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COLLISIONS**

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FORMATION TIME OF HADRONS AND DENSITY OF MATTER PRODUCED IN RELATIVISTIC HEAVY - ION COLLISIONS

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ABSTRACT

Density of matter produced in relativistic heavy - ion collisions depends substantially on the space-time evolution of the collision and on the formation time of hadrons produced. Interactions of hadrons younger than their formation time are attenuated with respect to their normal values (transparency of hadronic matter for newly formed hadrons). The system of secondary hadrons produced in a heavy-ion collision thus expands as a gas of almost noninteracting particles before hadrons reach their formation time. Densities of interacting hadronic matter produced in Oxygen-Lead and Sulphur-Lead collisions at 200 GeV/nucleon are estimated as a function of the formation time of hadrons. Uncertainties in our knowledge of the critical temperature T_c and of the formation time of hadrons τ_0 permit at present three scenarios: an optimistic one (QGP has already been produced in collisions of Oxygen

and Sulphur with heavy ions and will be copiously produced in Lead collisions), a pessimistic one (QGP cannot be produced at 200 GeV/nucleon) and an intermediate one (QGP has not been produced in Oxygen and Sulphur Interactions with heavy ions and will be at best produced only marginally in Pb-collisions). We find the last option as most probable.

1. INTRODUCTION

Discussions of heavy-ion collisions at ultrarelativistic energies are most frequently based on the Bjorken picture [1] of a rather rapid thermalization of matter produced in the collision. Lorentz contracted nuclei are considered as thin pancakes passing through each other and the matter is assumed to thermalize within 1 fm/c in its rest frame. The energy of final state particles within c.m.s. rapidity unit near $y \equiv y_{c.m.} = 0$ is then confined to a cylinder of the length of about 1 fm and of the transverse radius equal to the radius of the smaller of the two colliding nuclei. The energy density is then roughly given by the Bjorken's expression

$$\varepsilon = \frac{\Delta E / \Delta y}{\pi R^2 \tau} \quad (1)$$

Studies of transverse energy production in the Oxygen - and Sulphur - collisions with the heaviest ions performed by NA-35 [2], NA-38 [3] and Helios [4] collaborations has shown that the highest values of $\Delta E / \Delta y$ are around 90 GeV for Oxygen and 170 GeV for Sulphur beams incident on the heaviest ions. From Eq.(1) we then find that $\varepsilon(O - Pb) \sim 3.5 \text{ GeV}/\text{fm}^3$ and $\varepsilon(S - Pb) \sim 4.1 \text{ GeV}/\text{fm}^3$. These values are to be compared with the expected energy densities of the Quark -Gluon Plasma (QGP) at the temperature of the phase-transitions T_c .

Taking the transition temperature as

$$150 \text{ MeV} < T_c < 200 \text{ MeV} \quad (2a)$$

we obtain, for a two-flavour QGP, critical energy densities

$$0.8\text{GeV}/\text{fm}^3 < \epsilon_c < 2.5\text{GeV}/\text{fm}^3 \quad (2b)$$

According to these estimates QGP should have been produced in both O-Pb and S-Pb collisions at 200 GeV/nucleon. For Pb-Pb collision at 160 GeV/nucleon the experimental values of $\Delta E/\Delta y$ are not yet known, so in order to obtain an estimate of energy density we have to guess $\Delta E/\Delta y$. (The lowering of energy per nucleon in Pb-Pb collisions with respect to O-Pb and S-Pb is due to the fact that the neutron/proton ratio is larger in Pb than in O and S nuclei.) A very optimistic estimate is obtained by assuming that $\Delta E/\Delta y$ is proportional to the number of nucleon-nucleon collisions in a heavy-ion interaction. In this way we obtain from experimental values of $\Delta E/\Delta y$ in O-Pb and S-Pb $\Delta E/\Delta y = 870$ GeV for Pb-Pb. The real value will be almost surely considerably lower because in a Pb-Pb collision each nucleon interacts little less than $2R_{Pb}/\lambda$ times, where $R_{Pb} \sim 13.4\text{fm}$ and $\lambda \sim 2\text{fm}$ is the mean free path of a nucleon in nuclear matter. After a few collisions the energy of a beam-nucleon is lower than it had been at the beginning and nucleons in the target have also lower energy than at the beginning (in the c.m.s. of the collision). We shall therefore consider for Pb-Pb

$$600\text{GeV} < \Delta E/\Delta y < 870\text{GeV} \quad (3)$$

Energy densities for Pb-Pb near $y \sim 0$ then become 4,3 - 6,2 GeV/fm³, which is significantly higher than ϵ_c in Eq.(2b) and the formation of the QGP should be almost sure. These estimates depend crucially on the assumption that the thermalization time τ in Eq.(1) is 1 fm/c. In his talk at the Quark Matter 87 Conference Ruuskanen [5] has pointed out that $\tau = 1$ fm/c used in Eq.(1) might lead to an overestimate of the energy density. According to his argument the time of thermalization must be higher than the time interval needed for nuclei to pass through each other increased by the formation time of final state quanta. At CERN SPS heavy ion experiments the c.m.s. energy in a nucleon-nucleon collision is $\sqrt{s} = 20$

GeV and the Lorentz contraction factor in the c.m.s. of the collision is $\gamma = 10$ for both nuclei. The Pb nucleus with $2R_{Pb} \sim 13.4$ fm is then contracted to a pancake with the width of $2R_{Pb}/\gamma = 1.34$ fm and that of the Sulphur to $2R_S/\gamma = 0.72$ fm. The nuclei pass through each other in the c.m.s. of nucleon-nucleon collision within the time $(2R_{Pb} + 2R_S)/2\gamma \sim 1$ fm/c and taking into account also the formation time of about 1 fm/c, one should use according to Ruuskanen [5] $\tau \sim 2$ fm/c. This would lead via Eq.(1) to $\epsilon(O - Pb) \sim 1.75$ GeV fm^{-3} , $\epsilon(S - Pb) \sim 2.05$ GeV fm^{-3} and taking into account Eq.(3) to $2.14 < \epsilon(Pb - Pb) < 3.1$ GeV fm^{-3} . Comparing this with Eqs.(2a) and (2b) one can see that for $T_c \sim 200$ MeV only in Pb-Pb one could hope to see QGP being produced, whereas for critical temperatures near $T_c \sim 150$ MeV the production of QGP is possible in O-Pb, S-Pb and Pb-Pb. It is to be noted, and we shall discuss it in more detail below, that in the central collision with impact parameter $b = 0$, the maximum energy density is, because of the nuclear geometry, by about 50% higher than the average one for O-Pb and S-Pb and by 100% higher for Pb-Pb. Because of that even with $\tau = 2$ fm/c and $T_c = 200$ MeV one could expect the production of QGP in a narrow cylinder also in O-Pb and S-Pb and in a larger region in Pb-Pb collisions.

The purpose of the present paper is to show in a qualitative way that even the estimates based on using $\tau = 2$ fm/c in Eq.(1) are overestimates if the formation time of hadrons is taken into account. Our basis assumption can be phrased also as that of the **transparency of hadronic matter to newly formed hadrons**. The experimental support for this point of view and estimates of the formation time of hadrons comes from the absence of significant cascading due to newly formed (secondary) hadrons in hadron-nucleus collisions at energies of the order of 100 GeV in the laboratory frame. The analysis [6,7,8,9] of data [6,7] on multiparticle production in hadron-nucleus collisions indicates that hadrons (mostly pions) produced in individual nucleon-nucleon collisions are unable to interact immediately after their creation with nucleons in the nucleus. They are gaining this ability only after some formation time τ_0 , expressed in their rest frame, and Lorentz dilated according to their velocity. The same effect is to be expected in heavy ion collisions when secondary particles are created as newly formed hadrons in individual nucleon-nucleon collisions. **The newly formed hadrons will be able to interact with nucleons in nuclei, or with each other, only after their formation times.**

Thermalization of matter produced in heavy ion collision is due to the interaction of quanta of this matter and the time of thermalization must be larger than the formation time of the quanta of the matter.

Assuming that the matter is produced at 200 GeV/nucleon via hadrons, the thermalization time of matter must be larger than the formation time of hadrons.

The paper is organized as follows. In the next section we discuss briefly the concept of the formation time of hadrons and some of the available information on its value. Sect.3 contains the description of models we are using for estimating the density of matter produced in heavy ion collisions. Results following from these models are given in Sect.4. Analysis of results, comments and conclusions are deferred to Sect.5.

2. Formation time of hadrons

The question of the formation time of hadrons has been discussed frequently in the past three decades in connection with the experimental data on multiparticle production in hadron-nucleus collisions. The concept is to some extent model dependent, since the formation of a hadron depends both on the mechanism of its production and on the space-time evolution of the wave function of the newly created hadron after its "decoupling" from its source. In the simplest pictures [8,10,11,12,13] the dependence on the mechanism of production is neglected and the newly formed hadron is assumed to need the time τ_0 in its own rest system to be formed, in the sense of being able to interact with other hadrons with the normal cross-section. In the system in which the hadron moves with energy E , its formation time is Lorentz dilated to

$$\tau = \frac{E}{m} \tau_0 \quad (4)$$

where m is the rest mass of the hadron.

In the beginning of seventies L.Stodolsky [14] has proposed a model of multiparticle production in hadronic reactions close to the idea of Landau- Pomeranchuk mechanism [15] for emission of soft photons. The model has been developed and used e.g. in Refs.[16]. The parametrization of the formation

time of hadrons following from this model has been used by Ranft [9]. In this formulation the hadron is formed in its rest system at the time τ_s :

$$\tau_s = \tau_0 \frac{m_s^2}{m_s^2 + p_{T_s}^2} \quad (5)$$

where m_s is the mass of the secondary hadron and p_{T_s} is its transverse momentum. The time τ_s is Lorentz dilated by Eq.(4) when passing to another system. In numerical analyses [9] it has been found that the a suitable value of the parameter τ_0 is about 5 fm/c which for a pion with $p_T \sim 0.3$ GeV/c gives $\tau_s \sim 1$ fm/c, for a pion with $p_T \sim 0.4$ GeV/c we have $\tau_s \sim 0.5$ fm/c and for a K with $p_T \sim 0.4$ GeV/c about 2 fm/c.

Bialas and Gyulassy [17] and others [18] have studied the question of the formation time of hadrons in the Lund model of multiparticle production [19]. The situation here is rather peculiar since one has to introduce two formation times [17], the former corresponding to the cutting of the colour string, the so-called "constituent-time" τ_c , the latter to the meeting time of two constituents of a meson, so called "yo-yo time" τ_y . Whereas τ_y is closer to the space-time development of the inside-outside cascade [11,20] corresponding roughly to Eqs.(4) and (5) the τ_c is closer to the outside-inside picture of the formation of hadrons.

There exist numerous models of multiparticle production, using different pictures of the space-time evolution of multiparticle production, like cluster model [21], the valon model [22], QCD branching models [23], formation zone models preferring rather short formation times of hadrons [24], the model by Levchenko and Nikolaev [25], see also [26]. These models have not been compared with the same set of data on multiparticle production in hadron-nucleus interactions and it is rather difficult to asses both applicability of the way in which the concept of the formation time has been introduced and the value of the formation time found. The multitude of approaches, in our opinion, does not diminish in any aspect the fundamental importance of the concept of the formation time for the understanding of multiparticle production in hadron-nucleus interactions and for the understanding of the behaviour of matter produced in heavy-ion collisions. In the present situation, when both qualitative and quantitative understanding of the behaviour

of matter produced in heavy ion collisions is of primary importance, it is useful to start with simplest picture of the formation of hadrons. This we shall attempt to do in the next section.

3. MODELS OF THE FORMATION OF HADRONS USED IN THE PRESENT ANALYSIS

To estimate the density of matter produced in heavy ion collisions we shall use three models

- a) Free-Streaming Model (FSM)
- b) Cluster Model (CM)
- c) Monte Carlo Model (MCM) [8]

In all of them the formation of hadrons proceeds according to the inside-outside cascade. In FSM and MCM the formation time of hadron τ_0 is a free parameter and the Lorentz dilation is given by Eq.(4). In the CM the cluster is assumed to decay after the Lorentz dilated cluster decay time τ_c into already fully formed secondary hadrons. FSM and CM are strongly simplified to permit an insight into the estimates, the MC is more sophisticated and consequently less transparent. We shall now describe the three models in more detail.

3.1 Free-Streaming Model

The collision of two ions is considered in the nucleon-nucleon c.m.s. Production of secondary hadrons in a nucleon-nucleon collision is shown in Fig.1. We have assumed there that first nucleon-nucleon collision start at $z = 0$, $t = 0$. Pions created in this collision are unable to interact (are not yet formed) before they get above the hyperbola $t^2 - z^2 = \tau_0^2$ where τ_0 is the pion formation time. A collision of two ions A and B presents a more complicated picture. Due to the Lorentz contraction the z-dimensions of the two nuclei are $\Delta z_A = 2R_A/\gamma$ and $\Delta z_B = 2R_B/\gamma$. The nuclei pass through each other during the penetration time t_p ($c = 1$)

$$\Delta t \equiv t_p = \frac{\Delta z_A + \Delta z_B}{2} = \frac{R_A + R_B}{\gamma} \quad (6)$$

and positions of individual nucleon-nucleon collisions are spread over the penetration length l_p

$$\Delta z \equiv l_p = \frac{\Delta Z_A + \Delta Z_B}{2} = \frac{R_A + R_B}{\gamma} \quad (7)$$

In the transverse plane positions of nucleon-nucleon collisions will be distributed in the disc with radius R_A , where $R_A \leq R_B$. In each point where nucleon - nucleon collision takes place secondary hadrons are produced and their formation proceeds according to the scheme in Fig.1. In estimating the energy density of the gas of secondary hadrons, we shall make a few rather drastic approximations :

- all secondary hadrons are pions
- all produced pions have the same transverse momentum $p_T = \langle |\vec{p}_T| \rangle = 0.4 \text{ GeV}/c$ and the same $E_T = \langle E_T \rangle$
- directions of transverse momenta of secondary pions in the transverse plane are random and uncorrelated with the position of the nucleon-nucleon collision in the transverse plane in which the pion has been produced

These approximations are certainly oversimplified but they lead to simple and transparent expressions which can be checked and corrected by less transparent Monte Carlo calculations to be described later on. We consider the rapidity region $-0.5 < y < 0.5$. The time when pions of this region are formed is taken as

$$t_F = \frac{t_p}{2} + ch(0.5) \frac{\langle E_T \rangle}{m_\pi} \tau_0 \quad (8a)$$

Here t_p is the penetration time given by Eq.(6), $(\langle E_T \rangle / m_\pi) \tau_0$ is the time in which pion with $\langle E_T \rangle$ is formed at $y = 0.5$ and $ch(0.5)$ is the Lorentz dilation factor corresponding to this rapidity. Nucleon-nucleon collisions are distributed within the time interval $0 < t < t_p$ but due to the geometry of the nuclei most of them happen around $t_p/2$. For this reason we have

put in the r.h.s. of Eq.(8a) $t_p/2$ instead of a possible t_p . During the time interval $(0, t_F)$ a pion at $y = 0$ makes in the transverse plane a distance l where

$$l = \frac{p_T}{E_T} t_F \sim 3\tau_0 + \frac{t_p}{2}$$

where we have approximated $\text{ch}(0.5)\langle E_T \rangle / m_\pi$ by 3 and taken $p_T/E_T \sim 1$. A pion created at a point \vec{r} in the transverse plane will be formed at the position $\vec{r} + \vec{l}$. Since \vec{r} and \vec{l} are uncorrelated we have

$$|\vec{r}| \rightarrow \left\langle (\vec{r} + \vec{l})^2 \right\rangle^{1/2} = (r^2 + l^2)^{1/2}$$

In the transverse plane pions, at there formation time, will be therefore distributed in a disc with the radius R'_A , where

$$R'_A = (R_A^2 + l^2)^{1/2} = \left[R_A^2 + \left(3\tau_0 + \frac{t_p}{2} \right)^2 \right]^{1/2} \quad (8b)$$

The longitudinal dimension of the considered rapidity region becomes

$$L = l_p + [v(y = 0.5) - v(y = -0.5)] \left[\text{ch}(0.5) \frac{\langle E_T \rangle}{m_\pi} \tau_0 + \frac{t_p}{2} \right] \quad (9)$$

here l_p is given by Eq.(7), $v(y = 0.5)$ is the velocity corresponding to the rapidity $y = 0.5$, $\text{ch}(0.5) \langle E_T \rangle$ is the energy of a pion with the transverse energy $\langle E_T \rangle$ and rapidity $y = 0.5$. The average energy density $\langle \varepsilon \rangle$ of the hadronic matter at the time of its formation is then given as

$$\langle \varepsilon \rangle = \frac{\Delta E / \Delta y}{\pi R_A^2 L} \quad (10)$$

The resulting values will be presented in the next section as a function of the formation time τ_0 . Apart of the average energy density $\langle \varepsilon \rangle$ it is also interesting to know the maximal energy density ε_m . We shall estimate it for two special cases: a central collision of a small nucleus with a large one, like O-Pb or S-Pb collision, and a central collision of the same nuclei, like Pb-Pb. We consider first collision of a small nucleus with radius R_A with a larger nucleus. The distance in the transverse plane from the common centre of the two nuclei will be denoted as s . Note that for the central collision centres of the colliding nuclei coincide in the transverse plane.

Assuming that the energy produced at certain s is proportional to the number of nucleon-nucleon collisions and taking into account that in the large nucleus $\sqrt{R_B^2 - s^2}$ is for $s < R_A < R_B$ roughly equal to R_B we obtain

$$\frac{\varepsilon_m}{\langle \varepsilon \rangle} = \frac{2R_A}{\int_0^{R_A} 2\sqrt{R_A^2 - s^2} d^2s / \pi R_A^2} = \frac{3}{2} \quad (11)$$

where the maximal energy ε_m corresponds to $s = 0$. The numerator in Eq.(11) is proportional to the number of nucleons in the smaller A-nucleus at $s = 0$ colliding with the nucleons in the larger B nucleus, whereas the denominator is proportional to the average number of nucleons in A colliding with nucleons in B. We shall use Eq.(11) for O-Pb and S-Pb collisions. For the case of $R_A = R_B = R$ we have instead of Eq.(11) in the same approach

$$\frac{\varepsilon_m}{\langle \varepsilon \rangle} = \frac{2R \cdot 2R}{\int_0^R d^2s \cdot 2\sqrt{R^2 - s^2} \cdot 2\sqrt{R^2 - s^2} / \pi R^2} = 2 \quad (12)$$

This will be used in discussing Pb-Pb collisions.

3.2 Cluster model of hadron production

Cluster models (CM) have frequently been used for the description of multiparticle production in hadronic collisions [21]. Cluster models are close to string fragmentation if the string is first fragmented into larger pieces and to branching models if branching into more massive objects precedes branching into lighter ones. Production of pions by decays of heavier resonances corresponds also to a cluster model. In cluster models we have two formation times : the formation time τ_c of a cluster and the formation time of its decay product τ_h . We shall not try to discuss here possible general combinations of these two formation times but concentrate instead on that combination which is most distant from the Free-Streaming Model. Heavier clusters, even with transverse momenta of the order of 0.5 - 1 GeV/c move slower in the transverse direction and hadronic matter coming from decays of clusters will be concentrated in a cylinder with a smaller radius than in the case of the Free-Streaming Model. In principle one could assume that pions from cluster decay still need some time for their formation. We shall assume here that the situation correspond to that shown in Fig.2, where we have assumed that $\tau_h \ll \tau_c$. This assumption means that the radial expansion of pions is suppressed. The decay time τ_c of a cluster may be simply related to the formation time τ_0 used in the sense of the Free-Streaming Model. Let S_0 be the c.m. system of the nucleon-nucleon collision, z-axis being the collision axis. During the collision a cluster with rapidity y is produced. Its rest system is denoted as $S_c(y)$. This system moves with respect to S_0 with velocity $V = \tanh(y)$ along the z-axis. The motion of the cluster in the transverse direction is neglected. The cluster decays in the time τ_c in its rest frame and in this decay a pion is produced with longitudinal momentum $p'_L = 0$, transverse momentum p'_T and transverse energy E'_T , all expressed in the cluster rest frame $S_c(y)$. After the decay of the cluster the pion is formed in time τ_π in its own rest frame. The formation time of the pion is Lorentz dilated with respect $S_c(y)$ and the pion is formed in $S_c(y)$ in time

$$t' = \tau_c + \frac{E'_T}{m_\pi} \tau_\pi \quad (13)$$

in the position with longitudinal (z') and transverse (x') coordinates

$$z' = 0$$

$$x' = v'_T \frac{E'_T}{m_\pi} \tau_\pi = \frac{p'_T}{m_\pi} \tau_\pi \quad (14)$$

Lorentz transformation between S_0 and $S_c(y)$

$$z = z' ch(y) + t' sh(y)$$

$$t = t' ch(y) + z' sh(y) \quad (15)$$

enables us to express coordinates of the space-time point in which the pion has been formed in the S_0 frame as

$$t = ch(y) \left(\tau_c + \frac{E'_T}{m_\pi} \tau_\pi \right) \quad (16a)$$

$$z = sh(y) \left(\tau_c + \frac{E'_T}{m_\pi} \tau_\pi \right) \quad (16b)$$

$$x = \frac{p'_T}{m_\pi} \tau_\pi \quad (16c)$$

In the FSM pion with rapidity y and transverse energy E'_T (E'_T expressed in the system connected with this value of rapidity) is formed after the time

$t = (E_T/m_\pi)\tau_0 = (\text{ch}(y) E'_T/m_\pi)\tau_0$. Requiring that this time is the same in the cluster model we find from Eq.(16a) the condition

$$\tau_c + \frac{E'_T}{m_\pi}\tau_\pi = \frac{E'_T}{m_\pi}\tau_0 \quad (17)$$

Assuming $\tau_\pi = 0$ as discussed above we obtain $\tau_c = (E'_T/m_\pi)\tau_0$ which guarantees that pion is formed in the same time in S_0 in both FSM and CM. The rapidity region $-0.5 < y < 0.5$ in the collision of ions A and B is then formed in time

$$t = c\hbar(0.5) \frac{\langle E'_T \rangle}{m_\pi} \tau_0 \quad (18)$$

its length at this time is given by Eq.(9) and its radius is equal to the radius R_A of the smaller of the two nuclei. The average energy density at the time of formation of hadrons is given by Eq.(10) with $R'_A = R_A$.

3.3 Monte Carlo Model

The Monte Carlo Model we are using is described in detail in Ref.[8]. There is only one important modification for the nucleus-nucleus case, namely that the cascading of produced secondaries in the projectile nucleus is taken into account in a symmetric way to the cascading inside the target. Therefore the resulting algorithm is completely symmetric in respect to the exchange of projectile and target nuclei.

At the present stage MC has no interactions between pions in the final state. This should not significantly influence the maximal energy densities calculated but it underestimates time intervals over which these densities last. Here we are interested in maximal densities reached, when studying properties of hadron gas depending on hadron-hadron interactions, pion-pion interactions will have to be introduced. For present calculation we assume, that recoiled, secondary nucleons are

able to interact immediately after an elementary nucleon-nucleon collision. The formation time is required only for mesons. Mesons are allowed to

interact after passing formation length l_f expressed as ($c = 1$) :

$$l_f = \frac{p}{m_\pi} \cdot \frac{1}{\mu} \equiv \frac{p}{m_\pi} \tau_f \quad (19)$$

where p is the momentum of the pion in the rest frame of corresponding nucleus. Formation time is assumed to have an exponential probability distribution :

$$P(\tau_f) d\tau_f = \frac{1}{\tau_0} \exp(-\tau_f/\tau_0) d\tau_f \quad (20)$$

The mean formation time τ_0 corresponds to that in Eq.(4) and one can therefore expect that results of the MC model would be close to those of the FSM. Some differences are to be expected, because the MC model is free of rather strong and simplifying assumptions as the same formation time and the same transverse energy for all pions produced. In this sense Monte Carlo Model is more realistic.

Since the MC algorithm keeps the history of whole cascade pattern in the memory, after each event the density of pions (or pion energy) can be easily calculated at any place and time.

4. Estimates of energy density of hadronic matter produced in heavy ion collisions

We shall start with presenting the estimates of energy density of hadronic matter at the time of hadron formation in the Free-Streaming-Model. Average densities (ε) are calculated as function of the formation time according to Eq.(10) with L and R'_A given by Eqs.(8b) and (9). The maximal density ε_m is expressed in terms of $\langle \varepsilon \rangle$ by Eq.(11) for central O-Pb and S-Pb collisions and by Eq.(12) for the case of central Pb-Pb collisions.

In Fig.3 we present $\langle \varepsilon \rangle$ and ε_m for O-Pb collisions. In the figure we have also indicated energy densities corresponding to the phase transition temperatures of $T_c = 200$ MeV and $T_c = 150$ MeV . The former T_c has been accepted as a reasonable estimate over a couple of years, the latter seems to be preferred by recent computations [27]. It is to be hoped that recent lattice

calculations with staggered quarks [28] and with dynamic Wilson fermions [29] will narrow the region of possible values of T_c . Estimates of energy density in the same model for central S-Pb collisions are shown in Fig.4. The case of central Pb-Pb collisions is a bit more complicated due to the fact that the experimental information on $\Delta E/\Delta y$ is missing and extrapolations are model dependent. As given in Eq.(3) we shall present the estimates for $\Delta E/\Delta y = 600$ GeV and $\Delta E/\Delta y = 870$ GeV. The two are given in Figs.5a and 5b.

In calculations performed within the Cluster Model, the energy density is still given by Eq.(10) where L is given again by Eq.(9) but in contradistinction to the FSM, the radius R_A in Eq.(10) is now equal to the radius of the A-nucleus since no transverse expansion is assumed to be present. Estimates of energy density within the Cluster Model of central O-Pb and S-Pb collisions are given in Figs.6 and 7. For central Pb-Pb collisions we consider again two cases of $\Delta E/\Delta y$ for $y_{c.m.} \sim 0$, namely $\Delta E/\Delta y = 600$ GeV and $\Delta E/\Delta y = 870$ GeV. Energy densities for these two assumption are presented in Figs.8a and 8b.

Monte Carlo Model [8] is close in basic assumptions to the FSM. We have therefore presented estimates obtained within this model as dots in Figs.3,4 and 5a. The energy density within the MC Model has been obtained by tracing particles near the central region, so that these results correspond rather to ε_m than to $\langle \varepsilon \rangle$. The difference between rough estimates of ε_m within the FSM and results obtained within MC Model are to be ascribed to more elaborated dynamics of the MCM and to a somewhat different notion of the formation time.

In Figs.9a,b we have shown space-time development of pion energy density in Pb-Pb central collision calculated by MC for two input values of τ_0 . Horizontal axis corresponds to collision axis and density is calculated in a narrow cylinder (1.5 fm radius) around the axis. Time $t = 0$ corresponds to the moment, when both nuclei are completely overlapped.

5. Analysis of results, comments and conclusions

Models used above are admittedly rather strong simplifications of what is going on in heavy ions collisions. In all of them we have assumed that only pions are produced in nucleon-nucleon collisions although there are indications [30] that resonances are copiously produced and this is also included in currently used models [19] of multiparticle production.

Out of the models discussed above we are inclined to believe that the cluster model might be closer to reality than that of the Free Streaming. The most natural physical picture behind the Free-Streaming-Model would consist in creation of small pions which are gradually increasing in size and reaching their full cross-sections. Such a mechanism would probably require a presence of hard processes in the creation of "young" pions which is not very probable. This picture is natural for the charmonium production [12,13], but may not be quite appropriate for production of pions.

The estimates obtained above depend crucially on two not quite well known parameters : the critical temperature T_c of the phase transition and the formation time of secondary hadrons τ_0 . Depending on the assumed values of these parameters one can sketch three types of scenarios :

- i) optimistic
- ii) pessimistic
- iii) intermediate

In the optimistic one it is assumed that T_c is rather low, not much higher than 150 MeV and formation times are rather short, say somewhere in the region $0.5 < \tau_0 < 1$ fm/c. As seen from Figs.3-5b QGP could be formed in this case in the Free-Streaming-Model near the collision axis in central S-Pb and over larger volumes in Pb-Pb collisions. (see in particular Fig.5a as a more realistic situation of Pb-Pb collisions). In the case of the cluster-type mechanism of hadron production one could expect QGP being produced even in O-Pb and S-Pb collisions, not to speak about Pb-Pb ones.

The problem of this scenario is in the fact, that the formation time of hadrons is probably close to 1 fm/c [9,10] or higher than 1 fm/c [7,8]. To illustrate the situation we present in Table I a comparison of one result of our Monte Carlo calculation with data of Ref.[6]. In making this type of comparisons one should preferably compare a model with all the data of one experiment, to be sure that one good comparison is not made at the price of having disagreement in others. Such a work is in preparation [31]. With this caveat we present in Table I the comparison of Monte Carlo Model with a quantity which is rather sensitive to the value of the formation time, namely the average lab. rapidity of secondary particles. As indicated by Table I, values of the formation time at the level of $\tau_0 \sim 2 - 3$ fm/c or higher might be required by the data. A stronger statement can be hardly made since Monte Carlo Model calculations has been performed assuming that all secondary particles are pions and that in a nucleon-nucleon collision both nucleons are able

to interact with a full cross-section immediately after the interaction. The latter point is probably close to reality but it should be understood better along the lines of Ref.[32] and literature quoted there.

The optimistic scenario could receive support from two sources. First, if re-coiled nucleons in hadron-nucleus collisions need some time before being able to interact with full cross-section, a part of their role in producing intranuclear cascade should be played by secondary pions, what requires lower values of the formation time of secondary hadrons. Second, the nuclear stopping is not strong enough to bring a suitable fraction of nucleons into the central rapidity region in O-Pb and S-Pb collisions, whereas in Pb-Pb collisions this fraction becomes important. This contribution has not been included into our estimates of energy density in Pb-Pb collisions. The effect will be studied in more detail in the near future [31].

The inclusion of pion-pion interactions into the MC Model will show down the transverse expansion and keep larger energy densities for longer time intervals. This we shall study in detail in the near future.

In the pessimistic scenario one assumes that the critical temperature T_c of the phase transition is around 200 MeV and formation time of hadrons is of about 1 fm/c or higher. As seen from Fig.8a in this case QGP will not be produced in Pb-Pb collisions at 200 GeV/nucleon. The hadronic system produced in these collisions would be still extremely interesting since at $\tau_0 \sim 1$ fm/c the energy density would be around 1.2 GeV/fm³ which corresponds to 2 - 3 pions/fm³.

In the intermediate scenario one assumes that T_c is rather low, not much higher than 150 MeV and formation time τ_0 is between 2 and 3 fm/c. In this situation, as seen from Fig.8a which we consider more realistic than Fig.8b, the maximal energy density ε_m in Pb-Pb collisions at 200 GeV/nucleon would reach for $\tau_0 < 3$ fm/c values required for the formation of the mixed phase or QGP. For $\tau_0 > 1$ fm/c the average value of energy density $\langle \varepsilon \rangle$ stays below values necessary for the formation of mixed phase or QGP and that means that for 1 fm/c $< \tau_0 < 3$ fm/c one could expect QGP production only in a part of the volume with the radius of Pb. Note that because of rough approximations made, the exact values of $\tau_0 = 1$ fm/c or $\tau_0 = 3$ fm/c can be taken only as rough estimates.

To make a choice between the three scenarios is a matter of personal choice, depending mostly on weights assigned to different arguments and estimates. Our opinion is based on giving relatively higher weight to estimates

in [7,8] and in Table I leading to rather large values of the formation time. We expect therefore that a most probable scenario is the intermediate one. That means that QGP has not been produced in O-Pb and S-Pb interactions at the CERN SPS and that it will be produced only in a narrow region around the collision axis in Pb-Pb collisions.

To make better based predictions one would need to have more accurate information on

- critical temperature T_c
- formation time of hadrons τ_0 , and to have
- better understanding of the dynamics of multiparticle production, including a realistic model for the space-time evolution of multiparticle production, taking into account also production of particles heavier than pion,
- reasonable accurate estimate of the expected $\Delta E/\Delta y$ in Pb-Pb collisions.

An accurate estimate of the hadron formation time is difficult to obtain from hadron-nucleus collisions in particular if the formation time τ_0 is larger than 1 fm/c. In such a situation interactions of secondary pions with nucleons present in the nucleus give only a small correction to the multiparticle production due to interactions of incident hadron and of recoiled nucleons. Studies of hadronic systems produced in ion-ion collisions can provide much more accurate information on the formation time since outside of the target and beam fragmentation regions the interactions between secondary particles determine the behaviour of the hadronic system. All of the effects studied so far as signatures of QGP could give valuable information on the formation time of hadronic gas produced in ion-ion collisions. We shall now comment briefly on some of QGP signatures from this point of view.

J/ Ψ suppression suggested by Matsui and Satz [33] as a signature of QGP has later on been questioned [34] as an unambiguous signature of QGP and as an alternative a suppression by a dense gas of hadrons has been suggested. In a recent study Gavin, Satz, Thews and Vogt [35] have estimated the density of hadrons J/ Ψ should meet in order to be suppressed as observed experimentally [37]. They have found that the density of secondaries is at least 0.8 fm^{-3} , i.e., five times standard nuclear density, what agrees with earlier estimates [36] of 1 fm^{-3} . Taking these secondaries as pions each with 0.5 GeV of transverse energy we obtain as energy estimate 0.4 GeV fm^{-3} . If heavier secondary particles are present the energy density would be correspondingly higher. The energy density of 0.4 GeV fm^{-3} is plotted

for comparison in Figs.6 and 7. The analyses of Refs.[35,36] have taken into account the formation time of J/Ψ . When the formation time of hadrons is included into analysis the experimentally observed J/Ψ suppression might put constraints in forms of upper bounds on the formation time of hadrons. When the present paper has been in the completion phase an important paper by Kharzeev and Satz [38] has appeared. If the conclusions of Kharzeev and Satz become confirmed J/Ψ is not suppressed by the hadron gas, but only by interactions with nucleons in colliding nuclei and/or - by QGP. In such a situation J/Ψ suppression, being insensitive to the hadronic gas would give no information about its density.

The data on HBT interference of identical particles [39] bring information on the space-time extension of region with last interactions of secondary particles. The density of hadron gas at the time of its formation determines the mean free path of hadrons formed at various space-time points and thus also the dimensions of the space-time region of last interactions of individual hadrons.

Recent data [40] on dilepton production in O - and S - collisions with heavy nuclei have shown an excess of dileptons in the region around and below the rho meson mass and in the region around two $2 \text{ GeV}/c^2$. In the hadron gas model the region around and below the rho receives contributions from pion-pion annihilation and bremsstrahlung of internal legs [41] depending on the density and average energy of pions in the hadronic system, bringing thus also an information about the formation time of hadrons. The region $2 \text{ GeV}/c^2$ might be either due to the detailed shape of the pion formfactor or to the annihilations of higher mass particles in the hadronic system, in either case bringing an information about the density and the composition of the gas of secondary hadrons.

The increased production of strange mesons and baryons [42] can hardly be obtained in models of hadron gas taking into account the formation time of hadrons if hadrons with masses larger than pions are not present at the time of hadron gas formation. Some K-mesons could be produced in pion-pion collisions, but in order to obtain strange baryons some baryons should be present at the time when hadron interactions are starting.

To conclude

The hadron gas present at the time of hadron formation is a suitable definition of initial conditions for tracing the space-time evolution of the hadronic system formed in heavy-ion collisions. If this is what happens in hadronic

collision the analyses of data from this point of view will lead to the determination of the basic parameter - the formation time of hadrons.

On the basis of the available information on the formation time of hadrons we consider as most plausible the intermediate scenario according to which the QGP has not been produced in O-Pb and S-Pb collisions at 200 GeV/nucleon. In Pb-Pb collisions the QGP will perhaps be produced in a limited region close to the axis of collision in central collisions corresponding to the highest values of $\Delta E/\Delta y$. More definite statements could be made only on the basis of more accurate information on the critical temperature T_c and on the formation time of secondary hadrons τ_0 .

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Figure captions

Fig.1

Production of secondary particles (mostly pions) in a nucleon-nucleon collision. a , b - incoming nucleons, 1 , 2 - outgoing nucleons, π - outgoing pions, dotted parts of their trajectories correspond to pions not yet formed. The hyperbola H is given as $\tau_0^2 = t_2 - z_2$

Fig.2

Production of hadrons in a nucleon-nucleon collision via clusters. a , b - incoming, 1 , 2 - outgoing nucleons. Dots denote cluster decay. Hadrons produced in decays of clusters are assumed to be formed, τ_c - time of the decay of a cluster in its rest mass frame.

Fig.3

Estimates of energy density in O-Pb collisions in the Free-Streaming Model. Dashed line - ε_m , solid line - $\langle\varepsilon\rangle$. Dash-dot constants correspond to energy densities of QGP with phase transition temperatures of $T_c = 200$ MeV and $T_c = 150$ MeV. Black-circles: results of the Monte Carlo Model.

Fig.4

Estimates of energy density in S-Pb collisions in the Free-Streaming Model. Notation as in Fig.3.

Fig.5a

Estimates of energy density for Pb-Pb central collisions in the FSM under the assumption of $\Delta E/\Delta y = 600$ GeV. Notation the same as in Fig.3

Fig.5b

The same as Fig.5a under the assumption of $\Delta E/\Delta y = 870$ GeV.

Fig.6

Estimates of energy density in central O-Pb for the Cluster Model. Dashed line - ε_m , solid line - $\langle\varepsilon_m\rangle$, dash-dot lines - energy densities corresponding to critical temperatures $T_c = 200$ MeV and $T_c = 150$ MeV. The dotted line corresponds to the estimate of energy density by Gavin, Satz, Thews and Vogt, discussed below in Sect.5.

Fig.7

Estimates of energy densities for central S-Pb collisions in the Cluster Model, notation is the same as in Fig.6.

Fig.8a

Estimates of energy densities for central Pb-Pb collisions in the Cluster Model under the assumption of $\Delta E/\Delta y = 600$ GeV. Notation as in Fig.6.

Fig.8b

Estimates of energy densities for central Pb-Pb collisions in the Cluster Model under the assumption of $\Delta E/\Delta y = 870$ GeV. Notation as in Fig.6.

Fig.9a

Estimates of energy density evolution on the axis of Pb-Pb central collision in MCM for $\tau_0 = 0.5$ fm.

Fig.9b

The same as Fig.9a for $\tau_0 = 1.5$ fm.

References

- [1] J.D.Bjorken, Phys.Rev. D27 (1982) 140
- [2] W.Heck et al., Zeit.f.Phys. C38 (1988) 19
- [3] A.Bussiere et al., Zeit.f.Physik C38 (1988) 117
- [4] F.Corriveau et al., Zeit.f.Physik C38 (1988) 15
- [5] P.V.Ruuskanen, Zeit.f.Physik C38 (1988) 219
- [6] C.DeMarzo et al., Phys.Rev. D26 (1982) 1019, Phys.Rev. D29 (1984) 363 and Phys.Rev. D29 (1984) 2476
- [7] D.H.Brick et al., Phys.Rev. D39 (1989) 2484
- [8] P.Závada, Zeit.f.Phys. C32 (1986) 135, Phys.Rev. C40 (1989) 285 and Phys.Rev. C42 (1990) 1104
- [9] J.Ranft, Phys.Rev. D37 (1988) 1842, Zeit.f.Phys. C43 (1989) 439 and H.J.Moehring, J.Ranft, Zeit.f.Physik C52 (1991) 643
- [10] Z.Pengfei and C.Weiquin, Nucl.Phys. A552 (1993) 620
- [11] K.Kajantie, R.Raitio and P.V.Ruuskanen, Nucl.Phys. B222 (1983) 152
- [12] I.Horvath et al., Phys.Lett. B214 (1988) 237
V. Černý et al., Zeit.f.Physik C46 (1990) 481
- [13] J.Huefner and M.Simbel, Phys.Lett. B258 (1991) 465
J.Dolejší and J.Huefner, Zeit.f.Physik C54 (1992) 489
- [14] L.Stodolsky, Phys.Rev.Letters 28 (1972) 60,
L.Stodolsky, in Multiparticle Phenomena and Inclusive Reactions, Proceedings of the VII Rencontre de Moriond, Meribel - les Allues, France, 1972 edited by J.Tran Than Van, CNRS, Paris, 1972, Vol.II,

- L.Stodolsky, Proc. 5th International Colloq. on Multiparticle Reactions, Oxford, 1975, p.577
- [15] L.D.Landau, I.Pomeranchuk, Dokl.Akad.Nauk SSSR, 92 (1953) 535 and 735,
 A.B.Migdal, Dokl.Akad.Nauk SSSR 96 (1954) 49, Phys.Rev. 103 (1956) 1811,
 E.L.Feinberg, I.Ya.Pomeranchuk, Nuovo Cim.Suppl. 3, ser.10 (1956) 652,
 E.L.Feinberg, Sov.Phys.-JETP 23 (1966) 132,
 N.N.Nikolaev, Usp.Fiz.Nauk 134 (1981) 369, English translation, Soviet Physics, Uspekhi 24 (1981) 531
- [16] A.Bialas, I.Derado, L.Stodolsky, Phys.Letters B156 (1985) 421,
 A.Bialas, E.H.de Groot, T.Ruijgrok, Phys.Rev. D36 (1987) 752
- [17] A.Bialas, M.Gyulassy, Nucl.Phys. B291 (1987) 793
- [18] T.Chmaj, Acta Phys.Pol. B18 (1987) 1131,
 A.Bialas, J.Czyzewski, Z.Phys. C47 (1990) 133 and Phys.Letters B222 (1989) 133,
 J.Czyzewski, Acta Phys.Pol. B21 (1990) 41 and Phys.Rev. C43 (1991) 2426
- [19] B.Andersson, G.Gustafson, G.Engelman and T.Sjostrand, Phys.Reports 97 (1983) 31
- [20] J.D.Bjorken, in Current-Induced Interactions, Proc.Int.Summer Institute of Theoretical Physics, Hamburg, 1975, Lecture Notes in Physics, Springer Berlin, 1976, Vol.56, p.73,
 J.D.Bjorken, in Proc. 1979 SLAC Summer Institute on Particle Physics, ed. A.Mosher, SLAC Report 224, Stanford 1980,
 J.D.Bjorken, Proc. of the SLAC Summer Institute on Particle Physics, ed.M.Zipt, SLAC - 167, 1973

- [21] A.Giovannini and L.Van Hove, *Zeit.f.Physik* C30 (1986) 391
A.Giovannini and L.Van Hove, *Acta Phys.Polonica* B19 (1988) 495
- [22] R.C.Hwa, *Phys.Rev.* D22 (1980) 759 and 1593
- [23] A.Bassetto et al., *Phys.Reports* 100 (1983) 201,
L.Van Hove, A.Giovannini, *Acta Phys.Polonica* B19 (1988) 917 and *Acta Phys.Polonica* B19 (1988) 931
- [24] B.Z.Kopeliovich, F.Niedermayer, *Yad.Fizika* 42 (1985) 797,
B.Z.Kopeliovich, L.I.Lapidus, *Proc.6th Balaton Conf. on Nuclear Physics, Balatonfured, 1983*, p.73,
A.S.Bisadze et al., *Nucl.Phys.* B279 (1986) 770
- [25] B.B.Levchenko and N.N.Nikolaev, *Yad.Fiz.* 37 (1983) 1016 and *Yad.Fiz.* 42 (1984) 1255
- [26] M.Shabelsky, *Yad.Fiz.* 50 (1989) 239
- [27] F.Karsch, K.Redlich, L.Turko, *Zeit.f.Phys.* C60 (1993),
S.Gottlieb et al., *Phys.Rev.* D35 (1987) 3972
- [28] W.Lee, *Phys.Rev.* D49 (1994) 3563,
M.Fukugita et al., *Phys.Rev.* D47 (1993) 4739
- [29] K.M.Bitár et al., *Phys.Rev.* D49 (1994) 3546
- [30] G.Jancso et al., *Nucl.Phys.* B124 (1977)1
- [31] P.Závada, in preparation
- [32] B.K.Jennings, G.A.Miller, *Phys.Rev.* D44 (1991) 692
- [33] T.Matsui, H.Satz, *Phys.Lett.* B178 (1986) 416
- [34] J.Ftáčnik et al., *Phys.Lett.* B207 (1988) 194,

- J.Ftáčnik et al., *Zeit.f.Phys.* C42 (1988) 139,
 S.Gavin et al., *Phys.Lett.* B207 (1988) 257,
 R.Vogt et al., *Phys.Lett.* B207 (1988) 263 ,
 C.Gerschel and J.Huefner, *Phys.Lett.* B207 (1988) 254 and *Zeit.f.Phys.*
 C56 (1992) 171
- [35] S.Gavin, H.Satz, R.L.Thews and R.Vogt, *Zeit.f.Phys.* C61 (1994) 351
- [36] S.Gavin, R.Vogt, *Nucl.Phys.* B345 (1990) 104 , *Nucl.Phys.* A525
 (1991) 693c,
 R.Vogt, S.J.Brodsky, P.Hoyer, *Nucl.Phys.* B360 (1991) 97
- [37] C.Baglin et al., NA-38 Collab., *Phys.Lett.* B220 (1989) 471, *Phys.Lett.*
 B255 (1991) 459, *Phys.Lett.* B262 (1991) 362, *Phys.Lett.* B270 (1991)
 105 ,
 O.Drapier, NA-38 Collab., *Nucl.Phys.* A544 (1992) 209 ,
 C.Laurenco, NA-38 Collab., *Nucl.Phys.*
- [38] D.Kharzeev and H.Satz: Quarkonium interactions in hadronic matter,
 preprint CERN-TH.7274/94 and BI-TP 94/24, May 1994
- [39] T.Humanic, NA-44 Collab., *Nucl.Phys.* A566 (1994) 115c,
 G.Roland, NA-35 Collab., *Nucl.Phys.* A566 (1994) 527c
- [40] C.Lourenco, NA-38 Collab., *Nucl.Phys.* A566 (1994) 77c,
 A.Mazzoni, Helios/3 Collab., *Nucl.Phys.* A566 (1994) 95c,
 G.London, Dimuon and vector meson production in pW and SW inter-
 actions at 200 GeV/c, Helios/3 results, Helios Note and Proc. of the
 Workshop on Dilepton Production in Relativistic Heavy Ion Collisions,
 Darmstadt, March 2-4, 1994, Ed. H.Bokemeyer, GSI Darmstadt
- [41] J.Cleymans, V.V.Goloviznin, K.Redlich, *Phys.Rev.* D47 (1993) 173 ,
Phys.Rev. D47 (1993) 989 , *Zeit.f.Phys.* C59 (1993) 495

- [42] U.Heinz, Nuclear Physics A566 (1994) 205c ,
D.P.Morrison, E802 Collab., Nucl.Phys. A566 (1994) 457c ,
M.Gazdzicki, NA-35 Collab., Nucl.Phys. A566 (1994) 503c

Table I

Comparison of Monte Carlo model calculations of average lab.rapidity with data of De Marzo et al.[6] on pAr and pXe interactions

$\langle y_L \rangle, pp$	$\langle y_L \rangle, pAr$	$\langle y_L \rangle, pXe$	
3.047	2.607	2.415	Exper.Ref.[6]
3.05	2.64	2.51	MC $\tau_0 = \infty$
3.05	2.57	2.34	MC $\tau_0 = 3.5$
3.05	2.49	2.20	MC $\tau_0 = 1.5$
3.05	2.31	1.88	MC $\tau_0 = 0.5$

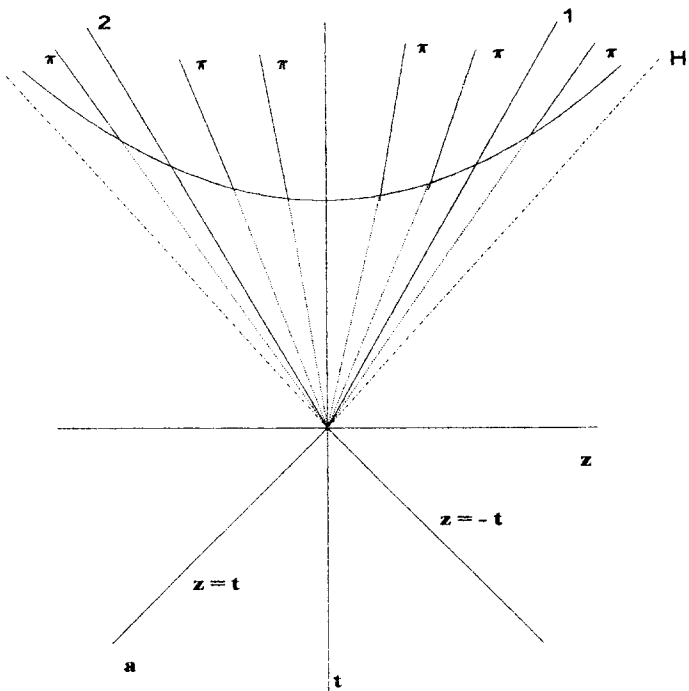


Fig.1

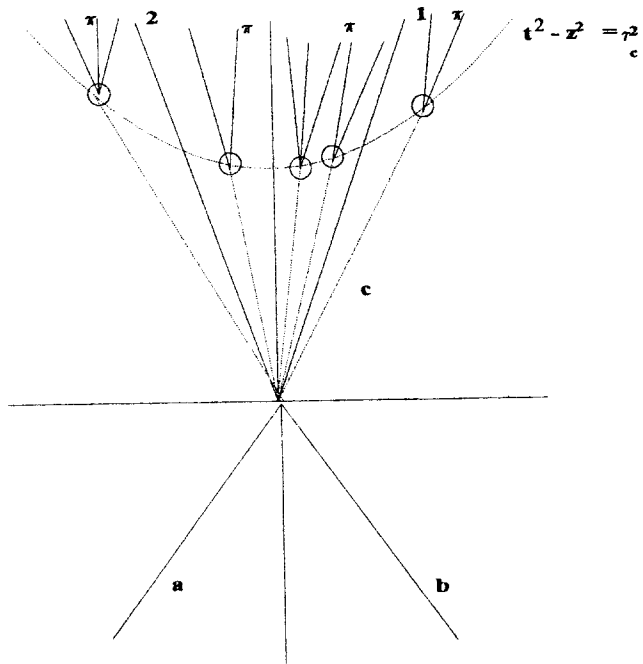


Fig.2

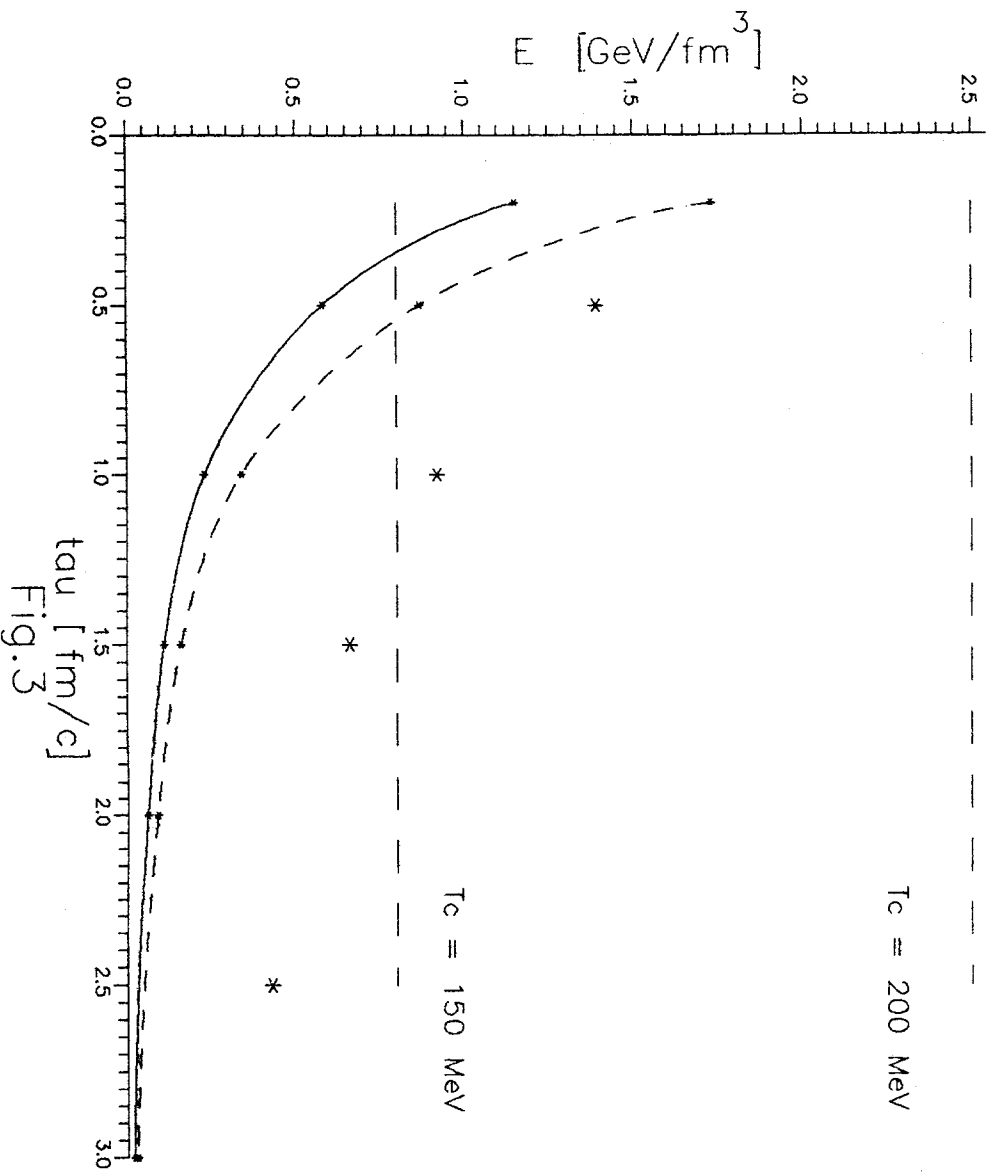


Fig. 3

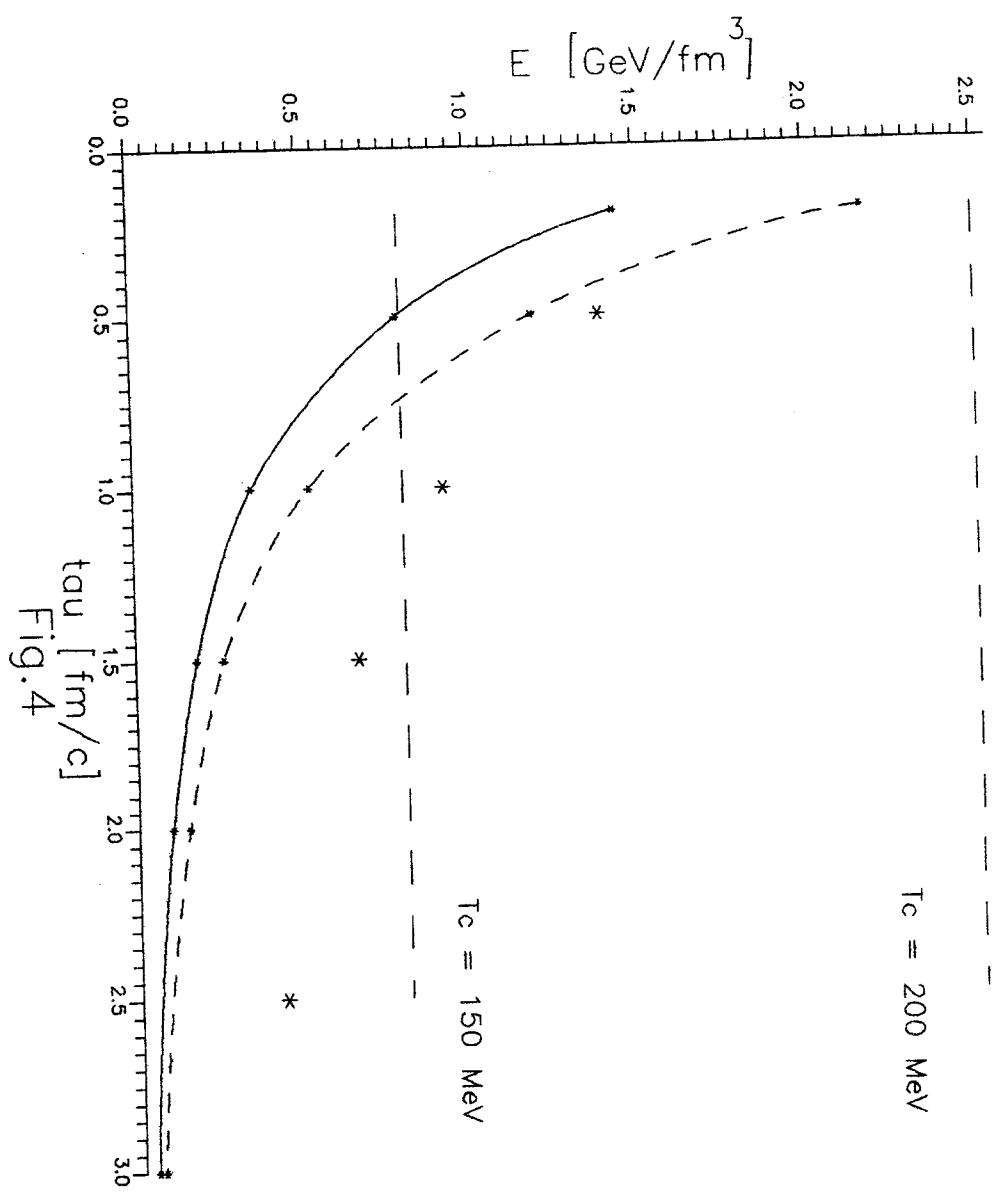


Fig.4

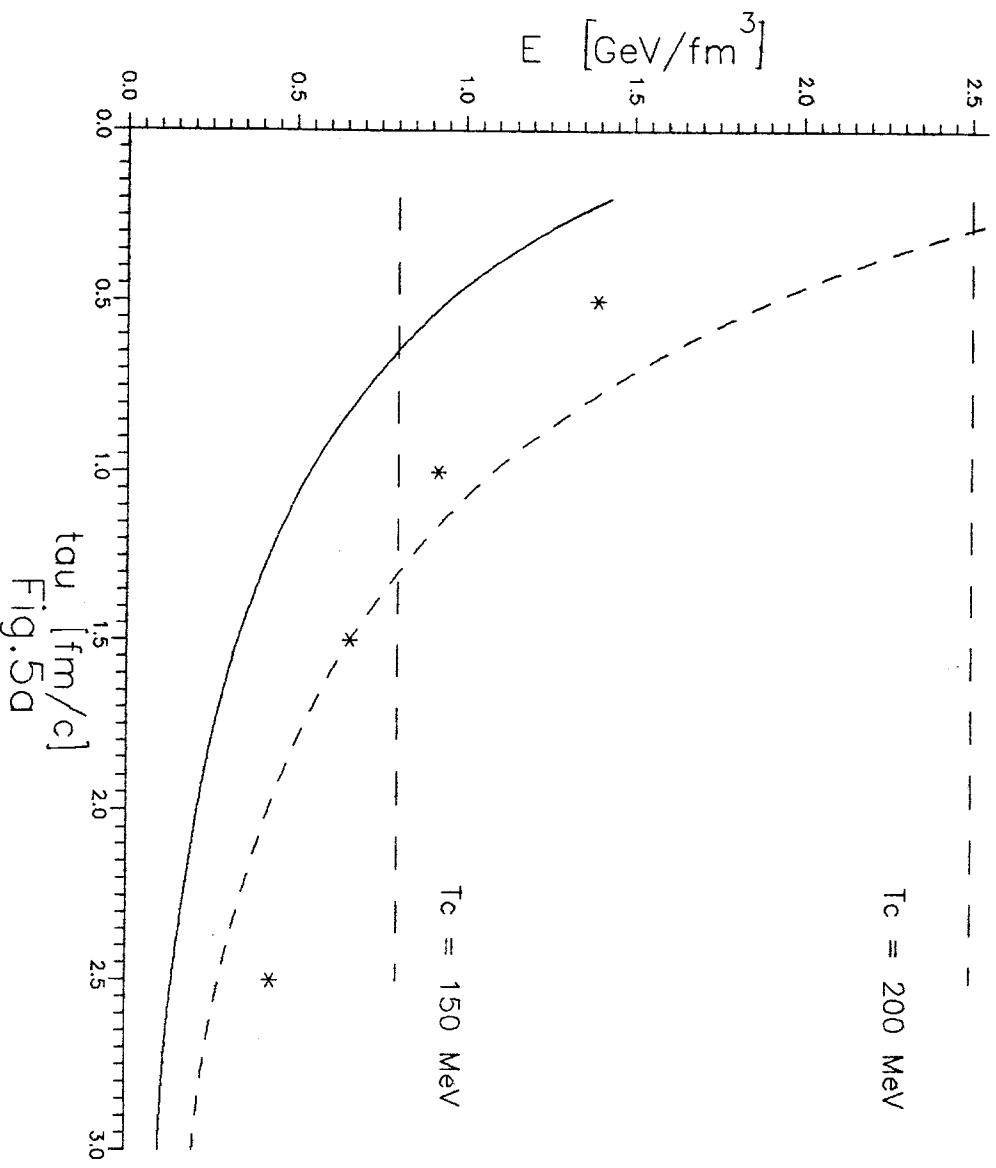
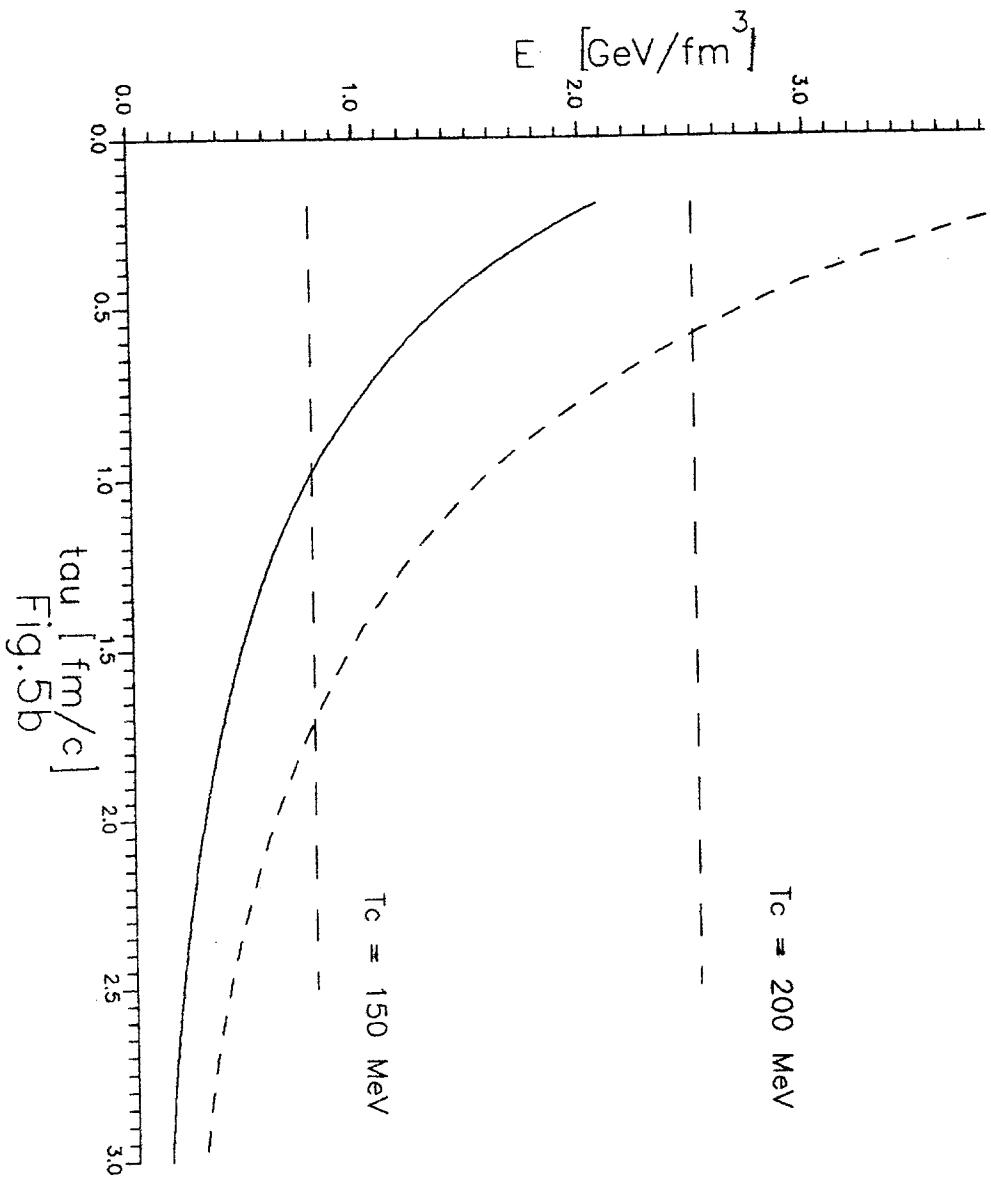


Fig.5d



tau [fm/c]
Fig. 5b

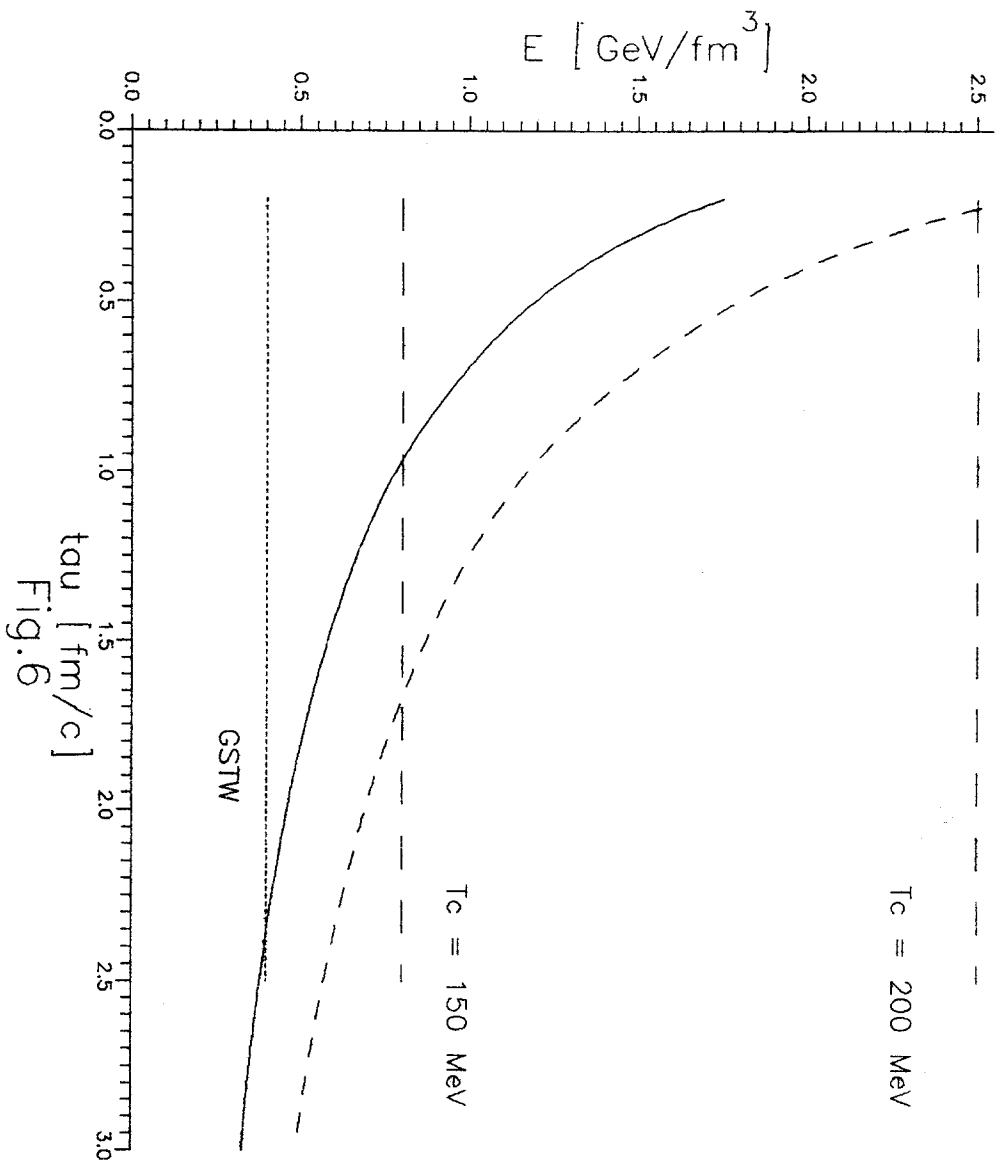


Fig. 6

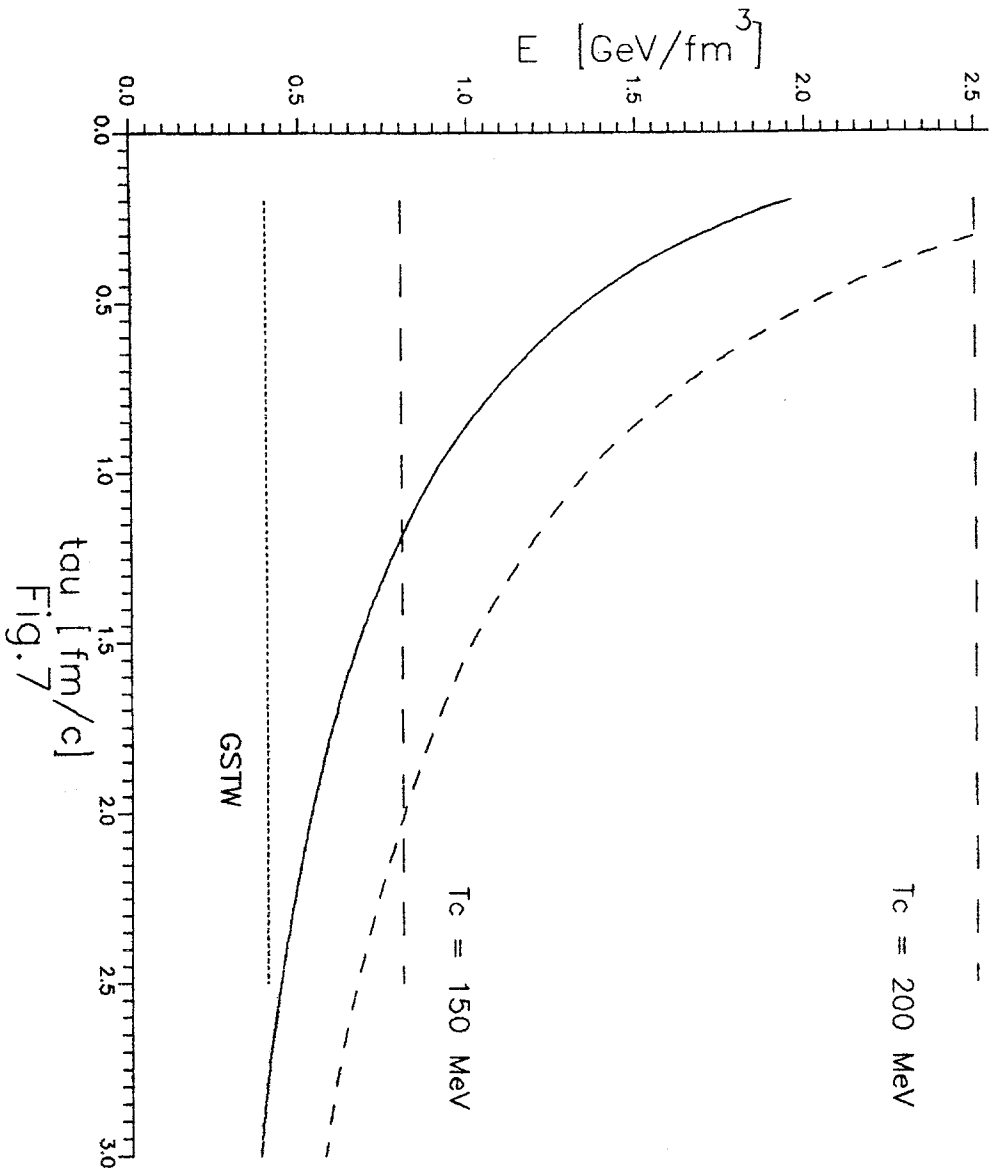
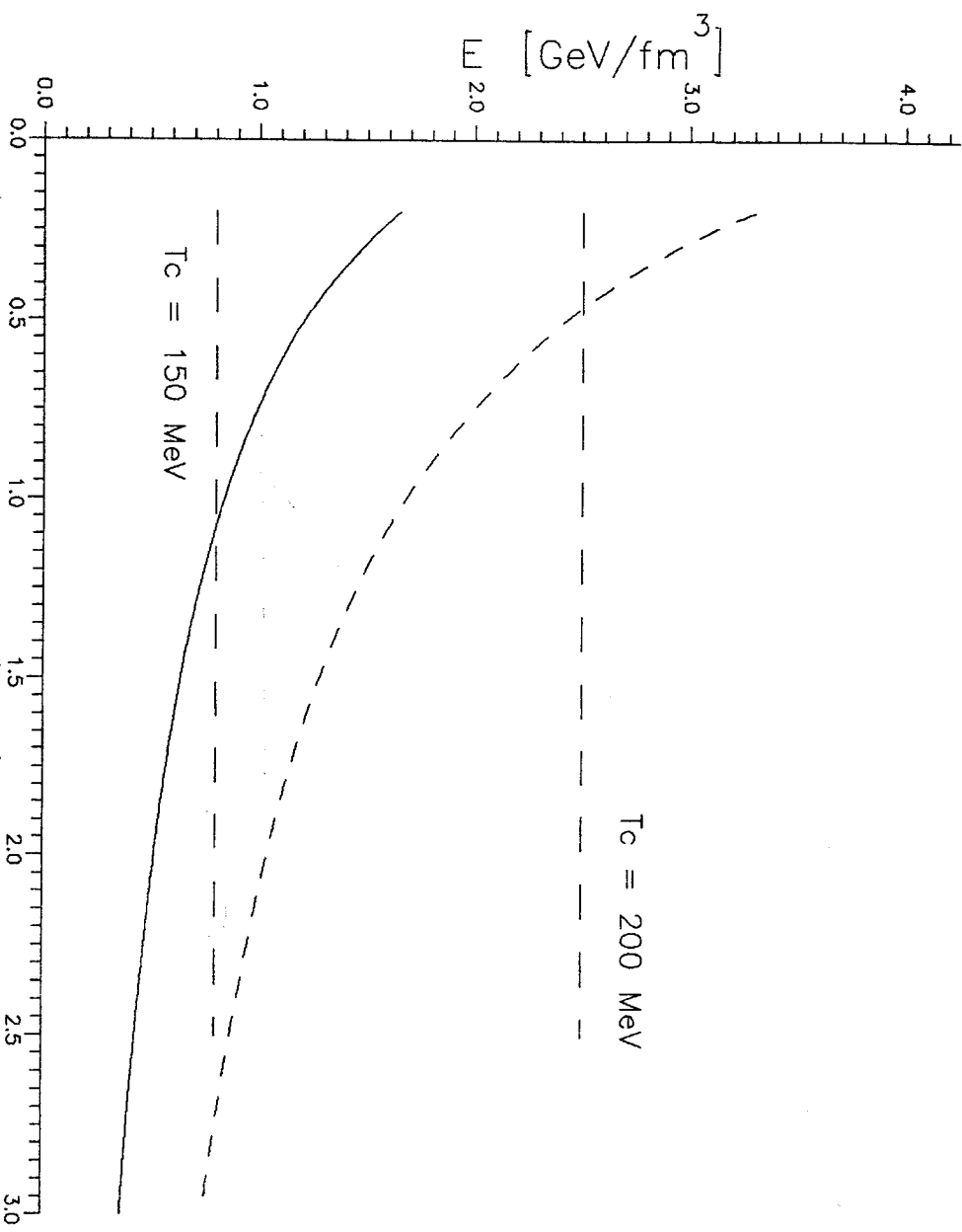
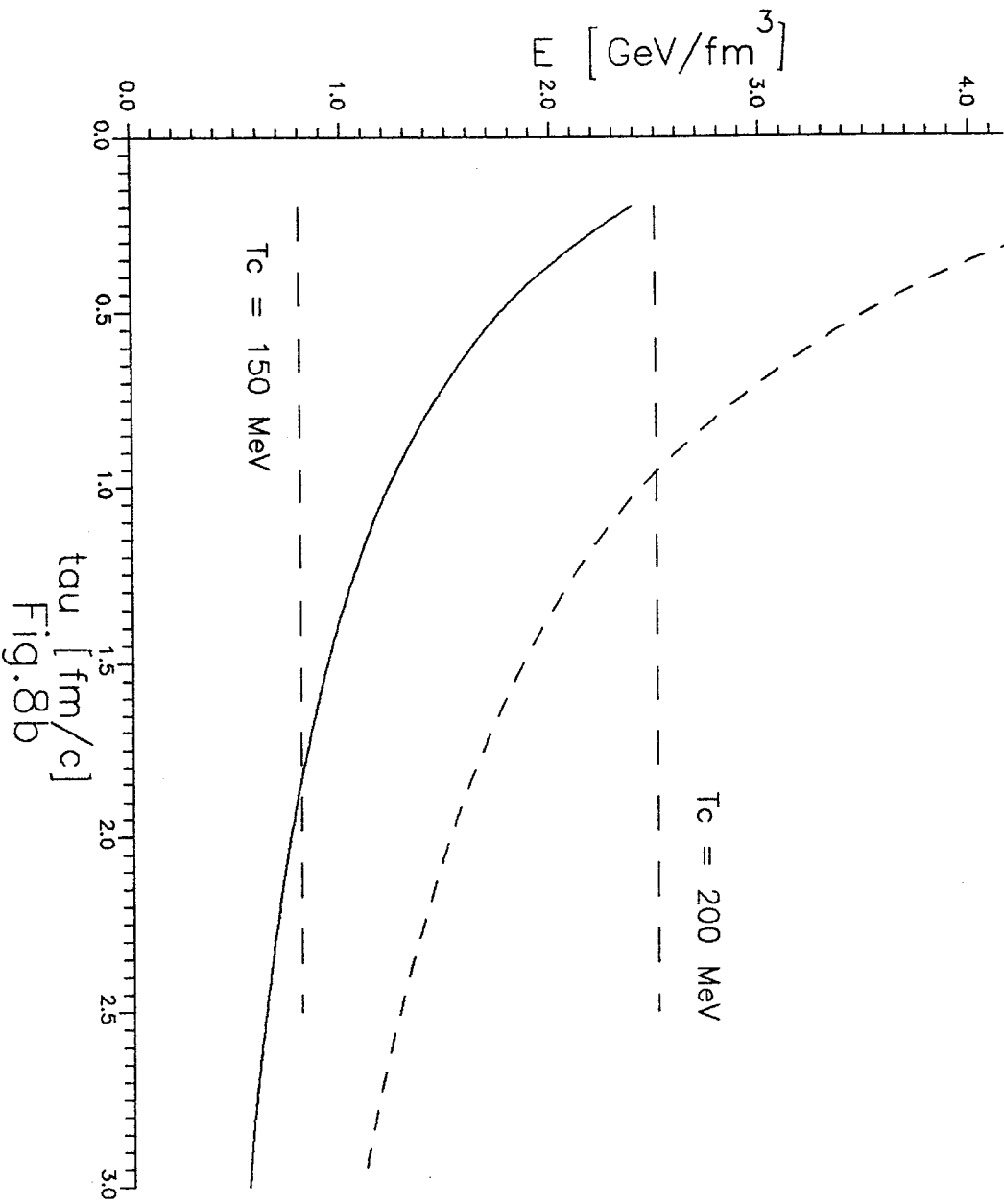


Fig. 7



tau [fm/c]
Fig. 8a



tau [fm/c]
Fig. 8b

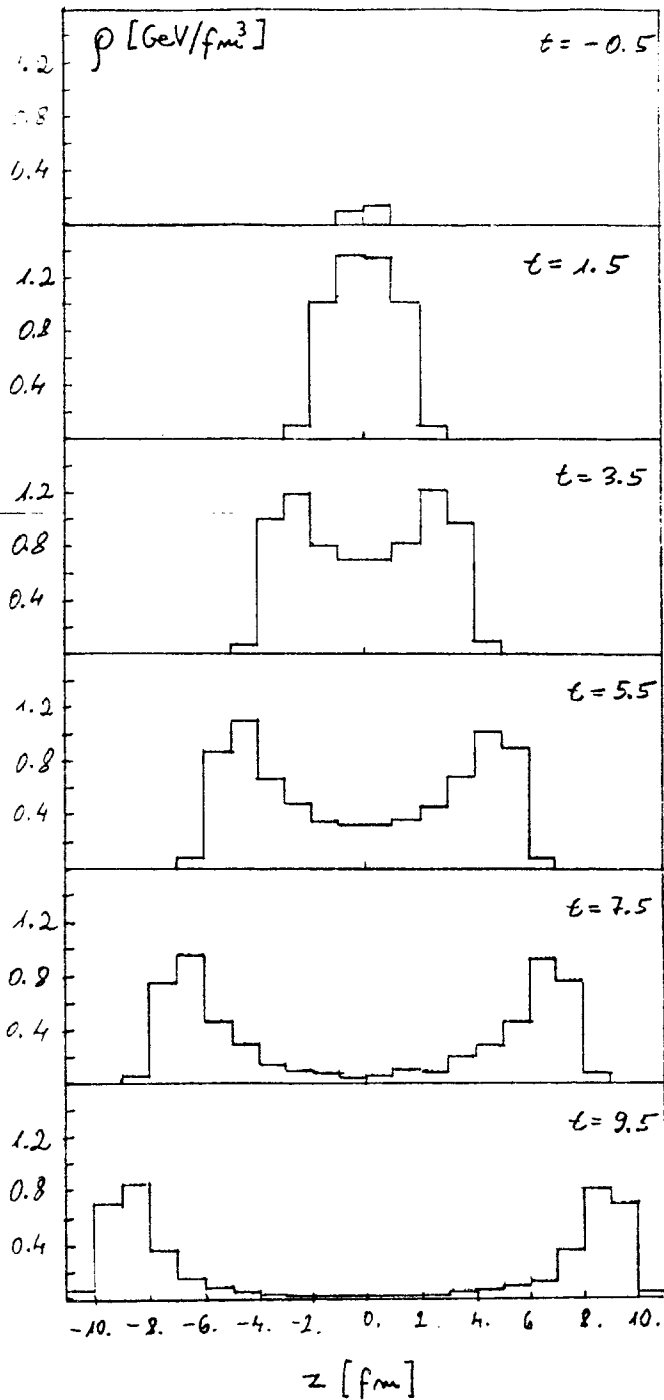


Fig. 9a

$\tau_0 = 0.5$ fm

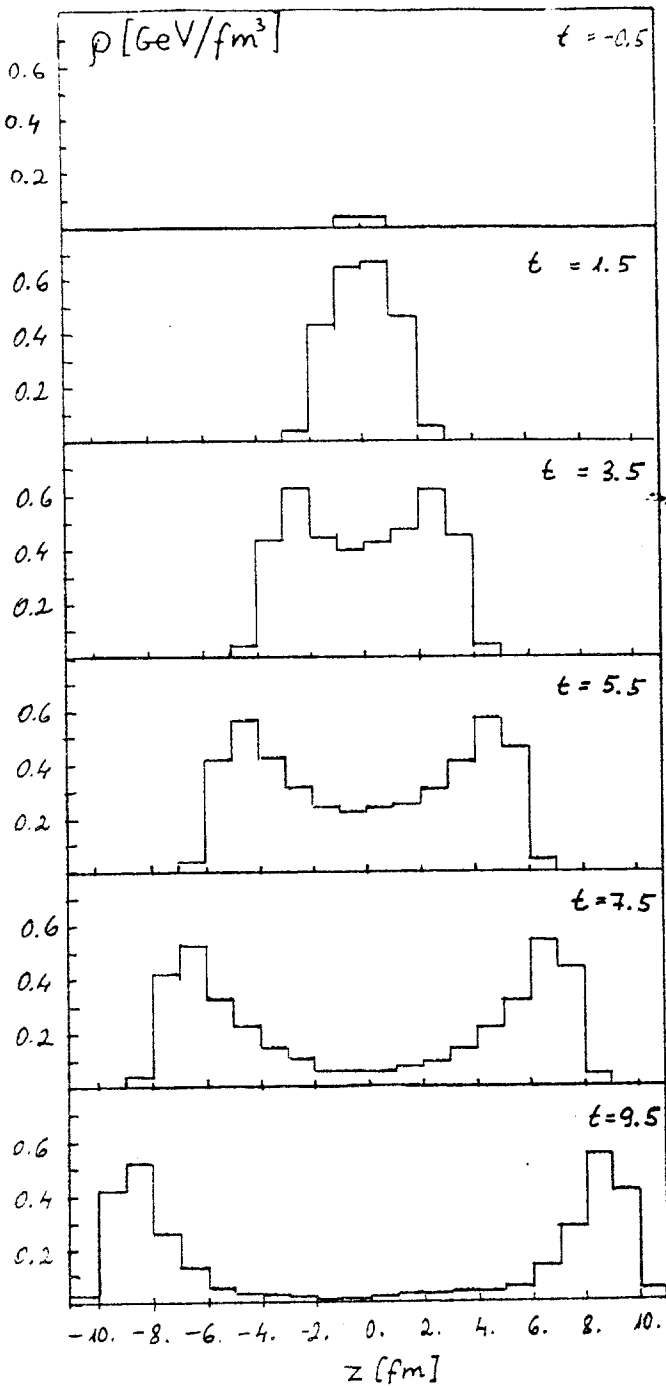


Fig. 9b

$\tau_0 = 1.5$ fm