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A Possible Explanation of the Negative Values of $m_{\nu_e}^2$ Obtained from the β -Spectrum Shape Analyses

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ABSTRACT

By comparing the results obtained using the same experimental data of CIAE but different theoretical formula fits it is pointed out that the negative value of m_{ν}^2 is most likely stemmed from inaccuracy of the theoretical formula of the β -spectrum.

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In the previous paper[1] we have reported on the ν_e -mass limit obtained by the CIAE(China Institute of Atomic Energy) group as $m_{\nu} < 12.4$ eV (95% C.L.). It can be compared with other results reported since 1986 by several other laboratories, namely

m_{ν}	<	12.4 eV	CIAE	(1992)	[1]
	<	11.0 eV	Zurich	(1992)	[2]
	<	9.3 eV	LANL	(1991)	[3]
	<	13.0 eV	INS	(1991)	[4]
	<	18.0 eV	Zurich	(1986)	[5]
	<	7.2 eV	Mainz	(1993)	[6]

and also the hotly disputed result by ITEP

$$17eV < m_{\nu} < 40eV(1987)[7]$$

However, a striking feature is that all the central values of m_{ν}^2 are negative, and it is hard to explain it merely by the experimental uncertainties. This can be seen in Table 1, where the collection of all these results are shown.

However, by re-analysing the data of CIAE, we shall show in the following that the negative central value of m_{ν}^2 should not be regarded as unbiased measurement resulted from statistical fluctuation, but rather is most likely stemmed from the inadaquacy of the theoretical formula used for describing the β -spectrum shape.

In fact, the theoretical β spectrum shape for the experimental data fit can be written down as

$$N_{\beta}(E) = AF(Z, E)pE_t \sum_n W_n(E_0 - E - E_{fn}) \times [(E_0 - E_{fn} - E)^2 - m_{\nu}^2]^{1/2} \quad (1)$$

where, A is the normalization constant; $F(Z, E)$ is the Fermi function, Z is the daughter nuclear charge; p , E and E_t are the momentum, kinetic and total energy of β rays, respectively; W_n and E_{fn} are, respectively, the relative

probability and the excitation energy of the final state with $E_{f0} = 0$ in our definition; E_0 is the end point of the β -spectrum. For simplicity we have omitted in Eq.(1) all corrections resulting from a given experiment.

As is well-known, W_n and E_{fn} must be calculated based on a specific molecular model, which should be chosen to be as faithful as possible to reproduce the source used in a given experiment. There are two different radioactive sources in the experiments of Ref[1-6]. For both LANL[3] and Mainz[6] groups, the T_2 (or TH) molecular sources were used. And therefore the theoretical formula is based on T_2 (or TH) molecule calculations. For the other groups, only the organic molecular sources were available. To be more precise: INS[4] used T-labelled arachidic acid, and a part of this molecule looks as $C_{18}H_{31}T_2C_2O_2CdO_2$. And the theoretical formula used is based on the CH_3T calculation. The Zurich[2] group used $C_{18}H_{31}T_8SiCl_3$ source, and the final result was obtained using C_3H_7T calculation. The radioactive source of CIAE experiment is 3T - labelled PAD($C_{14}H_{15}T_6O_2N_3$) with tritium sitting in the C-H covalent bond of the molecule. Therefore a realistic approach is also to approximate this big molecule to covalent bond such as CH_3T , CH_3-CH_2T , or $CH_3-CHT-CH_3$. We have used all these models for the CIAE data fits. In addition, we have also tried for comparison the T_2 molecule, the T-atom, the T-nucleus as well as Valine 2, where a theoretical calculation for the latter is available[8]. All the re-fitted results are shown in Table 2. One can, however, argue in advance that it is highly unlikely that 3T in PAD can be mimiced by a bare nucleus or an 3T atom. In accordance with this conjecture, the least-square fits presented in the last two lines of Table 2 clearly show that the bare nucleus model yields the negative m_e^2 as big as about 3σ away from zero, (and amazingly the smallest mass limit!) and the highest χ^2 value. And the atom model also does no good. Both these two

models can be rejected on the ground of the more negative m_e^2 and higher χ^2 - values. As for the results of other molecular models, the CH_3T model with 7 levels, and the C_3H_7T with 20 levels give the smallest negative value of m_e^2 as well as the smallest χ^2 - value. Thus the results present in Table 2 suggest strongly that there is a correlation between the negativeness of m_e^2 and the precision of the theoretical formulas and the corresponding model as well. This has led us to question the precision of the theoretical formulas that were used in getting the results of Table 2.

From the theoretical point of view, and for a many-electron system, the ground state wave functions of the parent and daughter molecules can be calculated, and are calculated with better precision for most models listed in Table 2. However, it is not always the case for E_{fn} and W_n when n lies highly above the ground state. It is difficult even for simple two-electrons' molecular system such as T_2 and $(HeT)^+$. In order to see this point clearly, let us recall that there is a theoretically rigorous criterion—the sum rule, which should be fulfilled in any theoretical calculation. Therefore what we have done is to construct the first and second order energy sum rules using the latest calculations of different models, where the spectra are given on the one hand, and calculate the same quantities using the wave functions of the parent molecules on the other hand, and then to see the difference. It is appropriate to notice here that, in principle, the higher order energy sum rules can also be constructed, but as we shall see later, the first and the second order energy sum rules enter the β -spectrum shape formula explicitly, we therefore concern only these two sum rules. In Table 3, Table 4, and Table 5 we have summarized respectively, the best calculated branching ratios and the excitation energies of 7 levels of $CH_3T \rightarrow CH_3He^+$, 20 levels of $C_3H_7T \rightarrow C_3H_7He^+$, and 12 levels of $T_2(TH) \rightarrow THe^+(HHe^+)$ known to us.

Now according to the definitions, the first and the second order energy sum rules can be written down as follows:

$$\langle \Psi_i | \Delta H | \Psi_i \rangle = \sum W_n (\Delta E_{fn} + E_{f0} - E_{i0}) \quad (2)$$

and

$$\langle \Psi_i | (\Delta H)^2 | \Psi_i \rangle = \sum W_n (\Delta E_{fn} + E_{f0} - E_{i0})^2 \quad (3)$$

with

$$\sum W_n = 1 \quad (4)$$

where ΔH is the difference of the Hamiltonians of the initial and final molecular systems RT and RHe^+ . Ψ_i is the ground state wave function of RT, ΔE_{fn} is the excitation energy of the n^{th} -state with respect to the ground state of the daughter molecule RHe^+ , so that it is identical with E_{fn} in Eq. (1) and in Tables 3 - 5. E_{i0} and E_{f0} are the ground state binding energies of RT and RHe^+ respectively.

The average excitation energy $\overline{\Delta E}$ is defined according to the following equation:

$$\overline{\Delta E} = \sum W_n \Delta E_{fn} \quad (5)$$

Then with the use of Eq. (2 - 5) one can construct the energy dispersion function σ^2 as the following:

$$\sigma^2 \equiv \langle \Psi_i | (\Delta H)^2 | \Psi_i \rangle - (\langle \Psi_i | \Delta H | \Psi_i \rangle)^2 = \overline{\Delta E^2} - (\overline{\Delta E})^2 \quad (6)$$

with

$$\overline{\Delta E^2} = \sum W_n (\Delta E_{fn})^2 \quad (7)$$

We have calculated directly the quantities $\overline{\Delta E}$ and σ^2 using the spectra presented in Tables 3 - 5 and compared them with that obtained from the definitions using the initial wave functions of the parent molecules. The results are presented in Tables 6 - 8.

It should be noted that since in practical calculations $\sum W_n$ does not equal to 1 exactly, this results in a small correction term in σ^2 . Therefore σ^2 in Tables 6 - 8 is calculated, if needed, using the following formula

$$\sigma^2 = \overline{\Delta E^2} - (\overline{\Delta E})^2 + (E_{i0} - E_{f0})^2 \Delta W_n \sum W_n - 2(E_{i0} - E_{f0}) \overline{\Delta E} \times \Delta W_n$$

with

$$\Delta W_n = 1 - \sum W_n$$

From Tables 6 - 8 it can be seen that although $\sum W_n$ in all calculations are very close to unit, and if the precision achieved in $\overline{\Delta E}$ computation is tolerable—less than a few percent—the precision in the calculated σ^2 is very poor, and it does not exceed $\approx 30\%$ to 40% , and hence is not acceptable. This shows clearly that it is really very hard to achieve reasonably high accuracy even for HT- HHe^+ or T_2-THe^+ molecular systems. In fact, the best and the most accurate calculation for T_2 and THe^+ so far was reported in Ref.[16] by W. Kolos et. al., and the 12 levels' formula was extracted based on this calculation. But if one is looking beyond the ground state, one immediately sees that the excited states were calculated not as precise as the former. The similar argument is also expressed in Ref.[14].

Then the question is: if it is sufficient to remain satisfied, as claimed in Ref.[9], with the first order energy sum rule, which is calculated with an accuracy of a few percent? Unfortunately it is not the case for β -spectrum shape. It is obvious by recalling that the β -spectrum including the final state

interaction is proportional to the following expression:

$$\sum W_n(E_0 - E - \Delta E_{fn})((E_0 - E - \Delta E_{fn})^2 - m_\nu^2)^{1/2}$$

Therefore the average spectrum shape is related directly to the first and second order energy sum rules:

$$N(E) \propto ((E_0 - E)^2 - 2(E_0 - E)\overline{\Delta E^2} + \overline{\Delta E^2})$$

for $m_\nu = 0$. This point has been unfortunately neglected in most of the literatures, and the theoretical formula with first and second order energy sum rules included was derived for small neutrino mass in Ref. [15] in 1982 and later was rewritten in Ref.[13] in 1984. Now since the second order energy sum rule is reproduced with rather poor precision for all models listed above, it is natural to inquire the reliability of the theoretical formulas used in the present data analysis concerning the m_ν determination.

In order to re-analyse the β -spectrum properly our principle is the following: since the ground state wave functions are calculated with highest precision, we therefore rely only on the ground state branching ratio, and the first and the second energy sum rules, which are evaluated using only the initial wave functions. The theoretical β -spectral shape is given as the following:

$$N(E) = AF(Z, E)pE_i \{W_1(E_0 - E)[(E_0 - E)^2 - m_\nu^2]^{1/2}\theta(E_0 - E - m_\nu) + (1 - W_1) \times [(E_0 + \langle \Delta H \rangle_1 - E)^2 + \langle \Delta H^2 \rangle_1 - \langle \Delta H \rangle_1^2 - m_\nu^2/2] \times \theta(E_0 + \langle \Delta H \rangle_1 - E - m_\nu)\} \quad (S)$$

where

$$\langle \Delta H \rangle_1 = \overline{\Delta E^2}/(1 - W_1)$$

and

$$\langle \Delta H^2 \rangle_1 = (\overline{\Delta E^2})^2/(1 - W_1)$$

Eq.(8) is the so-called two-levels' formula with closure with the ground state transition treated exactly while the contribution from all excited state transitions is estimated using the first and second energy sum rules. For a formula with more transitions treated exactly we refer the reader to Ref.[13].

Now using formula Eq.(8) the CIAE data are re-fitted. The results are shown in Table 9. Two remarkable features from Table 9 can be seen, namely, 1, in all models without exception the least square fits with closure formula (S) lead to smaller negative values of m_ν^2 and smaller χ^2 values, particularly for C_3H_7T , the m_ν^2 is $+4(eV)^2$ with the χ^2 equal to 1.091, the smallest one; and 2, all the obtained m_ν^2 values using formula (S) are compatible with zero within one standard deviation. This result clearly demonstrates that the quality of the fitting as well as the m_ν^2 value itself rely heavily on the precision of the theoretical spectrum. and the negativeness of m_ν^2 seems at least alleviated. It is also interesting and appropriate to note that the sum rule approach for both C_3H_7T and T_2 models here leads to very similar results. This is related to the fact that incidentally these two models have roughly the same values of W_1 and $\overline{\Delta E^2}$, as well as σ^2 . As for CH_3T , the more recent calculation [9] seems to favour a smaller branching ratio: $W_{00}=0.578$. And with this new value in mind, one will see that an improved CH_3T calculation should lead to a result, which is not very different from that of T_2 and C_3H_7T models, and therefore a positive value of m_ν^2 may also be obtained using the CIAE data. All these demonstrate that tritium in different covalent bond behaves more or less similarly as far as only the first and second order sum rules are considered.

*As a conclusion, we would like to point out that the analysis of CIAE data seems to indicate that the negativeness of m_ν^2 is most likely related to the inaccuracy of the theoretical formula. However, it is very important and

desirable to verify this suggestion by re-analysing the experimental data of different laboratories.

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Table Captions:

Table 1, m_ν^2 and m_ν - upper limit.

Table 2, m_ν^2 , m_ν -limit and χ^2 -Value for CIAE data fit.

Table 3, The W_n and E_{fn} in the 7 levels' transition of CH_3T molecule.

Table 4, The W_n and E_{fn} in the 20 levels' transition of C_3H_7T molecule.

Table 5, The W_n and E_{fn} in the 12 levels' transition of T_2 molecule.

Table 6, A comparison of $\overline{\Delta E^-}$ and σ^2 between the direct calculation and the sum rule approach for 7 levels in CH_3T model.

Table 7, A comparison of $\overline{\Delta E^-}$ and σ^2 between the direct calculation and the sum rule approach for 20 levels in C_3H_7T model.

Table 8, A comparison of $\overline{\Delta E^-}$ and σ^2 between the direct calculation and the sum rule approach for 12 levels in T_2 model.

Table 9, Re-fitted m_ν^2 using the closure formula (8) and the comparison with that using formula (1).

Table 1: m_ν^2 and m_ν - upper limit

Reference	m_ν^2 (eV ²)	Error (eV) ²		Upper limit for m_ν (95% C.L.)
		stat.	syst.	
CIAE -92 [1]	-31	± 75	± 48	12.4 eV
Zurich-92 [2]	-24	± 48	± 61	11.0 eV
LANL -91 [3]	-147	± 68	± 41	9.3 eV
INS -91 [4]	-65	± 85	± 65	13.0 eV
Zurich-86 [5]	-11	± 63	± 178	18.0 eV
Mainz-93 [6]	-39	± 34	± 15	7.2 eV
ITEP -87 [7]	919	± 60	± 150	17 < m_ν < 40

Table 2: m_ν^2 , m_ν -limit and χ^2 -value for CIAE data fit[1]

Model	No. of level	m_ν^2 (eV) ²	m_ν (eV)	χ^2	E_0 -18500 (eV)
CH ₃ T	7	-31± 75	12.4	1.141	78.3
CH ₂ =CHT	2	-51± 75	12.0	1.145	79.7
CH ₃ -CHT-CH ₃	2	-43± 75	12.2	1.144	79.9
CH ₃ -CHT-CH ₃	20	-9± 75	12.9	1.134	79.4
VALINE II	2	-141± 75	10.3	1.140	78.9
T-molecule	2	-68± 75	11.7	1.148	79.9
T-molecule	12	-177± 75	9.7	1.145	77.6
T-atom	2	-191± 75	9.4	1.146	75.2
T-nuclei	1	-237± 75	8.9	1.158	67.2

Table 3: The W_N and E_{FN} in the 7 levels transition of CH_3T molecule.[8]

W_n	$E_{fn}(eV)$	W_n	$E_{fn}(eV)$
0.6056	0.00	0.017	57.50
0.084	22.50	0.075	72.50
0.141	32.50	0.044	91.33
0.033	47.50		

Table 4: The W_N and E_{FN} in the 20 levels transition of C_3H_7T molecule.[9]

W_n	$E_{fn}(eV)$	W_n	$E_{fn}(eV)$
.571036	0.00	.045685	63.740
.117594	23.105	.012837	65.565
.073401	35.655	.007596	73.222
.012191	38.874	.053237	78.616
.008831	42.572	.005876	82.119
.020535	44.795	.002271	86.099
.007183	48.285	.001970	92.813
.012122	51.384	.003415	97.807
.008235	55.890	.011496	106.532
.015856	58.777	.001496	120.988

Table 5: The W_N and E_{FN} in the 12 levels transition of T_2 molecule.[10]

W_n	$E_{fn}(eV)$	W_n	$E_{fn}(eV)$
.5822	0.00	.0089	41.75
.1675	27.29	.0143	46.03
.0787	33.89	.0166	51.71
.0081	37.96	.0789	65.28
.0001	38.82	.0297	75.45
.0092	39.38	.0061	88.07

Table 6: A comparison of $\overline{\Delta E^2}$ and σ^2 between the direct calculation and the sum rule approach for 7 levels in CH_3T model.

	Direct calcul.	Sum rule	Deviation
W_n	1	1	very small
$\overline{\Delta E^2}(eV)$	18.51	18.98[11]	2.5%
$\sigma^2(eV)^2$	744.07	1207.6[11]	$\approx 40\%$

Table 7: A comparison of $\overline{\Delta E^-}$ and σ^2 between the direct calculation and the sum rule approach for 20 levels in C_3H_7T model.

	Direct calcul.	Sum rule	Deviation
W_n	.9929	1.00	.7%
$\overline{\Delta E^-}$ (eV)	20.56	19.1±.4[3,9]	7.5%
σ^2 (eV) ²	795.69	1231.14[12]	≈ 30%

Table 8: A comparison of $\overline{\Delta E^-}$ and σ^2 between the direct calculation and the sum rule approach for 12 levels in T_2 model.

	Direct calcul.	Sum rule(HT)	Sum rule(T_2)	Deviation
W_n	0.9997	1.00	1.00	.3%
$\overline{\Delta E^-}$ (eV)	17.67	18.62[11]	18.80[13]	5-6%
σ^2 (eV) ²	566.50	1109.5[11]	1045.9[13]	45-50%

Table 9: Re-fitted m_2^2 using the closure formula (8) and the comparison with that using formula (1).

Model	No. of levels	Formula(1) or(8)	W_1	$E_0-18500$ (eV)	m_2^2 (eV) ²	χ^2
CH ₃ T	2	(8)	0.6056	79.7	-21	1.128
CH ₃ T	7	(1)	0.6056	78.3	-31	1.141
CH ₃ -CHT-CH ₃	20	(1)	0.5710	79.4	-9	1.134
CH ₃ -CHT-CH ₃	2	(8)	0.5710	82.2	+4	1.091
T-molecule	2	(8)	0.5820	78.2	1	1.098
T-molecule	12	(1)	0.5822	77.6	-177	1.145