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Triviality Bounds in Next to Minimal Supersymmetric Standard Model *

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Disclaimer

Abstract

We study the implication of the triviality problem on the Higgs mass and other relevant parameters in the Next to Minimal Supersymmetric Standard Model. Through a full examination of the parameter space, we are able to derive triviality bounds on the heaviest Higgs, the soft-breaking parameters, and the Higgs singlet vacuum expectation value. Besides, an absolute upper bound of $2.8\,M_W$ on the lightest Higgs is established.

1 Introduction

There has been a lot of effort spent in finding the upper bound on the Higgs mass of the standard model (or the supersymmetric extension) [1, 2, 3]. One of the approaches is based on triviality of the ϕ^4 theory [4]. Due to triviality, the standard model is inconsistent as a fundamental theory but is a good effective theory with a cut-off momentum Λ. And, by requiring that Λ be larger than the Higgs mass for the consistency of the standard model as a cut-off theory, Dashen and Neuberger were the first to derive the triviality upper bound (about 800 GeV) on the Higgs mass in the minimal standard model [4]. Improvements on this triviality upper bound were then made in several aspects, including the non-perturbative calculations, or the inclusion of gauge couplings and the Yukawa coupling of top quark [5, 6, 7, 8]. So far, supersymmetry is the only viable framework where the Higgs scalar is natural [9, 10]. It is then interesting to know how this triviality bound on the Higgs mass behaves in the supersymmetric extension of the standard model. However, no similar discussion is available in the context of supersymmetry to the present. These considerations then lead to the idea of this paper: understand the implication of the triviality problem on the Higgs mass and other relevant parameters within the supersymmetric standard model.

Essentially, this paper extends the work of Dashen and Neuberger [4] to the supersymmetric standard model. Specifically, the supersymmetric standard model with two SU(2)×U(1) Higgs doublets and one Higgs singlet (usually named Next to Minimal Supersymmetric Standard Model, NMSSM) plus appropriate soft-breaking terms is considered [10, 11]. The inclusion of the Higgs singlet in NMSSM provides a natural explanation to the μ-problem of the minimal supersymmetric standard model(MSSM) [12]. In addition, the existence of the Higgs singlet is suggested in many superstring models [13, 14] and grand unified supersymmetric models [12]. These features make NMSSM an appealing alternative to MSSM. In considering the strong Higgs coupling limit of NMSSM, it is suggested that triviality still persists, at least at one-loop. This observation certainly implies an upper bound on the Higgs mass. Although a non-perturbative discussion of triviality is beyond the scope here, this does not stop us in understanding the behavior of the triviality bound on the Higgs mass in NMSSM.

In Section 2, the relevant NMSSM lagrangian and renormalization group equations are given. The observation of triviality is made in the strong Higgs coupling limit. This triviality observation then implies the Landau-pole behavior of the Higgs couplings. To facilitate the computations of triviality bounds later, an analytic expression of the Landau pole Λ_L is also derived. In Section 3, the parametrization of the NMSSM Higgs mass spectrum over the full parameter space is done. The determination of the full parameter space is a non-trivial one because the minimization of the scalar potential leads to several constraints on the parameters. In Section 4, the triviality bound is solved by requiring Λ_L larger than the mass of the heaviest Higgs in order to ensure NMSSM a valid cut-off theory, where $\tan\beta = 1$ is chosen for the sake of simplicity. The behavior of the triviality bound is then analyzed through the full parameter space. Combined with the present experimental knowledge of the lower bound on the Higgs mass, this analysis indicates that a very large portion of the parameter space is ruled out. For example, the vacuum expectation value v_3 of the Higgs singlet can be constrained to: $0.24M_W \leq |v_3| \leq 0.749M_W$, where M_W is the mass of W gauge boson. The soft-breaking parameters are constrained from above. And, if the experimental lower bound on the Higgs mass is raised, the above constraints will get even stronger, which implies a better understanding of the correct parameter ranges. In Section 5, an absolute upper bound of $2.8M_W$ on the lightest Higgs is established by a search through the full parameter space. This absolute upper bound is beyond the reach of LEP.

2 Indication of Triviality in NMSSM

The supersymmetric Higgs scalar potential for Next to Minimal Supersymmetric Standard Model (NMSSM) can be written as [10, 15]:

$$V = |hN|^2({\Phi^\dagger}_1\Phi_1 + {\Phi^\dagger}_2\Phi_2) + |h{\Phi^\dagger}_1\Phi_2 + \lambda N^2|^2 + \frac{1}{8}g_1^2({\Phi^\dagger}_1\Phi_1 - {\Phi^\dagger}_2\Phi_2)^2$$

$$+\frac{1}{8}g_2^2[(\Phi^\dagger{}_1\Phi_1+\Phi^\dagger{}_2\Phi_2)^2-4(\Phi^\dagger{}_1\Phi_2)(\Phi^\dagger{}_2\Phi_1)] \tag{1}$$

$\Phi_1 = (\phi^\dagger{}_1, \phi^0{}_1)$ and $\Phi_2 = (\phi^\dagger{}_2, \phi^0{}_2)$ are two SU(2)×U(1) Higgs doublets, and N is a complex singlet. It is assumed that only ϕ_1^0, ϕ_2^0 and N acquire vacuum expectation values v_1, v_2 and v_3 respectively. The scalar-quarks and scalar-leptons do not acquire vacuum expectation values, and we can ignore their contributions to the scalar potential when studying the Higgs mass spectrum. Note that the superpotential corresponding to (1) does not contain linear and bi-linear terms [10] because these terms lead to naturalness problems. Besides, these terms do not appear in a large class of superstring-inspired models. About the soft-breaking terms, we are going to confine ourselves to the following type:

$$V_{soft} = m_1^2\Phi^\dagger{}_1\Phi_1+m_2^2\Phi^\dagger{}_2\Phi_2-m_{12}^2\Phi^\dagger{}_1\Phi_2-m^{*2}_{12}\Phi^\dagger{}_2\Phi_1 \tag{2}$$

in order to have predictive power. However, this limited choice of V_{soft} actually does not destroy the generality of the conclusions made in this paper. Details on this will be given in the last section.

We assume three generations of quarks and leptons together with their supersymmetric partners. As for the renormalization group equations relevant to the Higgs couplings, all the Yukawa couplings to the Higgs bosons are neglected except for that of the top quark, whose coupling to Φ_2 is denoted by f_t. Then, the relevant renormalization group equations (one-loop) of NMSSM are given by [14, 15]:

$$8\pi^2 \frac{d\,g_1^2}{dt} = 11 g_1^4 \tag{3}$$

$$8\pi^2 \frac{d\,g_2^2}{dt} = g_2^4 \tag{4}$$

$$8\pi^2 \frac{d\,g_3^2}{dt} = -3 g_3^4 \tag{5}$$

$$8\pi^2 \frac{d\,f_t^2}{dt} = f_t^2 (6 f_t^2 + h^2 - \frac{13}{9} g_1^2 - 3 g_2^2 - \frac{16}{3} g_3^2) \tag{6}$$

$$8\pi^2 \frac{d\,h^2}{dt} = h^2 (4h^2 + 2\lambda^2 + 3 f_t^2 - g_1^2 - 3 g_2^2) \tag{7}$$

$$8\pi^2 \frac{d\,\lambda^2}{dt} = 6\lambda^2 (\lambda^2 + h^2) \tag{8}$$

where g_1, g_2 and g_3 are the gauge couplings associated with SU(3), SU(2) and U(1) gauge groups respectively. The parameter t is defined by:

$$\mathrm{t} = \frac{1}{2} \ln(\frac{-q^2}{M_W^2}) \tag{9}$$

where q^2 is the space-like effective square of the momentum at which these couplings are defined. At t=0, the gauge couplings can be determined from the experimentally derived inputs [16]:

$$g_1^2 = 0.126, \;\; g_2^2 = 0.446, \;\; g_3^2 = 1.257 \tag{10}$$

To understand the triviality problem, we assume strong Higgs coupling limit for h^2 and λ^2, where the Yukawa coupling f_t and all the gauge couplings can be neglected. This is actually not an unreasonable assumption due to the following observations: The RGEs (3)-(8) have been numerically studied by Babu and

Ma [15]. Their results indicate that gauge couplings are negligible if the ratio $\frac{h^2}{g_1^2} \geq 5$ or $\frac{\lambda^2}{g_1^2} \geq 2$ holds, where $\tan\beta$=1 and the top quark mass m_t=40~400 GeV. (See Fig.1 in [15] for a more precise description.) Therefore, it's certainly reasonable to expect $\frac{h^2}{g_1^2} \geq 5$ or $\frac{\lambda^2}{g_1^2} \geq 2$ holds in strong Higgs coupling limit even though triviality eventually will put an upper bound on the magnitude of the Higgs coupling. For the reason of self-consistency, this assumption of negligible gauge couplings in the strong Higgs coupling limit will be verified *a posteriori* using the numerical results of Sections 4 and 5. The assumption of negligible Yukawa coupling f_t^2 is a little obscure. There has been numerical evidence in the standard model [5] that the determination of triviality bounds on the Higgs mass is insensitive to the top quark mass m_t if $m_t \leq 200$ GeV, and it may be still true in NMSSM. The assumption of negligible f_t^2 will be checked for self-consistency within the calculations of Sections 4 and 5, and it turns out that f_t^2 is important only in certain extreme situations. A detailed discussion will be given in Section 4.

In the strong Higgs coupling limit with negligible f_t^2 and g_i^2, the RGEs for the Higgs couplings are:

$$8\pi^2 \frac{d\,h^2}{dt} = h^2(4h^2 + 2\lambda^2) \tag{11}$$

$$8\pi^2 \frac{d\,\lambda^2}{dt} = 6\lambda^2(\lambda^2 + h^2) \tag{12}$$

There is only one fixed point, the infrared stable fixed point at h^2=0, λ^2=0.

Therefore, similar to the Landau pole [10, 3] in the pure ϕ^4 theory, $h^2(\mathrm{t})$ and $\lambda^2(\mathrm{t})$ will diverge at some finite t=t_L (the Landau pole) unless the Higgs couplings vanish, where triviality is clearly indicated at one-loop. Note that the above conclusion is still valid even if the Yukawa coupling f_t^2 is included. In Section 3, we will use triviality of the Higgs couplings to establish an upper bound on the Higgs mass, and an analytic expression of the Landau pole t_L will be useful. The general solution of t_L can be found using the method of integrating factor [21]:

$$\mathrm{t}_L = \frac{2\pi^2 \lambda_0^{-\frac{2}{3}} \sqrt{1 + C\lambda_0^{-\frac{4}{3}}}}{C} - \frac{2\pi^2 \ln(\sqrt{1 + C\lambda_0^{-\frac{4}{3}}} + \sqrt{C\lambda_0^{-\frac{4}{3}}})}{C^{\frac{3}{2}}} \quad (13)$$

$$C = h_0^4 \lambda_0^{-\frac{8}{3}} + 2\, h_0^2 \lambda_0^{-\frac{2}{3}} \quad (14)$$

$$h_0 = h(\mathrm{t}=0), \;\; \lambda_0 = \lambda(\mathrm{t}=0), \;\; \Lambda_L \equiv M_W \cdot \exp(\mathrm{t}_L) \quad (15)$$

where Λ_L is the momentum corresponding to the Landau pole t_L.

Before going to the next section, note that the treatment of triviality here is of perturbative nature. Although the problem of triviality should be of non-perturbative nature, several non-perturbative numerical simulations have been performed [17, 18] and imply that the renormalized perturbative calculation gives essentially the correct triviality upper bound on the Higgs mass. This observation may justify our approach as a first approximation.

3 Parameter Space and Higgs Mass Spectrum

In this section, we describe the relevant parameter space and the Higgs mass spectrum. Consider $V'=V+V_{soft}$, and the relevant parameters are h, λ, v_1, v_2, v_3, m_1^2, m_2^2, m_{12}^2. Without loss of generality, the convention here is to take h, λ, m_1^2, m_2^2 to be real, and v_1, v_2, v_3, m_{12}^2 complex, that is, $v_1 = \tilde{v}_1 e^{i\phi_1}$, $v_2 = \tilde{v}_2 e^{i\phi_2}$, $v_3 = \tilde{v}_3 e^{i\phi_3}$, $m_{12}^2 = \tilde{m}_{12}^2 e^{i\phi_m}$. The minimization of the scalar potential V' leads to three complex constraints on these parameters:

$$h^2(|v_1|^2 + |v_2|^2)v_3 + 2\lambda(hv_1^* v_2 + \lambda v_3^2)v_3^* = 0 \tag{16}$$

$$\frac{1}{4}(g_1^2 + g_2^2)(|v_1|^2 - |v_2|^2) + h^2(|v_2|^2 + |v_3|^2) + m_1^2 = \frac{v_2}{v_1}(m_{12}^2 - h\lambda v^{*2}_3) \tag{17}$$

$$\frac{1}{4}(g_1^2 + g_2^2)(|v_2|^2 - |v_1|^2) + h^2(|v_1|^2 + |v_3|^2) + m_2^2 = \frac{v_1}{v_2}(m^{*2}_{12} - h\lambda v_3^2) \tag{18}$$

In addition, one has the following physical constraint:

$$M_W^2 = \frac{1}{2}g_2^2(|v_1|^2 + |v_2|^2) \tag{19}$$

Imaginary parts of constraints (16)-(18) fix the phase among complex parameters v_1, v_2, v_3, m_{12}^2, and (19) reduces ($\tilde{v}_1$, $\tilde{v}_2$) to a single parameter $\tan\beta = \frac{\tilde{v}_2}{\tilde{v}_1}$. Furthermore, ($h$, λ, $\tilde{v}_3$) can be expressed in terms of other parameters using the real parts of constraints (16)-(18). The resultant parameter space is then defined by the parameters (ϕ, $\tan\beta$, m_1^2, m_2^2, $\tilde{m}_{12}^2$):

$$0 \le \phi < 4\pi, \quad -\infty < \tan\beta,\ m_1^2,\ m_2^2,\ \tilde{m}_{12}^2 < \infty \tag{20}$$

The other dependent parameters can be expressed in terms of (20) as:

$$
\begin{aligned}
\tilde{v}_3 &= \frac{M_W}{g_2}\sqrt{\frac{2\,A\,B\,\tan\beta - B^2(1+\tan^2\beta)}{A^2(1+\tan^2\beta)}} \\
A &= \tilde{m}_{12}^2(\tan\beta - \frac{1}{\tan\beta}) - m_1^2 + m_2^2 + M_W^2(1+\frac{g_1^2}{g_2^2})\frac{\tan^2\beta - 1}{\tan^2\beta + 1} \\
B &= \frac{1}{\tan\beta}\{m_1^2 + \frac{1}{2}(1+\frac{g_1^2}{g_2^2})M_W^2\} - \tan\beta\{m_2^2 + \frac{1}{2}(1+\frac{g_1^2}{g_2^2})M_W^2\} \qquad (21) \\
v_1 &= \frac{M_W}{g_2}\frac{\sqrt{2}}{\sqrt{\tan^2\beta+1}},\ v_2 = \frac{M_W}{g_2}\frac{\sqrt{2}\tan\beta}{\sqrt{\tan^2\beta+1}}e^{i\phi},\ v_3 = \tilde{v}_3\, e^{i\frac{\phi}{2}} \qquad (22) \\
m_{12}^2 &= \tilde{m}_{12}^2\, e^{-i\phi} \qquad (23) \\
h^2 &= \frac{\tilde{m}_{12}^2\tan\beta - m_1^2 + \frac{M_W^2}{2}(1+\frac{g_1^2}{g_2^2})\frac{\tan^2\beta-1}{\tan^2\beta+1}}{\tilde{v}_3^2 + \tan\beta\,\frac{M_W^2}{g_2^2}(\frac{\tan\beta}{\tan^2\beta+1} \pm \sqrt{\frac{\tan^2\beta}{(\tan^2\beta+1)^2} - \frac{g_2^2\,\tilde{v}_3^2}{M_W^2}}\,)} \qquad (24) \\
\lambda &= h\,\frac{M_W^2}{g_2^2\,\tilde{v}_3^2}(-\frac{\tan\beta}{\tan^2\beta+1} \pm \sqrt{\frac{\tan^2\beta}{(\tan^2\beta+1)^2} - \frac{g_2^2\,\tilde{v}_3^2}{M_W^2}}\,) \qquad (25)
\end{aligned}
$$

$$\tilde{v}_3,\ h,\ \text{and}\ \lambda\ \text{are real.} \qquad (26)$$

$''\pm''$ in (24) and (25) indicates $(h,\ \lambda)$ has two solutions, where the upper one $(''+'')$ is denoted as $(h_A,\ \lambda_A)$ and the lower one $(''-'')$ is $(h_B,\ \lambda_B)$.

(20)-(26) specify the full parameter space. (26) is non-trivial because the square root is involved in (21), (24) and (25). An immediate consequence of (26) follows from the reality requirement on λ:

$$\tilde{v}_3 \leq \frac{|\tan\beta|}{g_2(\tan^2\beta+1)}M_W \qquad (27)$$

$\frac{|\tan\beta|}{tan^2\beta+1}$ has its maximum$=\frac{1}{2}$ at $\tan\beta = \pm 1$. Thus, we are able to establish an absolute upper bound on $\tilde{v}_3$ as:

$$\tilde{v}_3 \leq \frac{1}{2\,g_2} M_W \tag{28}$$

Given $M_W = 80\,\text{GeV}$ and $g_2^2 = 0.446$ from (10), this absolute upper bound on the magnitude of v_3 is 60 GeV. As $\tilde{v}_3 \to \frac{M_W}{2g_2}$, (27) implies $|\tan\beta| \to 1$. Therefore, in NMSSM, $\tan\beta$ is constrained by the determination of $|v_3|$, and vice versa.

For the sake of simplicity, we choose $\tan\beta = 1$ when studying (20)-(26). When $\tan\beta = 1$, $\tilde{v}_3$ becomes a free parameter, and $m_1^2 = m_2^2$ is required by the minimization of the scalar potential V': (17) and (18). On the whole, the number of free parameters is unchanged. The full parameter space ($\tan\beta = 1$) is then defined by the parameters (ϕ, $\tilde{v}_3$, $m_1^2 = m_2^2$, $\tilde{m}_{12}^2$) plus the following constraints:

$$0 \leq \phi < 4\pi, \;\; 0 < \tilde{v}_3 \leq \frac{1}{2\,g_2} M_W \tag{29}$$

$$-\infty < m_1^2 = m_2^2 < \tilde{m}_{12}^2 < \infty \tag{30}$$

There is no essential change to the expressions of dependent parameters except for h and λ:

$$h^2 = \frac{\tilde{m}_{12}^2 - m_1^2}{\tilde{v}_3^2 + \frac{M_W^2}{g_2^2}(\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{g_2^2\tilde{v}_3^2}{M_W^2}}\,)} \tag{31}$$

$$\lambda = h\,\frac{M_W^2}{g_2^2\,\tilde{v}_3^2}(-\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{g_2^2\tilde{v}_3^2}{M_W^2}}\,) \tag{32}$$

(29)-(32) specify the full parameter space and will be the main study later. Note that (31) and (32) imply:

$$h^2,\ \lambda^2 \ \propto \ (\tilde{m}_{12}^2 - m_1^2) \tag{33}$$

and $m_1^2 = m_2^2 < \tilde{m}_{12}^2$ in (30) is the consequence of the reality requirement on h.

Based on the parameter space described in (20) or (29), it is trivial to work out the spectrum of Higgs mass^2 from V'. In general, $\{\phi_1^0,\ \phi_2^0,\ N\}$ does not mix with $\{\phi_1^\dagger,\ \phi_2^\dagger\}$. The mass^2-matrix $[M_n^2]$ for $\{\phi_1^0,\ \phi_2^0,\ N\}$ is a 6×6 matrix, and the mass^2-matrix $[M_c^2]$ for $\{\phi_1^\dagger,\ \phi_2^\dagger\}$ is a 4×4 matrix. $[M_n^2]$ contains five neutral Higgs bosons and one massless particle. $[M_c^2]$ contains two charged Higgs bosons and two massless particles, as expected. Although the detailed expressions of $[M_n^2]$ and $[M_c^2]$ are not given here, there are several symmetries of $[M_n^2]$ and $[M_c^2]$:

$$[M_n^2] \text{ and } [M_c^2] \text{ are periodic in } \phi, \text{ of periodicity } \pi. \tag{34}$$

In addition, $[M_n^2]$ and $[M_c^2]$ are invariant under two discrete symmetries on the parameter space:

$$h \rightarrow -h,\ \ \lambda \rightarrow \lambda,\ \ \tan\beta \rightarrow -\tan\beta,\ \ m_{12}^2 \rightarrow -m_{12}^2 \tag{35}$$

$$h \rightarrow -h,\ \ \lambda \rightarrow -\lambda \tag{36}$$

4 Constraints on Higgs Mass and Soft-Breaking Parameters

Based on the parameter space $(\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2,\ \tilde{m}_{12}^2)$, (29)-(32), $\Lambda_L \geq m_{heaviest\,Higgs}$ is required by triviality and the triviality upper bound B_H on the heaviest Higgs is established by $\Lambda_L = m_{heaviest\,Higgs} \equiv B_H$, where Λ_L is defined in (13)-(15) and $m_{heaviest\,Higgs}$ is obtained numerically from $[M_n^2]$. Geometrically, the triviality upper bound B_H defines a surface in the parameter space by $\Lambda_L = m_{heaviest\,Higgs}$, and our convention is to parametrize triviality surface B_H in terms of $(\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2)$. Actually, at any point $(\phi,\ \tilde{v}_3,\ m_1^2 = m_2^2)$ of triviality surface B_H, there are seven triviality upper bounds obtained from $[M_n^2]$ and $[M_c^2]$. Each triviality upper bound corresponds to one of the seven physical Higgs bosons. So, studying triviality surface B_H will be the main interest.

A typical example is shown in Fig.1, where $(h,\ \lambda) = (h_A,\ \lambda_A)$, $\phi = 0$, $\tilde{v}_3 = 0.7\,M_W$, and $m_1^2\,(= m_2^2)$ is varied. This corresponds to a line lying within triviality surface. Several universal features about Fig.1 are important. First, in the small-m_1^2 regime, the soft-breaking terms are negligible and the determination of B_H is independent of m_1^2. Second, the B_H curve as in Fig.1 terminates at $m_1^2 \simeq -M_W^2$ because the mass^2-matrix $[M_n^2]$ or $[M_c^2]$ will develop negative eigenvalues if m_1^2 gets too negative. Together with (30), this observation indicates m_1^2, m_2^2, $\tilde{m}_{12}^2$ are bounded from below. Since nothing interesting happens for $m_1^2 < 0$, we will take $0 \leq m_1 = m_2 < \tilde{m}_{12}$ from now on.

The last and most important universal feature from Fig.1 is: In the large-m_1 regime, $B_H = \sqrt{2}\, m_1$ is approximately true. To explain this observation, note that (13) implies, on triviality surface, $h^2,\ \lambda^2 \rightarrow 0$ if $B_H\,(= \Lambda_L) \rightarrow \infty$. The structure of $[M_n^2]$ also implies $B_H \rightarrow \infty$ if $m_1 = m_2 \rightarrow \infty$. The above two facts then lead to:

$$\text{On triviality surface: } h^2,\ \lambda^2 \rightarrow 0 \ \text{ if } \ m_1 = m_2 \rightarrow \infty \tag{37}$$

Thus, in the limit $m_1 = m_2 \rightarrow \infty$ on triviality surface, $[M_n^2]$ and $[M_c^2]$ can be solved up to order $O(h^2)$:

$$\begin{aligned} \text{Eigenvalues of } [M_n^2] &= [2m_1^2 + O(h^2),\ 2m_1^2 + O(h^2),\ O(h^2),\ O(h^2),\ O(h^2),\ 0] \\ \text{Eigenvalues of } [M_c^2] &= [2m_1^2 + O(h^2),\ 2m_1^2 + O(h^2),\ 0,\ 0] \end{aligned} \tag{38}$$

(38) then explains why $B_H = \sqrt{2}\, m_1$ is valid up to order $O(h^2)$ in the large-m_1 regime. (38) also implies that there are exactly three light (neutral) Higgs bosons in the large-m_1 regime. In fact, triviality upper bounds on these three Higgs monotonically decrease to zero in the large-m_1 limit. According to the present experimental knowledge of the lower bound on the Higgs mass [19, 20], it is reasonable to require the seven triviality upper bounds on Higgs be larger than 1 M_W, which immediately leads to an upper bound on $m_1\,(= m_2)$ due to (38). A typical realization of this idea is given in Fig.2, where $(h,\ \lambda) = (h_A,\ \lambda_A)$,

$\phi = 0.6$, and $(\tilde{v}_3,\ m_1)$ is fully examined. The enclosed region in Fig.2 indicates the allowed range of $\tilde{v}_3$ versus m_1. An interesting quantity B_{soft} can be defined in such a way that the allowed range of $\tilde{v}_3$ shrinks to a single point at $m_1 = B_{soft}$. In Fig.2, $B_{soft} = 138\ M_W$, and there is no region allowed for $m_1 > B_{soft}$. The meaning of B_{soft} is then clear: For given ϕ, B_{soft} is the absolute upper bound on m_1 with respect to $0 < \tilde{v}_3 \leq \frac{M_W}{2g_2}$. That is, B_{soft} is the upper bound on m_1 when $\tilde{v}_3 = \frac{M_W}{2g_2}$, and the upper bound on m_1 gets smaller than B_{soft} when $\tilde{v}_3 < \frac{M_W}{2g_2}$. In fact, B_{soft} can be interpreted as the absolute upper bound (with ϕ fixed) on all the soft-breaking parameters: $m_1,\ m_2,\ \tilde{m}_{12}$, because $m_1 \simeq \tilde{m}_{12}$ is true in the large-m_1 regime of triviality surface. This can be easily understood by the fact that $h^2 \propto (\tilde{m}_{12}^2 - m_1^2)$, and the fact that h^2 becomes negligible in the large-m_1 regime.

The full dependence of B_{soft} on ϕ is displayed in Fig.3, where $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$ have identical results. Fig.3 together with Fig.2 forms the complete picture of the triviality upper bound on the soft-breaking parameters. For example, $B_{soft} = 2380\ M_W$ at $\phi = 0$, and $B_{soft} = 84.6\ M_W$ at $\phi = \frac{\pi}{2}$. Furthermore, all the conclusions about B_{soft} can be re-interpreted as the absolute triviality upper bound (with ϕ fixed) B_H on the heaviest Higgs by $B_H \simeq \sqrt{2}\, B_{soft}$, (38). Thus, Fig.3 also provides the complete picture of the absolute triviality upper bound on the heaviest Higgs. The determination of Fig.3 is very sensitive to the present knowledge of the lower bound on the Higgs mass. Fig.3 is obtained by requiring all the Higgs triviality bounds $\geq\ 1\ M_W$, and $B_{soft}\ =\ 2380\ M_W$ at $\phi\ =\ 0$. However, $B_{soft}\ =\ 4.88\ M_W$ at $\phi\ =\ 0$ will be obtained if one can require all the Higgs triviality bounds $\geq\ 2\ M_W$.

Inspired by Fig.2, we can define the absolute triviality lower bound B_N on $\tilde{v}_3$ for given ϕ. For example, $B_N\ =\ 0.7\ M_W$ in Fig.2. B_N then gives a modest measure of the constraint on $\tilde{v}_3$. Fig.4 displays the full dependence of B_N on ϕ for $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$. The dotted straight line corresponds to the absolute upper bound of $0.749\ M_W$ on $\tilde{v}_3$, (28). In the case $\phi\ =\ 0$ (i.e., no CP-violation in the scalar sector), we have $0.24\,M_W\ \leq\ \tilde{v}_3\ \leq\ 0.749\,M_W$. In the case $\phi\ =\ \frac{\pi}{2}$, $0.65\,M_W\ \leq\ \tilde{v}_3\ \leq\ 0.749\,M_W$. Thus, the consideration of triviality does lead us to a good understanding of $\tilde{v}_3$. In addition, if the experimental lower bound on the Higgs mass is raised in the future, all the bounds involved in Fig.3 and Fig.4 will only get stronger, which implies a better understanding of the heaviest Higgs mass, the Higgs singlet vacuum expectation value, and the soft-breaking parameters. However, NMSSM will not be consistent with an unlimited raise of the experimental lower bound on Higgs. In Section 5, we will derive an absolute upper bound of $2.8\ M_W$ on the lightest Higgs mass.

Finally, let's check the assumption of negligible f_t^2 and g_i^2. With the help of [15], the assumption of negligible g_i^2 is well satisfied by all the calculations involved in Figures 1-4. To check the assumption of negligible f_t^2, take the mass of top quark $m_t\ =\ 170$ GeV. Generally speaking, this assumption is reasonable in the small-m_1 regime, but needs modifications in the large-m_1 regime because h^2 and λ^2 are small, (37). About Fig.4, the calculations of B_N indicates f_t^2 is negligible. However, the calculations of B_{soft} in Fig.3 indicate f_t^2 is as important as h^2 and λ^2. To understand the effect of f_t^2 on B_{soft}, refer to (6)-(8). Because all the coefficients of f_t^2-terms in (6)-(8) are positive (assuming negligible g_i^2), the inclusion of f_t^2 will only make triviality even stronger. That is, the Landau pole t_L will get smaller if f_t^2 is included. Qualitatively, this implies that the calculated B_{soft} should get smaller (i.e., a stronger upper bound) if f_t^2 is included. Generally speaking, all the triviality bounds will get stronger when f_t^2 is included. In other words, the results of Fig.3 should be regarded as a weak absolute upper bound on the soft-breaking parameters and the heaviest Higgs boson.

5 Absolute Upper Bound on the Lightest Higgs

With the inclusion of the Higgs singlet in NMSSM, the tree-level upper bound of M_{Z^0} on the mass of the lightest Higgs of MSSM is no longer valid. It is, therefore, of considerable importance to study the triviality upper bound on the lightest Higgs in order to devise effective search strategies for the detection of Higgs particles. As before, for given ϕ, we search for the absolute triviality upper bound B_{LH} on the lightest Higgs through all possible m_1 $(= m_2)$ and $\tilde{v}_3$.

However, (38) implies a search in the small-m_1 regime is enough, and the resultant full dependence of B_{LH} on ϕ is displayed in Fig.5, where line A and line B correspond to (h_A, λ_A) and (h_B, λ_B). It is then verified that the assumption of negligible f_t^2 and g_i^2 is satisfied by Fig.5.

In the case $\phi = 0$ (i.e., no CP-violation in the scalar sector), the absolute upper bound $B_{LH} = 2.8\ M_W$ from line B. In the case $\phi = \frac{\pi}{2}$, $B_{LH} = 1.75\ M_W$ from line B. Thus, the absolute triviality upper bound on the lightest Higgs does lie outside the range of LEP.

6 Conclusion

By a complete study of triviality surface defined in the parameter space of NMSSM, we were able to derive the triviality bounds on the heaviest Higgs, the soft-breaking parameters, the Higgs singlet vacuum expectation value in Section 4, and the absolute upper bound on the lightest Higgs in Section 5. Essentially, all the triviality bounds are derived based on the observations (37) and (38), where the mass of the lightest Higgs monotonically decreases to zero in the large-m_1 limit.

The limited choice of the soft-breaking potential V_{soft} in (2) can be viewed as an unsatisfactory feature of the present formulation. However, the triviality bounds derived in Sections 4 and 5 persist even if a more general V_{soft} is considered. We begin the argument of the above statement with the most general V_{soft} [10, 15]:

$$\begin{aligned} V_{soft} &= m_1^2 \Phi^\dagger{}_1 \Phi_1 + m_2^2 \Phi^\dagger{}_2 \Phi_2 - m_{12}^2 \Phi^\dagger{}_1 \Phi_2 - m^{*2}_{12} \Phi^\dagger{}_2 \Phi_1 \\ &\quad + m_4^2 N^* N + m_5^2 N^2 + m^{*2}_5 N^{*2} \\ &\quad + h\, m_3 (A_1 \Phi_1^\dagger \Phi_2 N + A^*{}_1 \Phi_2^\dagger \Phi_1 N^*) \\ &\quad + \frac{1}{3}\, \lambda\, m_3 (A_2 N^3 + A^*{}_2 N^{*3}) \end{aligned} \tag{39}$$

Now, the relevant parameter space consists of:

$$(m_1^2,\ m_2^2,\ m_3,\ m_4^2,\ m_5^2,\ m_{12}^2,\ A_1,\ A_2) \tag{40}$$

plus three complex constraints on the parameters derived from the minimization of $V' = V + V_{soft}$. Choosing $\tan\beta = 1$, again we have $m_1^2 = m_2^2$ from V'-minimization. In the limit of large $\Lambda_L\,(= B_H)$, the Landau pole (13) always implies:

$$h^2 \to 0 \text{ if } \Lambda_L\,(= B_H) \to \infty \tag{41}$$

In the large-$m_{heaviest\,Higgs}$ limit (for example, in the large-m_1 limit), (41) then implies that the Higgs mass^2-matrices $[M_n^2]$ and $[M_c^2]$ can be solved up to order $O(h)$:

$$\begin{aligned} \text{Eigenvalues of } [M_n^2] &= [2m_1^2 + O(h),\ 2m_1^2 + O(h),\ m_{(+)} + O(h),\ m_{(-)} + O(h),\ O(h),\ 0] \\ \text{Eigenvalues of } [M_c^2] &= [2m_1^2 + O(h),\ 2m_1^2 + O(h),\ 0,\ 0] \end{aligned}$$

$$m_{(\pm)} = m_4^2 + 4\lambda^2|v_3|^2 \pm 2|m_5^2 + \lambda m_3 v_3 A_2 + \lambda^2 v_3^{*2}| \tag{42}$$

With the most general V_{soft} (39), there is, in general, exactly one Higgs staying light in the large-$m_{heaviest\ Higgs}$ limit. Similar to (38), the mass of this lightest Higgs is proportional to $O(h)$, and decreases to zero in the large-$m_{heaviest\ Higgs}$ limit. This observation then implies that the analysis made in Sections 4 and 5 still applies to the most general $V' = V + V_{soft}$. That is, the triviality bounds on the heaviest Higgs, the lightest Higgs, the singlet vacuum expectation value, and the soft-breaking parameters are not lost even in the largest possible parameter space of NMSSM.

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FIGURE CAPTIONS

Fig.1: A plot of the triviality upper bound B_H on the heavest Higgs versus m_1^2 for $\phi = 0$, $\tilde{v}_3 = 0.7\,M_W$, and $(h,\ \lambda) = (h_A,\ \lambda_A)$. The unit of the vertical axis is M_W, where M_W=80 GeV. The unit of the horizontal axis is M_W^2.

Fig.2: A plot of the allowed range of $\tilde{v}_3$ versus m_1 (the enclosed region) for $\phi = 0.6$, $(h,\ \lambda) = (h_A,\ \lambda_A)$. The allowed range of $\tilde{v}_3$ shrinks to a point at $m_1 = B_{soft}$=138 GeV. The unit of both the axes is M_W.

Fig.3: The plot of the absolute triviality upper bound B_{soft} versus ϕ, where the two solutions $(h_A,\ \lambda_A)$ and $(h_B,\ \lambda_B)$ have identical results. B_{soft} is periodic in ϕ of periodicity π. The unit of the vertical axis is M_W.

Fig.4: The plot of the absolute triviality lower bound B_N on $\tilde{v}_3$ versus ϕ, where the dashed line corresponds to $(h_A,\ \lambda_A)$ and the solid line corresponds to $(h_B,\ \lambda_B)$. B_N is periodic in ϕ of periodicity π. The dotted line corresponds to the absolute upper bound of $\frac{M_W}{2g_2}$ on $\tilde{v}_3$. The unit of the vertical axis is M_W.

Fig.5: The plot of the absolute triviality upper bound B_{LH} on the lightest Higgs versus ϕ, where the dashed line corresponds to $(h_A,\ \lambda_A)$ and the solid line corresponds to $(h_B,\ \lambda_B)$. B_{LH} is periodic in ϕ of periodicity π. The unit of the vertical axis is M_W.

Fig.1

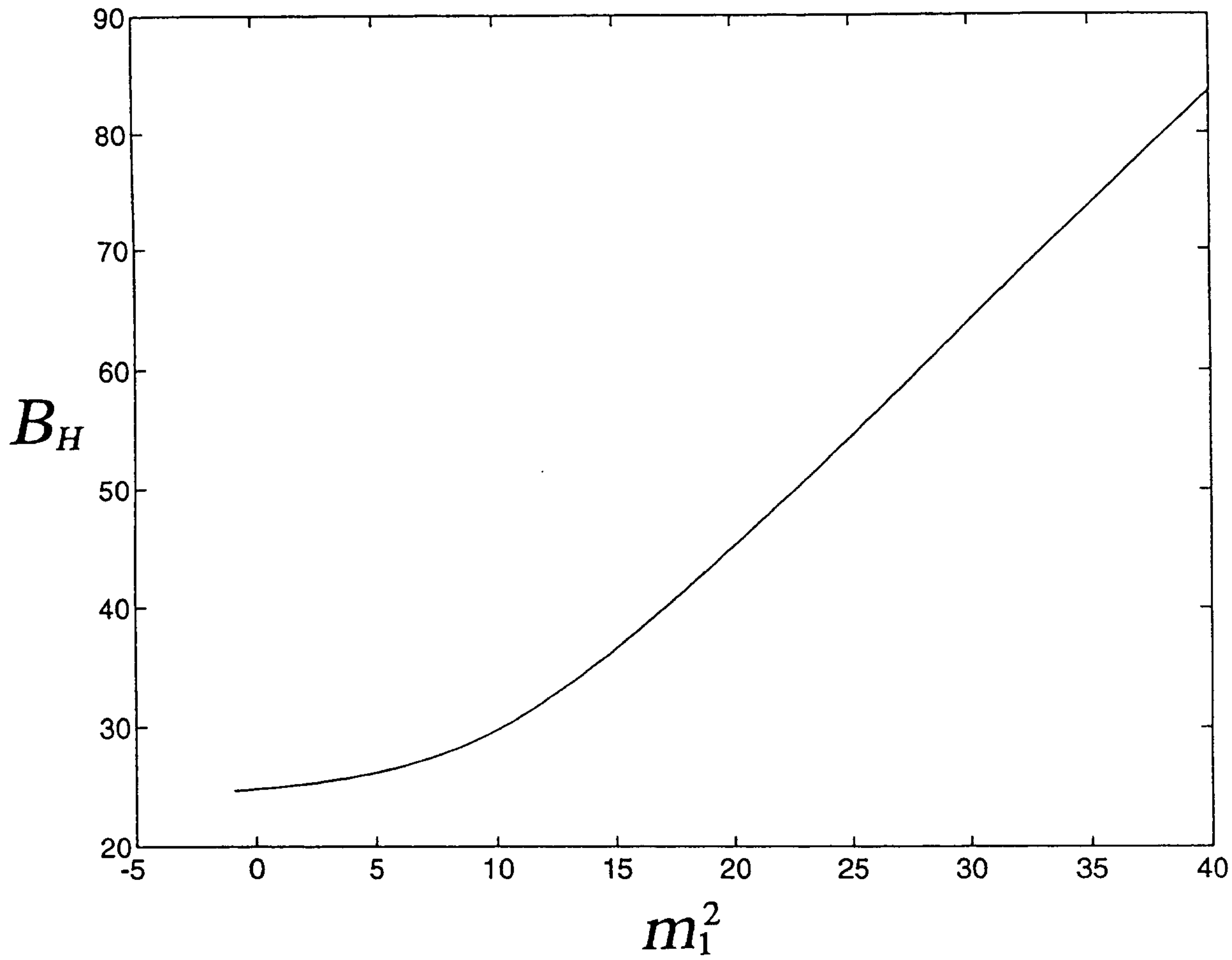

Fig.2

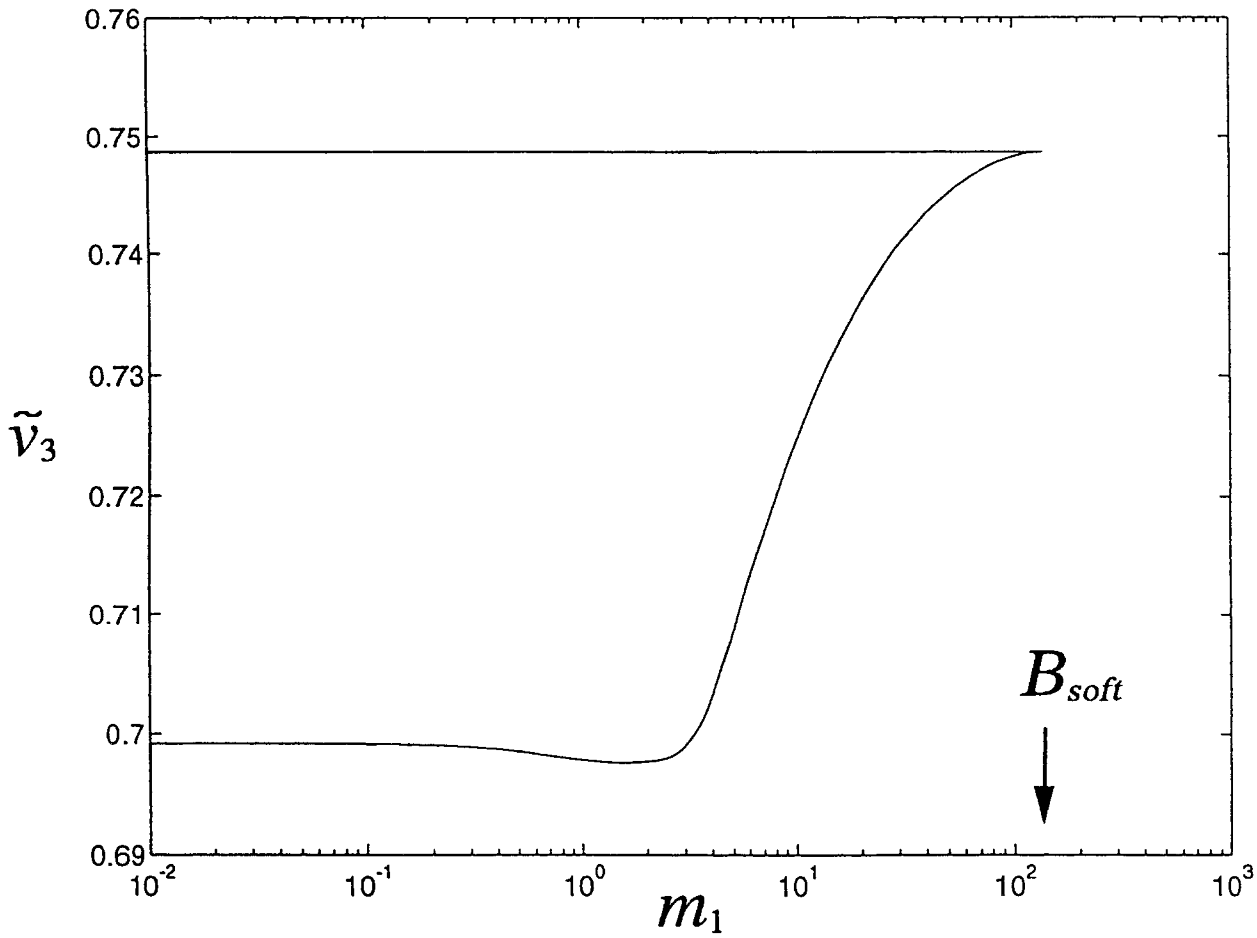
0.76
0.75
0.74
0.73
0.72
0.71
0.7
0.69
$\tilde{v}_3$
B_{soft}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
m_1

Fig.3

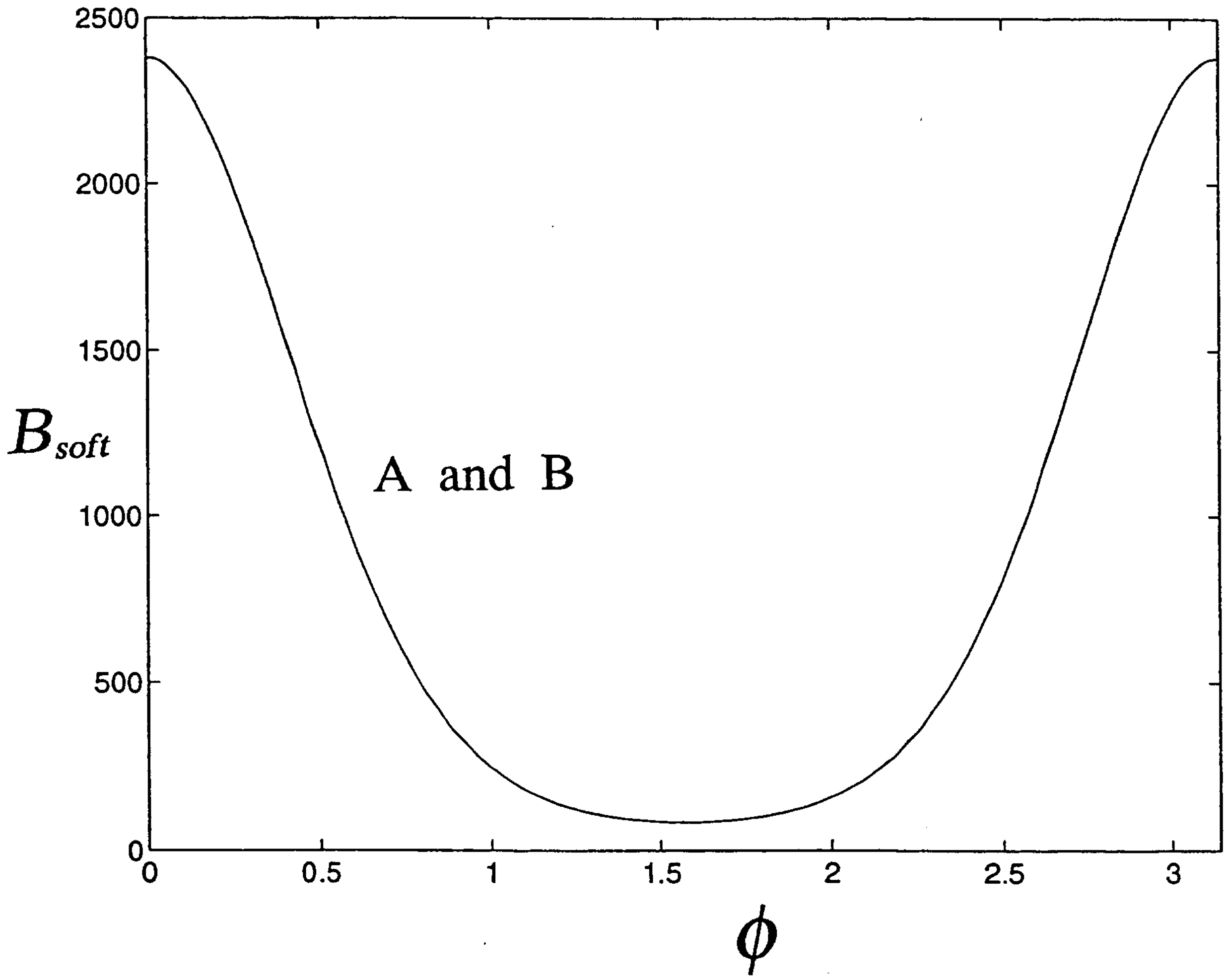

Fig.4

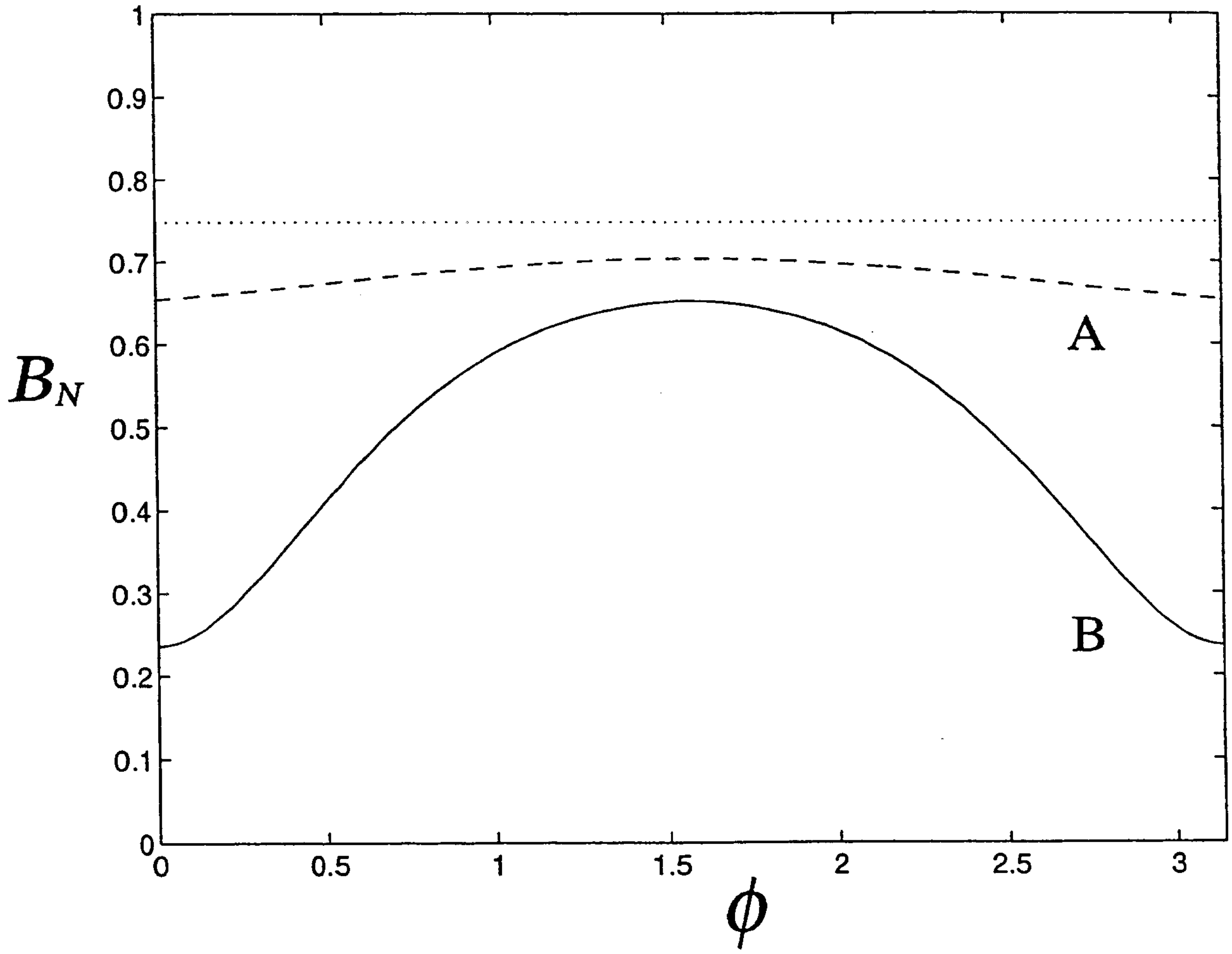

Fig.5