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A Study on the Applicability of Perturbative QCD to the Process $B \to \pi\pi$

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Abstract

The applicability of perturbative QCD (PQCD) to $B \to \pi\pi$ is discussed. By analyzing the previous approximate calculation we find that PQCD is less reliable due to the singularities and there is no good reason to say that PQCD can be applied legitimately to the process $B \to \pi\pi$.To estimate the higher Fock-state contributions is suggested.

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I Introduction

Recently, weak decays of heavy hadrons are under intense study. The main difficulty in weak decays of heavy hadrons is how to exactly calculate the hadronic matrix elements from the first principle.We have heavy quark symmetry theory [1] which leads to predictive relations between the form factors involving different heavy quarks in the initial and final states respectively. Heavy quark effective theory [2] provides a theoretical framework to include the leading symmetry breaking corrections in an expansion in $1/m_Q$. These relations are model-independent in the limit of infinite quark mass. Also we have chiral perturbation theory to deal with the light quark which has a low energy compared with the chiral symmetry breaking scale. However, in the case of heavy to light mesons, we can't apply the combined heavy quark symmetry-chiral symmetry theory (heavy meson chiral perturbation theory)[3] in most decays since the light meson has a large energy compared with the chiral symmetry breaking scale.

Now the decays of B into charmless hadrons are quite interesting for studying CKM matrix element and CP violation.At present,there are several approaches. For example,the BSW(Bauer-Stech-Wirbel)approach by assuming the nearest pole dominance and using the model wave function; Perturbative QCD approach by calculating the lowest order of valence-quark diagrams. The QCD exclusive theory which was developed by Lepage and Brodsky[4], is an elegant theory derivable from QCD. Therefore, it is very interesting to explore the possibility that the PQCD exclusive theory can be applied to weak decays of heavy hadrons.

In the last three years , there are some attempts [5-9] to apply the formalism to exclusive weak decays of B meson. As observed by Brodsky et.al [5], the mass of heavy meson establishes the relevant momentum scale $Q^2 \sim m_B^2$, so that the large internal momentum transfer is involved and the PQCD becomes applicable. Unfortunately, the numerical results in [5,6] show that the lowest nontrivial order perturbative amplitude is very small and far below that of the BSW approach [10,11]. As Chernyak and Donoghue et.al pointed out [6,12,13], too small results seem to imply that the perturbative hard scattering is sub- dominant and asymptopia far away. Since in [5], the on-shell heavy quark was recongnized as violation of the factorization , a cut-off has to be introduced to exclude the contribution from the singularity region of the on-shell heavy quark, the PQCD result is small. Later on, Carlson et. al [9] argue that the on-shell heavy quark in the hard scattering travels only a short distance and the factorization of the formalism still holds. Therefore, one can enhance the whole results up to the magnitude comparable to the result of the BSW approach[10,11] by using different distribution amplitude of B meson and the on-shell heavy quark, which produces a larger imaginary part. It seems to tell us that the perturbative result is dominant and the PQCD becomes applicable . In this paper, we will discuss the applicability of the PQCD to exclusive decays of B meson. To be convenient and specific, we will confine our attention to the process $B \to \pi\pi$.

This paper is organized as following: The section II gives a brief review of the previous analysis about applying PQCD to $B \to \pi\pi$ and their main approximations.Our analysis of the applicability of PQCD to this process is presented in Sec III. The summary and comments are given in the last section.

II Brief review [5,9]

In order to carry out our analysis , we should give the necessary review of the previous analysis and show the key issues in applying PQCD to the process $B \to \pi\pi$.

Ignoring the final state interactions and assuming the factorization of the nonleptonic decays $B \to \pi\pi$, which reduces the hadronic matrix element of four-quark operator to a product of current matrix elements, one can write down the decay amplitude of the process $B \to \pi\pi$ as:

$$M(B \to \pi\pi) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud} < \pi | j^\mu | 0 >< \pi | J_\mu | B > = i G_F V_{ub} V_{ud} f_\pi P_\pi^\mu < \pi | J_\mu | B >, \quad (1)$$

where

$$j^\mu = \bar{u}\gamma^\mu (1-\gamma_5) d, \quad (2)$$

$$J_\mu = \bar{u}\gamma^\mu (1-\gamma_5) b \quad (3)$$

$$< \pi | j^\mu | 0 > = i\sqrt{2} f_\pi P_\pi^\mu. \quad (4)$$

Following the notation of [10], we decomposite

$$< \pi | J_\mu | B > = (P_B^\mu + P_\pi^\mu - \frac{M_B^2 - m_\pi^2}{q^2} q^\mu) F_L(q^2) + \frac{M_B^2 - m_\pi^2}{q^2} q^\mu F_T(q^2). \quad (5)$$

As in the PQCD analysis of electromagnetic form factor of meson at large momentum transfer, the lowest order Feynman diagrams of $B \to \pi\pi$ are given in Fig.1 .

The kinematic factors in Fig.1 are

$$k_1 = q + yP_\pi, \tag{6}$$

$$k_2 = xP_B - q, \tag{7}$$

and

$$Q = (1-x)P_B - (1-y)P_\pi \tag{8}$$

The wavefunctions of B,π are respectively

$$\psi_B = (\not{P}_B + m_B)\gamma_5\phi_B(x) \tag{9}$$

and

$$\psi_\pi = \not{P}_\pi\gamma_5\phi_\pi(y), \tag{10}$$

where distribution amplitudes ϕ_B and ϕ_π are related to decay constants f_B and f_π respectively.

$$\int dx\phi_B(x) = \frac{1}{2\sqrt{3}}f_B, \int dy\phi_\pi(y) = \frac{1}{2\sqrt{3}}f_\pi. \tag{11}$$

Therefore, we can obtain from Fig.1

$$< \pi|J_\mu|B > = \frac{8\pi}{3}\cdot\int dxdy\frac{\alpha_s(\bar{Q^2})}{Q^2}\phi_B(x)\phi_\pi(y)[\frac{Tr(\not{P}_\pi\gamma_5\gamma_\mu(\not{k}_1+m_b)\gamma_\nu(\not{P}_B+m_B)\gamma_5\gamma^\nu)}{k_1^2-m_b^2} + \frac{Tr(\not{P}_\pi\gamma_5\gamma_\nu\not{k}_2\gamma_\mu(\not{P}_B+m_B)\gamma_5\gamma^\nu)}{k_2^2}]. \tag{12}$$

The form factor is given by

$$F_T(q^2) = \frac{32\pi\alpha_s(\bar{Q^2})}{3(m_B^2-m_\pi^2)}\int dxdy\frac{\phi_B(x)\phi_\pi(y)}{Q^2}T_H \tag{13}$$

where

$$T_H = \frac{[2m_Bm_b - m_B^2y + (y-1)m_\pi^2](m_B^2-2m_\pi^2) - 2m_\pi^4(1-y)}{ym_B^2-m_b^2} + \frac{(1-2x)m_\pi^2}{x(x-1)}, \tag{14}$$

and

$$Q^2 = (1-x)^2 m_B^2 - (1-x)(1-y)m_B^2. \tag{15}$$

In order to analyze the applicability of the PQCD theory, let's first consider the singularities in the $F_T(q^2)$. Obviously, one must eliminate or suppress the singularities associating with violation of the formalism factorization [4,14,15].There are the following singularities:

(1)The singularity produced by the on-shell heavy quark (b quark):$k_1^2 - m_b^2 = 0$, as argued by Carlson et.al.[9] , the on-shell heavy quark travels only a short distance and can't destroy the factorization, therefore, it only produces an imaginary part.

(2)The singularity produced by the on-shell light quark:$k_2^2 = 0$,which leads to $x \simeq 0, 1$;they are end points of $\phi_B(x)$. As in the analysis of the electromagnetic form factor of the pion [4,14,15], they can expect to be suppressed by distribution amplitudes of B meson.

(3)The singularity produced by the on-shell gluon: $Q^2 = 0$,which leads to $x = 1$ and $x = y$(see Fig.2).

Obviously, x=1 belongs to the end-point problem and can be eliminated as above. Because of the dominance of $x \to 1$ in B meson distribution amplitude, as suggested by Brodsky et.al.in [5], one can neglect the $(1-x)^2$ factors in Q^2 so that the singularity y=x reduces to $y \approx 1$ and can be suppressed by the pionic distribution amplitude.

In summary, in order for the PQCD to be applicable , one has to eliminate these singularities associating with the long distance . It requires the distribution amplitude of B meson must have the behaviors:

$$as\ x \to 1, \quad \phi_B \to (1-x)^n, \quad n \geq 2$$

$$and \tag{16}$$

$$as\ x \to 0, \qquad \phi_B \to x^m, \qquad m \geq 1 \tag{17}$$

The distribution amplitude of B meson given in [5] meets the above requirement.

$$\phi_B = \frac{A_B x^2(1-x)^2}{[\epsilon^2 x + (1-x)^2]^2}. \tag{18}$$

where the parameter ϵ is the fraction momentum of the light quark in B meson, we use the phenonmenlogical relation [11] :

$$\epsilon = \frac{m_q}{m_q + m_b}, \tag{19}$$

where m_q is the constituent mass of the light quark q in the B meson and $m_q \approx 0.35 Gev$ [10,11] . Obviously, ϵ is small and consistent with $\epsilon \sim 0.05 \sim 0.1$ in [5] . The parameter A_B is determined by

$$\int dx \phi_B(x) = \frac{f_B}{2\sqrt{3}}. \tag{20}$$

For the pionic distribution amplitude , asymptotic form ϕ_π^a [4] and Chernyak-Zhitnitsky form ϕ_π^{cz} [16] are taken:

$$\phi_\pi^a(y) = \sqrt{3}\, f_\pi y(1-y) \tag{21}$$

and

$$\phi_\pi^{cz}(y) = 5\sqrt{3} f_\pi y(1-y)(2y-1)^2. \tag{22}$$

All of these distribution amplitudes will be taken in the following analysis.

III Present analysis and Numerical results

The key point in Ref.[5] was to make an approximation:

$$Q^2 = (1-x)^2 M_B^2 - (1-x)(1-y)M_B^2 \approx Q_0^2 = -(1-x)(1-y)M_B^2 \tag{23}$$

to eliminate the singularity produced by the on-shell gluon so that the all of singularities occur only at the end-point regions and can be suppressed by the distribution amplitude of B and pion mesons. However , from Fig.2 , one can observe that this approximation completely eliminates the timelike gluon's contribution (in the region II) and enlarges the spacelike gluon's contribution from region I to region II.

In order to justify whether this approximation is reasonable , let's first come back to analyze the starting point of this formalism. The hadronic matrix element between B meson and π meson is obtained by evaluating the initial and final state B ,π meson's wavefunctions with a hard amplitude and the hard amplitude can be calculated in the collinear approximation. Therefore, the transverse momentum through gluon has been ignored and is regarded as a higher twist term.However , if taking account of them , one has

$$Q^2 = (1-x)^2 M_B^2 - (1-x)(1-y)M_B^2 - (K_{\perp\pi} - K_{\perp B})^2. \tag{24}$$

Actually, the transverse momentum in Q^2 can be ignored only if

$$|Q^2 = (1-x)^2 M_B^2 - (1-x)(1-y)M_B^2| >> < K_\perp^2 > = < (K_{\perp\pi} - K_{\perp B})^2 > = < K_\perp^2 >_\pi + < K_\perp^2 >_B . \tag{25}$$

As pointed out by Ref.[17,18,19], in the "soft" gluon's region : $|Q^2| \leq < K_\perp^2 >$, the higher twist terms enter and the coupling constant of the strong interaction is large and it seriously violates the assumption that the momentum transfer proceeds perturbatively , the calculation is unreliable .Therefore , if the main contribution comes from this region, the calculated branch ratio with this formalism will be unreliable and less believable. In order to justify whether the main contribution comes from this region , let's here assume the approximation $Q^2 \approx Q_0^2$ works well , and the branch ratio of the process $B \to \pi\pi$ is

$$Br(B \to \pi\pi) \sim |F_{Q_0}|^2 = |F_{Q_0}^{cut}|^2 + F_{Q_0}^{cut} \cdot F_{Q_0}^{s*} + F_{Q_0}^{cut*} \cdot F_{Q_0}^{s} + |F_{Q_0}^{s}|^2 \tag{26}$$

and

$$F_{Q_0} = \frac{32\pi\alpha_s(\bar{Q}^2)}{3(m_B^2 - m_\pi^2)} \int dx dy \; \frac{\phi_B(x)\phi_\pi(y)}{Q_0^2} \cdot T_H \tag{27}$$

with

$$F_{Q_0}^{cut} = F_{Q_0}|_{|Q_0^2| \geq < K_\perp^2 >} \tag{28}$$

$$F_{Q_0}^{s} = F_{Q_0}|_{|Q_0^2| \leq < K_\perp^2 >} \tag{29}$$

Then we can define the difference between $|F_{Q_0}|^2$ and $|F_{Q_0}^{cut}|^2$:

$$\Delta = |F_{Q_0}|^2 - |F_{Q_0}^{cut}|^2 \tag{30}$$

which contributes from the region where the higher twist terms enter and PQCD is seriously unreliable(hereafter,we will call Δ "unreliable" contribution).

As argued above , the only reliably calculated quantity is $|F_{Q_0}^{cut}|^2$ .i.e. the contribution of the "hard" gluon with $|Q_0^2| \geq < K_\perp^2 >$. However , this "hard" gluon's contribution is obtained under the approximation $Q^2 \approx Q_0^2$. In fact , the correct "hard" gluon's contribution should be

$$|F_Q^{cut}|^2 = |F_{T(|Q^2| \geq < K_\perp^2 >)}|^2 \tag{31}$$

where

$$F_T(q^2)|_{|Q^2| \geq < K_\perp^2 >} = \frac{32\pi\alpha_s(\bar{Q}^2)}{3(m_B^2 - m_\pi^2)} \int_{|Q^2| \geq < K_\perp^2 >} dxdy \frac{\phi_B(x)\phi_\pi(y)}{Q^2} \cdot T_H \tag{32}$$

As pointed out before , in order to eliminate the divergence caused by the on-shell gluon , one has to make the approximation $Q^2 \approx Q_0^2$ .But if this approximation is good , it shouldn't affect the "hard" gluon's contribution .e.g. the difference between the calculated contribution of "hard" gluon with Q^2 and Q_0^2 respectively

$$\delta F^{cut} = F_{Q_0}^{cut} - F_Q^{cut} \tag{33}$$

should be small enough , to be more specific , this requires the quantity

$$\Omega = |F_{Q_0}^{cut}|^2 - |F_Q^{cut}|^2 \tag{34}$$

should be small (obviously, this requirement is not stronger than the above requirement).

The form factors F_{Q_0},$F_{Q_0}^{cut}$,F_Q^{cut} and δF^{cut} are listed in the table 1 and the table 2, and the quantity $R_1 = \frac{|F_{Q_0}|^2 - |F_{Q_0}^{cut}|^2}{|F_{Q_0}|^2}$ gives the ratio of the "unreliable" contribution in the PQCD calculation to $Br(B \to \pi\pi)$, $R_2 = \frac{|F_{Q_0}^{cut}|^2 - |F_Q^{cut}|^2}{|F_{Q_0}^{cut}|^2}$ shows how large the difference between contributions of two "hard" gluon with Q^2 and Q_0^2 respectively is . From the table 1 , for $\phi_\pi = \phi_\pi^a$, the "unreliable" contribution in $Br(B \to \pi\pi)$ is less important. (for example , $|R_1| < 50\%$)

only as $m_b \leq 4.9Gev$. However, at the same time , if the approximation $Q^2 \approx Q_0^2$ is required to be good too(for example,$|R_2| < 50\%$) , the parameter m_b only can be taken,

$$m_b \sim 4.8 \sim 4.9Gev \tag{35}$$

Although the "unreliable" contribution is less important , it is still large ($R_1 \sim 40\%$ for $m_b \sim 4.8 \sim 4.9Gev$). For $\phi_\pi = \phi_\pi^{cz}$ in the table 2 ,it can be seen that there is no parameter m_b which makes the ratios R_1 and R_2 are simultaneously small . The reason is very simple, since the C-Z distribution amplitude of the pion has two peaks near the end-point region , its main contribution comes from the end-point region as in the pionic electromag netic form factor [17] although it produces a large branch ratio $Br(B \to \pi\pi)$ .The situation is even worse for the CZ distribution amplitude.

In the numerical estimate , for example, $\alpha_s = 0.4$, $m_B = 5.28Gev$, $f_\pi = 0.093Gev$ and $f_B = 0.1Gev$ are taken. For $< K_\perp^2 >_\pi$,$< K_\perp^2 >_B$, one inputs $< K_\perp^2 >_\pi = < K_\perp^2 >_B \sim (350MeV)^2$,and $< K_\perp^2 > = < K_\perp^2 >_B + < K_\perp^2 >_\pi = (500Mev)^2$. It should be pointed out that our conclusion is independent of input parameters . For example , by changing $< K_\perp^2 > = 2 \cdot (300Mev)^2$, in order to keep the "unreliable" contributions small ($|R_1| << 50\%$) and the approximation $Q^2 \approx Q_0^2$ is good , m_b can be taken value only at the narrow interval $4.9 \sim 5.0Gev$ ($R_1 \sim 30\%$) for $\phi_\pi = \phi_\pi^a$. As $\phi_\pi = \phi_\pi^{cz}$, PQCD is impossible to find the valid parameter m_b.

To sum up , only as m_b is taken some very special value and $\phi_\pi = \phi_\pi^a$, the "unreliable" contribution is less important. However , as pointed above , although the "unreliable" contribution is less important,it is still comparable to the contribution from the "hard" gluon . Second , as a reliable calculation, in principle , it doesn't require that m_b must be taken only for some very special value , in other words , its applicability shouldn't be so dependent on the input parameters such as m_b . Third , even through assuming PQCD is self consistent here as m_b is taken these very special value and $\phi_\pi = \phi_\pi^a$, the lowest order result of PQCD still is only $\frac{1}{2}$ or $\frac{1}{3}$ of values of the BSW approach [10,11] and QCD sum rules[20,21,22]. One can't prove that,in the leading order and the next-leading order, the formalism is self-consistent , and more importantly, one can't prove that the next-leading order is smaller than the leading order,and the higher twist terms can be ignored , and ther is no evidence that the the higher Fock state contributions can be ignored .In contrary , the results of the BSW approach and QCD sum rules are far larger than the lowest order PQCD result , which obviously tell us the contribution of "soft" wavefunctions are possible dominate over the perturbative one[23].

Finally, let's ask a question: if treating the on-shell heavy quark as in [5], how about the applicability of PQCD to $B \to \pi\pi$? In [5], the on-shell heavy quark was recongnized as violation of the factorization. One has to make an approximation $m_b \approx M_B$ and introduce a parameter ϵ to cut off this singularity at $y \sim 1$. At this time, first, one doesn't know how good the approximation $m_b \approx M_B$ is , here let's assume it good. Same as above, in the table 3, we calculate all of the form factors and ratios with the cut-off parameter ϵ as in [5]. One can observe the approximation $Q^2 \approx Q_0^2$ is poor for $\phi_\pi = \phi_\pi^{cz}$ and the "unreliable" contribution for both ϕ_π^a and ϕ_π^{cz} still be sizable , especially for small ϵ . However, for large ϵ , F_{Q_0} will become very small, for example, $\epsilon = 0.1$, $F_{Q_0} = 0.0377$, far below that of other methods such as the BSW approach and QCD sum rules. As argued above, we conclude that there is no good reason to expect that PQCD is applicable.as the results are sensitive to the cutoff parameter ϵ.

IV. Summary and Comments

In summary,it is interesting to study the heavy meson decay into light mesons for extrating the information about the CKM matrix element and the CPviolation.The applicability of perturbative QCD to $B \to \pi\pi$ is studied in this paper.For applying PQCD to the process $B \to \pi\pi$, one has to make an approximation $Q^2 \approx Q_0^2$ to eliminate the divergence caused by the on-shell gluon with the distribution amplitudes of mesons. But for applying PQCD legitimately , one should keep the approximation reasonable and the "unreliable"contribution small , which comes from the region where higher twist terms enter and PQCD is unreliable . By the numerical analysis ,we find that it is difficult to expect that one can apply PQCD legitimately to the process $B \to \pi\pi$. As in the pionic electromagnetic form factor, the C-Z distribution amplitude of the pion can produce a large branch ratio for the process $B \to \pi\pi$ but its main contribution comes from the region where perturbative theory is a priori unreliable. Finally, we give a few comments.

i) From tables 1-2, one can find PQCD results in the lowest order is very sensitive to the m_b :the b quark mass. This remind us suspect the travel of the on-shell heavy quark is not stable because this sensitivity is caused by the on-shell heavy quark to produce a large imaginary part . If the on-shell heavy quark travel only at a short distance , its travel (or its interaction) shouldn't be so dependent on its mass , the result of PQCD shouldn't keep relatively stable against vary of the m_b.

ii) It should be emphased that the situation here is very different from in the pionic electromagnetic form factor. In the case of the pionic electro magnetic form factor , the initial and final states are same , therefore , the exchange gluon in the hard scattering kernel is only spacelike , and the on-shell gluon occurs only at the end-points of pionic distribution amplitude. However,the exchange gluon can be spacelike and timelike in the $B \to \pi\pi$ since the initial and final states are quite different(the heavy and light mesons) .The on-shell gluon can't occur naturally at the end-point region of meson distribution amplitudes so that PQCD analysis is unreliable for the process $B \to \pi\pi$.The reason is that Q^2 is not large enough.It may be expected to be valid for the top quark meson.

iii) One should find a reliable model-independent way to calculate the heavy meson into light mesons in the future. At present, one only consider the valence -state contribution both in the BSW approach and PQCD calculation.However PQCD calculation doesn't dominate this kind of processes and the BSW approach by only using the valence-state contribution doesn't dominate too[10,11]. To estimate the higher Fock-state contributions is a key point to understand the heavy meson decay into light mesons,since the probability of finding the valence-state in the light meson is much less than unity. Ref.[10]and [11] have different ansatz to take into account for the higher Fock-state contributions.

Notes added:

While this paper was being written, we became aware of the work by Ward [24] on $B \to \pi\pi$. As in [9] , by the on-shell heavy quark to produce a large imaginary part and the approximation $Q^2 \approx Q_0^2$ to eliminate the singularities, he calculated the contribution from tree diagrames and penguin diagrams with enough accurancy . We disagree with his conlusion. As one see above, PQCD can't be legitimately applied to $B \to \pi\pi$ for one can't keep the approximation $Q^2 \approx Q_0^2$ good and the "unreliable" contribution small.

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Figure Captions

Fig.1(a),(b): The lowest order Feynman diagrams for $B \to \pi\pi$ in PQCD.

Fig.2: The integration domain for x,y (Region I+II): Region I represents spacelike gluon , Region II represents timelike gluon ; $x = 1$ and $y = x \to Q^2 = 0$ (the on-shell gluon).

Table 1 : Form factors and ratios with ϕ_π^a for the finite heavy quark (form factors in the unite of 10^{-2})

$m_b(Gev)$	F_{Q_0}	$F_{Q_0}^{cut}$	F_Q^{cut}	δF^{cut}	$R_1 = 1 - \frac{\lvert F_{Q_0}^{cut}\rvert^2}{\lvert F_{Q_0}\rvert^2}$	$R_2 = 1 - \frac{\lvert F_Q^{cut}\rvert^2}{\lvert F_{Q_0}^{cut}\rvert^2}$
5.2	9.17i+8.60	0.130i+4.80	-0.396i+3.19	0.527i+1.61	85%	55%
5.1	8.68i+5.77	1.10i+5.62	-0.652i+3.79	1.76i+1.83	70%	55%
5.0	8.20i+4.17	2.42i+5.71	-0.724i+4.55	3.15i+1.15	55%	45%
4.9	7.74i+3.05	3.53i+5.21	-0.695i+5.57	4.22i-0.356	43%	21%
4.8	7.29i+2.20	4.26i+4.45	-0.621i+7.42	4.88i-2.97	35%	-46%

Table 2 : Form factors and ratios with ϕ_π^{cz} for the finite heavy quark (form factors in the unite of 10^{-2})

$m_b(Gev)$	F_{Q_0}	$F_{Q_0}^{cut}$	F_Q^{cut}	δF^{cut}	$R_1 = 1 - \frac{\lvert F_{Q_0}^{cut}\rvert^2}{\lvert F_{Q_0}\rvert^2}$	$R_2 = 1 - \frac{\lvert F_Q^{cut}\rvert^2}{\lvert F_{Q_0}^{cut}\rvert^2}$
5.2	40.5i+14.8	0.576i+7.53	-1.75i+1.96	2.33i+5.56	97%	88%
5.1	32.5i+0.843	4.14i+8.85	-2.45i+3.02	6.59i+5.83	91%	84%
5.0	25.8i-5.41	7.63i+7.11	-2.28i+3.88	9.91i+3.23	84%	81%
4.9	20.2i-8.48	9.21i+4.01	-1.81i+5.06	11.0i-1.05	79%	71%
4.8	15.5i-9.79	9.07i+1.08	-1.32i+7.46	10.4i-6.38	75%	31%

Table 3 : Form factors and ratios with a cut-off to cut the singularity produced by the on-shell heavy quark as in [5].(form factors in the unite of 10^{-2}).

ϵ	F_{Q_0}	$F_{Q_0}^{cut}$	F_{Q}^{cut}	$R_1 = 1 - \frac{\|F_{Q_0}^{cut}\|^2}{\|F_{Q_0}\|^2}$	$R_2 = 1 - \frac{\|F_{Q}^{cut}\|^2}{\|F_{Q_0^{cut}}\|^2}$
$\phi_\pi = \phi_\pi^a$					
0.05	9.06	4.23	3.53	78%	30%
0.075	5.46	3.69	3.06	54%	31%
0.1	3.77	3.08	2.69	33%	24%
$\phi_\pi = \phi_\pi^{cz}$					
0.05	18.6	5.17	1.93	92%	86%
0.075	9.32	4.86	1.67	73%	88%
0.1	5.42	3.90	1.49	48%	85%

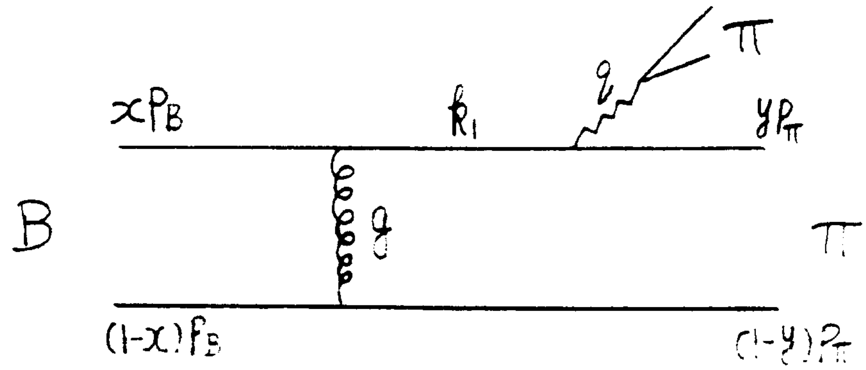

Fig.1(a)

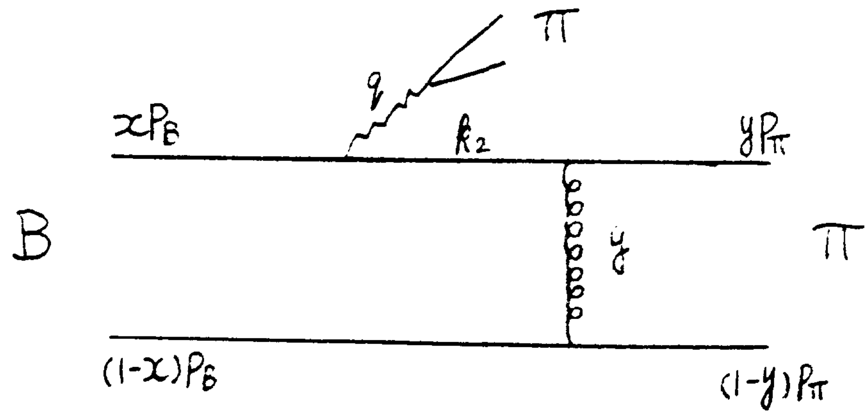

Fig.1(b)

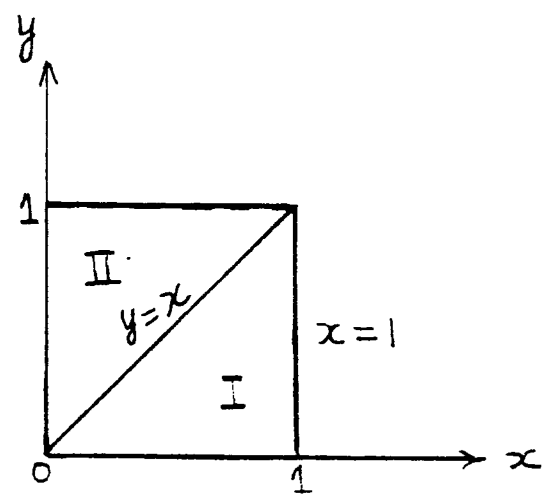

Fig.2