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APPROXIMATION OF ISOSURFACE IN THE MARCHING CUBE METHOD: AMBIGUITY ON THE FACES

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Abstract

Matveyev S.V. Approximation of isosurface in the marching cube method: ambiguity on the faces: IHEP Preprint 93-133. – Protvino, 1993. – p. 5, figs. 5, refs.: 4.

The purpose of the present article is the consideration of the problem of ambiguity over the faces [3] arising in the Marching Cube algorithm. The article shows that for unambiguous choice of the sequence of the points of intersection of the isosurface with edges confining the face it is sufficient to sort them along one of the coordinates.

Аннотация

Матвеев С.В. Аппроксимация изоповерхности в методе движущегося кубика: неоднозначность на гранях : Препринт ИФВЭ 93-133. – Протвино, 1993. – 5 с., 5 рис., библиогр.: 4.

Целью данной статьи является рассмотрение проблемы неоднозначности на гранях [3], возникающих в алгоритме движущегося кубика. В работе показано, что для однозначного выбора последовательности соединения точек пересечения изоповерхности с ребрами ограничивающими грань достаточно провести процедуру их сортировки по одной из координат.

Introduction

Let there be a rectilinear volume grid whose nodes contain the values of the function $F_{ijk} = F(x, y, z)$. The problem is to approximate the isosurface

$$S_\beta = \{(x, y, z) : F(x, y, z) = \beta\}. \tag{1}$$

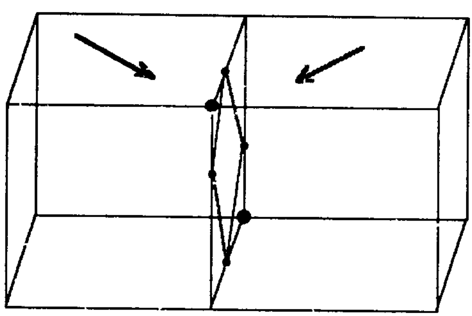

Figure 1. Ambiguity at the Edge

In the MC algorithm [1] [2] the isosurface is approximated sequentially in all cells comprising the volume grid and intersecting the specified surface.

In this case the coordinates of the points of of edges intersecting the isosurface are computed. Then the part of the surface intersecting the given cell is constructed at the points obtained.

In virtue of symmetry there are only 15 possible types of intersection of the isosurface and the cubic cell.

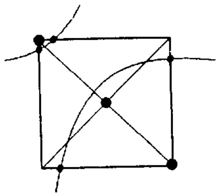

Figure 2. Example of wrong connection.

The problem is that during approximation of the isosurface there is a possibility for "holes" to appear inside the cells of the volume grid as a result of the wrong connection of the points on the edges of the cells (see fig.1). Here and in what follows the black points denote the edges outside the isosurface $F_{node} > \beta$. In the MC method this problem is solved for each cell separately without taking into account the effect of the adjacent cells. Now the problem is to connect the points at the cell edges correctly, in which case the problem of "holes" appearing at the cell edges is solved.

To connect the points correctly one may use the value of the function at the edge center [2].

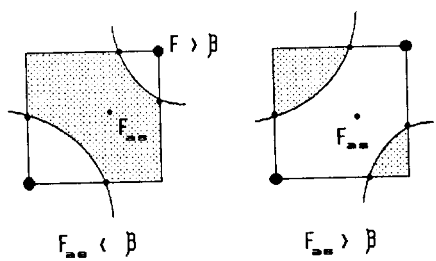

Figure 3. Two Ways of Connecting the Points at the Edge.

The comparison of the value at the point with the one on the isosurface allows one to conclude whether the given point is inside or outside the isosurface. However this solution does not always yield the correct result (see fig. 2).

For the solution of this problem Nielson and Hamann [4] proposed to use a bilinear representation of the function. In this case the curve describing the intersection of the isosurface with the edge will be a hyperbolic one.

Defining the value of the function at the point of the intersection of the hyperbola asymptotes we may tell in what sequence it is necessary to connect points at the edge (see fig. 3) because this point lies between the isolines at the edge.

Solution at the Cell Edges

To determine the point of the intersection of the isosurface with the cell edge the MC method uses the linear interpolation. The bilinear interpolation will be a natural representation of the function at the edge. When analyzing the function behaviour at the edge we will use the edge projection onto a unity square. Then the the function behaviour at the edge is described by the equation in local coordinates **u**, **v**:

$$F(\mathbf{u},\mathbf{v}) \;=\; a + b\mathbf{u} + c\mathbf{v} + d\mathbf{uv}, \tag{2}$$

where $0 \leq \mathbf{u} \leq 1$, $0 \leq \mathbf{v} \leq 1$.

For the straight line $\mathbf{u} = const$ equation (2) will depend only on one variable

$$F(\mathbf{u} = const, \mathbf{v}) \;=\; F(\mathbf{v}), \tag{3}$$

and, hence, will have not more than one solution on the section from 0 to 1, i.e. not *more than one intersection with the isosurface.*

Let us sort the points of the intersection of the edges with the isosurface with respect to **u** and connect them in pairs (see fig.4), in which case the condition of "one intersection" will be satisfied.

In this case for $\mathbf{v} = const$ this rule is satisfied automatically.

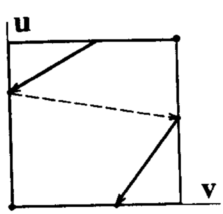

Figure 4. Sorting points at the edge.

Let us assume that this is not so and the case shown in fig. 5 is possible. Let the isosurface with the function value $S_0 = \{(x,y,z) : F(x,y,z) = 0\}$ be approximated. As to other cases, they are reduced by the transfer of the coordinates. Then for the points of the intersection of the isosurface with the face edges the following inequalities should hold true:

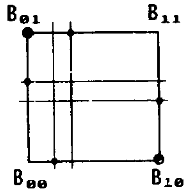

Figure 5. Inadmissible intersection of the isosurface with edge

$$\begin{aligned}
\frac{B_{01}}{B_{01}-B_{11}} &> \frac{B_{00}}{B_{00}-B_{10}}, \\
\frac{B_{00}}{B_{00}-B_{01}} &> \frac{B_{10}}{B_{10}-B_{11}}, \\
\frac{B_{10}}{B_{10}-B_{00}} &> \frac{B_{11}}{B_{11}-B_{01}}, \\
\frac{B_{11}}{B_{11}-B_{10}} &> \frac{B_{01}}{B_{01}-B_{00}}.
\end{aligned} \tag{4}$$

We obtain from the 1st pair that

$$\frac{B_{01}}{B_{10}} > \frac{(B_{00}-B_{01})(B_{01}-B_{11})}{(B_{00}-B_{10})(B_{10}-B_{11})}, \tag{5}$$

and from the 2nd one that

$$\frac{B_{01}}{B_{10}} < \frac{(B_{01}-B_{00})(B_{11}-B_{01})}{(B_{10}-B_{00})(B_{11}-B_{10})}. \tag{6}$$

Hence, we have proved that this case is impossible and, consequently, to connect the points at the edge correctly it is sufficient to sort them along one of the coordinates.

Conclusions

This article offers a new approach to the solution of the problem of ambiguity at the cell edge using the MC algorithm. It has been shown that for such a solution it is sufficient to sort the points of the intersection of the isosurface with the edges confining the given face along one of the coordinates and then to connect them in pairs.

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