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# A BIASED DOUBLE-WELL HETEROSTRUCTURE IN A FREE-ELECTRON LASER

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# A biased double-well heterostructure in a free-electron laser

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**Abstract**

The eigenvalue problem of electrons in a biased double-well heterostructure driven by a laser field is studied. The closed-form solutions for the quasienergy and the Floquet states are obtained with the help of $SU(2)$ symmetry. A remarkable feature is found to be that of the occurrence of the quasienergy crossing, which implies that the coherent tunneling of electrons between two wells can be suppressed almost completely through the choice of the field parameters. Such a coherent destruction of tunneling should be observed in experiments by using free-electron laser facilities.

## I. Introduction

The interesting effects have recently been discovered in the study of interaction of electrons in symmetric double-well heterostructures with laser fields. Among these are the coherent destruction of tunneling [1,2], the exact level crossing [3], the induction of a static dipole moment [4], and low-frequency generation [5]. The observation of such effects can be fulfilled in quantum-well heterostructures due to their large dipole moments ($10 - 10^3$ large than that of molecular systems) and adjustable small splitting between levels [6]. The intensities required can be obtained by using a free-electron laser which produces intense $20-\mu s$ square pulses that are tunable in the region of $6 - 170\ cm^{-1}$. This means that only the low-lying states of the system are involved in the consideration.



It has been shown that the low-lying states of a double quantum wells can be described approximately as a two-level system [7]. Such a system, as well known, is of great practical importance in physics. Hence, there is a considerable growing interest on it. Very recently, the present author find that the problem can be transformed into the study of a first-differential equation or a Neumann-Liouville expansion, and gives out general closed-form solutions of the evolution operator with the help of $SU(2)$ symmetry [2,7,8]. In this paper, by applying our method, we will study the quasienergy spectrum resulting from the interaction of an electron in a biased quantum double-well heterostructure under the influence of a laser field. The Hamiltonian we considered here is

$$H(t) = \frac{\omega_0}{2}\sigma_z + \mu[E_0 + E\cos\omega t]\sigma_x\ , \qquad (1)$$

where $\sigma_x$ and $\sigma_y$ (as well as $\sigma_z$ used below) are the Pauli matrices, $(\omega_0/2)\sigma_z$ is the unpertured Hamiltonian of the system with eigenvalues ($E_\alpha = -\omega_0/2$ , $E_\beta = \omega_0/2$) and eigenvectors ($|\alpha>$ , $|\beta>$), $\mu$ is the transition dipole between two levels, $E_0$ is a constant field for breaking the symmetry of the double-well, and $E$ and $\omega$ are, respectively, the amplitude and the frequency of the driving laser field. This model was studied recently for the special case of weak leterlevel coupling and high-frequency driving field ($\omega_0/\omega << 1$) [9]. Here, we focus on the general case in which the ratio $\omega_0/\omega$ can be arbitrarily large or small.

The rest of this paper is set out as follows. In Sec. II, we give a brief presentation for the derivation of the eigenvalue equation of the evolution operator, from which the closed-form solutions of the quasienergy and the Floquet states are obtained (Sec. III). These are followed by a discussion and conclusion in Sec. IV.

## II. Eigenvalue Equation

In this and next sections, we outline the procedure to describe the derivation of the quasienergy and the Floquet states for the periodically driven two-level system.

Since the Hamiltonian (1) is periodic in time with period $\tau = 2\pi/\omega$, Floquet's theorem asserts that the wave functions of $H(t)$ can be written as $|\psi(t)>= \exp(-i\epsilon t)|\phi(t)>$ with the time-periodic Floquet functions $|\phi(t+\tau)>=|\phi(t)>$, where $\epsilon$ is the quasienergy. The evolution operator $U(t,t_0)$ of the system, defined through the Floquet function $|\phi(t)>$ as

$$|\phi(t)>= e^{i\epsilon t}U(t,t_0)e^{-i\epsilon t_0}|\phi(t_0)>\ , \qquad (2)$$

satisfies the evolution equation

$$i\frac{\partial}{\partial t}U(t,t_0)=H(t)U(t,t_0) \tag{3}$$

with the initial condition $U(t_0,t_0)=1$, here we put $\hbar=1$ throught this paper. Setting $t=t_0+\tau$ in Eq. (2) and using the periodicity of the Floquet states, $|\phi(t+\tau)>=|\phi(t)>$, we get

$$\left[U(t_0+\tau,t_0)-e^{-i\epsilon\tau}\right]|\phi(t_0)>=0\ . \tag{4}$$

Note that $U(t,t_0)$ is the $2\times 2$ matrix, therefore, the requirement of the non-trivial solution for the Floquet state $|\phi(t_0)>$ leads to the eigenvalue equation

$$\det\left[U(t_0+\tau,t_0)-e^{-i\epsilon\tau}\right]=0\ . \tag{5}$$

Using the properties of the evolution operator $U(t,t_0)=U(t,t')U(t',t_0)$, $U(t,t_0)=U^{\dagger}(t_0,t)$, we can rewrite Eq. (5) as

$$\det\left[U(\tau,0)-e^{-i\epsilon\tau}\right]=0\ , \tag{6}$$

where the result $U(t+\tau,t_0+\tau)=U(t,t_0)$ for the time-periodic Hamiltonian (1) has been used. Thus the quasienergy can be determined from Eq. (6) directly so long as we obtain the evolution operator in one period.

## III. Quasienergy Spectrum and Wave Functions

It has been shown that the evolution equation (3) can be solved analytically in the closed-form solution [2,7]. To do that, we introduce the unitary transformation

$$U_0(t)=e^{-i\sigma_z\omega_0 t/2}\ , \tag{7}$$

letting

$$U(t,t_0)=U_0(t)U_I(t,t_0)U_0^{\dagger}(t_0)\ , \tag{8}$$

which result in Eq. (3) to be

$$i\frac{\partial}{\partial t}U_I(t,t_0)=H_I(t)U_I(t,t_0) \tag{9}$$

where

$$H_I(t)=X(t)\sigma_x+Y(t)\sigma_y \tag{10}$$



with

$$X(t)=\mu[E_0+E\cos(\omega t)]\cos(\omega_0 t)\ ,\quad Y(t)=-\mu[E_0+E\cos(\omega t)]\sin(\omega_0 t)\ . \tag{11}$$

The solution of the evolution operator $U_I(t,t_0)$ can be expressed as

$$U_I(t,t_0)=T\exp(-i\int_{t_0}^{t}H_I(t')dt')\ , \tag{12}$$

where $T$ denotes time ordering. The use of the Neumann-Liouville expansion yields explicit form of Eq. (12)

$$U_I(t,t_0)=\sum_{m=0}^{\infty}R_x^{(2m)}(t,t_0)+i\sigma_z\sum_{m=0}^{\infty}R_y^{(2m)}(t,t_0)+i\sigma_x\sum_{m=0}^{\infty}R_x^{(2m+1)}(t,t_0)+i\sigma_y\sum_{m=0}^{\infty}R_y^{(2m+1)}(t,t_0)\ , \tag{13}$$

with

$$R_x^{(2m)}(t,t_0)=(-1)^m\left(\prod_{l=1}^{2m}\int_{t_0}^{t}dt_l\right)\ \theta(t_1-t_2)\dots\theta(t_{2m-1}-t_{2m})X(t_1t_2\dots t_{2m})\ , \tag{14}$$

$$R_y^{(2m)}(t,t_0)=(-1)^m\left(\prod_{l=1}^{2m}\int_{t_0}^{t}dt_l\right)\ \theta(t_1-t_2)\dots\theta(t_{2m-1}-t_{2m})Y(t_1t_2\dots t_{2m})\ , \tag{15}$$

$$R_x^{(2m+1)}(t,t_0)=(-1)^{m+1}\left(\prod_{l=1}^{2m+1}\int_{t_0}^{t}dt_l\right)\ \theta(t_1-t_2)\dots\theta(t_{2m}-t_{2m+1})X(t_1t_2\dots t_{2m+1})\ , \tag{16}$$

$$R_y^{(2m+1)}(t,t_0)=(-1)^{m+1}\left(\prod_{l=1}^{2m+1}\int_{t_0}^{t}dt_l\right)\ \theta(t_1-t_2)\dots\theta(t_{2m}-t_{2m+1})Y(t_1t_2\dots t_{2m+1})\ , \tag{17}$$

$$R_x^{(0)}(t,t_0)=1\ ,\qquad R_y^{(0)}(t,t_0)=0\ , \tag{18}$$

where $\theta(t)=1$ for $t>0$ and 0 otherwise, $X(t_1t_2\dots t_m)$ and $Y(t_1t_2\dots t_m)$ satisfy the following recurrence formulae

$$X(t_1t_2\dots t_m)=X(t_1t_2\dots t_{m-1})X(t_m)+Y(t_1t_2\dots t_{m-1})Y(t_m),\quad (m\geq 2)\ , \tag{19}$$

$$Y(t_1t_2\dots t_m)=X(t_1t_2\dots t_{m-1})Y(t_m)-Y(t_1t_2\dots t_{m-1})X(t_m),\quad (m\geq 2)\ . \tag{20}$$

Substituting Eqs. (7), (8) and (13) into Eq. (6), we get the quasienergy

$$\epsilon_{\pm}=\pm\frac{\omega}{2\pi}\Theta(\tau)\ \mathrm{mod}(\omega)\ , \tag{21}$$

where

$$\Theta(\tau)=\cos^{-1}\left\{\cos(\pi\frac{\omega_0}{\omega})\sum_{m=0}^{\infty}R_x^{(2m)}(\tau,0)+\sin(\pi\frac{\omega_0}{\omega})\sum_{m=0}^{\infty}R_y^{(2m)}(\tau,0)\right\}\ . \tag{22}$$

The corresponding wave functions can also be obtained as

$$|\phi(0)>_\pm = a_\pm|\alpha> + b_\pm|\beta> \tag{23}$$

with

$$a_\pm = \frac{1}{\sqrt{2}}\Big\{\Big[\Big(\sum_{m=0}^{\infty} R_x^{(2m+1)}(\tau,0)\Big)^2 + \Big(\sum_{m=0}^{\infty} R_y^{(2m+1)}(\tau,0)\Big)^2\Big]\times \Big[1-\cos((\epsilon_\pm - a)\tau)\sum_{m=0}^{\infty} R_x^{(2m)}(\tau,0) + \sin((\epsilon_\pm - a)\tau)\sum_{m=0}^{\infty} R_y^{(2m)}(\tau,0)\Big]^{-1}\Big\}^{1/2}, \tag{24}$$

$$b_\pm = \Big(\sum_{m=0}^{\infty} R_y^{(2m+1)}(\tau,0) + i\sum_{m=0}^{\infty} R_x^{(2m+1)}(\tau,0)\Big)^{-1}\times \Big[e^{-i(\epsilon_\pm - a)\tau} - \Big(\sum_{m=0}^{\infty} R_x^{(2m)}(\tau,0) + i\sum_{m=0}^{\infty} R_y^{(2m)}(\tau,0)\Big)\Big]a_\pm . \tag{25}$$

From these results, we can get any evolution states for the system by means of a linear superposition of the Floquet states $|\psi(t)>_\pm$.

## IV. Concluding Remarks

From Eq. (21) it is clearly seen that the quasienergy spectrum is that of two discrete levels. Note that from Eq. (22) we have always $|\Theta(\tau)| \leq \pi$ , therefore the quasienergy must be in the range $-\omega/2 \leq \epsilon < \omega/2$ , or equivalently the quasienergy gap $\Delta\epsilon = \epsilon_+ - \epsilon_- \leq \omega$ . This means that the length of a "Brillouin zone" in the quasienergy space is $\omega$ . We call the range $-\omega/2 \leq \epsilon < \omega/2$ the first "Brillouin zone". Other "Brillouin zones" can be obtained by adding integral multiples of $\omega$ to the quasienergy.

In principle, Eq. (22) can hold exactly. However, this requires us to calculate infinite integrals, which, obviously, is impossible. Hence to obtain an explicit expression for the quasienergy one needs to make some approximation. Here, as an example, we consider the situation of weak interlevel coupling, so that we can cut off the series in Eq. (22).

As the zero-order approximation, we obtain

$$\sum_{m=0}^{\infty} R_x^{(2m)}(\tau,0) \simeq R_x^{(0)}(\tau,0) = 1 , \quad \sum_{m=0}^{\infty} R_y^{(2m)}(\tau,0) \simeq R_y^{(0)}(\tau,0) = 0 . \tag{26}$$

This leads to $\Theta(\tau) = (\omega_0/\omega)\pi$ , therefore

$$\epsilon_\pm = \pm\frac{\omega_0}{2} \bmod(\omega) , \tag{27}$$



i.e., the eigenvalues of the unpertured Hamiltonian.

For the first-order approximation, we have

$$\sum_{m=0}^{\infty} R_x^{(2m)}(\tau,0) \simeq 1 + R_x^{(2)}(\tau,0) = 1 - \mu^2\Big[\frac{E_0}{\omega_0} + \frac{E}{2}\Big(\frac{1}{\omega+\omega_0} - \frac{1}{\omega-\omega_0}\Big)\Big]^2\Big[1-\cos(2\pi\frac{\omega_0}{\omega})\Big] , \tag{28}$$

$$\sum_{m=0}^{\infty} R_y^{(2m)}(\tau,0) \simeq R_y^{(2)}(\tau,0) = \mu^2\Big\{\Big[\frac{E_0}{\omega_0} + \frac{E}{2}\Big(\frac{1}{\omega+\omega_0} - \frac{1}{\omega-\omega_0}\Big)\Big]^2 \sin(2\pi\frac{\omega_0}{\omega}) - \frac{2\pi}{\omega}\Big[\frac{E_0^2}{\omega_0} + \frac{E^2}{2}\frac{\omega_0}{\omega_0^2-\omega^2}\Big]\Big\} . \tag{29}$$

Substituting Eqs. (28) and (29) into Eqs. (21) and (22), we obtain the quasienergy

$$\epsilon_\pm = \pm\frac{\omega}{2\pi}\cos^{-1}\Big\{\cos(\pi\frac{\omega_0}{\omega}) - 2\pi(\frac{\mu}{\omega})^2\Big[\frac{E_0^2}{\omega_0/\omega} - \frac{E^2}{2}\frac{\omega_0/\omega}{1-(\omega_0/\omega)^2}\Big]\sin(\pi\frac{\omega_0}{\omega})\Big\} \bmod(\omega) . \tag{30}$$

In this expression, $\omega_0/\omega$ may be any real number. This gives us more freedom to discuss various situations of transition process between $|\alpha>$ and $|\beta>$. For example, when $\omega_0/\omega = 1/2$, the quasienergy reads

$$\epsilon_\pm = \pm\frac{\omega}{4}\Big\{1 + (\frac{2\mu}{\omega})^2\Big[2E_0^2 - \frac{E^2}{3}\Big]\Big\} \bmod(\omega) . \tag{31}$$

It is interesting to note that if $E = \sqrt{6}E_0$, the quasienergy becomes

$$\epsilon_\pm = \pm\frac{\omega}{4} \bmod(\omega) , \tag{32}$$

which is obviously independent of the magnitude of driving laser field.

Another remarkable feature we will see is that of the appearance of quasienergy crossing in the case of $\mu << \omega$ and $\omega_0 << \omega$, where

$$\Theta(\tau) = \cos^{-1}\Big\{1 - \frac{\pi^2}{2}(\frac{\omega_0}{\omega})^2 - 2(\frac{\pi\mu}{\omega})^2\Big[E_0^2 - \frac{E^2}{2}\frac{(\omega_0/\omega)^2}{1-(\omega_0/\omega)^2}\Big]\Big\} . \tag{33}$$

This yields

$$\Theta(\tau) = \pi\frac{\omega_0}{\omega}\sqrt{1 - 4\mu^2\Big[\frac{1}{2}(\frac{E}{\omega})^2 - (\frac{E_0}{\omega_0})^2\Big]} \simeq \pi\frac{\omega_0}{\omega}\Big\{1 - \mu^2\Big[(\frac{E}{\omega})^2 - 2(\frac{E_0}{\omega_0})^2\Big]\Big\} . \tag{34}$$

Therefore the quasienergy becomes

$$\epsilon_\pm = \pm\frac{\omega_0}{2}\Big\{1 - \mu^2\Big[(\frac{E}{\omega})^2 - 2(\frac{E_0}{\omega_0})^2\Big]\Big\} \bmod(\omega) . \tag{35}$$

Notice that under the condition $\mu << \omega$, the ordinary Bessel function of order zero $J_0(2\mu\sqrt{(E/\omega)^2 - 2(E_0/\omega_0)^2})$ can be written as

$$J_0(2\mu\sqrt{(E/\omega)^2 - 2(E_0/\omega_0)^2}) \simeq 1 - \mu^2\Big[(\frac{E}{\omega})^2 - 2(\frac{E_0}{\omega_0})^2\Big] , \quad (36)$$

we obtain that

$$\epsilon_\pm = \pm\frac{\omega_0}{2}J_0(2\mu\sqrt{(E/\omega)^2 - 2(E_0/\omega_0)^2}) \bmod(\omega) . \quad (37)$$

This result shows that the doublet of quasienergy can approaches each other more and more closely with increasing the argument of the Bessel function $J_0$. Such a phenomenon of quasienergy crossing means that the coherent tunneling of electrons between the biased two wells may completely turn to that of localized motion when $J_0 = 0$. This coherent destruction of tunneling can be fulfilled through the choice of the field parameters.

In summary we have investigated the eigenvalue problem of the time-periodic driving two-level system, focusing on the dynamic effects of an electron in a biased double-well heterostructure under the action of a laser field. We addressed detailed algebraic structure of the closed-form solutions for the quasienergy and the Floquet states, from which some special cases of the ratio of the energy gap of the undriven system to the laser frequency have been discussed. The remarkable feature was found to be that the emergence of quasienergy crossing, which implies that the coherent tunneling of electrons between the biased two wells can be fully suppressed by the choice of the field parameters. Such an effect of dynamic localization should be observable in quantum-well heterostructures by using free-electron laser facilities.

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