks on the noise spectrum and alternative noise injection schemes remain to be studied for the FCC in the future.

4.2 Counteraction via phase modulation

The bunch profile could also be shaped using RF phase modulation[8, 9], where the offset added to the RF phase is

$$\Delta\phi_{\mathsf{rf}}(t) = \phi_m \sin{(\omega_m t)}, \tag{8}$$

where the amplitude of the modulation is ϕ_m and ω_m is the frequency of the modulation. First, preliminary simulations show that a single tone is insufficient, but using multiple discrete lines in the desired bandwidth to compensate synchrotron radiation loss are a potential alternative to the band-limited noise injection.

4.3 Ramp

In a next step, the FCC-hh energy ramp will be studied, where controlled emittance blow-up is required to prevent from loss of Landau damping. Similar to the LHC, the ramping time will be around 20 minutes, but from 3.3 TeV to 50 TeV. The energy ramp combined with synchrotron radiation is calling for an even stronger the controlled emittance blow-up, where potentially a bunch length feedback will be necessary.

5 Conclusions

In the FCC-hh, the significant synchrotron radiation loss at top energy requires measures to continuously counteract the shrinkage of the longitudinal emittance. In this note, continuous phase noise injection has been studied as a counteraction method. The bunch length can indeed be levelled to a constant value, with a phase noise power spectral density that matches estimated values. During the continuous blow-up, the bunch length has significant fluctuations of 200 ps peak to peak. The steady-state bunch profile has been determined and can be described with a binomial distribution with an exponent of $n = 2.5$ and a full bunch length of 1.26 ns. A refinement of the steady-state bunch profile is required including quantum excitation, which has been neglected for these first studies for runtime speed-up.

Further studies are foreseen for the future on alternative blow-up methods, the effect of beam feedbacks, and the controlled emittance blow-up during the energy ramp.

6 Acknowledgements

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