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INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 93-134

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Resolving the Topological Ambiguity in Approximating Scalar Function Isosurface

Protvino 1993

UDK 519.62.622.2:539.18.182 M–24

Abstract

Matveyev S.V. Resolving the Topological Ambiguity in Approximating Scalar Function Isosurface: IHEP Preprint 93-134. – Protvino, 1993. – p. 8, figs. 5, refs.: 7.

The purpose of the present paper is the consideration of the problem of topological ambiguities arising in the Marching Cube algorithm. It also presents the solution of this problem inside the cube. The graph theory methods are used to approximate the isosurface inside the cell.

Аннотация

Матвеев С.В. Решение проблемы топологической неоднозначности при аппрокси- мация изоповерхности скалярной функции.: Препринт ИФВЭ 93-134. – Протвино, 1993. – 8 с., 5 рис., библиогр.: 7.

Целью данной статьи является рассмотрение проблемы топологической неоднозначности возникающей в алгоритме движущегося кубика. Решается проблема неоднозначности внутри ячейки. Для аппроксимации изоповерхности внутри ячейки используются методы теории графов.

Introduction

The Marching Cube algorithm [1] was developed for visualization of three-dimensional data sets $F_{ijk} = F(x, y, z)$ specified in the nodes of a rectilinear volume grid in the form of isosurfaces

$$S_\beta = \{(x, y, z) : F(x, y, z) = \beta\}. \tag{1}$$

In the MC algorithm the isosurface is approximated in the form of polygons at each point sequentially and independently of other cells constituting the volume grid, which intersect the specified surface.

The coordinates of the points of the intersection of the cell edges are computed using the linear interpolation of the function on the edges. The points obtained are used to construct the surface lying inside the given cell.

The problem is that during the isosurface approximation inside the cells of a volume grid there is a possibility for "holes" to appear as a result of the wrong connection of the points at the cell edges [7].

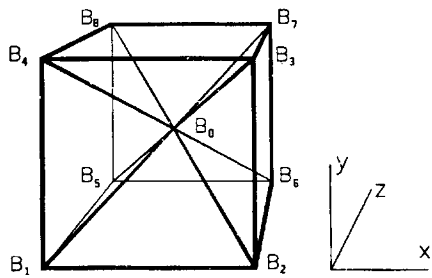

Figure 1. Cell.

Another problem is that inside the cell it is necessary to obtain the isosurface topologically equivalent to the given one [2], [3]. The solution to this problem consists in the correct connection of the points in the cell volume and in separating triangles (in the general case of polygons) approximating the isosurface correctly. The possible cases are analyzed in the paper by Nielsen and Hamann [3].

Let 8 values of the function B_i (see fig. 1) be specified in the cube nodes. A trilinear interpolation will be a natural description of the function inside the cell. In this case when going over to the face we obtain a bilinear description and when going over to the edge we obtain a linear one. Then using a unity cube for the description of the cell we obtain the following equation:

$$F(\mathbf{x},\mathbf{y},\mathbf{z}) = a + b\mathbf{x} + c\mathbf{y} + d\mathbf{z} + e\mathbf{xy} + f\mathbf{xz} + g\mathbf{yz} + h\mathbf{xyz}, \quad (2)$$

where $0 \le \mathbf{x} \le 1$, $0 \le \mathbf{y} \le 1$, $0 \le \mathbf{z} \le 1$.

The constants are defined as

$$\begin{aligned} a &= B_1, \\ b &= B_2 - B_1, \\ c &= B_4 - B_1, \\ d &= B_5 - B_1, \\ e &= B_3 + B_1 - B_2 - B_4, \\ f &= B_6 + B_1 - B_2 - B_5, \\ g &= B_8 + B_1 - B_4 - B_5, \\ h &= B_7 + B_5 + B_4 + B_2 - B_1 - B_3 - B_6 - B_8, \end{aligned} \quad (3)$$

Obtaining Points inside the Cell

Now let us consider the function behaviour inside the cell. As a complexity criteria we use the number of the isosurface intersections with the cube diagonals (see fig.2).

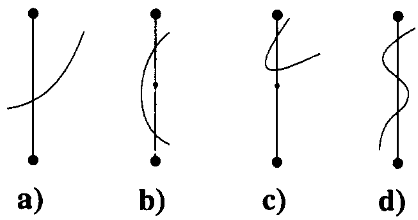

Figure 2. Types of intersections with the diagonal.

In the case of one intersection it is sufficient to have the values of the function in the cell nodes, whereas in the case of two intersections one may introduce an additional point inside the cell that can be the point of the intersection of the diagonals as it was offered in the work by E. Chernyaev and S. Matveyev [4]. However, using the technique offered it is impossible to determine the intersection points for the case presented in fig.2. The technique offered in the paper by Nielsen and Hamann [3] that consists in obtaining an additional point belonging to the isosurface inside the cell will yield the correct result in cases $2a, b, c$.

In the case of three intersections it becomes impossible to reconstruct the topology using the technique offered. The point is that it is necessary to obtain *additional points* belonging to the isosurface inside the cube.

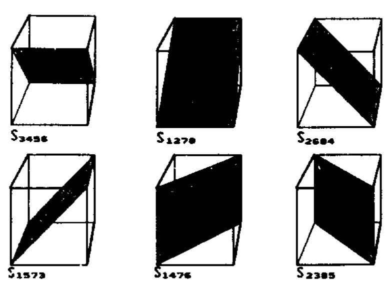

Figure 3. Configuration of slices.

Let us construct on the diagonals of the cube six rectilinear slices, each determined by two diagonals (see fig. 3):

S_{1476} (points B_1, B_4, B_7, B_6), S_{2385}, S_{1278}, S_{3456}, S_{1573} и S_{2684}.

Let us introduce a local variable ξ connected with the position of the point on the diagonal. Then the equations describing the function behaviour on the diagonals $B_1 - B_7$, $B_2 - B_8$, $B_4 - B_6$, $B_5 - B_3$ are equal to

$$\begin{aligned}
F(\xi,\xi,\xi) &= a+(b+c+d)\xi+(e+f+g)\xi^2+h\xi^3, \\
F(1-\xi,\xi,\xi) &= a+b+(-b+c+d+e+f)\xi \\
&\quad +(-e-f+g+h)\xi^2-h\xi^3, \\
F(\xi,1-\xi,\xi) &= a+c+(b-c+d+e+g)\xi \\
&\quad +(-e+f-g+h)\xi^2-h\xi^3, \\
F(\xi,\xi,1-\xi) &= a+d+(b+c-d+f+g)\xi \\
&\quad +(e-f-g+h)\xi^2-h\xi^3,
\end{aligned} \tag{4}$$

respectively, where $0 \le \xi \le 1$.

Hence, on the diagonals the function is specified by a cubic equation and there is a possibility to find the coordinates ξ_i of three intersections with the diagonal of the surface approximated.

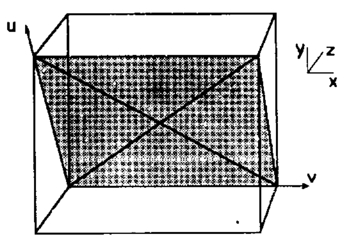

Figure 4. Slice S_{3456} in local coordinates.

Two cube edges and two diagonals of the cube faces form the edges confining each slice (see fig. 3). The points of the isosurface intersection with the diagonals of the cube edges are found from equations (2) becoming a bilinear one on the edge [7] by introducing the parameter ξ. On the diagonals of the cube edges the function will have a quadratic dependence on ξ and, hence, will have not more than two intersections with the isosurface.

The next step is the correct connection of the obtained points in the planes of the slices.

Let us consider the slice S_{3456} shown in fig. 4. Let us go over to the local coordinate system (**u**,**v**) related to this slice. Then for the straight line $\mathbf{u} = const$ equation (2) will have a linear dependence,

$$F(\xi, const, const) = F(\xi), \tag{5}$$

and for the line $\mathbf{v} = const$ it will have a quadratic one,

$$F(const, 1 - \xi, \xi) = F(\xi^2). \tag{6}$$

Hence, the points sorting on the slice with respect to **u** specifies the correct sequence for their connection. In this case the points lying on the edges (boundary) of the slice should be marked as boundary ones at which there is the transition from one isoline to another one. On the sorted list these points should be at the adjacent places. In the general case, the list is as follows:

$\{b, i, ..., b, b, i, ..., i, b, b, ..., b\}$,
where b(oundary) means boundary points
i(nside) means inner points

and the set of the isolines is presented in the form

$\{b, i, ..., b\}, \{b, i, ..., i, b\}, \{b, ..., b\}$.

As a result of previous steps we obtained a large set of points and segments constructed at these points.

We will consider these points to be the nodes of a graph and segments to be its arcs (see fig. 5). Here b_j denote the points belonging to the boundary of the cell (lying on the faces) and i_j denote those inside it.

Let us construct the adjacency matrix of this graph:

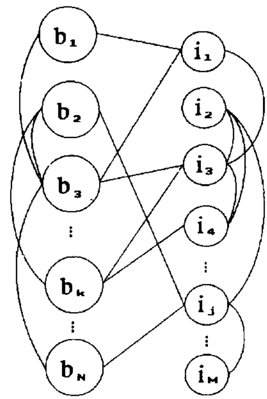

Figure 5. Graph for the cell points

$$\mathbf{adj} = \begin{pmatrix} 0 & \dots & 1 & \dots & 1 & \dots \\ 0 & 0 & \dots & 1 & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \dots & 0 \end{pmatrix} \qquad (7)$$

Here 0 means the absence of the connection between points i and j and 1 implies presence of such a connection or the path equal to 1. In this case the elements of the main diagonal and those below it are equal to 0 because it is sufficient for use to take into account only once that the node i is connected with that of j.

Now consider the expression [5]

$$(adj(i,1) \textbf{ and } adj(1,j)) \textbf{ or } (adj(i,2) \textbf{ and } adj(2,j)) \textbf{ or } \dots \textbf{ or } (adj(m,1) \textbf{ and } adj(m,j)). \qquad (8)$$

The value of this expression is equal to 1 if there is a path of length 2 from the node i to the node j. From here one can find out through which node they are connected.

It is seen from the matrix that element (ij) of matrix adj_2 is equal to *Boolean product of adjacency matrix* by itself:

$$\mathbf{adj_2} = \mathbf{adj} \otimes \mathbf{adj}.$$

Now let us find the logical sum of matrixes **adj** and adj_2:

$$\mathbf{adj_{12}} = \mathbf{adj} \quad \mathbf{and} \quad \mathbf{adj_2}.$$

The element (ij) of the matrix adj_{12} obtained is equal to 1 if the nodes i and j are the vertices of a triangle. The third node can be found from the expression (see above) used for the construction of the matrix adj_2.

The matrix of paths equal to 3 is obtained as a result of Boolean product of the adjacency matrix by the one of paths equal to 2:

$$\mathbf{adj_3} = \mathbf{adj} \otimes \mathbf{adj_2}.$$

If matrix adj_{13},

$$\mathbf{adj_{13}} = \mathbf{adj} \quad \mathbf{and} \quad \mathbf{adj_3}$$

contains elements equal to 1 we may mark the nodes approximating the isosurface by quadrangles. Their triangulation is carried out following, for example, the criterion offered by Choi and his co-authors[6]. If all elements of matrix adj_{13} are equal to 0 this means that triangles are sufficient for approximation and there is no need in further calculations.

The given procedure will be carried out m times until all the elements of the matrix $\mathbf{adj_{1m}}$ are equal to 0.

Conclusions

The present work offers the procedure of approximating the surfaces of a complicated configurations inside a volume cell. It also shows the technique for obtaining inside it the points lying on the surface and for connecting them in the correct sequence. The obtained points and connections between them are presented in the form of a graph. To approximate the isosurface, the graph theory methods are used.

To conclude, the author would like to express his sincere gratitude to S. Klimenko, V. Zhilchenkov for their support of this work, to V. Gusev for fruitful discussions and also to V. Yankova for her assistance in the preparation of the text of this paper.

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Received 9 November, 1993.