BB

GSI-94-54
PREPRINT
September 1994

NEUTRON REMOVAL IN PERIPHERAL RELATIVISTIC HEAVY ION COLLISIONS

T. Aumann, C.A. Bertulani, K. Sümmerer

Gesellschaft für Schwerionenforschung mbH
Postfach 11 05 52 · D-64220 Darmstadt · Germany

Neutron Removal in Peripheral Relativistic Heavy Ion Collisions

T. Aumann[(a)], C.A. Bertulani[(b,c)], and K. Sümmerer[(b)]

(a) Institut für Kernchemie, Universität Mainz
D-55099 Mainz, Germany

(b) Gesellschaft für Schwerionenforschung,
Planckstr. 1, D-64291 Darmstadt, Germany

(c) Instituto de Fisica, Universidade Federal do Rio de Janeiro
21945-970 Rio de Janeiro, RJ, Brazil

ABSTRACT

We investigate the relativistic Coulomb fragmentation of ^{197}Au by heavy ions, leading to one-, two- and three-neutron removal. To resolve the ambiguity connected with the choice of a specific minimum impact parameter in a semiclassical calculation, a microscopic approach is develloped based on nucleon-nucleon collisions ("soft-spheres" model). This approach is compared with experimental data for ^{197}Au at 1 GeV/nucleon and with a calculation using the "sharp-cutoff" approximation. We find that the harmonic-oscillator model predicting a Poisson distribution of the excitation probabilities of multiphonon states gives a good agreement with one-neutron removal cross sections but is unable to reach an equally good agreement with three-neutron removal cross sections.

1 Introduction

Experiments with relativistic heavy-ion beams have accumulated evidence for the population of two-phonon giant-dipole resonances (GDR) both directly via the observation of neutron [1, 2] or γ-decay [3, 4] and indirectly via the measurement of neutron-emission cross sections [5]. Theoretical descriptions of these processes were based on a semiclassical approach [6]. In the present communication we review some specific aspects of semiclassical Coulomb-excitation calculations: First, we discuss the differences between perturbation theory and a harmonic oscillator model in calculating two-phonon-GDR excitation cross sections. We then show how one can avoid the ambiguity connected with the choice of a specific lower integration limit in the semiclassical calculation when integrating over impact parameter. This is done by performing a Glauber type transparency calculation. To check the validity of our calculations, we compare our results to measured $1n$- to $3n$- removal cross sections from ^{197}Au bombarded with several projectiles at relativistic energies [5]. The key question that we try to answer is whether our improved calculations are able to solve the puzzle contained in the fact that $1n$- removal cross sections could be reproduced reasonably well, whereas $3n$-removal cross sections (which are dominated by 2-phonon GDR excitation) were underestimated. This amounted to a deficit in the 2-phonon GDR of roughly a factor of two [5]. Similar deficits were observed also in the exclusive experiment (see Ref.[7] for a detailed discussion of this subject).

2 Semiclassical descriptions of relativistic Coulomb excitation

Two different approaches have been used in the literature to describe the excitation probabilities and cross sections in relativistic Coulomb excitation, namely first- and second-order perturbation theory [8, 9, 10], and a harmonic oscillator model [6, 11]. As we will show in more detail below, the two models represent two extreme assumptions: in perturbation theory, the multi-phonon excitations are assumed to be completely independent from each other, whereas in the harmonic oscillator model they are assumed to be coupled up to infinite order. Our choice of one of these extremes will be guided by comparison with experimental data for ^{197}Au.

2.1 Perturbation theory

If first-order perturbation theory is valid, one can show that the excitation of a GDR-state with energy E yields for the differential excitation probability the expression [6]

$$\mathcal{P}(E,\ b) \equiv \frac{dP(E,\ b)}{dE} = \frac{1}{E}\ N(E,\ b)\sigma_\gamma(E)\ , \tag{1}$$

where b is the impact parameter, $\sigma_\gamma(E)$ is the photo-nuclear cross section for a photon with energy E, and $N(E,\ b)$ is the number density of equivalent photons with energy E. A semiclassical calculation yields [6]

$$N(E,\ b) = \frac{Z^2\alpha}{\pi^2 b^2}\left(\frac{c}{v}\right)^2 x^2 \left[K_1^2(x) + \frac{1}{\gamma^2}\ K_0^2(x)\right]\ , \tag{2}$$

where $x = Eb/\gamma\hbar v$, v is the projectile velocity, K_0 (K_1) is the modified Bessel function of order zero (one), $\alpha = 1/137$ is the fine structure constant, and Z is the charge number of the projectile (target) for target (projectile) excitations.

To calculate the probability to excite a double-phonon state, i.e. a state composed of two GDR-states, one can use second order perturbation theory. Apart from a small interference term, the excitation probability of a state with energy E is a simple product of the probability to excite an intermediate state with energy E' and the probability to go from this state to the final state, summed over all intermediate states. A drawback of this method is that for small impact parameters b, for which the probability is large, the loss of probability for one-phonon excitation due to the two-phonon (and higher order) excitation is not accounted for, i.e. unitarity is violated. This problem can be eliminated by incorporating higher-order corrections, but a proper treatment of this procedure depends strongly on the model assumed for the nuclear states. As a conclusion from the above considerations, we expect perturbation theory to **overestimate** single-phonon excitation probabilities.

2.2 Harmonic-oscillator model

A simple and transparent result is obtained under the assumption of a harmonic vibrator model. In this case, the excitation probability of multiphonon states is given in terms of a Poisson distribution of the probabilities obtained in first order perturbation theory. This is shown in the appendix of this article. In this approximation the excitation of the one-phonon state is modified to yield

$$\mathcal{P}^{(1)}(E,\ b) = \mathcal{P}(E,\ b)\ \exp\Big\{ - \mathcal{P}(E,\ b)\Big\}\ . \tag{3}$$

The exponential on the right hand side takes care of the flux of probability to higher order excitations. Of course, the harmonic vibrator model is only a rough approximation to the nuclear states. To obtain Eq.(3) it is implicitly assumed (see appendix) that all states contribute equally to the unitarity condition. In other words, even a multiphonon state with an energy equal to, e.g., five times the one-phonon GDR state is considered to take out flux (although very small) from the probability to excite the one-phonon state. Such pure high-lying states are however very unlikely to exist. Thus, while the second order perturbation theory is expected to overestimate the excitation probabilities, the harmonic vibrator model is likely to **underestimate** them.

The excitation of the double-phonon state in the harmonic vibrator model is given by

$$\mathcal{P}^{(2)}(E,\ b) = \frac{1}{2!}\ \int dE'\ \mathcal{P}(E-E',\ b)\ \mathcal{P}(E',\ b)\ \exp\Big\{ - \mathcal{P}(E',\ b)\Big\} \tag{4}$$

where the integral is over the energies of all 1^- intermediate states. An extra assumption is used here: the excitation operator (the dipole operator) does not connect the intermediate states due to spin and parity selection rules. The double-phonon states are considered as $L^\pi = 0^+, 2^+$ states [8], the ground-state and the GDR-state are $L^\pi = 0^+$ and 1^-, respectively. Moreover, direct excitations of the double-phonon state are neglected since they are accomplished by means of quadrupole excitations, which are much smaller than the dipole ones (monopole states are not accessible with Coulomb excitation). But, besides these assumptions, we note that Eq.(4) goes beyond the validity of the harmonic oscillator model. As shown in the appendix it is fundamental for the proof of Eq.(4) that $E - E' = E'$ for all intermediate states, as in a harmonic oscillator. This is not the case when one assumes that the giant dipole resonance is a mixture of several states the envelope of which is a broad bump around a centroid energy. It should be considered as a good approximation to the harmonic oscillator only when the width of the giant resonance states are small compared to its centroid energy, and when $E \approx 2E'$.

Another argument used frequently to support the use of the Poisson distribution for the calculation of the excitation probabilities for multi-GDR states in nuclei is the so-called *equivalent photon* picture. This is based on the fact that, due to the requirement that a pure Coulomb excitation can occur only when the nuclei do not penetrate, the matrix elements for Coulomb excitation are exactly proportional to the matrix elements for photo-nuclear excitations. The probabilities and cross sections can always be written as a product of a factor and the photo-nuclear cross sections, independent of the multipolarity of the excitation. The proportionality factor is conveniently denoted as the equivalent photon number. One has to keep in mind that this is only a terminology, useful for pedagogical purposes. Moreover, the direct relationship between the Coulomb excitation and the photo-nuclear cross section is only valid for first-order processes. Higher-order processes will depend on the intermediate nuclear states, and consequently on the model used for the purpose. One might realize a bunch of virtual photons hitting a target, out of which n of them are taken randomly and sequentially excite the nucleus. Such a hypothesis, which may be used to justify Eq.(4), is at best pedagogical. The number of virtual, or equivalent, photons with the energy of a giant resonance obtained by the factorization procedure is only a few (or even less than one). A statistical argument is therefore not valid.

Despite the above criticisms semiclassical calculations using one of the approaches mentioned are useful as simple and transparent models. A choice between the two is hampered by the fact, however, that the usual way of semiclassical calculations involves a more or less free parameter, namely the minimum impact parameter, the choice of which can yield agreement with experimental data for either model. In the following we use a prescription that avoids this free parameter.

3 Calculation of neutron-removal cross sections

3.1 Sharp-cutoff approximation

In the semiclassical approach the total cross section for relativistic Coulomb excitation is obtained by integrating the excitation probabilities over impact parameter, starting from a minimum value. It is assumed that below this minimum value the interaction is exclusively nuclear, whereas above pure Coulomb interactions occur ("sharp-cutoff" approximation). It has been found that with this approximation the Coulomb cross sections are very sensitive to the parameterization of the minimum impact parameter [1, 3, 5, 8]. One commonly used parameterization at relativistic energies is that of Benesh et al. [12], fitted to Glauber-type calculations of total reaction cross sections and reading

$$b_{min}^{BCV} = 1.35 \cdot (A_p^{1/3} + A_t^{1/3} - 0.75 \cdot (A_p^{1/3} + A_t^{1/3}))\ fm, \tag{5}$$

which we refer to hereafter as "BCV". In ref. [12] a detailed study has been made concerning the parameterization procedure of the minimum impact parameter. It was also found that the nuclear contribution to the neutron removal channels in peripheral collisions has a negligible interference with the Coulomb excitation mechanism. This is a very useful result since the Coulomb and nuclear part of the cross sections may be treated separately.

Another parametrization is that of Kox et al. [13] which reproduced well measured total reaction cross sections of light and medium-mass systems:

$$b_{min}^{Kox} = 1.1\ \left(A_p^{1/3} + A_t^{1/3} + 1.85 \frac{A_p^{1/3}\ A_t^{1/3}}{A_p^{1/3} + A_t^{1/3}} - 1.9\right)\ fm \tag{6}$$

We have used this parametrization previously [5] and found reasonable agreement with the measured data for $1n$ cross sections. It should be noted, however, that the Kox parametrization of total interaction cross sections has been derived mainly from experiments with light projectiles and that its application to heavy systems involves an extrapolation into a region where no data points are available.

3.2 Competition of nuclear and Coulomb processes

It is well known that at relativistic energies and grazing impact parameters nuclei are partly transparent to each other and that it is much better to replace the sharp-cutoff approximation by a smooth transition from purely nuclear collisions at $b << b_{min}$ to pure Coulomb collisions at $b >> b_{min}$ [11]. Such a "soft spheres" model can be derived from Glauber theory and can be incorporated in our semiclassical calculation by rewriting the number of equivalent photons, Eq.(2), as

$$\begin{aligned} N(E,\ b) &= \frac{Z^2\alpha}{\pi^2 b^2} \left(\frac{c}{v}\right)^2 x^2 \left[K_1^2(x) + \frac{1}{\gamma^2}\ K_0^2(x)\right] \\ &\times\ \exp\left\{ -\sigma_{NN} \int dz\ \int d^3r\ \rho_p(r)\ \rho_t(R-r)\right\}, \end{aligned} \tag{7}$$

where $R \equiv (b,\ z)$ with z being the coordinate along and **b** perpendicular to the beam direction. The quantity σ_{NN} is the nucleon-nucleon cross section, and $\rho_{p,t}$ are the ground state nuclear densities of projectile and target, respectively. The parametrization of the nuclear densities has been taken from the droplet model [14] in accordance with Shen et al. [15].

Since we are dealing with nucleus-nucleus collisions at energies of the order of one GeV/nucleon, we adopt a value of $\sigma_{NN} = 40$ mb in our calculations.

3.3 Coulomb dissociation of ^{197}Au

In the following we will apply the soft-spheres approximation to the case of ^{197}Au where inclusive $1n-$ to $3n-$ removal cross sections have been measured by Aumann et al. [5]. Apart from the exponential function at the right hand side of Eq.(7) which accounts for nuclear transparency in near-grazing collisions, the calculation is identical to the one described in [5].

As input to our calculations we will use the experimental photo-neutron emission cross sections from Ref. [16]. A Lorentzian fit to the (γ, xn)-data is used to parametrize the GDR in ^{197}Au. The parameters are an excitation energy of 13.72 MeV, a width of 4.61 MeV, and a strength of 128% of the dipole sum rule [16]. The Lorentz parameters for the isoscalar (isovector) GQR are taken as 10.8 (23.0) MeV for the excitation energy, 2.9 (7.0) MeV for the width; we assume 95% exhaustion of the respective sum rules [17]. With these parameters we calculate the excitation cross sections $d\sigma(E)/dE$ for one- and two-phonon dipole- and quadrupole-excitations. The respective neutron emission cross sections are given by $\sigma_{xn} = \int \frac{d\sigma(E)}{dE} f_{xn}(E) dE$, where $f_{xn}(E)$ is the probability to evaporate x neutrons at excitation energy E. For excitation energies below 27 MeV, $f_{xn}(E)$ is taken from the experimental (γ, xn)-data [16], and for higher energies from a statistical decay calculation with the code HIVAP [18]. Since the three-neutron emission threshold in gold is above the energy of the GDR state, this channel is fed mainly by the two-phonon excitation mechanism, while the $1n$ cross section is dominated by the excitation of the GDR.

In Fig. 1 we plot the one- and two-phonon excitation probabilities for gold-gold collisions at 1 GeV/nucleon as a function of the impact parameter using the harmonic-oscillator model (Eqs.(7) and (8)). The solid curve is the result of the soft-spheres model. We observe that this model gives an excitation probability which is a smoothly increasing function of b up to a maximum value, after which it decreases exactly as the sharp-cutoff approximation (dashed curve). For this latter approximation, we have taken b_{min}^{BCV} from Eq.(5). We expect that the BCV parametrization of b_{min} should yield similar results as the soft-spheres calculation since it was derived in fitting the complementary process, the nuclear interaction,

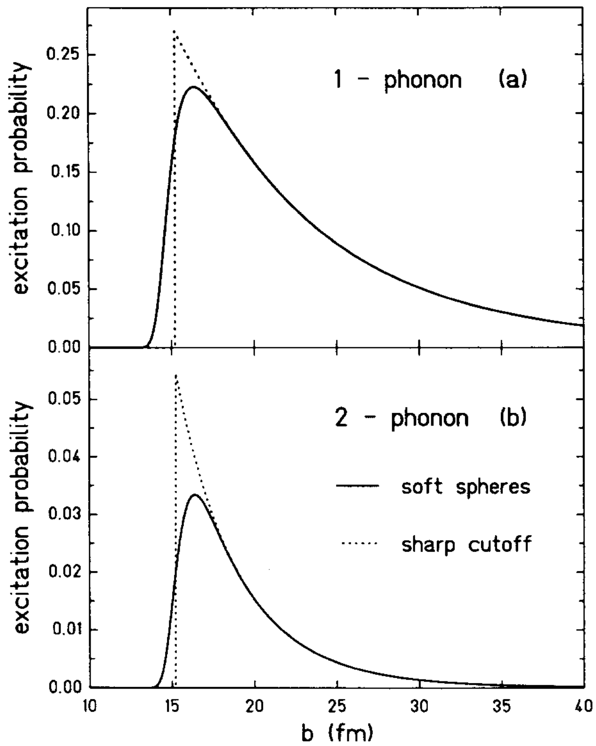

Figure 1: Excitation probabilities of one-phonon (a) and two-phonon (b) GDR states in ^{197}Au due to relativistic Coulomb excitation by a gold projectile at 1 GeV/nucleon, as a function of the impact parameter. The full (dashed) curves are obtained using a "soft-spheres" ("sharp-cutoff") model as described in the text.

calculated also with Glauber theory.

In Fig.2 we examine how well we can reproduce experimental $1n$- and $3n$- removal cross sections with our model. Again, the solid curve denotes the soft-spheres calculation using the harmonic-oscillator model. This calculation is in good agreement with the $1n$ cross sections. The dotted curve, which is from a sharp-cutoff calculation with b_{min}^{BCV} from Eq.(5), deviates only insignificantly from the soft-spheres result, as expected. This remarkable agreement tells us that for practical purposes we can avoid the extra numerical complication connected with the use of Eq.(7) and corroborates the use of b_{min}^{BCV} in sharp-cutoff calculations in the earlier work [11, 19]. It also indicates that, contrary to our previous choice [5], the use of b_{min}^{Kox} is physically less justified.

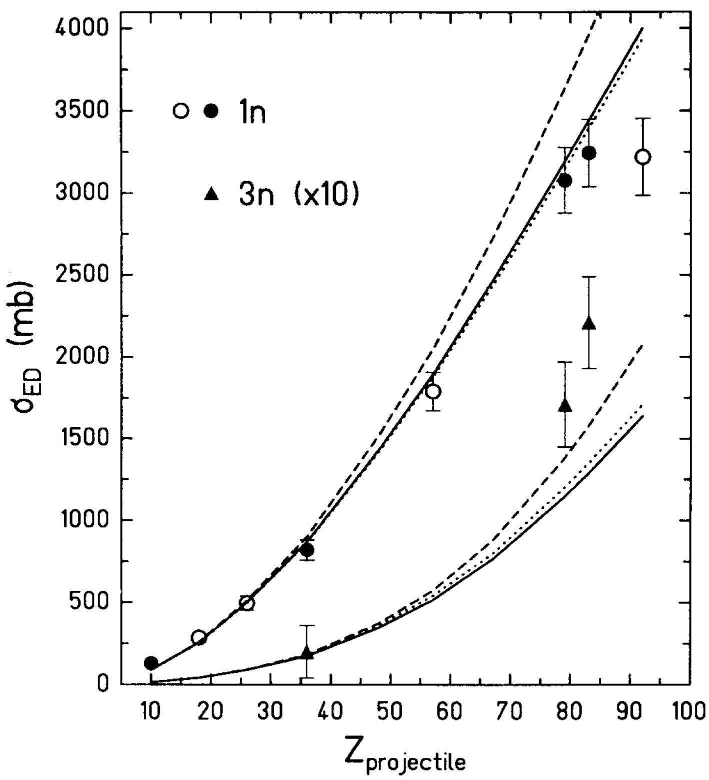

Figure 2: Experimental $1n$- and $3n$- removal cross sections for ^{197}Au bombarded with relativistic projectiles (from Ref.[5, 19]) in comparison with theoretical calculations from this work (solid curve: "soft- spheres" calculation with the harmonic-oscillator model; dashed curve: same for perturbation theory; dotted curve: "sharp-cutoff" calculation with the harmonic-oscillator model using b_{min}^{BCV} from Eq.(5)).

The dashed curve in Fig. 2 denotes a soft-spheres calculation with perturbation theory used to calculate the excitation probabilities. As expected from the discussion in Section 1, the $1n$ cross sections are much higher than with the harmonic oscillator approach and deviate considerably from the data. Since our model avoids an arbitrary choice of b_{min} as in Ref.[8], we conclude that the harmonic oscillator model of multiple giant resonances is appropriate for the case of large-Z systems and that the violation of the unitarity condition in the perturbation theory approach leads to discrepancies with the experimental data far beyond the error bars.

The lower set of curves in Fig. 2 shows the results for the $3n$ channel using the same models as in the upper part of the figure. We note that, as expected, perturbation theory yields higher cross sections, which in this case are closer to the measured data than those calculated with the harmonic-oscillator model. Since we have chosen, however, to use the $1n$ cross sections as the test case, where the statistical accuracy is better and the nuclear contribution can be neglected completely, we are left with the conclusion that it is not possible to reproduce $1n$ and $3n$ cross sections (i.e. one- and two-phonon excitation) with the same model and that the lack of two-phonon excitation probability observed previously [1, 3, 5] is not connected with an improper choice of b_{min}. Adjusting slightly the density radius parameter in the soft-spheres calculation to reproduce better the upper data points in Fig. 2 would worsen the agreement with the lower data points, i.e., the lower solid curve would be pushed downwards. Including the three-phonon excitation probabilities would not explain these discrepancies, since they are very small.

As a final proof that there are only minor differences between a soft-spheres approach and a sharp-cutoff calculation using b_{min}^{BCV}, we show in figure 3 the differences that one obtains for the excitation energy spectrum of one- and two-phonon states with the two models. The excitation spectra of the one-phonon states are indistin-

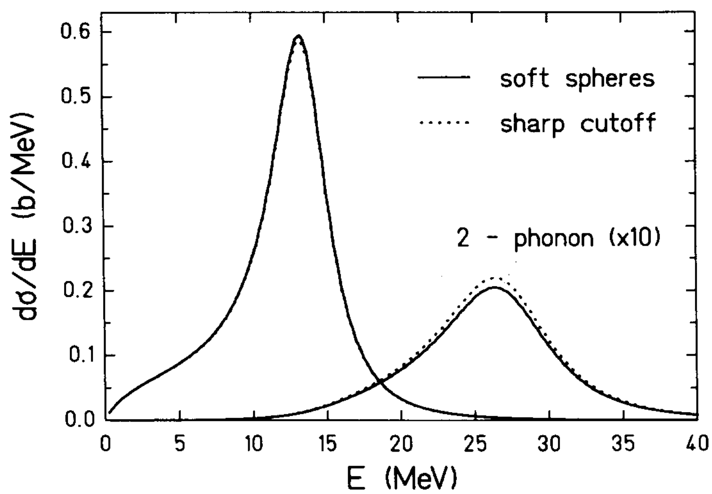

Figure 3: The energy spectrum of relativistic Coulomb excitation to one-phonon and two-phonon GDR states obtained in the harmonic-oscillator approach with the "soft-spheres" model (solid curves) and with the sharp-cutoff model using the BCV parameterization of b_{min} (dotted curves). The two-phonon spectrum is multiplied by a factor of 10.

guishable in both approaches. Only a minor difference between the two models is visible for the double-phonon excitations. This is due to the stronger dependence of the double-phonon probability on small impact parameters ($\sim 1/b^2$ for one-phonon excitation, and $\sim 1/b^4$ for double-phonon excitation). One could expect that the spectrum would be somewhat distorted by the stronger dependence of the virtual photon spectrum on "hard photons" which originate from small impact parameter collisions. But, apart from a small difference in the area below the spectrum, its form is not appreciably modified.

3.4 Nuclear-plus-Coulomb interactions

Up to now it was tacitly assumed that in a peripheral nuclear collision either a nuclear interaction or, in the case of transparency, a Coulomb interaction may take place. It is conceivable, however, that in the same collision both processes occur. As an estimate of the contribution of such processes to the $1n$ to $3n$ channels studied in the present work, we have modified our intranuclear-cascade calculations of the nuclear processes to take into account also possible electromagnetic excitations. We note that the only channel that needs to be studied is the one-neutron knock-out in the intranuclear-cascade step of the collision. The ^{196}Au prefragment formed in this process then feeds the $2n$ and $3n$ channels by evaporation. The inclusion of Coulomb processes proceeds in our estimate in the same way as in the soft-spheres calculation of the total nuclear interaction probability: The impact-parameter distribution of ^{196}Au-formation from the cascade calculation (upper part in Fig.4) has to be multiplied by the probability of Coulomb excitation (dashed curve in Fig.1). As a result, about 30% of the ^{196}Au prefragments are Coulomb excited and are thus shifted towards higher excitation energies which are obtained by folding the nuclear-excitation energy distribution taken from the cascade calculation with the Lorentz curve of GDR excitation (lower part of Fig.4).

The net effect of the inclusion of nuclear-plus-Coulomb processes is small: on the one hand the nuclear part of the $1n$ to $3n$ channels is depleted by 30%, since the corresponding ^{196}Au prefragments have been shifted to a different excitation energy distribution. On the other hand the $2n$ and $3n$ channels are fed by evaporation from this very distribution, yielding a net increase e.g. in the $3n$ channel of about 10 mb - a value that is less than the accuracy of the cross sections of Ref. [5] and also much less than the deficit found in the theoretical cross section for the $3n$ channel as compared to the experimental one of about 100 mb.

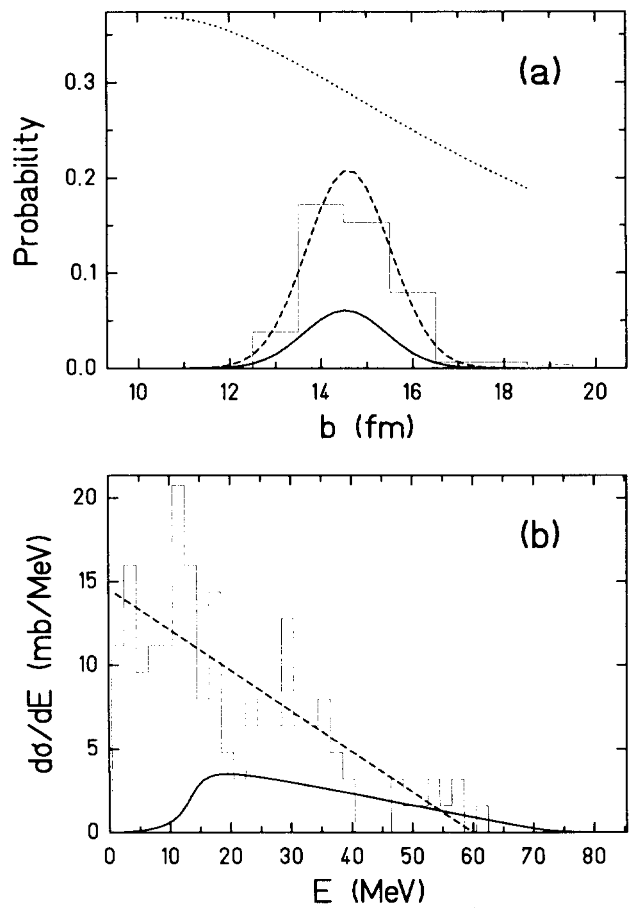

Figure 4: (a) Upper part: Impact-parameter distribution from an intranuclear-cascade calculation [5] for the formation of ^{196}Au without (dashed curve) and with Coulomb excitation (full curve). The latter is obtained by multiplying the former distribution with the Coulomb excitation probability (dotted curve). (b) Lower part: Excitation-energy distribution obtained by folding the nuclear excitation-energy spectrum of ^{196}Au from the cascade calculation (histogram, approximated by the dashed line) with the Lorentz curve representing the GDR excitation of ^{196}Au. Note that only 30% of all nuclear events leading to ^{196}Au are calculated to undergo also Coulomb excitation.

4 Conclusion

We conclude that an obvious modification of the semiclassical theory of relativistic Coulomb excitation, namely the transition from a "sharp-cutoff" to a "soft-spheres" model, resolves ambiguities connected with the choice of a specific expression for the minimum impact parameter necessary in previous calculations. Once this previously free parameter is fixed, one can make a decision which of two rather extreme assumptions, namely complete independence or complete coupling of the multi-phonon excitations, is more appropriate. Our calculations show that the harmonic oscillator model describes $1n$-removal cross sections for the interactions of relativistic heavy ions with ^{197}Au with good accuracy. The basic discrepancy, however, that we and others have noted earlier, which lies in a good description of one-phonon excitation and a large deficit in the calculated two-phonon excitation, persists. In a simple estimate we have shown that this deficit cannot be attributed to a neglect of nuclear-plus-Coulomb interactions. It is possible that a coupled-channels calculation could be able to remove this dicrepancy. More generally, a truly microscopic description of multi-phonon excitations would be desirable, that circumvents the problems connected with the inadequacies of the presently used harmonic-oscillator and perturbation-theory models.

The authors acknowledge gratefully stimulating discussions with T. Brohm and K.H. Schmidt concerning the parametrization of nuclear density distributions and the aspect of nuclear-plus-Coulomb interactions.

Appendix - Dipole excitations of an oscillator

We give here a proof that the dipole excitations of a harmonic oscillator are given by a Poisson distribution of the excitation probabilities obtained in first-order perturbation theory. For simplicity we consider a one-dimensional harmonic oscillator with a time-dependent hamiltonian in the form of

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_0 + \lambda(t) \cdot \hat{x} \\ &= \mathcal{H}_0 + \Lambda(t)\ (a + a^\dagger)\ , \end{aligned} \tag{8}$$

where $\lambda(t)\hat{x}$ is a time-dependent perturbation, e.g., the dipole field of a relativistic heavy ion. In the last step of the above equation we expressed $\hat{x}$ in terms of the creation and annihilation operators for the harmonic oscillator states. Besides the strength of the time-dependent field, $\Lambda(t)$ incorporates the strength of dipole excitations in the harmonic oscillator model. The above perturbation links neighbouring states only. This is important for the proof.

If the perturbation $\Lambda(t)$ is very weak, on may use the first-order perturbation result to calculate the excitation of dipole states, i.e.,

$$\chi \equiv a_{0\to 1} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} \exp\Big\{ i(E_f - E_i)t/\hbar \Big\}\ \Lambda(t)\ dt\ . \tag{9}$$

The problem is best solved in the interaction representation [20]. Then one can show that the amplitude for the system going from the ground state to the n-th state is given by

$$a_n = < n \Big| \mathcal{T} \exp\Big\{ \frac{1}{i\hbar} \int_{-\infty}^{\infty} \mathcal{V}(t)\ dt \Big\} \Big| 0 >\ , \tag{10}$$

where $\mathcal{T}$ is the time-ordering operator and

$$\mathcal{V}(t) = \exp\Big\{ i\mathcal{H}_0 t/\hbar \Big\}\ \Lambda(t)\ (a + a^\dagger)\ \exp\Big\{ -i\mathcal{H}_0 t/\hbar \Big\}\ . \tag{11}$$

Since for the harmonic oscillator only unit steps of energy $E_f - E_i = \hbar\omega$ are possible, we get $\mathcal{V}(t) = e^{i\omega t}\ \Lambda(t)\ (a + a^\dagger)$. The transition operator commutes at different times, and

$$\mathcal{T} \exp\Big\{ \frac{1}{i\hbar} \int_{-\infty}^{\infty} \mathcal{V}(t)\ dt \Big\} = e^{\chi a^\dagger}\ e^{\chi a}\ e^{-\chi^2/2}\ , \tag{12}$$

where we used that for two operators A and B the following idendity holds: $e^{A+B} = e^A\ e^B\ e^{-[A,B]/2}$ [20]. Moreover, since $e^{\chi a}\Big|0 >= 0$, we get

$$a_n = < n \Big| e^{\chi a^\dagger}\ e^{-\chi^2/2} \Big| 0 > \tag{13}$$

Only the part of the expansion $e^{\chi^2 a^\dagger}\Big|0>$ with the n-th term has to be considered. The other terms do not contribute since $< n\Big|n' >= 0$ for $n \neq n'$. Thus, from the equation above we get

$$\begin{aligned} a_n &= < n\Big|e^{-\chi^2/2}\, \frac{\chi^n}{n!}(a^\dagger)^n\Big|0 > \\ &= \frac{1}{\sqrt{n!}}\, < n\Big|e^{-\chi^2/2}\, \chi^n\Big|n > \\ &= \frac{1}{\sqrt{n!}}\, e^{-\chi^2/2}\, \chi^n \,. \end{aligned} \tag{14}$$

The excitation probability of the n-th state is thus given by the Poisson distribution

$$P^{(n)} = \frac{1}{n!}\, e^{-\mathcal{P}}\, \mathcal{P}^n \tag{15}$$

where $\mathcal{P} = \chi^2$ is the first-order probability.

References

1. R. Schmidt, Th. Blaich, Th.W. Elze, H. Emling, H. Freiesleben, K. Grimm, W. Henning, R. Holzmann, J.G. Keller, H. Klingler, R. Kulessa, J.V. Kratz, D. Lambrecht, J.S. Lange, Y. Leifels, E. Lubkiewicz, E.F. Moore, E. Wajda, W. Prokopowicz, Ch. Schütter, H. Spies, K. Stelzer, J. Stroth, W. Walus, H.J. Wollersheim, M. Zinser, and E. Zude (LAND Collaboration), Phys. Rev. Lett. **70** (1993) 1767.

2. E. Wajda, J. Stroth, Th. Blaich, Th.W. Elze, H. Emling, H. Freiesleben, K. Grimm, W. Henning, R. Holzmann, H. Klingler, R. Kulessa, J.V. Kratz, D. Lambrecht, Y. Leifels, E. Lubkiewicz, E.F. Moore, K. Stelzer, W. Walus, M. Zinser, and E. Zude (LAND Collaboration), Nucl. Phys. **A569** (1994) 141c.

3. J. Ritman, F.-D. Berg, W. Kühn, V. Metag, R. Novotny, M. Notheisen, P. Paul, M. Pfeiffer, O. Schwalb, H. Löhner, L Venema, A. Gobbi, N. Herrmann, K.D. Hildenbrand, J. Mösner, R.S. Simon, K. Teh, J.P. Wessels, and T. Wienold, Phys. Rev. Lett. **70** (1993) 533, erratum, *ibid.*, 2659.

4. J.R. Beene, Nucl. Phys. **A569** (1994) 163c.

5. T. Aumann, J.V. Kratz, E. Stiel, K. Sümmerer, W. Brüchle, M. Schädel, G. Wirth, M. Fauerbach, and J.C. Hill, Phys. Rev. C **47** (1993) 1728, Nucl. Phys. **A569** (1994) 157c.

6. C.A. Bertulani and G. Baur, Phys. Reports **163** (1988) 299.

7. H. Emling, GSI Preprint 94-16, to be published in "Progr. in Nucl. Phys.".

8. C.A. Bertulani and V. Zelevinsky, Phys. Rev. Lett. **71** (1993) 967; Nucl. Phys. **A568** (1994) 931.

9. V.Yu. Ponomarev, E. Vigezzi, P.F. Bortignon, R.A. Broglia, G. Colò, G. Lazzari, V.V. Voronov, and G. Baur, Phys. Rev. Lett. **72** (1994) 1168.

10. L.F. Canto, A. Romanelli, M.S. Hussein, and A.F.R. de Toledo Piza, Phys. Rev. Lett. **72** (1994) 2147.

11. W.J. Llope and P. Braun-Munzinger, Phys. Rev. C **41** (1990) 2644; *ibid.* C **45** (1992) 799.

12. C.J. Benesh, B.C. Cook, and J.P. Vary, Phys. Rev. C **40** (1989) 1198.

13. S. Kox, A. Gamp, C. Perrin, J. Arvieux, R. Bertholet, J.F. Bruandet, M. Buenerd, R. Cherkaoni, A.J. Cole, Y. El-Masri, N. Longequene, J. Menet, F. Merchez, and J.B. Viano, Phys. Rev. C **35** (1987) 1678.

14. W.D. Myers and W.J. Swiatecki, Ann. of Phys. **55** (1969) 395; *ibid.* **84** (1974) 186.

15. Shen Wen-Qing, Wang Bing, Feng Jun, Zhan Wen-Long, Zhu Yong-Tai, and Feng En-Pu, Nucl.Phys. **A491** (1989) 130.

16. A. Veyssière, H. Beil, R. Bergère, P. Carlos, and A. Leprêtre, Nucl. Phys. **A159** (1970) 561; B.L. Berman, R.E. Pywell, S.S. Dietrich, M.N. Thompson, K.G. McNeill, and J.W. Jury, Phys. Rev. C **36** (1987) 1286.

17. F.E. Bertrand, Annu. Rev. Nucl. Sci. **26** (1976) 457.

18. GSI version of the code ALICE (M. Blann and F. Plasil, Phys. Rev. Lett. **29** (1972) 303); W. Reisdorf and M. Schädel, Z. Phys. A - Hadrons and Nuclei **343** (1992) 47.

19. J.C. Hill, F.K. Wohn, D.D. Schwellenbach, and A.R. Smith, Phys. Lett. B **273** (1991) 371, and references therein.

20. E. Merzbacher, "Quantum Mechanics", 2nd Ed., (Wiley, New York, 1970).