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Vacuum wormholes in Brans-Dicke theory

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Abstract

Two kinds of vacuum wormhole solutions are found in Brans-Dicke (BD) theory with a negative-definite coupling constant ω. One kind of the solution is a usual wormhole with two asymptotically euclidean regions, and another is a new type wormhole which has three asymptotically euclidean regions. We compare the results to the re-scaled BD theory in which no vacuum wormholes exist for $-\frac{3}{2} < \omega \leq 0$.

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Wormholes are Riemannian manifolds which have two or more asymptotically Euclidean regions. It is believed that wormholes have some important consequences such as quantum decoherence [1] and making the cosmological constant vanishing [2]. Recently the Brans-Dicke (BD) theory [3] has attracted some attention in the inflationary universe [4]. It is known that vacuum wormhole solutions do not exist in BD theory with a positive coupling constant ω. If the Brans-Dicke scalar field is extended as a complex scalar field, one may construct vacuum wormholes [5]. More recently, Ref. [10] shows the possibility of constructing vacuum wormhole solutions in the BD theory with a negative ω. In this paper we construct vacuum wormhole solutions in BD theory with a negative-definite coupling constant, and compare with the re-scaled BD theory or the generalized Brans-Dicke (GBD) theory [7]. The result of this paper is just a powerful example to show that the GBD theory with the Pauli metric do differ from the original BD theory with the Jordan metric: Vacuum wormholes exist in the BD theory, but not in the GBD theory for $-\frac{3}{2} < \omega \leq 0$.

The Brans-Dicke Lagrangian [3] in the absence of matter is

$$\mathcal{L} = -\sqrt{-\gamma}\left[\phi R + \omega\gamma^{\mu\nu}\frac{\partial_\mu\phi\partial_\mu\phi}{\phi}\right], \tag{1}$$

where $\gamma = det\gamma_{\mu\nu}$, $\gamma_{\mu\nu}$ is called the Jordan metric, ϕ is the Brans-Dicke scalar field, and ω is the Brans-Dicke coupling constant.

Let us consider a closed Robertson-Walker universe. After euclidization the metric for such an universe is given by

$$ds^2 = d\tau^2 + R^2(\tau)\left[\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right]. \tag{2}$$

Field equations are followed from Eqs. (1) and (2)

$$\frac{R'^2}{R^2} + \frac{R'}{R}\frac{\phi'}{\phi} = \frac{1}{R^2} + \frac{\omega}{6}\frac{\phi'^2}{\phi^2}, \tag{3}$$

$$\phi'' + 3\frac{R'}{R}\phi' = 0, \tag{4}$$

where the prime denotes $d/d\tau$, the scale factor R and the scalar field ϕ are functions of the euclidean time τ. Integrating Eq. (4), one arrives at

$$\phi'(\tau)R^3(\tau) = a, \tag{5}$$

where $a \neq 0$ is a integration constant (note that $a = 0$ would lead to $\phi(\tau) =$ constant, or $R(\tau) = 0$, and hence no wormholes exist). Eq. (5) indicates that $\phi'(\tau)$ is positive-definite (or negative-definite), and hence $\phi(\tau)$ is a monotonic function of τ. Furthermore (4) implies

$$\phi''(\tau_0) = 0, \quad for\ R'(\tau_0) = 0\ \ or\ \ R(\tau_0) \to \infty\ (\phi'(\tau_0) = 0), \tag{6}$$

i.e. $\phi''(\tau)$ would change its sign as τ acrosses τ_0.

Let $y = R^2\phi$, Eq. (3) becomes

$$\frac{dy}{y\sqrt{y^2 + \frac{\lambda a^2}{4}}} = \pm\frac{2}{|a|}\frac{d\phi}{\phi} \tag{7}$$

with $\lambda \equiv 1 + \frac{2}{3}\omega$, where Eq. (5) is used. Solutions to Eq. (7) are given by

$$\frac{1}{R^2} = \alpha\left(|\phi|^{1-\sqrt{\lambda}} - |\phi|^{1+\sqrt{\lambda}}\right), \qquad for\ \ \lambda > 0\left(\omega > -\frac{3}{2}\right), \tag{8}$$

$$\frac{1}{R^2} = -\beta|\phi|\ln|\phi|, \qquad for\ \ \lambda = 0\left(\omega = -\frac{3}{2}\right), \tag{9}$$

$$\frac{1}{R^2} = \gamma|\phi|\sin\left(\sqrt{|\lambda|}\ln|\phi|\right), \qquad for\ \ \lambda < 0\left(\omega < -\frac{3}{2}\right), \tag{10}$$

where ϕ is re-scaled by some constants under which Eqs. (3) and (4) are invariant, and $\alpha^2 = 1/(a^2\lambda)$, $\beta^2 = 4/a^2$, $\gamma^2 = 4/(a^2|\lambda|)$. In principle, $\phi(\tau)$ and hence $R(\tau)$ may be explicitly expressed as functions of τ by making use of Eqs. (8)-(10) and (5). In fact, integrating Eq. (5) a relationship between τ and ϕ can be expressed as

$$\tau = \frac{1}{a}\int R^3 d\phi + const \tag{11}$$

where $R(\phi)$ is given by Eqs. (8)-(10).

Our purpose is to search for wormhole solutions of $R(\tau)$, which have at least one non-zero minimum (throat) for a finite value of τ. Setting $R'(\tau_i) = 0$ in Eq. (3), one arrives at

$$\frac{1}{R^2(\tau_i)} = -\frac{\omega}{6}\frac{\phi'^2(\tau_i)}{\phi^2(\tau_i)}. \tag{12}$$

Using Eq. (5), Eq. (12) is reduced to

$$R^4(\tau_i)\phi^2(\tau_i) = -\omega a^2/6. \tag{13}$$

That the left-hand side of (13) is not negative implies the following requirement:

$$\omega \leq 0. \tag{14}$$

Taking derivative of Eq. (3), and then using $R'(\tau_i) = 0$ and Eq. (5), we obtain

$$R''(\tau_i) = \frac{2}{R(\tau_i)} \geq 0. \tag{15}$$

This implies that all finite values of $R(\tau_i)$ are the minima of $R(\tau)$. Consider next asymptotic behavior of R, ϕ, and τ using Eqs. (8)-(11).

Case (A): $-\frac{3}{2} < \omega \leq 0$ $(0 < \lambda \leq 1)$. Solutions to (7) are given by (8). If $\alpha < 0$, (8) shows $|\phi| \geq 1$. We can get from (11) and (8) that $\tau \to -\infty$ as $|\phi| \to 1$. However, as $|\phi| \to \infty$ or a finite value, $\tau \to$ a finite constant. Therefore α in (8) should be a positive and non-zero constant, and hence $|\phi| \leq 1$.

(i) $\omega = 0$ $(\lambda = 1)$. $R \to \infty$ as $\phi \to \pm 1$. Putting (8) with $\lambda = 1$ in (11), we have

$$R^2 = \frac{1}{\alpha}\left(1 + b\tau^2\right), \qquad \phi^2 = \frac{b\tau^2}{1 + b\tau^2} \tag{16}$$

with $b \equiv a^2\alpha^3$. This gives $R^2(\tau) \sim \tau^2 \to \infty, \phi \to \pm 1$, as $\tau \to \pm\infty$. This is a Tolman wormhole [6].

(ii) $-\frac{3}{2} < \omega < 0$ $(0 < \lambda < 1)$. It is easy known from (8) with $\alpha > 0$ that $R \to \infty$ as $\phi \to 0, \pm 1$. In order to consider behavior of τ, substituting (8) into (11) we have

$$\tau = \alpha^{-3/2}\int\left[|\phi|^{1-\sqrt{\lambda}} - |\phi|^{1+\sqrt{\lambda}}\right]^{-3/2} d\phi + const. \tag{17}$$

Consider the following limits: (a) $\phi \to \pm 1$. Setting $\phi = \pm(1-\epsilon)$ with $\epsilon << 1$ in (17), one arrives at $\tau \sim \pm\epsilon^{-1/2} \to \pm\infty$ and $R^2(\tau) \sim \tau^2 \to \infty$ as $\epsilon \to 0$; (b) $\phi \to 0$. Putting $\phi = \pm\epsilon$ with $\epsilon << 1$ in (17), we get $\tau \sim \mp\epsilon^{(3\sqrt{\lambda}-1)/2}$ and $R^2 \sim \tau^{2(\sqrt{\lambda}-1)/(3\sqrt{\lambda}-1)}$ for $\lambda \neq 1/9$ $(\omega \neq -4/3)$. Then making the limit $\epsilon \to 0$, we have the results: $\tau \to 0$ and $R \to \infty$ for $-4/3 < \omega < 0$ $(1/9 < \lambda < 1)$; $\tau \to \mp\infty$ and $R \to \infty$ for $-2/3 < \omega < -4/3$ $(0 < \lambda < 1/9)$. In case of $\lambda = 1/9$ $(\omega = -4/3)$, Eq. (17) gives $\tau \sim \pm\ln\epsilon \to \mp\infty$ and $R^2 \sim exp(2|\tau|/3) \to \infty$ as $\epsilon \to 0$. The $\tau-$dependences of R and ϕ are drawn in Figs. 1 and 2, where the constant a is chosen to be positive, i.e. $\phi'(\tau) > 0$.

Case (B): $\omega = -\frac{3}{2}$ ($\lambda = 0$). Eq. (9) is a limit case of (8) as $\lambda \to 0$. The τ−dependences of $R(\tau)$ and $\phi(\tau)$ are similar with those as shown in Fig. 2.

Case (C): $\omega < -\frac{3}{2}$ ($\lambda < 0$). Analysis performed for the case is just as it does for case (A). However since the right-hand side of Eq. (10) is a sin function, two zero values of $1/R^2$ correspond to two non-zero values of ϕ. With some calculation, we find from Eqs. (11) and (10) $\tau \to \pm\infty$ as $R \to \infty$. The τ−dependences of $R(\tau)$ and $\phi(\tau)$ are similar to those as shown in Fig. 2.

In short, when $\omega = 0$ and $\omega \leq -\frac{4}{3}$, the scale factor $R(\tau)$ has its minimum at a finite value of τ, and goes to infinity as $\tau \to \pm\infty$. Thus these are wormholes with two asymptotically euclidean regions corresponding to $\tau \to \pm\infty$ (see Fig. 2). However in case of $-\frac{4}{3} < \omega < 0$, the $R(\tau)$ has its two minima at two finite values of τ, and goes to infinity as $\tau \to 0, \pm\infty$. Therefore this is a new kind of wormhole with three asymptotically euclidean regions corresponding to $\tau = 0, \pm\infty$ (see Fig. 1).

Table 1. Comparison of existence of wormholes in the BD and GBD theories.

	$\omega < -\frac{3}{2}$	$-\frac{3}{2} < \omega \leq 0$	$0 < \omega$
BD theory	yes	yes	no
GBD theory	yes	no	no

In the above, we find two kinds of vacuum wormholes in the BD theory with a negative-definite coupling constant ω. It is well-known that the BD theory with a negative coupling constant would lead to a negative kinetic energy for the dilaton field. One reason of considering this type theory is responsible for a relationship to the re-scaled BD theory (the generalized BD theory). An example of a negative ω is the Kaluza-Klein unification. It has been claimed that the BD theory can be derived from such an unification theory in which the Jordan metric couples to the Kaluza-Klein scalar field with a negative ω. A conformal transformation of the metric would restore the positive kinetic energy of dilaton [7]. In fact, introducing a dilaton field σ and a conformal transformation of the Jordan metric by definitions $\phi = \frac{1}{2\kappa^2}e^{a\sigma}$ and $g_{\mu\nu} = e^{a\sigma}\gamma_{\mu\nu}$, the BD Lagrangian (1) is transformed into

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right]. \qquad (18)$$

This is called the re-scaled BD theory or the generalized BD (GBD) theory in the absence of matter (i.e. vacuum) where $a^2 = 2\kappa^2/(2\omega+3)$, $g_{\mu\nu}$ is referred as the Pauli metric, the dilaton field σ is real if $\omega > -\frac{3}{2}$, but pure imaginary if $\omega < -\frac{3}{2}$. The Lagrangian (18) is just a Einstein's Lagrangian with a massless scalar field. Field equation for a closed universe is obtained from a euclidean metric and (18)

$$R'^2 = 1 + \frac{\kappa^2}{6}R^2\sigma'^2. \qquad (19)$$

It is easy to know from Eq. (19) that no wormholes exist for $\omega > -\frac{3}{2}$ (in this case, σ is real) [8], but wormhole solutions may be constructed for $\omega < -\frac{3}{2}$ (in this case, σ is pure imaginary) [9]. Therefore it is strongly shown from the above results that the GBD theory with the Pauli metric do differ from the original BD theory with the Jordan metric when the coupling constant ω takes a value in the domain $(-\frac{3}{2}, 0)$: the GBD theory has no vacuum wormholes, but the BD theory exists two kinds of vacuum wormholes (comparison between the BD and GBD theories is given in Table 1)

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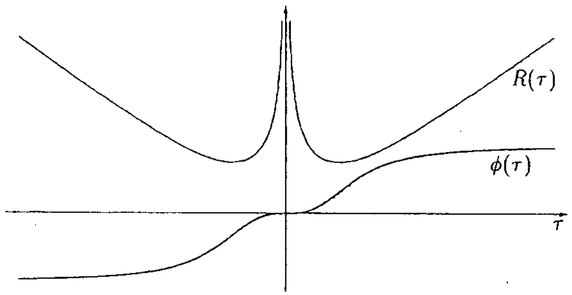

Fig. 1. Wormhole solutions for $-4/3 < \omega < 0$. There three asymptotically euclidean regions corresponding to $\tau \to 0, \pm\infty$.

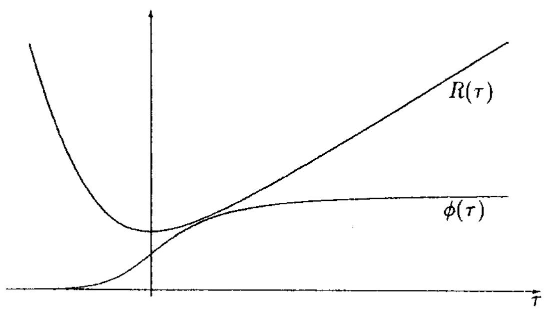

Fig. 2. Wormhole solutions for $-3/2 < \omega \leq -4/3$ (It is similar for $\omega = 0$ and $\omega \leq -3/2$). There are two asymptotically euclidean regions corresponding to $\tau \to \pm\infty$.