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Abstract

In the most general two-Higgs doublet model, with an assumption of an approximate family symmetry, lepton family number nonconservation processes occur via neutral scalar exchanges, but at a level that is naturally superweak. As a result of the Bjorken-Weinberg two-loop mechanism, the present experimental upper limit on the $\mu \to e\gamma$ decay provides a stringent constraint on the lepton family-changing Yukawa couplings. A modest improvement in the precision of the present experiment for $\mu \to e\gamma$ decay might yield a first evidence of lepton family number nonconservation.

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The simplest extension of the standard model is the addition of a second Higgs doublet. The most general two-Higgs doublet model (2HDM) has the feature that there exist neutral Higgs bosons with flavor-changing couplings. As a result there exist, at tree level, processes induced by flavor-changing neutral exchange (FCNE). To meet the stringent limits on FCNE most versions of the 2HDM forbid the flavor-changing couplings by means of a discrete symmetry suggested by Glashow and Weinberg [1].

However the possibility of this FCNE is very interesting because it provides a simple version [2–4] of the superweak interaction [5]. Furthermore it has been emphasized by a number of authors [6,7] that because of the small masses that enter Higgs boson couplings the experimental constraints on FCNE are not so severe. In fact it becomes reasonable to assert that the necessary suppression of flavor-changing couplings arises from some kind of approximate global U(1) symmetries (which act only on the fermions) required to explain the small mixing angles in the CKM matrix. Here we follow this general approach but not the specific ansatze that have been given in [6,7].

From this point of view it becomes very interesting to look for signals of FCNE. As is well-known in non-leptonic processes these show up as superweak contributions in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixings. However at present one cannot distinguish these from standard model effects. A detailed analysis of future experiment on CP violation in B^0 physics could conceivably do this [8]. In semi-leptonic processes an interesting possibility is the decay $B_s \to \mu^+\mu^-$ [9]. Here we focus on purely leptonic processes. These have been discussed recently by Antaramian et al [10] but we do not agree with their conclusions.

For simplicity we will concentrate on the lightest neutral scalar boson H_1^0. The generalization to a sum over all three bosons is straightforward and leads to no new quantitative results. The couplings of H_1^0 to leptons is written

$$L_Y = (\sqrt{2}G_F)^{1/2} H_1^0 \{ \sum_i \bar{e}_{Li}\; m_i \eta_i\; e_{Ri} + \sum_{i\neq j} \bar{e}_{Li}\; \sqrt{m_i m_j} \zeta_{ij}\; e_{Rj} + H.C. \} \quad (1)$$

The flavor-changing couplings from i to j contain the suppression factor $\sqrt{m_i m_j}$ previously suggested in [6,7] multiplied by the factor ζ_{ij}, which we assume may be of order 0.1

to 1. The value in any model will depend on many details including the ratio of vacuum expectation values and the Higgs mass matrix. The η_i are expected to be of order unity although in some models they may be enhanced by a factor of order $\tan\beta$, where $\tan\beta$ is the ratio of two vacuum expectation values, i.e., $\tan\beta = v_2/v_1$. For our calculations we also need the couplings of H_1^0 to W

$$\frac{1}{2}g^2(\sqrt{2G_F})^{-1/2}\beta_1 H_1^0 W_\mu^+ W^{\mu-} \tag{2}$$

and the diagonal coupling to the top quark

$$(\sqrt{2G_F})^{1/2}\{H_1^0\bar{t}_L m_t \eta_t t_R + H.C.\} \tag{3}$$

Here β_1 measures the overlap of the mass eigenstate H_1^0 with the "real" Higgs boson, that is, the scalar state that has the couplings of the Higgs boson in the standard model. The quantity $\eta_t \equiv \beta_1 + \eta_t'$ with η_t' being expected to be of order unity.

The simplest tree-level processes such as $\mu \rightarrow 3e$ or $\tau \rightarrow 3\mu$ have extremely small branching ratios because of the dependence on the small lepton masses. For example, the order of magnitude of the branching ratio for $\mu \rightarrow 3e$ is 10^{-19}. There is one decay, however, that could be detectable in the near future. This is the decay $\mu \rightarrow e\,\gamma$, which has a branching ratio limit of 5×10^{-11} and is the subject of experiments aiming below 10^{-12}. It is for this decay that we differ from the results in Ref. [10].

There are two contributions of interest. The first is a single loop diagram. Assuming the product $\zeta_{\mu\tau}\zeta_{\tau e}$ is not too small the major contribution comes from the loop with an intermediate τ- lepton. However, as emphasized by Bjorken and Weinberg [11] two loop diagrams are generally more important. Fig. (1a) shows the two-loop diagram with a W-loop which they considered. The advantage of the two loop diagram is that only one of the Higgs couplings is suppressed by the light masses. There is also a comparabale contribution from the t-quark loop shown in Fig. (1b). There are still other contributions, such as that with a charged Higgs boson in the loop, that could be important, however, we will limit the present discussion to the W and t loops. The importance of these two loop diagrams was rediscovered in the context of the electric dipole moment of the electron by Barr and Zee [12], some of whose results we use.

The decay rate is given by

$$\Gamma(\mu \to e\gamma) = \frac{G_F^2 m_\mu^5 \alpha}{32\pi^4} \frac{m_e}{m_\mu} C^2 \tag{4}$$

and the branching ratio

$$\frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = 6.6 \times 10^{-5} C^2 \tag{5}$$

with $C^2 = |C_L|^2 + |C_R|^2$, here $C_{L,R} = (C^{(1)}_{L,R} + (C^{(2)}_{L,R})_{t-loop} + (C^{(2)}_{L,R})_{W-loop})$. The one loop contribution due to the intermediate τ is

$$C_L^{(1)} = \zeta^*_{\mu\tau}\zeta^*_{\tau e}\frac{1}{2}E(z_\tau); \qquad C_R^{(1)} = \zeta_{e\tau}\zeta_{\tau\mu}\frac{1}{2}E(z_\tau) \tag{6}$$

where $z_\tau = m_\tau^2/m_{H_1^0}^2$ and

$$E(z) = \frac{1}{2}\frac{z}{(z-1)^2}[\frac{1}{2}z - \frac{3}{2} + \frac{\ln z}{z-1}] \tag{7}$$

The two loop contributions are

$$\begin{aligned}
(C_L^{(2)})_{t-loop} &= \frac{4\alpha}{3\pi}\zeta^*_{\mu e}\{[f(z_t) + g(z_t)]\eta_t + [f(z_t) - g(z_t)]\eta_t^*\} \\
(C_R^{(2)})_{t-loop} &= \frac{4\alpha}{3\pi}\zeta_{e\mu}\{[f(z_t) + g(z_t)]\eta_t + [f(z_t) - g(z_t)]\eta_t^*\} \\
(C_L^{(2)})_{W-loop} &= \frac{5\alpha}{\pi}\zeta^*_{\mu e}[\frac{3}{5}f(z_W) + g(z_W)]\beta_1; \qquad (C_R^{(2)})_{W-loop} = \frac{5\alpha}{\pi}\zeta_{e\mu}[\frac{3}{5}f(z_W) + g(z_W)]\beta_1
\end{aligned} \tag{8}$$

where $z_i = m_i^2/m_{H_1^0}^2$. $f(z)$ and $g(z)$ are the integral functions which also appeared in the analyses of the electric dipole moment of the electron [12]. $f(1) \sim 0.8$, $g(1) \sim 1.2$; for large z, $f(z) \sim \frac{1}{3}\ln z + \frac{13}{18}$, $g(z) \sim \frac{1}{2}\ln z + 1$; and for small z, $f(z) \sim g(z) \sim (z/2)(\ln z)^2$.

To see the relative importance of the different terms consider the case that ζ_{ij} and η_i are all real and $\zeta_{ij} = \zeta_{ji}$; the result for C can be written

$$C_L = C_R = \frac{1}{\sqrt{2}}C = 10^{-2}\zeta_{e\mu}\{a\eta_t + b\beta_1 + c\ \zeta_{e\tau}\zeta_{\mu\tau}/\zeta_{e\mu}\} \tag{9}$$

For $m_{H_1^0} = 50$ GeV

$$a \simeq 1.0 \ , \qquad b \simeq 2.4 \ , \qquad c \simeq 0.16 \tag{10}$$

and for $m_{H_1^0} = 200$ GeV

$$a \simeq 0.5 \ , \qquad b \simeq 0.46 \ , \qquad c \simeq 0.02 \tag{11}$$

Here a corresponds to the t-loop, b to the W-loop, and c to the one loop contributions. As noted above, $\eta_t = \beta_1 + \eta_t'$, where β_1 is the "real" Higgs contribution. We expect η_t' to be of order unity unless H_1^0 is predominantly the neutral Higgs in which case β_1 is close to unity. In either case, barring accidental cancellations, we expect η_t of order unity. It follows that for masses of H_1^0 of order the t mass or greater we expect the t-loop to be at least as important as the W-loop, while the one-loop contribution is unimportant.

There is an important difference between the t-loop and W-loop contributions that has the consequence that in certain cases the W-loop contribution becomes small while the t-loop does not. Higgs boson mixing plays an essential role for the W-loop because the W coupling β_1 is due to the "real" Higgs boson whereas the flavor-changing coupling $\zeta_{e\mu}$ is entirely due to the extra Higgs bosons. As a result in the decoupling limit in which the "real" Higgs boson is approximately a mass eigenstate the W-loop contribution approaches zero. On the other hand the t-loop contribution proportional to $\eta_t'\zeta_{e\mu}$ remains when one considers the other mass eigenstates. When one sums over the three mass eigenstates the three contributions to the W-loop tend to cancel, as noted by Bjorken and Weinberg, because of the orthogonality condition on the mixing. Thus in the limit that the three masses become degenerate the W-loop contribution vanishes. Again this is not true for the t-loop contribution.

From eqs. (9) and (11) we conclude that the present limit on the branching ratio for $\mu \to e\gamma$ requires $\zeta_{e\mu}$ to be less than 0.1 even if $m_{H_1^0}$ is of order 200 GeV. In previous discussions [7] the strongest limit on FCNE in the 2HDM seemed to come from the CP-violating parameter ϵ due to $K^0 - \bar{K}^0$ mixing. If this were not to become too large one requires either that ζ_{sd} (the parameter analogous to $\zeta_{e\mu}$) be between 10^{-1} to 10^{-2} or that the relevant CP-violating phase be close to zero or $\pi/2$ or that the relevant scalar boson mass be much larger than 200 GeV. From the present discussion it appears that the decay $\mu \to e\gamma$ provides a similar limit independent of CP violation.

If these general considerations are correct, we would expect $\zeta_{e\mu}$ to lie not far below 0.1. In this case there would be a good chance that the ongoing search for $\mu \to e\gamma$ would detect this decay with a branching ratio of order 10^{-12}.

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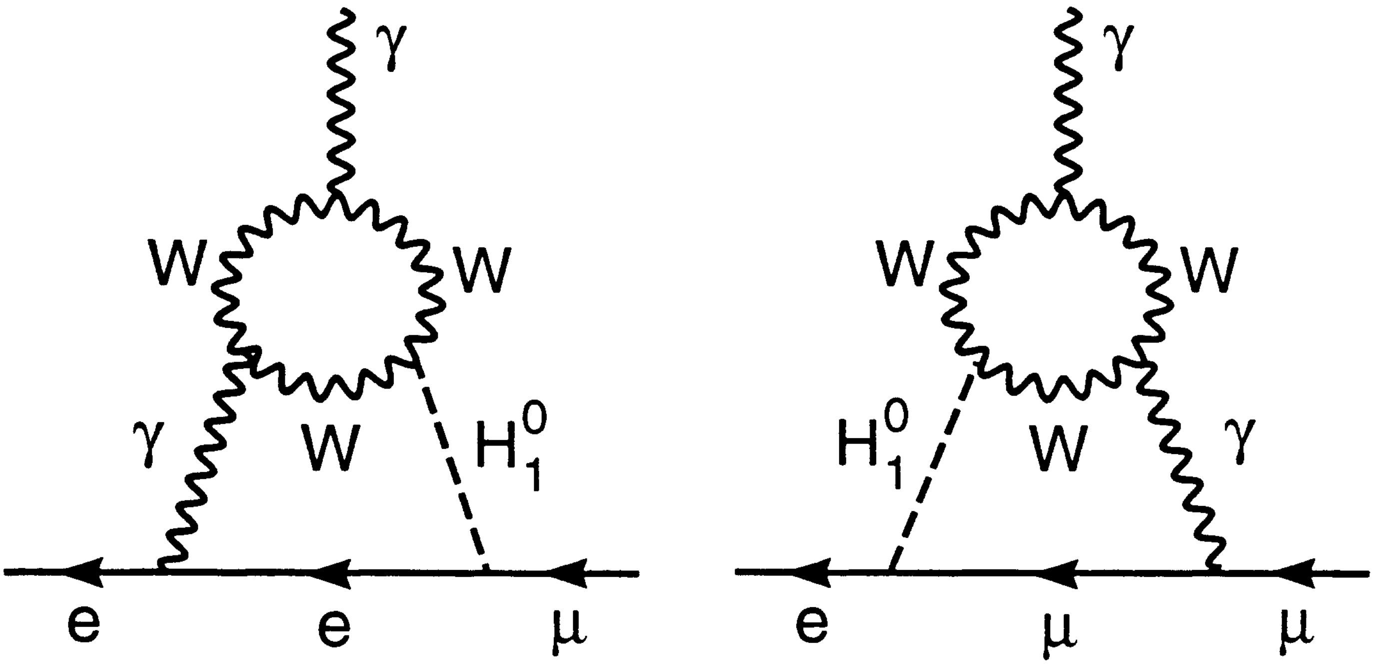

Figure 1(a)

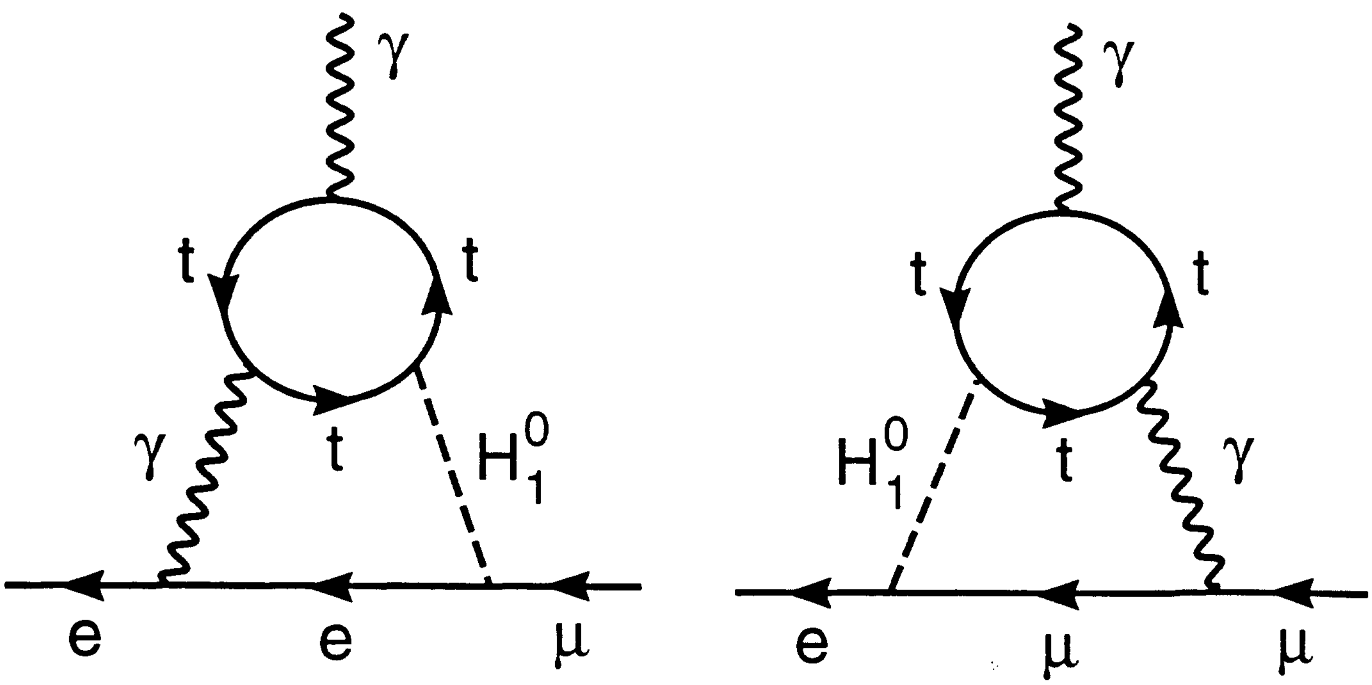

Figure 1(b)