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Chiral Meson Lagrangians from the QCD Based NJL Model Modified by Nonlocal Effects

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Abstract

Starting from a QCD inspired bilocal quark interaction we obtain a local effective meson lagrangian. In contrast to previous local (NJL-like) approaches, we include nonlocal corrections related to the finite meson size which we characterize by a small parameter. After bosonization using the heat-kernel method we predict the structure coefficients of the Gasser-Leutwyler p^4-lagrangian up to first order in this parameter. The modifications for the L_i coefficients are typically of the order 15-20%, except for L_5, where we find a stronger nonlocal influence.

This report is based on the work [1], where we have studied the bosonization of an effective QCD-inspired quark interaction, including nonlocal effects. First we have considered a "fixed-distance" approximation and introduced a small parameter α characterizing the size of the nonlocal corrections in order to obtain a quantitative estimate. The result was used as an input into a more general dynamical separation ansatz which then allowed to predict the modifications of the structure constants of the effective Gasser-Leutwyler p^4-lagrangian [2] to first order in α. Of course, in the limit $\alpha \to 0$, our method reproduces the results of the bosonization in the usual local NJL model [3] (see refs.[4] and references therein). We briefly review the standard method of transforming the QCD lagrangian into an effective 4-quark lagrangian and estimate the size of the $\bar{q}q$ system within a nonrelativistic Schrödinger approach for constituent quarks. We use the result for a quantitative estimate of the nonlocal effects and then describe a fixed-distance approximation. Further we present the bosonization in the more general dynamical bilocal approach, where we consider a separable ansatz for the bilocal collective fields. Solving the heat-kernel equation, we calculate the modified structure constants L_i of the chiral p^4-lagrangian.

The starting point of our consideration is the QCD action in Minkowski space,

$$S[\bar{q}, q, A] = \int d^4x \Big[\bar{q}(i\hat{D} - m_0)q - \frac{1}{4}\sum_{a=1}^{8} G^a_{\mu\nu}G^{a\,\mu\nu}\Big],$$

and ξ, $\bar{\xi}$, J^a_μ are the external sources associated with the fields $\bar{q}$, q, A^a_μ; q is the quark field; m_0 is the current quark mass matrix; A^a_μ represents a gluon with color index a, and λ^a_C are $SU(3)_C$ matrices. We have used the common definitions for the covariant derivative and the gluon field-strength tensor,

$$D_\mu = \partial_\mu - ig\sum_{a=1}^{8}\frac{\lambda^a_C}{2}A^a_\mu\,, \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu\,.$$

Further g is the QCD coupling constant, and f_{abc} are the $SU(3)$ structure constants. We use $\hat{D}$ for $\gamma_\mu D^\mu$.

Using standard techniques of path integration [5], after integration over the gluon fields, neglecting triple and higher-order gluon vertices, the effective action can be written,

$$S_{int} = -i\frac{g^2}{2}\iint d^4x\, d^4y\, j^{\mu a}(x)\, D^{ab}_{\mu\nu}(x-y) j^{\nu a}(y) \qquad (1)$$

where $j^a_\mu(x) = \bar{q}(x)\gamma_\mu\frac{\lambda^a_c}{2}q(x)$.In the Feynman gauge the nonperturbative gluon propagator is defined as $D^{ab}_{\mu\nu}(x) = g^{-2}\delta^{ab}g_{\mu\nu}D(x)$. As the behavior of the Green's function $D(x)$ is unknown for large distances, a specific ansatz has to be used.

Here we will outline the ideas which motivate the bilocal fixed-distance approximation. Basing on the assumption of the dominant role of equal-time interactions in the formation of bound states [6], the Schwinger-Dyson equations of the bilocal meson theory reduce to the nonrelativistic Schrödinger approach corresponding to the description of $\bar{q}q$-pairs interacting via some effective gluonic potential. Studies of the $\bar{q}q$ system have shown, that it can be approximated by a linear one $V(r) = \sigma \cdot r$ with $\sigma \approx 0.27\ GeV^2$. Using the results of [7] one can estimate the characteristic distance between the quark and antiquark,

$$< r > \equiv h = \frac{2E_1}{3\sigma} \approx 0.68\,fm,$$

where $E_1 = 2.238(\sigma^2/2\mu)^{1/3}$ is the ground state energy and $\mu = 0.336\ GeV$ the reduced mass of the system.

In the following we only consider the ground state ($l = 0$), i.e., neglect excited mesonic states such as π^*, K^* etc. . In order to obtain a qualitative estimate of the nonlocal corrections from the bilocal effective action, we make the following ansatz. We consider the case where the constituent quarks in the meson are localized at the scale h. This we do by including a delta function $\delta((x-y)^2 - h^2)$ into the integrand S_{int}, Eq. (1),

$$S_{int} = -i\frac{\kappa^2}{2}\iint d^4x\, d^4y\; j^a_\mu(x)j^{\mu a}(y)D(x-y)\delta\left((x-y)^2 - h^2\right) \qquad (2)$$

The correct dimension is obtained by introducing a constant κ ($([\kappa] = m^{-1})$. After shifting the argument y by a Lorentz-invariant operator $K(h,x)$, the effective action, Eq. (2), becomes

$$S_{int} = -i\frac{\kappa^2 D(h)}{2}\int d^4x\; j^a_\mu(x)K(h,x)\,j^{a\,\mu}(x)\,,$$

with

$$K(h,x) = \int d^4y \exp((y-x)_\mu\partial^\mu)\delta((x-y)^2 - h^2) = \pi^2h^2\left[1 + \frac{1}{8}\frac{\Box}{\Lambda^2} + O\left(\frac{\Box^2}{\Lambda^4}\right)\right],$$

where $\Lambda = h^{-1}$. Introducing $G = \frac{8}{9}\pi^2\kappa^2h^2D(h)$ and performing a Fierz transformation the action, Eq. (2), reads

$$S_{int} = i\frac{9G}{16}\int d^4x\left(\bar{q}(x)\frac{\mathcal{M}^\theta}{2}q(x)\,\bar{q}(x)\frac{\mathcal{M}^\theta}{2}q(x) + \frac{1}{8\Lambda^2}\bar{q}(x)\frac{\mathcal{M}^\theta}{2}\Box[q(x)\bar{q}(x)]\frac{\mathcal{M}^\theta}{2}q(x)\right) \quad (3)$$

where $\mathcal{M}^\theta$ are tensor products of Dirac, flavor and color matrices of the type

$$\left\{1\,, i\gamma_5\,, i\sqrt{\frac{1}{2}}\gamma^\mu\,, i\sqrt{\frac{1}{2}}\gamma_5\gamma^\mu\right\}^D\left\{\frac{1}{2}\lambda^a_F\right\}^F\left\{\frac{4}{3}\mathbf{1}\right\}^C.$$

Here we consider the $SU(3)_F$ flavor group with flavor matrices λ^a_F, and we restrict ourselves to the color-singlet $\bar{q}q$ contributions.

Before considering the physical results of the bilocal fixed-distance approximation, let us study a more general approach using a dynamical bilocal model. Starting with the effective action, Eq. (1), and performing a Fierz transformation one finds

$$S_{int} = \frac{i}{2}\iint d^4x\, d^4y\; D(x-y)\,\bar{q}(x)\frac{\mathcal{M}^\theta}{2}q(y)\,\bar{q}(y)\frac{\mathcal{M}^\theta}{2}q(x)\,.$$

Introducing bilocal collective meson fields in a standard way [5] leads to an effective action which is bilinear in the quark fields,

$$S_{int} = \iint d^4x\, d^4y\left\{-\frac{9}{8D(x-y)}\,\mathrm{tr}\left[\tilde{S}^2 + \tilde{P}^2 + 2\tilde{V}^2_\mu + 2\tilde{A}^2_\mu\right] + \bar{q}(x)\tilde{\eta}(x,y)q(y)\right\},$$

with $\tilde{\eta}(x,y) = -\tilde{S} - i\gamma^5\tilde{P} + i\gamma^\mu\tilde{V}_\mu + i\gamma^\mu\gamma^5\tilde{A}_\mu$ and $S, P, V, A \equiv S, P, V, A(x,y)$.

Following ref. [8], we assume a strong localization of the bilocal fields and make the ansatz

$$\tilde{\eta}(x,y) \rightarrow \tilde{\eta}(z,t) = \eta(z)f(t) + \eta_\mu(z)t^\mu g(t) + \cdots\,, \qquad (4)$$

where $z = (x+y)/2$, $t = (y-x)/2$ are the global and relative coordinates, and $\eta(z) = -S(z) - i\gamma^5P(z) + i\gamma^\mu V_\mu(z) + i\gamma^\mu\gamma^5A_\mu(z)$ combines the local collective fields of the composite operators $\bar{q}(z)q(z)$, $\bar{q}(z)i\gamma^5q(z)$, $\bar{q}(z)\gamma_\mu q(z)$ and $\bar{q}(z)\gamma_\mu\gamma^5q(z)$, corresponding to the lowest meson excitations 0^{++}, 0^{-+}, 1^{--}, 1^{++}. The next order term of Eq. (4), proportional to η_μ, can be identified with the excitations 1^{--}, 1^{+-}, 2^{++}, 2^{--} which will be neglected. The functions $f(t)$ and $g(t)$ rapidly decrease for $|t^2| \gg h^2$ and strongly localize the bilocal fields $\tilde{\eta}(x,y)$ to the effective size of the collective meson $h \equiv 1/\Lambda$.

Expanding $q(y)$ and $\bar{q}(x)$ in a Taylor series about z, and using Eq. (4) we obtain

$$\iint d^4x\, d^4y\; \bar{q}(x)\tilde{\eta}(x,y)q(y) \;=\; 2\int d^4z\, \bar{q}(z)\eta(z)q(z)\int d^4t\, f(t) + 2\int d^4z \partial^\mu \bar{q}(z)\eta(z)\partial_\mu q(z)\int d^4t\, t^2 f(t)$$

Then, for the first generation of mesons corresponding to the (0^{++}, 0^{-+}, 1^{--}, 1^{++}) multiplets, the generating action is

$$S_{int} \;=\; \int d^4z\Big[-\frac{1}{4G_1}\mathrm{tr}[(\Phi(z)-m_0)^\dagger(\Phi(z)-m_0)] - \frac{1}{4G_2}\mathrm{tr}(V_\mu^2(z)+A_\mu^2(z)) + \bar{q}(z)i\widehat{\mathrm{D}}q(z) - \frac{\alpha}{\Lambda^2}\partial^\mu\bar{q}(z)\,\eta(z)\,\partial_\mu q(z)\Big]\Big\}\,, \tag{5}$$

where $\widehat{\mathrm{D}}$ is the Dirac operator in the presence of local collective meson fields,

$$i\widehat{\mathrm{D}} = [i(\hat{\partial}+\hat{A}^{(+)}) - (\Phi+m_0)]P_R + [i(\hat{\partial}+\hat{A}^{(-)}) - (\Phi^\dagger+m_0)]P_L. \tag{6}$$

Here $\Phi = S + iP$, $\hat{V} = V_\mu\gamma^\mu$, $\hat{A} = A_\mu\gamma^\mu$; $P_{R/L} = \frac{1}{2}(1\pm\gamma_5)$ are chiral right/left projectors; $\hat{A}^{(\pm)} = \hat{V}\pm\hat{A}$ are right and left combinations of fields. The parameter α is defined as

$$\frac{\alpha}{\Lambda^2} = \frac{1}{2}\int d^4t\, t^2 f(t)\,,$$

where $f(t)$ is normalized as $2\int d^4t f(t) = 1$.

Performing a partial integration and dropping the surface term, the last term in Eq. (5) can be rewritten in the form

$$\int d^4z\partial^\mu\bar{q}(z)\,\eta(z)\,\partial_\mu q(z) \;=\; -\int d^4z\bar{q}(z)[\partial^\mu\eta(z)\,\partial_\mu + \eta(z)\,\partial^2]q(z)\,. \tag{7}$$

Of course, we do not know the explicit form of the function $f(t)$. However, we can now use the fixed-distance approximation of Eq. (2) to estimate the parameter α. Indeed, the second term in Eq. (3) can be transformed into the form

$$S^2_{int} \;=\; \frac{i}{16\Lambda^2}\int d^4x\bar{q}(x)[\partial^\mu\eta(x)\,\partial_\mu + \eta(x)\,\partial^2]q(x)\,, \tag{8}$$

where $\eta(x)$ is the combination of the local collective fields, which now are defined by the local quark bilinears.

Comparing Eqs. (7) and (8) we can fix the value α, $\alpha = 1/16$, corresponding to the naive bilocal fixed-distance approximation. Using the values $\Lambda = 0.28\,\mathrm{GeV}$ the ratio is estimated to be of the order

$$\frac{\alpha\mu^2}{\Lambda^2} \approx 0.09\,.$$

This value will be used as a small parameter for further numerical estimates of nonlocal effects.

After integration over the quark fields, the full action arising from the generating functional, Eq. (5), is

$$S(\Phi,\Phi^\dagger,V,A) \;=\; \int d^4z\Big[-\frac{1}{4G_1}\mathrm{tr}[(\Phi-m_0)^\dagger(\Phi-m_0)] - \frac{1}{4G_2}\mathrm{tr}(V_\mu^2+A_\mu^2)\Big] - i\,\mathrm{Tr}'[\log(i\widehat{\widehat{\mathrm{D}}})]\,. \tag{9}$$

Here the second term is the quark determinant of the Dirac operator $i\widehat{\widehat{\mathrm{D}}}$ which is extended to the case of nonlocality. It is obtained from the usual operator, Eq. (6), by the following replacement,

$$A_\mu^{(\pm)} \to A_\mu^{(\pm)}\Big(1+\frac{\alpha}{\Lambda^2}\partial^2\Big) + \frac{\alpha}{\Lambda^2}\big(\partial_\nu A_\mu^{(\pm)}\big)\,\partial^\nu\,, \Phi \to \Phi\Big(1+\frac{\alpha}{\Lambda^2}\partial^2\Big) + \frac{\alpha}{\Lambda^2}\big(\partial_\nu\Phi\big)\,\partial^\nu\,.$$

The modulus of the quark determinant in Eq. (9) corresponds to non-anomalous part of the effective meson action. Using the heat-kernel technique [9] one can calculate the quark determinant in a momentum expansion.

We will follow here the standard notations [2] for the chiral lagrangian, where F_0 is the bare π decay constant and L_i are the dimensionless p^4 structure constants. Our calculation predicts the following expression for F_0,

$$F_0^2 = \frac{N_c\mu^2}{4\pi^2}\Big[y - \frac{4\pi^2 <\bar{q}q>}{\mu^3 N_c}\frac{\alpha\mu^2}{\Lambda^2}\Big]\,, \tag{10}$$

where $y = \Gamma(0,\mu^2/\tilde{\Lambda}^2)$. In Eq. (10) the first term is the standard prediction of the local limit and the second corresponds to the nonlocal correction. For the meson mass matrix $M = diag(\chi_u^2, \chi_d^2, ..., \chi_n^2)$ we obtain

$$\chi_i^2 = \frac{N_c\mu m_i^0}{2\pi^2 F_0^2}\Big(\tilde{\Lambda}^2 e^{-\mu^2/\tilde{\Lambda}^2} - \mu^2 y\Big) = -\frac{2m_i^0 <\bar{q}q>}{F_0^2}\,.$$

Moreover, the coefficients L_i are given by $L_1 - L_2/2 = L_4 = 0$ and

$$
\begin{aligned}
L_2 &= \frac{N_c}{16\pi^2}\frac{1}{12}\left(1 + 2\frac{\alpha\mu^2}{\Lambda^2}\right), \quad L_3 = -\frac{N_c}{16\pi^2}\frac{1}{6}\left(1 + 5(1-y)\frac{\alpha\mu^2}{\Lambda^2}\right), \\
L_5 &= \frac{N_c}{16\pi^2}x\left[y - 1 - \frac{28}{3}\frac{\alpha\mu^2}{\Lambda^2}\right], \quad L_9 = \frac{N_c}{16\pi^2}\frac{1}{3}\left(1 + \frac{21y-26}{6}\frac{\alpha\mu^2}{\Lambda^2}\right), \\
L_{10} &= -\frac{N_c}{16\pi^2}\frac{1}{6}\left(1 + \frac{15y-10}{3}\frac{\alpha\mu^2}{\Lambda^2}\right).
\end{aligned}
$$

where $x = -\mu F_0^2/(2 <\bar{q}q>)$. We made use of the approximation $\Gamma(k, \mu^2/\tilde{\Lambda}^2) \approx \Gamma(k)$ for $k \geq 1$ and $\mu^2/\tilde{\Lambda}^2 \ll 1$.

In Eq. (10) we use $<\bar{q}q> = -(0.25\,\mathrm{GeV})^3$ and $F_0 = 92\,MeV$ to fix the value of y. In comparison with the local limit it changes from $y_{loc} \sim 1$ to $y_{nonloc} \approx 0.5$. It is worth to be mentioned here that the mass matrix is not affected by nonlocal corrections. We found that the nonlocal corrections to the L_i coefficients were typically of the order 15-20%, except for L_5 which is stronly modified by nonlocal effects. Thus, the local NJL model has turned out to be a reasonable approximation for bosonization of low-energy quark interactions and deriving the effective meson lagrangian at $O(p^4)$-level. Of course, our analysis could be extended to any order in the momentum expansion. Given the fact that nonlocal corrections to the coefficients L_i are of the order 20 %, it is to be expected that these corrections are of the same order of magnitude as local p^6 contributions.

The uncertainties arising from nonlocality make it difficult to distinguish between next to leading order effects of the momentum expansion and nonlocal contributions to the leading order. As an example where this is not the case we suggest to investigate the processes $\eta \to \pi^0\gamma\gamma$ and $\gamma\gamma \to \pi^0\pi^0$ where the nonzero *Born contributions* to the amplitudes appear only at $O(p^6)$-level.

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