Estimation of attractive and repulsive interactions from the fluctuation observables at RHIC using van der Waals hadron resonance gas Model

Subhasis Samanta[1,*] and Bedangadas Mohanty[1,2,†]

[1]School of Physical Sciences, National Institute of Science Education and Research, HBNI, Jatni - 752050, India
[2]Experimental Physics Department, CERN, CH-1211, Geneva 23, Switzerland

Experimental data on the moments of net-proton distribution in central Au-Au collisions for various center of mass energies ($\sqrt{s_{NN}}$) measured by the STAR collaboration at the Relativistic Heavy-Ion Collider (RHIC) are compared to the corresponding results from a van der Waals type interacting hadron resonance gas (VDWHRG) model. The parameters representing the attractive and repulsive interactions in the VDWHRG model have been extracted by fitting the σ^2/M, $S\sigma$ and $\kappa\sigma^2$, where M is the mean, σ is the standard deviation, S is the skewness and κ the kurtosis of the net-proton distribution. Considering all the three moment products we observe that the strength of the repulsive interactions increases with decrease in $\sqrt{s_{NN}}$ = 200 to 19.6 GeV while the strength of the attractive interaction is of the similar magnitude. For $\sqrt{s_{NN}}$ = 11.5 and 7.7 GeV there is drop in the strength of both attractive and repulsive interactions relative to $\sqrt{s_{NN}}$ = 19.6 GeV. On the other hand, if we consider only the higher order moment products, $S\sigma$ and $\kappa\sigma^2$, which are more sensitive to critical point physics, a segregation with respect to the strength of the attractive parameter is observed. The data for $\sqrt{s_{NN}}$ = 19.6 and 27 GeV supports a larger attractive strength compared to other energies. For this latter case, the repulsive interaction values are of similar order for most of the beam energies studied, except for 7.7 GeV where the parameter value is not well constrained due to large uncertainties.

I. INTRODUCTION

One of the major goals of high energy heavy-ion collisions is to explore the phase diagram of strongly interacting nuclear matter at high temperature and density. At large temperature T and zero baryon chemical potential μ_B, the lattice quantum chromodynamics (LQCD) predicts a smooth cross-over transition from hadronic to a quark gluon plasma (QGP) phase [1]. While at large μ_B a first order phase transition is expected [2–5]. Therefore, there should be a critical point (CP), the end point of the first order phase boundary towards the crossover region [6–8]. The beam energy scan (BES) program is currently ongoing at the Relativistic Heavy-Ion Collider (RHIC) facility at Brookhaven National Laboratory to locate the QCD CP.

Fluctuations of conserved charges in heavy-ion collisions like baryon number, electric charge, strangeness quantum number are sensitive observables for the CP search [9–12]. The non-monotonic variation of these quantities with the colliding beam energy is regarded as one of the characteristic signature in presence of the CP. The STAR collaboration at RHIC has measured the fluctuation observables related to the net-proton (a proxy for net-baryon), net-charge and net-kaon (a proxy for net-strangeness) distribution [13–15]. The product of $S\sigma$ and $\kappa\sigma^2$ for the measured net-proton distribution in central Au-Au collisions qualitatively show non-monotonic dependence on the beam energy. Since uncertainties in the measurements at lower $\sqrt{s_{NN}}$ are large, the evidence for the existence of a CP is not yet conclusive. The product of moments of the net-number distributions are also related to the susceptibilities of conserved charges which can be computed in LQCD [12, 16–20] and in the hadron resonance gas (HRG) models [21–28].

HRG model has several varieties. Some of the HRG models consider interaction among the constituent hadrons and some do not. Different versions of this model and some of the recent work using these models may be found in Refs. [22–24, 26, 29–79]. The HRG model is successful in describing the zero chemical potential LQCD data on bulk properties of the QCD matter up to temperatures $T \sim 150$ MeV [18, 80–83]. This model is also successful in describing the hadron yields, created in central heavy ion collisions at different $\sqrt{s_{NN}}$ [33, 34, 37, 43, 84]. Recently, van der Waals (VDW) type interaction with both attractive and repulsive parts have been introduced in HRG model [63–65, 68, 69, 85]. This model predicts a first order liquid-gas phase transition along with a CP. Further, this model can describe Lattice QCD data of different thermodynamical quantities satisfactorily. For the study of fluctuation of conserved charges, the HRG model is generally used to obtain a baseline for CP search in the experiments. In this regard, the VDWHRG model may bring in additional dimensions, as it has the capability to capture both the attractive interactions (important for CP physics) and the repulsive interactions due to finite size of the hadrons.

In the present work, we have extracted attractive and repulsive parameters in the VDWHRG model by fitting the experimental data on net-proton number fluctuation measured by the STAR collaboration in BES program at RHIC to the corresponding model results. This then allows for the first time to get an estimate of the changes in the relative contributions of the strength of repulsive and attractive interactions with $\sqrt{s_{NN}}$ in Au-Au collisions at RHIC.

The paper is organized in the following manner. In the next section we discuss ideal (non interacting) HRG and the VDWHRG models. In Sec. III, we discuss the experimental observables related to the fluctuation of conserved charge. Then in the Sec. IV we discuss the methodology to extract van der Waals parameters of the VDWHRG model and the results. Finally in Sec. V we summaries our findings.

* subhasis.samant@gmail.com
† bedanga@niser.ac.in

II. MODEL

In this section we will briefly discuss the ideal HRG (IHRG) and the VDWHRG models.

A. IHRG model

In the ideal HRG model, the thermal system consists of non-interacting points like hadrons and resonances. The logarithm of the partition function of a hadron resonance gas in the grand canonical ensemble can be written as

$$\ln Z^{id} = \sum_i \ln Z_i^{id}, \tag{1}$$

where the sum is over all the hadrons and resonances. *id* refers to ideal *i.e.*, non-interacting HRG model. For particle species i,

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \tag{2}$$

where V is the volume of the system, g_i is the degeneracy, $E_i = \sqrt{p^2 + m_i^2}$ is the single particle energy, m_i is the mass of the particle and $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$ is the chemical potential. The B_i, S_i, Q_i are respectively the baryon number, strangeness and electric charge of the particle, μ's are the corresponding chemical potentials. The upper and lower signs of $\pm$ in Eq. 2 correspond to fermions and bosons, respectively. We have incorporated all the hadrons and resonances listed in the particle data book up to a mass of 3 GeV [86]. Once we know the partition function of the system we can calculate other thermodynamic quantities. The pressure is related to the partition function by the following relation:

$$\begin{aligned} p^{id} &= \sum_i \frac{T}{V} \ln Z_i^{id} \\ &= \sum_i (\pm) \frac{g_i T}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \end{aligned} \tag{3}$$

The n$^{\text{th}}$ order susceptibility can be calculated once we know the pressure using the formula,

$$\chi_q^n = \frac{\partial^n((p/T^4))}{\partial(\frac{\mu_q}{T})^n} \tag{4}$$

where μ_q is the chemical potential for the charge q which can be any conserved quantity like B (baryon), S (strangeness), and Q (electric charge) etc.

B. VDWHRG model

In IHRG model, hadrons and resonances are point-like non-interacting particles. However, interaction is needed especially at very high temperature or chemical potential where ideal gas assumption becomes inadequate. Further hadrons physically have finite sizes. To catch the basic qualitative features of a strongly interacting system of gas of hadrons, van der Waals type interaction is incorporated in the VDWHRG model. The van der Waals equation in the canonical ensemble is given by [87]

$$\left(p + \left(\frac{N}{V}\right)^2 a\right)(V - Nb) = NT, \tag{5}$$

where p is the pressure of the system, N is the number of particles and a, b (both positive) are the van der Waals parameters. The parameters a and b describe the attractive and repulsive interaction respectively. Higher the value of a, the greater the attraction between hadrons and more probable for a phase transition. The Eq. 5 can be written as

$$p(T,n) = \frac{NT}{V - bN} - a\left(\frac{N}{V}\right)^2 \equiv \frac{nT}{1 - bn} - an^2, \tag{6}$$

where $n \equiv N/V$ is the number density of particles. The first term in the right-hand side of the Eq. 6 corresponds to the excluded volume correction where the system volume is replaced by the available volume $V_{av} = V - bN$, where $b = \frac{16}{3}\pi r^3$ is the proper volume of particles with r being corresponding hard sphere radius of the particle. The second term in Eq. 6 corresponds to the attractive interaction between particles. The importance of van der Waals equation is that this analytical model can describe first-order liquid-gas phase transition of a real gas which ends at the critical point. Such a feature is also an expectation for the QCD phase diagram at larger $\mu_{\rm B}$ which corresponds to lower $\sqrt{s_{\rm NN}}$ in the experiments.

The van der Waals equation of state in the Grand canonical ensemble can be written as [63, 64]

$$p(T,\mu) = p^{id}(T,\mu^*) - an^2, \;\; \mu^* = \mu - bp(T,\mu) - abn^2 + 2an, \tag{7}$$

where $n \equiv n(T,\mu)$ is the particle number density of the van der Waals gas:

$$n \equiv n(T,\mu) \equiv \left(\frac{\partial p}{\partial \mu}\right)_T = \frac{n^{id}(T,\mu^*)}{1 + bn^{id}(T,\mu^*)}. \tag{8}$$

Susceptibilities in the VDWHRG model can be calculated by putting pressure of Eq. 7 into the Eq. 4. In the VDWHRG model, the interactions exist between all pairs of baryons and all pair of anti-baryons only. The mesons are non-interacting in this model.

Several methods have been proposed to fix the van der Waals parameters in VDWHRG model. In the Ref. [64] the VDW parameters a and b (or r) have been fixed by reproducing the saturation density $n_0 = 0.16$ fm^3 and binding energy $E/N = 16$ MeV of the ground state of nuclear matter. The parameters thus obtained in this method are $a = 329$ MeV fm^3 and $r = 0.59$ fm [64]. The model predicts a first order liquid-gas phase transition which has a CP at $T = 19.7$ GeV and $\mu_B = 908$ MeV [64]. While in Ref. [85], a and r have been fixed by fitting the LQCD data at zero chemical potential. The values of the VDW parameters in this model are

VDW parameter	Experimental data used
Set-1	$\sigma^2/M, S\sigma, \kappa\sigma^2$
Set-2	$S\sigma, \kappa\sigma^2$

TABLE I. Sets of van der Waals parameters and the corresponding experimental data of net-proton fluctuation used.

$a = 1250 \pm 150$ MeV fm^3 and $r = 0.7 \pm 0.05$ fm. A liquid-gas phase transition is observed in this model as well with a CP at $T = 62.1$ MeV and $\mu_B = 708$ MeV.

III. FLUCTUATION OBSERVABLES

Experimentally fluctuations of conserved charges are obtained by measuring the conserved number (net-charge or net-baryon or net-strangeness) on the event-by-event basis within a certain rapidity y and transverse momentum p_T acceptance. The net-number of the conserved quantity takes different values for each event and hence gives a distribution when measured for a large number of events. Mean of the distribution is the event average of the net-number of the conserved charge. i.e.,

$$M_q = \langle N_q \rangle . \tag{9}$$

The n$^{\text{th}}$ order central moment is defined as

$$\delta N_q{}^n = \left\langle \left(N_q - \langle N_q \rangle\right)^n \right\rangle . \tag{10}$$

The mean (M_q), variance (σ_q^2), skewness (S_q) and kurtosis (κ_q) of distribution of the conserved charge are related to the central moments of the distribution and also to different order of the corresponding susceptibilities by the following relations:

$$M_q = VT^3\chi_q^1, \tag{11}$$

$$\sigma_q^2 = \langle (\delta N_q)^2 \rangle = VT^3\chi_q^2, \tag{12}$$

$$S_q = \frac{\langle (\delta N_q)^3 \rangle}{\sigma_q^3} = \frac{VT^3\chi_q^3}{(VT^3\chi_q^2)^{3/2}}, \tag{13}$$

$$\kappa_q = \frac{\langle (\delta N_q)^4 \rangle}{\sigma_q^4} - 3 = \frac{VT^3\chi_q^4}{(VT^3\chi_q^2)^2}. \tag{14}$$

The mean, variance, skewness are respectively estimations of the most probable value, width, asymmetry and the peakedness of the distribution, respectively. The kurtosis indicates the sharpness of a distribution compared with the Gaussian distribution (for which all the moments higher than second order are zero). From Eqs. 11 - 14, volume independent ratios can be defined as:

$$\sigma_q^2/M_q = \chi_q^2/\chi_q^1, \tag{15a}$$

$$S_q\sigma_q = \chi_q^3/\chi_q^2, \tag{15b}$$

$$\kappa_q\sigma_q^2 = \chi_q^4/\chi_q^2. \tag{15c}$$

The left-hand side quantities in Eqs. 15 can be measured in the experiments while the right hand-side quantities can be calculated in models like VDWHRG. As already mentioned, non-monotonic variations of these quantities with beam energy ($\sqrt{s_{\rm NN}}$) are believed to be good signatures of a phase transition and CP. The STAR collaboration has published all of the above-mentioned observables for the net-proton, net-charge and net-kaon distributions at different energies ranging from 7.7 GeV to 200 GeV and at various centralities [13–15]. Similar observables for net-charge have been reported by the PHENIX collaboration [88]. For the current study, we use only the published results for the net-proton number distribution in central Au-Au collisions from the STAR experiment and not the subsequent preliminary unpublished results (as these could be subject to changes). We have not considered the net-kaon and net-charge results as they have large uncertainties and the later has dominant contributions from resonance decay effects [22]. We have also not used the net-charge results from the PHENIX experiment as the acceptance for those measurements ($|y| < 0.35$) are much smaller compared to STAR ($|y| < 0.5$). Studies suggest, to capture the relevant physics processes like CP it is ideally required to have acceptance of at least 1 unit in rapidity [89]. Further, it has been suggested that net-proton number is a sensitive observable for CP physics [90].

IV. EXTRACTION OF VAN DER WAALS PARAMETERS FROM EXPERIMENTAL DATA

In this work, we have extracted the attractive and repulsive parameters, a and r, of the VDWHRG model from the fluctuation observables of net protons measured by the STAR collaboration [13]. The T and the μ_B at the chemical freeze-out can be parametrized by the following functions [91]:

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \tag{16}$$

where $a = (0.166 \pm 0.002)$ GeV, $b = (0.139 \pm 0.016)$ GeV^{-1}, $c = (0.053 \pm 0.021)$ GeV^{-3} and

$$\mu_B(\sqrt{s_{\rm NN}}) = \frac{d}{1 + e\sqrt{s_{\rm NN}}}, \tag{17}$$

where $d = 1.308 \pm 0.028$ GeV and $e = 0.273 \pm 0.008$ GeV^{-1}. These chemical freeze-out parameters provide a good quantitative description of the hadronic yields over a wide range of $\sqrt{s_{\rm NN}}$. At different $\sqrt{s_{\rm NN}}$ we have used the chemical freeze-out T and μ_B from the above mentioned parametrized equations. To extract the van der Waals parameters a and r at a particular $\sqrt{s_{\rm NN}}$ we have used a χ^2 minimization technique where χ^2 is defined as

$$\chi^2 = \frac{1}{N}\sum_i^N \frac{\left(O_i^{expt} - O_i^{model}\right)}{\left(\sigma_i^{expt}\right)^2} \tag{18}$$

where O_i^{model} is the i$^{\text{th}}$ observable calculated in model whereas O_i^{expt} and σ_i^{expt} are its experimental value and uncertainty in

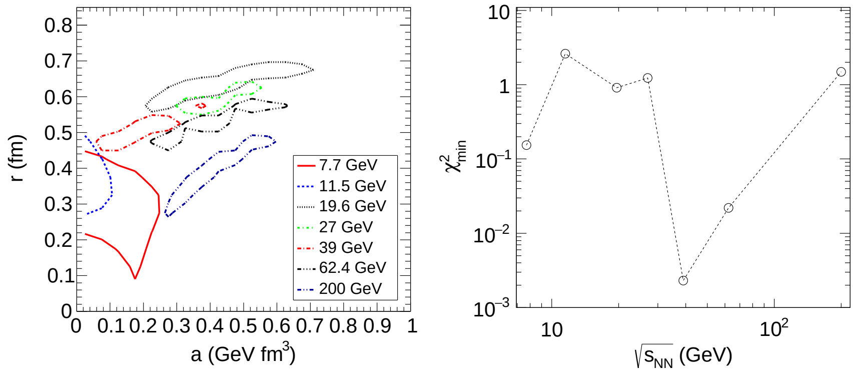

FIG. 1. (Left) 1σ contours of the van der Waals parameters in r, a plane, at different $\sqrt{s_{NN}}$ extracted from the experimental data of σ^2/M, $S\sigma$ and $\kappa\sigma^2$ of net-proton (VDW parameter Set-1), (Right) minima of the χ^2 at different $\sqrt{s_{NN}}$.

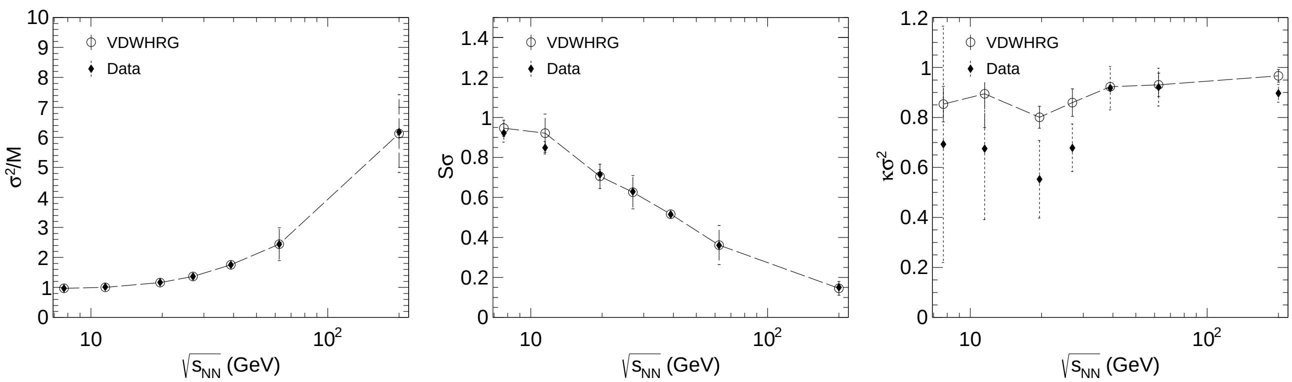

FIG. 2. σ^2/M, $S\sigma$ and $\kappa\sigma^2$ of net-proton in the VDWHRG model with the parameter Set-1. Results are compared with the experimental data of net-proton fluctuations for $0-5\%$ Au + Au collisions measured by the STAR collaboration at RHIC [13]. Errors on data points are systematic and statistical errors added in quadrature and errors on model results are obtained by considering the values at $\chi^2_{min}+1$.

the measurement respectively. For the experimental uncertainties, we have quadratically added the statistical and the systematic errors for the observables. N in the above equation is the number of observables used to calculate χ^2. The 1σ error of the extracted parameters correspond to $\chi^2_{min}+1$.

We have to extract values of two parameters and we have three experimental observables $\sigma^2/M, S\sigma$ and $\kappa\sigma^2$ of net-proton distribution for 7 different $\sqrt{s_{NN}}$. Two sets of van der Waals parameters (a and r) have been extracted by fitting the fluctuation observables in our present analysis which are listed in the Table. I. For the first set (Set-1), we use $\sigma^2/M, S\sigma$ and $\kappa\sigma^2$ of net-proton distributions at 0-5 % centrality in Au + Au collisions measured by the STAR Collaboration at RHIC [13]. For the second set (Set-2) we have used the $S\sigma$ and $\kappa\sigma^2$ data. The $S\sigma$ and $\kappa\sigma^2$ are common for both the sets. It may be noted that these two observables are expected to be more sensitive to the CP physics [10]. Net-proton fluctuations were experimentally measured at mid-rapidity ($|y| < 0.5$) and within the transverse momentum range $0.4 < p_T < 0.8$ GeV. To incorporate acceptance cuts in our model, we have written the d^3p and the single particle energy E as $d^3p = 2\pi\ p_T\ m_T\ \cosh\ y\ dy\ dp_T$, and $E = m_T\ \cosh y$ where $m_T = \sqrt{p_T^2+m^2}$. Then the interaction ranges in y and p_T are chosen as -0.5 to 0.5 and 0.4 to 0.8 GeV respectively to make the model results comparable with the experimental data.

The left plot of Fig. 1 shows the 1σ contours of the van der Waals attractive and repulsive parameters, a, r, for Set-1 at different $\sqrt{s_{NN}}$ from 7.7 GeV up to 200 GeV. We find that both a and r shows a possibility of reaching maxima at $\sqrt{s_{NN}} = 19.6$ GeV. It may be noted that the experimental data of $\kappa\sigma^2$ shows a minimum around this energy. We also find that the repulsive parameter strength increases as we go down from $\sqrt{s_{NN}} = 200$ GeV to 19.6 GeV. The attractive parameter strength ranges are of similar order for these energies. However for $\sqrt{s_{NN}}$ = 7.7 GeV to 11.5 GeV both the strengths of a and r shows a decrease relative to 19.6 GeV. Right plot of Fig. 1 shows the minima of the χ^2 at different $\sqrt{s_{NN}}$.

In Fig. 2 we show the variations of $\sigma^2/M, S\sigma$ and $\kappa\sigma^2$ from the VDWHRG model using the parameter obtained by fitting the net-proton experimental data Set-1. We also compare the model results with the experimental data. We observe that with this parameter set, VDWHRG model can describe σ^2/M and $S\sigma$ at all the energies within the uncertainties. In addition, the measured $\kappa\sigma^2$ of net-proton distribution can also be described qualitatively. At $\sqrt{s_{NN}} = 19.6$ and 27 GeV, VD-

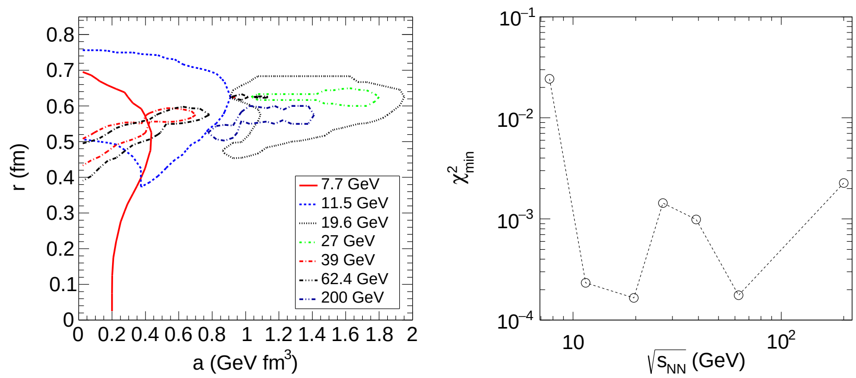

FIG. 3. (Left) 1σ contours of the van der Waals parameter in r, a plane at different $\sqrt{s_{\rm NN}}$ extracted from $S\sigma$ and $\kappa\sigma^2$ of net-proton (Set-2), (Right) minima of χ^2 at different $\sqrt{s_{\rm NN}}$.

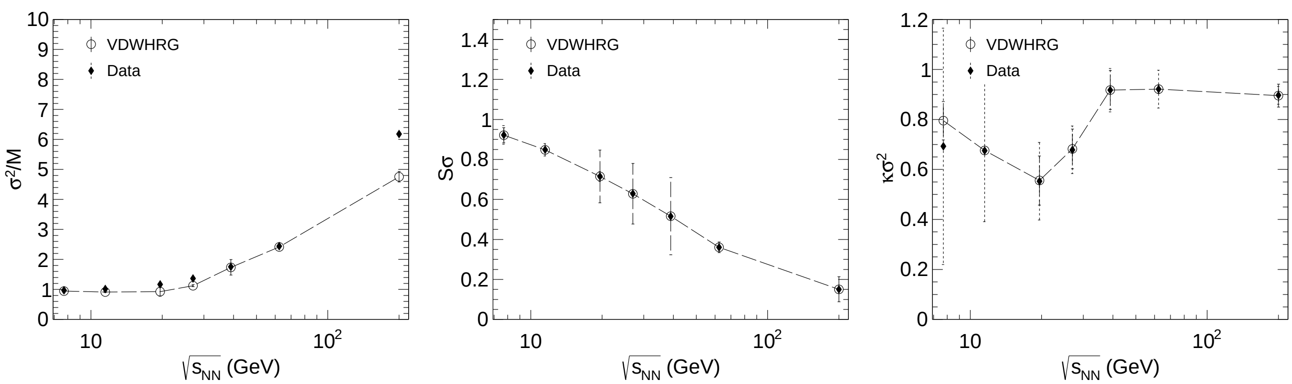

FIG. 4. Same as Fig. 2 but for the parameter Set-2.

WHRG model with parameters obtained using the data in Set-1 slightly overestimate the experimental values of $\kappa\sigma^2$ and at all other energies model and data agree within the uncertainties.

1σ contours of the VDW parameters using the data in Set-2 for different $\sqrt{s_{\rm NN}}$ are shown in left plot of Fig. 3. The data in Set-2 are more sensitive to CP physics. Here we observe that the attractive VDW parameter strength becomes large for $\sqrt{s_{\rm NN}}$ = 19.6 and 27 GeV relative to the values obtained at $\sqrt{s_{\rm NN}}$ = 7.7, 11.5, 39, 62.4 GeV. The values of the attractive VDW parameter in Set-2 are relatively larger compared to Set-1. Particularly, at $\sqrt{s_{\rm NN}} = 19.6$ GeV, value of a in Set-2 is double compared to that for Set-1. The larger values of a for the Set-2 indicates that, the higher order fluctuation observables, $S\sigma$ and $\kappa\sigma^2$ are more sensitive to the attractive interaction than σ^2/M. Further, large increase of the attractive interaction near 19.6 GeV might be due to the existence of a physics processes where attractive interactions are dominant such as a CP. The repulsive VDW parameter varies approximately within 0.4 to 0.75 fm at all the $\sqrt{s_{\rm NN}}$ except 7.7 GeV where the value r is not well constrained due to large uncertainties in the measurement. The maxima of r observed at $\sqrt{s_{\rm NN}} = 19.6$ GeV in Set-2 is comparable to that from the Set-1. This indicates that sensitive to the repulsive interaction is of the same order for the various choice of observables. Right plot of Fig. 3 shows the minima of the χ^2 at different $\sqrt{s_{\rm NN}}$.

Figure 4 shows the variations of $\sigma^2/M, S\sigma$ and $\kappa\sigma^2$ of net-proton in VDWHRG model using the parameters obtained from the fit to data given in Set-2. We compare the model results with the experimental data. It can be seen that, with this parameter set, σ^2/M of net-proton distribution can be described in all the energies except for 200 GeV where model underestimates the data. Further, this parameter set can describe $S\sigma$ and $\kappa\sigma^2$ of net-proton distribution measured in all the energies from 7.7 to 200 GeV.

V. SUMMARY

We have extracted the van der Waals attractive and repulsive parameters, a and r, by fitting the experimental data of fluctuation observables of net protons measured by the STAR Collaboration using VDWHRG model. The variation of VDW parameters reflective of the attractive and repulsive nature of the interactions for the QCD matter produced in heavy-ion collisions with collision energy has been discussed. Two sets of (a, r) parameters have been extracted. In the first set (Set-1), we have used $\sigma^2/M, S\sigma$ and $\kappa\sigma^2$ of net-proton. In another set (Set-2) we have used only $S\sigma$ and $\kappa\sigma^2$ of net-proton. We have incorporated the proper experimental acceptances in our calculation. We have observed that the higher order fluctuation observables $S\sigma$ and $\kappa\sigma^2$ are more sensitive to the attractive interaction. Large increases in the attractive interaction (i.e., a) is observed near $\sqrt{s_{NN}}$ = 19.6 GeV, indicates change in equation-of-state near this energy. Further, we observe that with the parameter Set-1, VDWHRG model can describe σ^2/M and $S\sigma$ of net-proton in all the energies studied in this work. The $\kappa\sigma^2$ of net-proton can also be described qualitatively. On the other hand, with the parameter Set-2, VDWHRG model can describe σ^2/M of net-proton from $\sqrt{s_{NN}}$ = 7.7 to 62.4 GeV. Not only that, the $S\sigma$ and $\kappa\sigma^2$ of net-proton can be described within the error bars by VDWHRG model in all the energies.

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