An Analysis of Cabibbo-Favored Two-body Decays of Charmed Mesons with η or η' in the Final State

R. C. Verma, A. N. Kamal* and M. P. Khanna
Centre for Advanced Study in Physics, Department of Physics,
Panjab University, Chandigarh -160 014 India

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Abstract

We have carried out a study of Cabibbo-favored two-body decays of D^0 and D_s^+ involving η and η' in the final state, $D^0 \to (\eta, \eta')\bar{K}^0$, $D^0 \to (\eta, \eta')\bar{K}^{*0}$ and $D_s^+ \to (\eta, \eta')\pi^+$, $D_s^+ \to (\eta, \eta')\rho^+$. We have introduced an annihilation term whereever admissible, and investigated its size if it were to bridge the gap between theory and experiment in each case. We have also related the semileptonic rates for $D_s^+ \to (\eta, \eta')e^+\nu$ to those of the hadronic rates for $D_s^+ \to (\eta, \eta')\pi^+$ and unveiled a puzzle. We offer a possible solution.

* Present Address: Laboratoire de Physique Theorique et Hautes Energies, Univesite Paris VII, 4 Place Jussieu, 75252 Paris Cedex 05, France.
Permanant address: Department of Physics, University of Alberta, Edmonton, Alberta, T6G2J1, Canada

1 Introduction

In some ways two-body Cabibbo-favored decays of D^0 and D_s^+ involving η or η' in the final state are cleaner systems as the final state involves only one isospin, yet these decays have proven to be problematic for the theory. For example, the measured branching ratios [1] for $D_s^+ \to \eta\rho^+$ and $D_s^+ \to \eta'\rho^+$ are considerably larger than those predicted in the spectator model [2, 3, 4]. Similarly, the recently measured [5] branching ratios for $D^0 \to \bar{K}^0\eta$ and $\bar{K}^0\eta'$ are much higher than those predicted by the spectator model; $D^0 \to \bar{K}^0\eta'$ especially so.

In Ref. [6] it was shown that factorization hypothesis related the rates for $D_s^+ \to (\eta, \eta')\rho^+$ to the semileptonic rates for $D_s^+ \to (\eta, \eta')e^+\nu$. With the new E-653 data on the semileptonic rates, it was shown [3, 6] that factorization hypothesis supported the experimentally measured rates for $D_s^+ \to (\eta, \eta')\rho^+$, though not separately. E- 653 collaboration has not as yet measured $B(D_s^+ \to \eta e^+\nu)$ and $B(D_s^+ \to \eta' e^+\nu)$ separately, which, of course, would be desirable.

In this paper we have undertaken a systematic study of all the two-body Cabibbo-favored decays of D^0 and D_s^+, and some Cabibbo-suppressed decays of D_s^+ that were relevant to our discussion, involving η or η' in the final state. We have varied the parameters of the theory such as the mixing angle and the q^2-dependence of the form factors. We have introduced an annihilation term whereever admissible in the factorization hypothesis and investigated its sign and size, if it were to bridge the gap between theory and experiment. Factorization hypothesis is expected to work better for decays involving QCD coefficient a_1, that indeed seems to be the case. Transition amplitudes involving a_2 are smaller, sensitive to further QCD corrections and may be affected by multichannel final state interactions. We have not carried out an inelastic final state interaction calculation largely because the input parameters for such an analysis are unknown.

We point out that factorization hypothesis can also be used to relate the rates for $D_s^+ \to (\eta, \eta')\pi^+$ to those for the semileptonic decays, $D_s^+ \to (\eta, \eta')e^+\nu$. However, the test of factorization fails in this case. Hence, if factorization holds then a consistent picture of $D_s^+ \to (\eta, \eta')\pi^+$ and $D_s^+ \to (\eta, \eta')\rho^+$ does not emerge. In the final section we discuss a possible resolution of this difficulty.

After introducing some definitions in section II, we discuss $D^0 \to \bar{K}^0\eta$ and $\bar{K}^0\eta'$ and $D_s^+ \to \eta\pi^+$, $\eta'\pi^+$ decays in section III. In section IV we study $D^0 \to \eta\bar{K}^{*0}$ and $\eta'\bar{K}^{*0}$ and $D_s^+ \to \eta\rho^+$, $\eta'\rho^+$ decays. In section V we relate $B(D_s^+ \to (\eta, \eta')\pi^+)$ to $B(D_s^+ \to (\eta, \eta')e^+\nu)$ through factorization and unveil a puzzle. In section VI we discuss our findings and possible solutions to various problems and paradoxes.

2 Definitions

The mixing angle, θ_P, believed to be $\approx -20^0$ [1], is defined by

$$\eta = \eta_8 \cos\theta_P - \eta_1 \sin\theta_P,$$
$$\eta' = \eta_8 \sin\theta_P + \eta_1 \cos\theta_P \tag{1}$$

where η_1 and η_8 are SU(3) flavor-singlet and octet respectively.

In the discussion to follow we prefer to work in a different basis with a mixing angle θ' defined by,

$$\eta = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\theta' - (s\bar{s})\cos\theta',$$
$$\eta' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\theta' + (s\bar{s})\sin\theta' \tag{2}$$

with $\theta' = 35.26^0$ - θ_P.

The effective weak Hamiltonian in the notation of [2] is

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \{a_1(\bar{s}c)_H(\bar{u}d)_H + a_2(\bar{s}d)_H(\bar{u}c)_H\} \tag{3}$$

For definiteness, we adopt the following value of a_1 and a_2 [7],

$$a_1 = 1.26, \qquad a_2 = -0.51.$$

These values of a_1 and a_2 have been particularly selected by $D \to \pi\pi$ [8] and $D \to \bar{K}\pi$ data [1] as has been discussed in [7, 9]. For the form factors we use (in the notation of [2]),

$$F_0^{DK}(0) = 0.76, \qquad F_0^{D\pi}(0) = 0.83$$
$$F_0^{D\eta}(0) = 0.68, \qquad F_0^{D\eta'}(0) = 0.65 \tag{4}$$
$$F_0^{D_s\eta}(0) = 0.72, \qquad F_0^{D_s\eta'}(0) = 0.70$$

These form factors are then extrapolated in q^2 using a monopole form for $F_0(q^2)$ using $J^P = 0^+$ masses , m($c\bar{u}$) = 2.47 GeV and m($c\bar{s}$) = 2.60 GeV.

In (4) we have used the experimental value [3] of $D \to K$ form factor, and for the $D \to \pi$ form factor a value consistent with data [3] but also one that has helped solve problems elsewhere [7, 9]. The remaining form factors involving $D \to \eta$ and $D \to \eta'$ transitions are taken from [2] for want of better sources.

For form factors $F_1(q^2)$ of [2] that enter the description of decays such as $D_s^+ \to \eta\rho^+$, $\eta'\rho^+$ and $D^0 \to \eta\bar{K}^{*0}$, $\eta'\bar{K}^{*0}$ we use $F_1(0) = F_0(0)$ and both monopole and dipole extrapolations in q^2 using $J^P = 1^-$ masses $m(c\bar{u})$ =2.01 GeV and $m(c\bar{s})$ = 2.11 GeV.

3 Cabibbo-favored $D^0 \to (\eta,\ \eta')\bar{K}^0$ and $D_s^+ \to (\eta,\ \eta')\pi^+$

We define the amplitude for $D \to PP$ (P $\equiv$ Pseudoscalar meson) through the rate formula which for Cabibbo-favored transitions reads,

$$\Gamma(D \to PP) = |\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^*|^2 \frac{|A|^2 |\mathrm{p}|}{8\pi m_D^2} \tag{5}$$

$|A|$ so defined has dimension GeV^3. In Table I we have listed the magnitudes of the decay amplitudes for $D^0 \to \bar{K}^0\eta$, $\bar{K}^0\eta'$ and $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$, obtained from the then measured branching ratios [1, 5]. In the folowing we discuss these decays case-by-case.

A. $D^0 \to \bar{K}^0\eta$

The spectator and the annihilation terms in the factorization scheme are given by,

$$A_{spec.}(D^0 \to \bar{K}^0\eta) = a_2 \frac{sin\theta'}{\sqrt{2}} (m_D^2 - m_K^2) f_K F_0^{D\eta}(m_K^2) \tag{6}$$

$$A_{ann.}(D^0 \to \bar{K}^0\eta) = a_2 (\frac{\sin\theta'}{\sqrt{2}} + \beta\cos\theta')(m_\eta^2 - m_K^2) f_D F_0^{K\eta}(m_D^2) \tag{7}$$

where β is the ratio of the probabilities to excite a $s\bar{s}$-pair from the vacuum to that of a $u\bar{u}$- or a $d\bar{d}$-pair. We use $\beta = 1/3$ throughout.

We have chosen to present numbers for three mixing angles, $\theta_P = -10^0$, -19^0 and -23^0 so as to make the trend with mixing angle evident. Numerically, with $f_K = 161 MeV$, we obtain, in units of GeV^3,

$$A_{spec.}(D^0 \to \bar{K}^0\eta) = \begin{cases} -0.093 & \text{for } \theta_P = -10^0 \\ -0.106 & \text{for } \theta_P = -19^0 \\ -0.111 & \text{for } \theta_P = -23^0 \end{cases} \tag{8}$$

Comparing (8) with the experimental value of the decay amplitude for $D^0 \to \bar{K}^0\eta$ one finds that there is a clear short fall which the annihilation term is very unlikely to provide due to the suppression factor $(m_\eta^2 - m_K^2)$ in (7). An alternative would be for $F_0^{D\eta}(m_K^2)$ to be larger by about 40 %.

Despite the comments above, we calculated the size of the annihilation term that would be needed to bridge the gap between theory and experiment since the same annihilation term enters the description of the Cabibbo-suppressed decay $D_s^+ \to K^+\eta$ albeit at $q^2 = m_{D_s}^2$. We could then check if the impact of this annihilation term on $D_s^+ \to K^+\eta$ is within reasonable bounds.

First, $F_0^{K\eta}(m_D^2)$ *must* be complex as it is evaluated in the physical region of $\bar{K}^0\eta$ scattering. With this in mind we tried two scenarios: One where F_0 is purely real and the other where it is imaginary. With $f_D \approx 0.3GeV$ we found that if purely real, $F_0^{K\eta}(m_D^2)$ is needed to be 12.0 and 11.0 for $\theta_P = -19^0$ and -23^0 respectively and for these form factors (ignoring the extrapolation from m_D^2 to $m_{D_s}^2$) we find $B(D_s^+ \to K^+\eta) = 0.18\%$ and 0.14% respectively. Though $F_0^{K\eta}(m_D^2) \sim 10$ is most probably too large its impact on $B(D_s^+ \to K^+\eta)$ is not so drastic. In fact, a branching ratio of 0.14 % for a Cabibbo-suppressed process is quite acceptable.

If $F_0^{K\eta}(m_D^2)$ is assumed to be imaginary then for $\theta_P = -19^0$ or -23^0 one needs $\mathrm{im}F_0^{K\eta}(m_D^2) \approx 23$ with little dependence on θ_P. This leads to $B(D_s^+ \to K^+\eta) \approx 0.35\%$. Though these branching ratios appear not to be unduly large we find it hard to accept such large values of the form factor $F_0^{K\eta}$ at $q^2 = m_D^2$ as appear to be needed to close the gap between theory and experiment.

B. $D^0 \to \bar{K}^0\eta'$

The spectator and the annihilation amplitudes are

$$A_{spec.}(D^0 \to \bar{K}^0\eta') = a_2\frac{\cos\theta'}{\sqrt{2}}(m_D^2 - m_{\eta'}^2)f_K F_0^{D\eta'}(m_K^2) \quad (9)$$

$$A_{ann.}(D^0 \to \bar{K}^0\eta') = a_2(\frac{\cos\theta'}{\sqrt{2}} - \beta sin\theta')(m_{\eta'}^2 - m_K^2)f_D F_0^{K\eta'}(m_D^2) \quad (10)$$

With $\beta = 1/3$, the spectator amplitude, (10) in units of GeV^3, is,

$$A_{spec.}(D^0 \to \bar{K}^0\eta') = \begin{cases} -0.072 & \text{for } \theta_P = -10^0 \\ -0.060 & \text{for } \theta_P = -19^0 \\ -0.054 & \text{for } \theta_P = -23^0 \end{cases} \quad (11)$$

As is seen from Table I these amplitudes are too small by a factor of 5 to 6.

If we now require the annihilation term to provide the balance of the amplitude we find that with $\beta = 1/3$ if the annihilation amplitude is purely real then $F_0^{K\eta'}(m_D^2)$ is needed to be (9.5, 18.3, 30.2) for $\theta_P =$ $(-10^0, -19^0, -23^0)$. All these values are large. If we assume $F_0^{K\eta'}(m_D^2)$ to be purely imaginary then we need im$F_0^{K\eta'}(m_D^2) =$ (11.9, 22.0. 35.6) for $\theta_P = (-10^0, -19^0, -23^0)$, again rather large values.

The same form factor appears in the annihilation term of the Cabibbo-suppressed decay $D_s^+ \to K^+\eta'$ though at $q^2 = m_{D_s}^2$. We can calculate its effect on $D_s^+ \to K^+\eta'$. If we ignore the extrapolation in q^2 from m_D^2 to $m_{D_s}^2$, the choice of real annihilation form factor results in $B(D_s^+ \to K^+\eta') = (0.30, 0.34, 0.37)\%$ for $\theta_P = (-10^0, -19^0, -23^0)$. These branching ratios are not too large, however, if we choose purely imaginary form factor we get $B(D_s^+ \to K^+\eta') = (1.28, 1.30, 1.31)\%$ for $\theta_P = (-10^0, -19^0, -23^0)$. These branching ratios are too large. We return to a discussion of these results towards the end of this section.

C. $D_s^+ \to \eta\pi^+$

The spectator amplitude is

$$A_{spec.}(D_s^+ \to \eta\pi^+) = -a_1\cos\theta'(m_{D_s}^2 - m_\eta^2)f_\pi F_0^{D_s\eta}(m_\pi^2). \quad (12)$$

The annihilation term vanishes in the factorization approximation by the conserved vector current (CVC) hypothesis. Numerically, with the form factors of (4) and $f_\pi = 132MeV$, we obtain, in units of GeV^3,

$$A_{spec.}(D_s^+ \to \eta\pi^+) = \begin{cases} -0.30 & \text{for } \theta_P = -10^0 \\ -0.25 & \text{for } \theta_P = -19^0 \\ -0.23 & \text{for } \theta_P = -23^0 \end{cases} . \quad (13)$$

The value of the amplitude with $\theta_P = -23^0$ is in a particularly good agreement with experiment.

D. $\quad D_s^+ \to \eta'\pi^+$

The spectator amplitude is

$$A_{spec.}(D_s^+ \to \eta'\pi^+) = a_1 \sin\theta'(m_{D_s}^2 - m_{\eta'}^2) f_\pi F_0^{D_s\eta'}(m_\pi^2). \qquad (14)$$

The annihilation term vanishes again by CVC in the factorization model. Numerically, we obtain in the units of GeV^3,

$$A_{spec.}(D_s^+ \to \eta'\pi^+) = \begin{cases} 0.25 & \text{for } \theta_P = -10^0 \\ 0.28 & \text{for } \theta_P = -19^0 \\ 0.30 & \text{for } \theta_P = -23^0 \end{cases} . \qquad (15)$$

Again comparing (15) with the amplitude for $D_s^+ \to \eta'\pi^+$ in Table I, we find that the value with $\theta_P = -23^0$ is in agreement with experiment, however, the value with $\theta_P = -19^0$ falls somewhat short of the experiment.

Another way to confront data is to evaluate the ratios of the amplitudes to eliminate the dependence on a_1 and a_2. Since CVC does not allow annihilation terms in $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ decays, it is meaningful to calculate,

$$\left|\frac{A(D_s^+ \to \eta\pi^+)}{A(D_s^+ \to \eta'\pi^+)}\right|_{spec.} = 1.21 \cot\theta' \frac{F_0^{D_s\eta}(0)}{F_0^{D_s\eta'}(0)}$$
$$= \begin{cases} 1.20 & \text{for } \theta_P = -10^0 \\ 0.87 & \text{for } \theta_P = -19^0 \\ 0.75 & \text{for } \theta_P = -23^0 \end{cases} . \qquad (16)$$

Experimentally the ratio of these two amplitudes is (0.59 ± 0.12). A mixing angle of $\theta_P = -23^0$ or slightly larger, would not only fit the ratio but also the individual amplitudes. In the final section of this paper we propose an alternative solution to this problem.

To summarise this section, we find that $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ decays, where annihilation terms are not allowed in factorization approximation, come close to being described well by theory. Two factors would help; first $F_0^{D_s\eta}(0)$ be lowered by (10 - 15)% and $F_0^{D_s\eta'}(0)$ be raised by about the same amount and/or θ_P be closer to -23^0.

Theory without the annihilation term fares poorly in explaining $D^0 \to \bar{K}^0\eta$ and $\bar{K}^0\eta'$, especially the latter where the spectator amplitude accounts for only $\sim 20\%$ of the total amlpitude at $\theta_P = -19^0$. If a real annihilation term is invoked to achieve agreement with experiment one needs $F_0^{K\eta'}(m_D^2) \approx 20$ for $\theta_P = -19^0$. Such an annihilation would also effect the Cabibbo- suppressed process $D_s^+ \to K^+\eta'$ and generate

$B(D_s^+ \to K^+\eta') \approx 0.31\%$ for $\theta_P = -19^0$ which though large is not unacceptable. However, if we assume the annihilation form factor to be purely imaginary, we need $im F_0^{K\eta'}(m_D^2) \approx 22$ for $\theta_P = -19^0$ and the effect on $D_s^+ \to K^+\eta'$ is to generate $B(D_s^+ \to K^+\eta') \approx 1.3\%$ which is unacceptably high.

The situation with $D_0 \to \bar{K}^0\eta$ is not as bad as in $D_0 \to \bar{K}^0\eta'$. Here the spectator term is $\sim 60\%$ of the needed amplitude. Due to the fact that the annihilation term appears with a kinematic factor $(m_\eta^2 - m_K^2)$, rather a large value of $F_0^{K\eta}(m_D^2)$ is needed to bridge the gap between experiment and theory. Though the effect of such a large annihilation form factor on Cabibbo-suppressed $D_s^+ \to K^+\eta$ is not unacceptable we consider such large form factors improbable.

4 Cabibbo-favored $D^0 \to \eta\bar{K}^{*0}$, $\eta'\bar{K}^{*0}$ and $D_s^+ \to \eta\rho^+$, $\eta'\rho^+$.

We begin with the definition of the decay amplitude, $\tilde{A}$, through the decay rate formula for $D \to PV (V \equiv \text{Vector})$ decays,

$$\Gamma(D \to PV) = |\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^*|^2 \frac{|\mathbf{p}|^3}{2\pi} |\tilde{A}|^2. \qquad (17)$$

$|\tilde{A}|$ has the dimension of GeV and is obtained by writing down the decay amplitude and removing from it a factor $2m_V(\epsilon^*.P_D)\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^*$, where m_V is the vector particle mass and ϵ^* its polarization vector, P_D is the D meson four momentum.

In Table II we have listed the magnitude of the amplitudes for $D^0 \to \eta\bar{K}^{*0}$, $\eta'\bar{K}^{*0}$ and $D_s^+ \to \eta\rho^+$, $\eta'\rho^+$ decays. We now proceed to discuss these decays one-by-one.

A. $\mathbf{D^0 \to \eta\bar{K}^{*0}}$

The spectator and the annihilation amplitudes are

$$\tilde{A}_{spec.} = a_2 \frac{\sin\theta'}{\sqrt{2}} f_{K^*} F_1^{D\eta}(m_{K^*}^2), \qquad (18)$$

$$\tilde{A}_{ann.} = a_2 (\frac{\sin\theta'}{\sqrt{2}} - \beta\cos\theta') f_D A_0^{K^*\eta}(m_D^2). \qquad (19)$$

The form factors F_1 and A_0 are as defined in [2]. In decays of B mesons such as $B^0 \to \psi K^0$, where $F_1^{BK}(q^2)$ enters the spectator description, a dipole extrapolation of this form factor seems to be required [10]. Since D meson is not as heavy as B, we have chosen to work with both a monopole and a dipole extrapolation of $F_1(q^2)$ to see if data prefer one form over the other.

The spectator amplitude for $\theta_P = (-10^0, \ -19^0 \ \ -23^0)$ in units of GeV is,

$$\tilde{A}_{spec.}(D^0 \to \eta \bar{K}^{*0}) = \begin{cases} -0.048 & \text{for } \theta_P = -10^0 \\ -0.055 & \text{for } \theta_P = -19^0 \\ -0.057 & \text{for } \theta_P = -23^0 \end{cases} , \qquad (20a)$$

for n=1,

$$\tilde{A}_{spec.}(D^0 \to \eta \bar{K}^{*0}) = \begin{cases} -0.060 & \text{for } \theta_P = -10^0 \\ -0.068 & \text{for } \theta_P = -19^0 \\ -0.072 & \text{for } \theta_P = -23^0 \end{cases} , \qquad (20b)$$

for n=2, where n=1, 2 stand for monopole and dipole forms respectively. From Table II we find that for $\theta_P = -19^0$ these amplitudes are too small by a factor of approximately two.

If we require the annihilation term to make up the short fall, we need the annihilation form factor $A_0^{K^*\eta}(m_D^2)$ to have the following sizes (we have used $\beta = 1/3$ and $f_D \approx 0.3$ GeV and assumed a real annihilation term),

$$A_0^{K^*\eta}(m_D^2) = \begin{cases} 2.04 & \text{for } \theta_P = -10^0 \\ 1.33 & \text{for } \theta_P = -19^0 \\ 1.13 & \text{for } \theta_P = -23^0 \end{cases} , \qquad (21a)$$

for n=1,

$$A_0^{K^*\eta}(m_D^2) = \begin{cases} 1.75 & \text{for } \theta_P = -10^0 \\ 1.09 & \text{for } \theta_P = -19^0 \\ 0.92 & \text{for } \theta_P = -23^0 \end{cases} , \qquad (21b)$$

for n=2. These form factors are large but not unduly so. If we had assumed the form factor to be purely imaginary, its magnitude would be larger as the annihilation term would then not interfere with the real spectator term and would have to provide $\approx 75\%$ of the rate, the remaining coming from the spectator term. For purely imaginary annihilation term one gets:

$$im A_0^{K^*\eta}(m_D^2) = \begin{cases} 3.00 & \text{for } \theta_P = -10^0 \\ 2.06 & \text{for } \theta_P = -19^0 \\ 1.82 & \text{for } \theta_P = -23^0 \end{cases} , \qquad (22a)$$

for n=1,

$$im A_0^{K^*\eta}(m_D^2) = \begin{cases} 2.82 & \text{for } \theta_P = -10^0 \\ 1.94 & \text{for } \theta_P = -19^0 \\ 1.70 & \text{for } \theta_P = -23^0 \end{cases} , \qquad (22b)$$

for n=2.

We conclude that a significant annihilation term is needed. If assumed real it would have to account for about 50% of the total amplitude to explain data. However, two other factors help in this regard: A larger mixing angle, $\theta_P = -23^0$ does better than $\theta_P = -19^0$, and a dipole extrapolation of the form factor $F^{D\eta}(q^2)$ also appears to be favored.

B. $D^0 \to \eta' \bar{K}^{*0}$

Very little phase space is available to this decay mode and only an upper limit to the branching ratio exists [1]. The spectator and annihilation terms are,

$$\tilde{A}_{spec.}(D^0 \to \eta' \bar{K}^{*0}) = a_2 \frac{\cos\theta'}{\sqrt{2}} f_{K^*} F_1^{D\eta'}(m_{K^*}^2) \tag{23}$$

$$\tilde{A}_{ann.}(D^0 \to \eta' \bar{K}^{*0}) = a_2 (\frac{\cos\theta'}{\sqrt{2}} + \beta \sin\theta') f_D A_0^{K^*\eta'}(m_D^2) \tag{24}$$

Numerically, the spectator amplitude is,

$$\tilde{A}_{spec.}(D^0 \to \eta' \bar{K}^{*0}) = \begin{cases} -0.046 & \text{for } \theta_P = -10^0 \\ -0.038 & \text{for } \theta_P = -19^0 \\ -0.034 & \text{for } \theta_P = -23^0 \end{cases}, \tag{25a}$$

for n= 1,

$$\tilde{A}_{spec.}(D^0 \to \eta' \bar{K}^{*0}) = \begin{cases} -0.057 & \text{for } \theta_P = -10^0 \\ -0.047 & \text{for } \theta_P = -19^0 \\ -0.044 & \text{for } \theta_P = -23^0 \end{cases}, \tag{25b}$$

for n= 2. All these amplitudes are an order of magnitude below the upper bound for the experimental amplitude. With only an upper bound on the experiment amplitude we do not see any useful purpose in investigating the size of annihilation amplitude.

C. $D_s^+ \to \eta\rho^+$

The spectator and the annihilation terms are ,

$$\tilde{A}_{spec.}(D_s^+ \to \eta\rho^+) = -a_1 \cos\theta' f_\rho F_1^{D_s\eta}(m_\rho^2), \tag{26}$$

$$\tilde{A}_{ann.}(D_s^+ \to \eta\rho^+) = a_1 \sin\theta' \sqrt{2} f_{D_s} A_0^{\eta\rho}(m_{D_s}^2). \tag{27}$$

The values of the spectator term in units of GeV are

$$\tilde{A}_{spec.}(D_s^+ \to \eta\rho^+) = \begin{cases} -0.163 & \text{for } \theta_P = -10^0 \\ -0.136 & \text{for } \theta_P = -19^0 \\ -0.122 & \text{for } \theta_P = -23^0 \end{cases}, \tag{28a}$$

for n=1,

$$\tilde{A}_{spec.}(D_s^+ \to \eta\rho^+) = \begin{cases} -0.189 & \text{for } \theta_P = -10^0 \\ -0.156 & \text{for } \theta_P = -19^0 \\ -0.141 & \text{for } \theta_P = -23^0 \end{cases}, \qquad (28b)$$

for n=2. Here, the trend is to favor smaller mixing angles, but with $\theta_P = -19^0$ a dipole form gets the amplitude within bounds of the error (see Table II). Again, a dipole form for $F_1(q^2)$ appears to fare better.

As the amplitudes in (28) are quite close to the experimental amplitudes a small annihilation term can easily generate the balance of the rate. If we assume $f_{D_s} = 0.34$ GeV then the following values of the annihilation form factor, $A_0^{\eta\rho}(m_{D_s}^2)$, assumed real, are needed

$$A_0^{\eta\rho}(m_{D_s}^2) = \begin{cases} -0.028 & \text{for } \theta_P = -10^0 \\ -0.081 & \text{for } \theta_P = -19^0 \\ -0.103 & \text{for } \theta_P = -23^0 \end{cases}, \qquad (29a)$$

for n=1,

$$A_0^{\eta\rho}(m_{D_s}^2) = \begin{cases} +0.030 & \text{for } \theta_P = -10^0 \\ -0.038 & \text{for } \theta_P = -19^0 \\ -0.067 & \text{for } \theta_P = -23^0 \end{cases}, \qquad (29b)$$

for n=2. These numbers are generated using the central values of the entry in Table II for $\tilde{A}(D_s^+ \to \eta\rho^+)$. Such small values of the annihilation form factor are within the realm of possibility for both monopole and dipole forms for $F_1(q^2)$.

D. $D_s^+ \to \eta'\rho^+$

The spectator and the annihilation terms for $D_s^+ \to \eta'\rho^+$ are,

$$\tilde{A}_{spec.}(D_s^+ \to \eta'\rho^+) = a_1 sin\theta' f_\rho F_1^{D_s\eta'}(m_\rho^2) \qquad (30a)$$

$$\tilde{A}_{ann.}(D_s^+ \to \eta'\rho^+) = a_1 cos\theta' \sqrt{2} f_{D_s} A_0^{\eta'\rho}(m_{D_s}^2) \qquad (30b)$$

The magnitude of the spectator term in units of GeV, is,

$$\tilde{A}_{spec.}(D_s^+ \to \eta'\rho^+) = \begin{cases} 0.160 & \text{for } \theta_P = -10^0 \\ 0.183 & \text{for } \theta_P = -19^0 \\ 0.192 & \text{for } \theta_P = -23^0 \end{cases}, \qquad (31a)$$

for n=1,

$$\tilde{A}_{spec.}(D_s^+ \to \eta'\rho^+) = \begin{cases} 0.185 & \text{for } \theta_P = -10^0 \\ 0.212 & \text{for } \theta_P = -19^0 \\ 0.222 & \text{for } \theta_P = -23^0 \end{cases}, \qquad (31b)$$

for n=2. Comparing with the entry in Table II for the experimental value of the amplitude one finds that the values in (31a) are about one-half of what is needed, nevertheless, again a larger mixing angle fares better as also does a dipole extrapolation of $F_1(q^2)$.

If now we require the annihilation term to provide the balance of the amplitude, assuming it to be real and with $f_{D_s} = 0.34GeV$, we get the following annihilation form factors,

$$A_0^{\eta'\rho}(m_{D_s}^2) = \begin{cases} 0.48 & \text{for } \theta_P = -10^0 \\ 0.52 & \text{for } \theta_P = -19^0 \\ 0.55 & \text{for } \theta_P = -23^0 \end{cases}, \tag{32a}$$

for n=1,

$$A_0^{\eta'\rho}(m_{D_s}^2) = \begin{cases} 0.43 & \text{for } \theta_P = -10^0 \\ 0.44 & \text{for } \theta_P = -19^0 \\ 0.46 & \text{for } \theta_P = -23^0 \end{cases}, \tag{32b}$$

for n=2. These form factors are quite reasonable in size.

From (29) and (32) we notice that the annihilation form factor for $D_s^+ \rightarrow \eta'\rho^+$, is a factor of (5- 10) larger than the form factor needed for $D_s^+ \rightarrow \eta\rho^+$, and opposite in sign. There is no compelling reason for the two form factors to have the same size and sign since though the continuum in $\eta\rho$ and $\eta'\rho$ channels have the same threshold ($\eta\rho$ channel being coupled to $\eta'\rho$ channel), the states contributing to spectral functions (imaginary parts of the form factors) are different. We return to a fuller discussion of this decay mode in the final section of this paper.

5 Relating $\Gamma(D_s^+ \rightarrow (\eta, \eta')e^+\nu)$ *to* $\Gamma(D_s^+ \rightarrow (\eta, \eta')\pi^+)$ through factorization– A Puzzle?

In [6] it was shown that $\Gamma(D_s^+ \rightarrow (\eta, \eta')e^+\nu)$ was natuarally related to $\Gamma(D_s^+ \rightarrow (\eta, \eta')\rho^+)$ since both decays invoke the same form factor, $F_1(q^2)$. For instance, the differential and total decay rates for $D_s^+ \rightarrow \eta e^+\nu$ are given by

$$\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow \eta e^+\nu) = \frac{G_F^2}{192\pi^3}\frac{|V_{cs}|^2C_\eta^2}{m_{D_s}^3}\lambda^3(m_{D_s}^2, m_\eta^2, q^2)|F_1^{D_s\eta}(q^2)|^2, \tag{33}$$

$$\Gamma(D_s^+ \rightarrow \eta e^+\nu) = \frac{G_F^2}{192\pi^3}\frac{|V_{cs}|^2C_\eta^2}{m_{D_s}^3}\Lambda_1^4 I(m_{D_s}, m_\eta, \Lambda_1)|F_1^{D_s\eta}(0)|^2, \tag{34}$$

where

$$C_\eta = \sqrt{\frac{2}{3}}(\cos\theta_P + \frac{1}{\sqrt{2}}\sin\theta_P),$$

$$I(m_{D_s}, m_\eta, \Lambda_1) \equiv \int_0^{(m_{D_s}-m_\eta)^2} \frac{\lambda^3(m_{D_s}^2, m_\eta^2, q^2)}{(q^2-\Lambda_1^2)^2} dq^2,$$

and

$$\lambda(x, y, z) = (x^2+y^2+z^2-2xy-2xz-2yz)^{1/2}.$$

$\Lambda_1 = 2.11$ GeV. is the mass of D_s^*. The hadronic rate for $D_s^+ \to \eta\rho^+$ in the factorization scheme is given by

$$\Gamma(D_s^+ \to \eta\rho^+) = \frac{G_F^2}{32\pi} \frac{|V_{cs}|^2|V_{ud}|^2\, a_1^2\, f_\rho^2 C_\eta^{\prime 2}}{m_{D_s}^3} \lambda^3(m_{D_s}^2, m_\eta^2, m_\rho^2)|F_1^{D_s\eta}(m_\rho^2)|^2. \tag{35}$$

From (33) and (35) one obtains

$$\Gamma(D_s^+ \to \eta\rho^+) = 6\pi^2\, |V_{ud}|^2\, a_1^2\, f_\rho^2 \frac{d\Gamma}{dq^2}(D_s^+ \to \eta e^+\nu)|_{q^2=m_\rho^2}, \tag{36}$$

and from (34) and (35) we get

$$\frac{\Gamma(D_s^+ \to \eta e^+\nu)}{\Gamma(D_s^+ \to \eta\rho^+)} = \frac{I(m_{D_s}, m_\eta, \Lambda_1)}{6\pi^2\, |V_{ud}|^2\, a_1^2\, f_\rho^2} \frac{(\Lambda_1^2 - m_\rho^2)^2}{\lambda^3(m_{D_s}^2, m_\eta^2, m_\rho^2)}. \tag{37}$$

An analogous treatment can be performed for the η' modes. In [6] $a_1 = 1.2$ with a 10% uncertainty was used. If we revise their numbers using a fixed value, $a_1 = 1.26$, the results of [6] can be restated as follows (n =1, 2 refer to monopole and dipole extrapolation of $F_1(q^2)$),

$$\frac{B(D_s^+ \to \eta e^+\nu)}{B(D_s^+ \to \phi\pi^+)} = \begin{cases} (0.91 \pm 0.19), & \text{for n} = 1 \\ (0.97 \pm 0.20), & \text{for n} = 2 \end{cases}, \tag{38}$$

$$\frac{B(D_s^+ \to (\eta')e^+\nu)}{B(D_s^+ \to \phi\pi^+)} = \begin{cases} (1.09 \pm 0.25), & \text{for n} = 1 \\ (0.98 \pm 0.22), & \text{for n} = 2 \end{cases}. \tag{39}$$

The errors here are from the measurements [11] of $B(D_s^+ \to \eta\rho^+)/\ B(D_s^+ \to \phi\pi^+)$ and $B(D_s^+ \to \eta'\rho^+)/B(D_s^+ \to \phi\pi^+)$. We have combined the statistical and systematic errors in quadrature. We note that the use of a dipole form factor does not alter results very much since it enters in both hadronic and semileptonic rates. The sum of the semileptonic ratios in (38) and (39) is,

$$\frac{B(D_s^+ \to (\eta+\eta')e^+\nu)}{B(D_s^+ \to \phi\pi^+)} = \begin{cases} (2.01 \pm 0.31), & \text{for n} = 1 \\ (1.95 \pm 0.30), & \text{for n} = 2 \end{cases}. \tag{40}$$

E-653 collaboration has measured [3].

$$\frac{B(D_s^+ \to (\eta+\eta')\mu^+\nu)}{B(D_s^+ \to \phi\mu^+\nu)} = 3.9 \pm 1.6. \tag{41}$$

(40) is consistent with (41) once one recognises that the ratio of the rates for $D_s^+ \to \phi e^+ \nu$ to that of $D_s^+ \to \phi\pi^+$ is ≈ 0.5 [1]. Thus, if factorization holds, this agreement could be viewed as providing support for the measured branching ratios for $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$ relative to $D_s^+ \to \phi\pi^+$, however large as they are. Also, we note that monopole and dipole extrapolations of $F_1(q^2)$ fare equally well.

However, it is also possible to relate the rates for $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ to the semileptonic rates for $D_s^+ \to \eta e^+ \nu$ and $\eta' e^+ \nu$ despite the fact that the hadronic modes invoke the form factors $F_0(q^2)$ and the semileptonic modes $F_1(q^2)$. The reason is that $F_0(q^2)$ is needed at $q^2 = m_\pi^2$ and if we use the fact that $F_0(0) = F_1(0)$, then $F_0(m_\pi^2) \approx F_0(0) = F_1(0)$. One is then led through an analysis similar to that in [6] to (we are now using $a_1 = 1.26$),

$$\frac{\Gamma(D_s^+ \to \eta e^+ \nu)}{\Gamma(D_s^+ \to \eta\pi^+)} = \begin{cases} 0.62 & \text{for n= 1} \\ 0.88 & \text{for n= 2} \end{cases}, \tag{42}$$

and

$$\frac{\Gamma(D_s^+ \to \eta' e^+ \nu)}{\Gamma(D_s^+ \to \eta'\pi^+)} = \begin{cases} 0.29 & \text{for n=1} \\ 0.35 & \text{for n = 2} \end{cases}, \tag{43}$$

where the two entries are for monopole and dipole extrapolations of $F_1(q^2)$. Now, using [12],

$$\frac{B(D_s^+ \to \eta\pi^+)}{B(D_s^+ \to \phi\pi^+)} = 0.54 \pm 0.09 \pm 0.06 \tag{44}$$

$$\frac{B(D_s^+ \to \eta'\pi^+)}{B(D_s^+ \to \phi\pi^+)} = 1.2 \pm 0.15 \pm 0.11. \tag{45}$$

We get, with errors added in quadrature,

$$\frac{B(D_s^+ \to (\eta + \eta')e^+ \nu)}{B(D_s^+ \to \phi\pi^+)} = \begin{cases} 0.68 \pm 0.09 & \text{for n = 1} \\ 0.90 \pm 0.11 & \text{for n = 2} \end{cases}. \tag{46}$$

Comparing this result with (40), obtained using factorization and $D_s^+ \to (\eta, \eta')\rho^+$ data, one finds that there is a problem- the predicted relative branching ratio for the combined semileptonic processes $D_s^+ \to (\eta + \eta')e^+\nu$ is too low by a factor of 2 when $D_s^+ \to (\eta, \eta')\pi^+$ data are used. However the dipole extrapolation of $F_1(q^2)$ fares better than the monopole. We discuss the issue of the form factor more completely in the Discussion and propose a solution to this puzzle.

6 Discussion

We start with a discussion of $D_s^+ \to (\eta, \eta')\pi^+$ and $D_s^+ \to (\eta, \eta')\rho^+$. To begin with, there is every indication at present that if factorization is correct then the experimentally measured rates for $D_s^+ \to (\eta,\eta')\rho^+$ and $D_s^+ \to (\eta,\eta')e^+\nu$ are mutually consistent. Yet, when we compute $B(D_s^+ \to (\eta,\eta')\rho^+)$ using spectator model, which is required for the validity of the test of factorization using semileptonic decays, with the form factors of Ref. [2] and monopole or dipole extrapolations for $F_1(q^2)$, we come up short of the experiments. We found that a small annihilation term sets things right but this is contrary to the spirit of the relation between $D_s^+ \to (\eta,\eta')e^+\nu$ and $D_s^+ \to (\eta,\eta')\rho^+$ which requires the hadronic modes to be given by the spectator model.

We have also found that the spectator model with the form factors of Ref. [2] comes very close to fitting $D_s^+ \to (\eta,\eta')\pi^+$ data. Here, annihilation terms are not allowed in the factorization scheme. Assuming factorization we have related $B(D_s^+ \to (\eta,\eta')\pi^+)$ to B($D_s^+ \to (\eta,\eta')e^+\nu$) and this relation appears to fail the test of experiments for monopole and dipole extrapolations of $F_1(q^2)$.

Perhaps, the puzzle posed by $D_s^+ \to (\eta,\eta')\rho^+$ and $D_s^+ \to (\eta,\eta')\pi^+$ decays are due to the use of incorrect form factors and, perhaps, factorization works well in all these hadronic decays . To demonstrate a possible way out, consider a different parametrization of the form factor $F_1(t)$, t = q^2,

$$F_1^{D_s\eta}(t) = \frac{0.58(1+.75t)}{(1-\frac{t}{\Lambda_1^2})^2}, \tag{47}$$

and

$$F_1^{D_s\eta'}(t) = \frac{0.83(1+.75t)}{(1-\frac{t}{\Lambda_1^2})^2}, \tag{48}$$

with $\Lambda_1 = 2.11$ GeV.

These form factors are a particular linear superposition of a monopole and a dipole form. With these form factors (recall, $F_0(m_\pi^2) \approx F_0(0) = F_1(0)$), we get,

$$\begin{aligned}
\tilde{A}_{spec.}(D_s^+ \to \eta\pi^+) &= \begin{cases} -0.24 & \text{for } \theta_P = -10^0 \\ -0.20 & \text{for } \theta_P = -19^0 \\ -0.18 & \text{for } \theta_P = -23^0 \end{cases}, \\
\tilde{A}_{spec.}(D_s^+ \to \eta'\pi^+) &= \begin{cases} 0.30 & \text{for } \theta_P = -10^0 \\ 0.33 & \text{for } \theta_P = -19^0 \\ 0.35 & \text{for } \theta_P = -23^0 \end{cases}, \\
\tilde{A}_{spec.}(D_s^+ \to \eta\rho^+) &= \begin{cases} -0.22 & \text{for } \theta_P = -10^0 \\ -0.18 & \text{for } \theta_P = -19^0 \\ -0.16 & \text{for } \theta_P = -23^0 \end{cases},
\end{aligned} \tag{49}$$

$$\tilde{A}_{spec.}(D_s^+ \to \eta'\rho^+) = \begin{cases} 0.32 & \text{for } \theta_P = -10^0 \\ 0.36 & \text{for } \theta_P = -19^0 \\ 0.38 & \text{for } \theta_P = -23^0 \end{cases} .$$

For $\theta_P = -19^0$, or -23^0 these amplitudes are in agreement with data and generate a value of 0.59 for the ratio in (16) for $\theta_P = -19^0$. Next we use these form factors to relate $B(D_s^+ \to (\eta, \eta')e^+\nu)$ to $B(D_s^+ \to (\eta, \eta')\pi^+)$ with the following results

$$\Gamma(D_s^+ \to \eta e^+\nu) = 0.46\Gamma(D_s^+ \to \eta\rho^+), \tag{50}$$

$$\Gamma(D_s^+ \to \eta' e^+\nu) = 0.23\Gamma(D_s^+ \to \eta'\rho^+). \tag{51}$$

Dividing by $\Gamma(D_s^+ \to \phi\pi^+)$ and using $B(D_s^+ \to \eta\rho^+)/B(D_s^+ \to \phi\pi^+)$ and $B(D_s^+ \to \eta'\rho^+)/(D_s^+ \to \phi\pi^+)$ from [11],

$$\frac{B(D_s^+ \to \eta\rho^+)}{B(D_s^+ \to \phi\pi^+)} = 2.86 \pm 0.38 \begin{array}{l} +0.36 \\ -0.38 \end{array}, \tag{52}$$

$$\frac{B(D_s^+ \to \eta'\rho^+)}{B(D_s^+ \to \phi\pi^+)} = 3.44 \pm 0.62 \begin{array}{l} +0.44 \\ -0.46 \end{array}. \tag{53}$$

We get (with $a_1 = 1.26$)

$$\frac{B(D_s^+ \to (\eta + \eta')e^+\nu)}{B(D_s^+ \to \phi\pi^+)} = 2.12 \pm 0.31, \tag{54}$$

which is not much different from the results using a dipole form for $F_1(t)$, given in (40) and is consistent with experiment.

If we redo the calculation of the previous section relating $B(D_s^+ \to (\eta, \eta')e^+\nu)$ to $B(D_s^+ \to (\eta, \eta')\pi^+)$ with the form factors of (47) and (48), we obtain,

$$B(D_s^+ \to \eta e^+\nu) = 2.24B(D_s^+ \to \eta\pi^+), \tag{55}$$

$$B(D_s^+ \to \eta' e^+\nu) = 0.55B(D_s^+ \to \eta'\pi^+). \tag{56}$$

Using the data of [12] these then lead to,

$$\frac{B(D_s^+ \to (\eta + \eta')e^+\nu)}{B(D_s^+ \to \phi\pi^+)} = 1.87 \pm 0.26. \tag{57}$$

This is an improvement on (46) and makes E-653 data consistent with $(D_s^+ \to (\eta, \eta')\pi^+)$ data. The puzzle alluded to in section V is, for all intents and purposes, resolved. Better statistics data on $(D_s^+ \to (\eta, \eta')e^+\nu)$ with a separation of η and η' will help a great deal in understanding D_s^+ decays into two- body channels involving η and η'.

Among $D^0 \to \eta K^0$, $\eta' K^0$, ηK^{*0}, and $\eta' K^{*0}$, the understanding of $D^0 \to \eta' K^0$ is by far the poorest where the spectator term provides only $\sim 20\%$

of the amplitude and a rescaling of the form factor is hardly likely to be the answer. The annilhilation term needed to fit data requires an unlikely large annihilation form factor, though its impact on the Cabibbo-suppressed process, $D_s^+ \rightarrow \eta' K^+$, does not appear to be out of the ordinary for a real form factor.

The situation with $D^0 \rightarrow \eta \bar{K}^0$ is not so desperate -the spectator term accounts for $\sim 60\%$ of the experimental amplitude. However, if the rest is to be found in the annihilation term, one needs an unlikely large anihilation form factor though its impact on Cabibbo-suppressed $D_s^+ \rightarrow \eta K^+$ is quite acceptable. Here a rescaling of the form factor might be what is needed.

In $D^0 \rightarrow \eta \bar{K}^{*0}$, the spectator model provides $\sim 50\%$ of the amplitude. Here also a scaling upward of $F_1^{D\eta}(0)(= F_0^{D\eta}(0))$ consistent with what is needed for $D^0 \rightarrow \bar{K}^0 \eta$ and an extrapolation of $F_1^{D\eta}(t)$ akin to (47) and (48) could bridge the gap between theory and experiment.

Not much can be said about $D^0 \rightarrow \eta' \bar{K}^{*0}$ as only an upper limit on its branching ratio exists and the spectator model yields a value well inside this limit.

We would advocate measurements of $D_s^+ \rightarrow (\eta,\ \eta')e^+\nu$ with a separation of η and η', and also $D^+ \rightarrow \eta e^+ \nu$ and $\eta' e^+ \nu$ which would give us the form factors needed to understand $D^0 \rightarrow \bar{K}^0 \eta$, $\bar{K}^0 \eta'$ and $D^0 \rightarrow \eta \bar{K}^{*0}$ decays.

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Table I

Magnitudes of decay amplitudes for charm $\rightarrow$ PP extracted from data [1, 5]. Amplitudes in units of GeV^3 as defined in (6).

Process	BR in %	$\lvert A\rvert$ in GeV^3
$D^0 \rightarrow \bar{K}^0\eta$	1.12 ± 0.22^a	0.18 ± 0.018
$D^0 \rightarrow \bar{K}^0\eta'$	2.71 ± 0.42^a	0.33 ± 0.025
$D_s^+ \rightarrow \eta\pi^+$	1.5 ± 0.4^b	0.20 ± 0.026
$D_s^+ \rightarrow \eta'\pi^+$	3.7 ± 1.2^b	0.34 ± 0.055

[a] Source Ref. [5]
[b] Source Ref. [1]

Table II

Magnitudes of decay amplitudes for charm $\rightarrow$ PV extracted from data [1, 5]. Amplitudes $\tilde{A}$ in units of GeV as defined in (17).

Process	BR in %	$\lvert A\rvert$ in GeV^3
$D^0 \rightarrow \eta\bar{K}^{*0}$	2.1 ± 1.2	0.13 ± 0.038
$D^0 \rightarrow \eta'\bar{K}^{*0}$	< 0.13	< 0.35
$D_s^+ \rightarrow \eta\rho^+$	7.9 ± 2.11	0.18 ± 0.023
$D_s^+ \rightarrow \eta'\rho^+$	9.5 ± 2.7	0.37 ± 0.052

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