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May 1994

UCT-TP 211/94



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VCT TP 94-211
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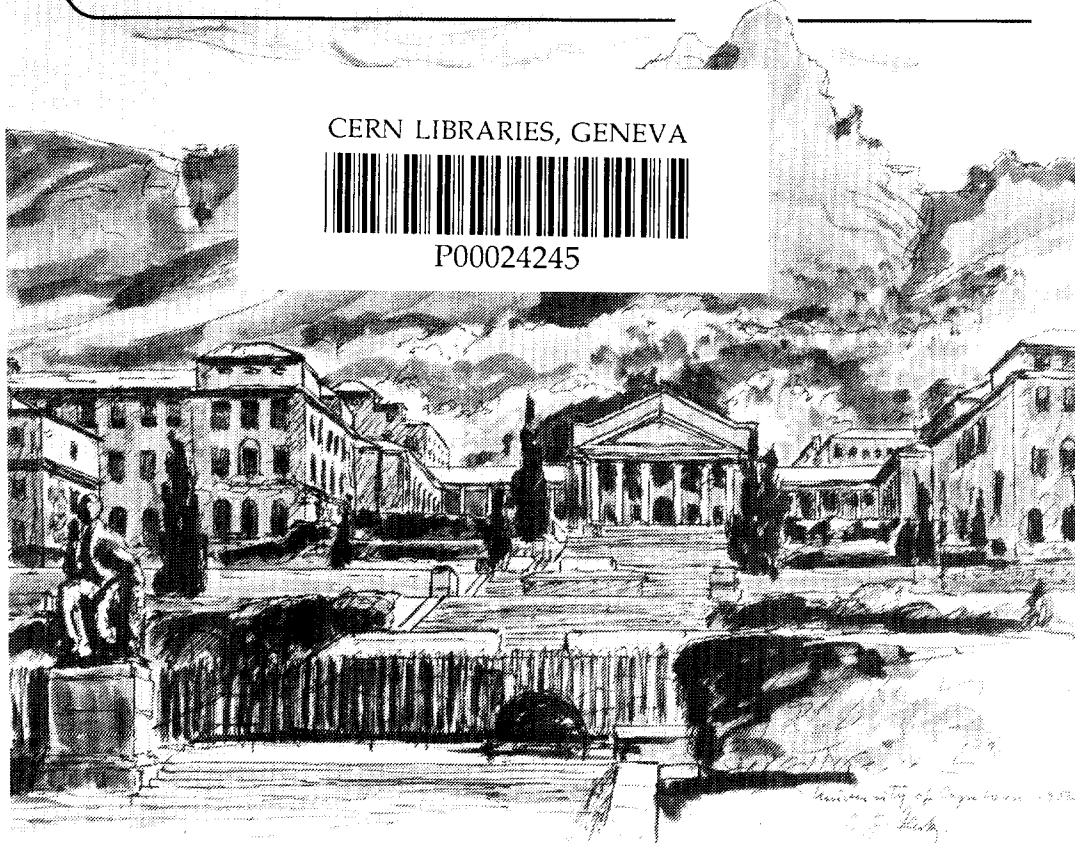
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(to be published in Phys. Lett. B.)

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COUPLING THE SCALAR-ISOSCALAR ' $q^2\bar{q}^2$ ' - MESON' TO THE NUCLEON

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ABSTRACT

We investigate the coupling of the nucleon to the lowest scalar-isoscalar $q^2\bar{q}^2$ state, that is predicted at $670 \text{ MeV}/c^2$ in the framework of cavity QCD to order α_s . The $q^2\bar{q}^2$ - nucleon vertex is evaluated in the 3P_0 $q\bar{q}$ pair creation model. Adjusting the 3P_0 strength to fit the $NN\pi$ coupling constant, the calculated $q^2\bar{q}^2$ - nucleon vertex agrees very well with the $NN\sigma$ vertices used in one-boson-exchange models of the nucleon-nucleon force.

Quantum Chromodynamics (QCD) is generally accepted as the theory of the strong interaction of the fundamental quarks and gluons [1]. Although confinement still needs to be proven within this framework, QCD provides an excellent qualitative understanding of the properties of the low-lying baryons and mesons [2,3]. Attempts to describe the nucleon-nucleon (NN) interaction in terms of nearly massless quarks and gluons only, have met with considerably less success than the well-established effective baryon-meson picture of the NN interaction [4-6]. This has led some groups to consider hybrid models [7-9], in which ideas borrowed from both pictures have been merged. These models, however, are in general plagued with inconsistencies, as both, elementary quarks and mesons, appear simultaneously in the formalism, although the mesons can also be represented by quark-antiquark ($q\bar{q}$) bound states.

The purpose of this paper is to reconcile the fundamental, but much less successful quark-gluon picture with the less fundamental, but highly successful baryon-meson picture of the NN interaction. If both descriptions are valid, it must be possible to reproduce within the more fundamental QCD the pattern of meson-exchanges between nucleons that has been successfully established over the last few decades [4-6]. Proceeding along these lines, we immediately face the problem that not all the mesons, that are necessary to obtain a good phenomenological description of the NN interaction, can be easily understood in terms of $q\bar{q}$ bound states. While the pseudoscalar and vector mesons π , η , ρ and ω play an important role in the long-range attraction, short-range repulsion and spin-isospin dependence of the NN interaction, it is in particular the experimentally elusive scalar-isoscalar σ meson with mass around $m_\sigma = 500-700 \text{ MeV}/c^2$ that provides the bulk of the intermediate-range attraction of the NN force. In meson-exchange models the σ meson is introduced to substitute for two-pion exchange processes in which the pion pair is coupled to spin and isospin $J=I=0$ [4]. However, Jaffe [2] has pointed out that a large width^{††}

^{††} The scalar-isoscalar $q^2\bar{q}^2$ -state can decay into $\pi\pi$. Since the state is well above threshold it is very broad with a decay width $\Gamma \approx 640 \text{ MeV}/c^2$.

scalar-isoscalar meson with a mass of around $660 \text{ MeV}/c^2$ could be understood in terms of the lowest $q^2\bar{q}^2$ -state, where quarks and antiquarks are interacting through one-gluon exchange. Thus in this paper we would like to explore the possibility that the exchange of such a dynamically correlated scalar-isoscalar $q^2\bar{q}^2$ - system can indeed generate the intermediate-range attraction that is needed for the NN force.

For the description of the interacting $q^2\bar{q}^2$ system, we rely on the framework of perturbative cavity QCD to order α_s [3,10], excluding the large quark self-energies [11]. Here massless up and down quarks and antiquarks, residing in the $1s_{1/2}$ mode of a spherical cavity, interact through one-gluon-exchange and one-gluon-annihilation diagrams. As this model provides a satisfactory description of the low-lying baryon (q^3) and meson ($q\bar{q}$) sector, we are confident that it will also describe the spectrum and quantum numbers of the strongly interacting $q^2\bar{q}^2$ system reliably [2,10]. Using the parameter set of the original M.I.T. bag model, which fits the ordinary hadron sector reasonably well, we arrive at a spectrum in which the lowest of the $q^2\bar{q}^2$ states carries the quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$ with a mass of $m=668 \text{ MeV}/c^2$ [10]. The wave function resulting from this dynamical calculation can be expanded in terms of an overcomplete di-meson basis as

$$|(q^2\bar{q}^2)_\sigma\rangle = \sum_{a,b} C_{ab}^\sigma |[(q\bar{q})_a \otimes (q\bar{q})_b]_\sigma\rangle, \quad (1)$$

where a, b stand for a complete set of quantum numbers (total spin, isospin and color) of the $q\bar{q}$ mesons. The coefficients of fractional parentage C_{ab}^σ of the interacting $q^2\bar{q}^2$ system with $I^G(J^{PC}) = 0^+(0^{++})$ denoted by σ are given in Table 1. The σ -meson consists with probability 35.3% of $\pi\pi$, 20.4 % of $\eta\eta$ and 0.5 % of $\rho\rho$, the remaining probabilities residing in the color-octet channels. Thus, in this framework, the lowest scalar-isoscalar $q^2\bar{q}^2$ state does not correspond purely to a correlated $\pi\pi$ state.

The coupling of a $q\bar{q}$ or a $q^2\bar{q}^2$ meson to the nucleon can be calculated in the non-relativistic $q\bar{q}$ pair-creation 3P_0 model. Thereby $q\bar{q}$ pairs are created or destroyed with vacuum quantum numbers ($I^G(J^{PC}) = 0^+(0^{++})$; 3P_0 in LS-coupling). This model has

been successful in the phenomenology of hadronic coupling constants, particularly baryon and meson decays [12], and it also has been successfully applied to nucleon-antinucleon annihilation [13]. The 3P_0 model has a solid foundation in strong coupling QCD [14], and it was shown that quark and gluon condensates provide the nonperturbative dynamical origin for the strength of this transition vertex [15].

Let us consider the coupling of the pion and the sigma ($q^2\bar{q}^2$) to the nucleon in the 3P_0 model corresponding to the effective quark line diagrams of fig.1. Using harmonic oscillator wave functions for the internal part of the quark clusters, we have in momentum space

$$\Psi_{\vec{p}_\pi}^\pi(\vec{p}_1, \vec{p}_2) = \left(\frac{R_\pi^2}{\pi}\right)^{3/4} \exp\left\{-\frac{R_\pi^2}{8}(\vec{p}_1 - \vec{p}_2)^2\right\} \delta\left(\sum_i \vec{p}_i - \vec{p}_\pi\right) \chi_\pi \quad (2a)$$

$$\Psi_{\vec{p}_N}^N(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \left(\frac{3R_N^4}{\pi^2}\right)^{3/4} \exp\left\{-\frac{R_N^2}{6}\sum_{i<j}(\vec{p}_i - \vec{p}_j)^2\right\} \delta\left(\sum_i \vec{p}_i - \vec{p}_N\right) \chi_N \quad (2b)$$

$$\Psi_{\vec{p}_\sigma}^\sigma(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4) = \left(\frac{4R_\sigma^6}{\pi^3}\right)^{3/4} \exp\left\{-\frac{R_\sigma^2}{8}\sum_{i<j}(\vec{p}_i - \vec{p}_j)^2\right\} \delta\left(\sum_i \vec{p}_i - \vec{p}_\sigma\right) \chi_\sigma, \quad (2c)$$

where χ stands for the various spin-flavor-color wave functions. In the case of the scalar-isoscalar $q^2\bar{q}^2$ state, χ_σ is given by the expression (1) as determined in the calculation of the interacting $q^2\bar{q}^2$ system. The rms radii of the hadrons are given by $\langle r^2 \rangle_\pi = \frac{3}{8}R_\pi^2$, $\langle r^2 \rangle_N = R_N^2$ and $\langle r^2 \rangle_\sigma = \frac{9}{8}R_\sigma^2$, respectively.

Let us first consider the $NN\pi$ vertex. The T-matrix element for the quark line process of fig. 1(a) is defined as

$$\langle N'\pi|T|N \rangle = 3 \langle \Psi_{\vec{p}_N}^{N'}(1, 2, 4) \Psi_{\vec{p}_\pi}^\pi(5, 3) | V_{sP_0}^{4,5} | \Psi_{\vec{p}_N}^N(1, 2, 3) \rangle. \quad (3)$$

Here the numbers refer to the individual quarks and antiquarks. The factor 3 takes into account the number of possible rearrangements: any of the three quarks of the nucleon can be used to form the pion. The 3P_0 vertex is given by

$$V_{sP_0}^{i,j} = \lambda \frac{1}{\sqrt{3}} \sum_\mu \sigma_{-\mu}^{ij} \mathcal{Y}_{1,\mu}(\vec{q}_i - \vec{q}_j) \delta(\vec{q}_i + \vec{q}_j) (-)^{1+\mu} \mathbf{1}_F^i \mathbf{1}_C^j, \quad (4)$$

where $\mathcal{Y}_{1\mu}(\vec{q}) = qY_{1\mu}(\hat{q})$ and $1_{F,C}$ are the unit matrices in flavor and color space, respectively. The dimensionless parameter λ corresponds to the strength of the transition¹². The T-matrix element of eq.(3) can be identified with that obtained from the effective Lagrangian

$$\mathcal{L}_{NN\pi} = ig_{\pi NN} (\bar{\psi}_N \gamma_5 \vec{\tau} \psi_N) \cdot \vec{\phi}_\pi \quad (5)$$

involving elementary hadron fields. For a detailed derivation see Ref.(12). The $NN\pi$ vertex form factor is then given in terms of the geometrical and strength parameters as

$$g_{NN\pi}(q^2) = \lambda 10\pi^{1/4} m_N m_\pi^{1/2} \frac{R_N^3 R_\pi^{3/2} (4R_N^2 + R_\pi^2)}{(3R_N^2 + R_\pi^2)^{5/2}} \exp \left\{ -\frac{R_N^2 (12R_N^2 + 5R_\pi^2)}{24(3R_N^2 + R_\pi^2)} q^2 \right\}, \quad (6)$$

where m_N and m_π are the nucleon and pion mass, respectively.

In analogy to the derivation of $g_{NN\pi}$ in the 3P_0 model we consider the coupling of the $q^2 \bar{q}^2$ sigma meson to the nucleon according to fig. 1(b). The corresponding elementary T-matrix element is defined as

$$\langle N' \sigma | T | N \rangle = 6 \langle \Psi_{\bar{P}_N'}^{N'}(1, 4, 5) \Psi_{\bar{P}_\sigma}^\sigma(6, 7, 2, 3) | \mathcal{O} | \Psi_{\bar{P}_N}^N(1, 2, 3) \rangle \quad (7)$$

with the transition operator given by

$$\mathcal{O} = V_{sP_0}^{4,6} \frac{1}{\Delta E} V_{sP_0}^{5,7}. \quad (8)$$

The energy denominator $(\Delta E)^{-1}$, describing the propagation of the intermediate state $q^3 \bar{q} \bar{q}$, is in the static limit momentum independent and, since we are only interested in a rough estimate for the $NN\sigma$ coupling constant, ΔE is chosen to be constant. The factor 6 in eq.(7) arises from the different possible arrangements in the graph of fig. 1(b). Due to the property of the 3P_0 vertex, graphs with one or two created $q\bar{q}$ pairs contracted with the valence quarks of the sigma state are zero. Therefore, elementary two- $q\bar{q}$ -meson emission

¹² Note that the present definition of the 3P_0 vertex is related to that of the Orsay group [12] using the strength parameter γ via $\lambda\sqrt{2} = \gamma$.

in the framework of the 3P_0 model requires each of these mesons to couple to different quark lines.

The transition matrix element of (7) is defined as

$$\begin{aligned} & \langle \Psi_{\vec{p}'_N}^{N'} = -\bar{q}(1, 4, 5) \Psi_{\vec{p}_\sigma}^\sigma = \bar{q}(6, 7, 2, 3) | \mathcal{O} | \Psi_{\vec{p}'_N=0}^N(1, 2, 3) \rangle = \\ & = \frac{\lambda^2}{\Delta E} \sum_{\mu\nu} (-)^{\mu+\nu} T_{spatial}^{\mu\nu} \cdot T_{SFC}^{\mu\nu} \end{aligned} \quad (9)$$

where we have separated the spatial part $T_{spatial}^{\mu\nu}$ and the spin-flavor-color part $T_{SFC}^{\mu\nu}$. The indices μ, ν specify the helicity components of the respective 3P_0 -vertices. In the center-of-mass frame of the initial nucleon the spatial part is defined as

$$\begin{aligned} T_{spatial}^{\mu\nu} = \int \Pi_{i=1..7} d^3 p_i \Psi_{-\vec{q}}^{N'}(1, 4, 5) \Psi_{\vec{q}}^\sigma(6, 7, 2, 3) \frac{1}{3} \mathcal{Y}_{1\mu}^*(\vec{p}_4 - \vec{p}_6) \delta(\vec{p}_4 + \vec{p}_6) \\ \mathcal{Y}_{1\nu}^*(\vec{p}_5 - \vec{p}_7) \delta(\vec{p}_5 + \vec{p}_7) \Psi_0^N(1, 2, 3) . \end{aligned} \quad (10)$$

Evaluating the integral (10) we arrive at

$$\begin{aligned} T_{spatial}^{\mu\nu} = \delta^3(0) \exp \left\{ -\frac{R_N^2 (3R_N^2 + 5R_\sigma^2)}{24 (3R_N^2 + R_\sigma^2)} q^2 \right\} \frac{3\pi^{7/2}}{\sqrt{2}} \\ \cdot \frac{(5R_N^2 + R_\sigma^2)}{(3R_N^2 + R_\sigma^2)^{5/2} (R_N^2 + R_\sigma^2)^4} (-)^\mu \delta_{\mu, -\nu} , \end{aligned} \quad (11)$$

where the contraction of the helicity components μ and ν is described by the Kronecker delta. Thus the spin-flavor-color part of the transition matrix element is given by

$$\begin{aligned} T_{SFC} & = \sum_{\mu\nu} (-)^\mu \delta_{\mu, -\nu} T_{SFC}^{\mu\nu} \\ & = \sum_{a,b} C_{ab}^\sigma \hat{T}_{SF} \hat{T}_C , \end{aligned} \quad (12)$$

where \hat{T}_{SF} and \hat{T}_C are the spin-flavor and color matrix elements in the di-meson basis.

The spin-flavor part is then evaluated as

$$\begin{aligned} \hat{T}_{SF} & = \langle N(1, 4, 5) [(q\bar{q})_{t_1}^{s_1}(2, 6) \otimes (q\bar{q})_{t_2}^{s_2}(3, 7)]_\sigma | \sum_{\mu} (-)^\mu \sigma_{-\mu}^{46\dagger} \sigma_{\mu}^{57\dagger} \mathbf{1}_F^{46} \mathbf{1}_F^{57} | N(1, 2, 3) \rangle \\ & = \langle \mathbf{1}_S \mathbf{1}_F \rangle F(s_1, t_1; s_2, t_2) , \end{aligned} \quad (13)$$

where the reduced matrix element F is given by

$$F(s_1, t_1; s_2, t_2) = \delta_{s_1, s_2} \delta_{t_1, t_2} \frac{3}{2} \{(2s_1 + 1)(2t_1 + 1)\}^{1/2} (-1)^{s_1 + t_1} \cdot \sum_{J_{23}=0,1} (-1)^{J_{23}} \frac{1}{(2J_{23} + 1)} \left\{ \begin{matrix} 1/2 & 1/2 & s_1 \\ 1/2 & 1/2 & J_{23} \end{matrix} \right\} \left\{ \begin{matrix} 1/2 & 1/2 & t_1 \\ 1/2 & 1/2 & J_{23} \end{matrix} \right\}. \quad (14)$$

Here $a(s_1, t_1, c_1)$, $b(s_2, t_2, c_2)$ represent the various spin, flavor and color quantum numbers of the $q\bar{q}$ pair in the $q^2\bar{q}^2$ state. The values for the reduced matrix element $F(s, t; s_1, t_1; s_2, t_2)$ are given in Table 2.

The evaluation of the color matrix element yields

$$\begin{aligned} \hat{T}_c &= \langle qq\bar{q}(1, 4, 5) [(q\bar{q})^{c_1}(2, 6) \otimes (q\bar{q})^{c_2}(3, 7)]^1 | \mathbf{1}_C^{46} \mathbf{1}_C^{57} | qq\bar{q}(1, 2, 3) \rangle = \\ &= \left\{ \begin{array}{l} \frac{1}{3} \quad c_1 = c_2 = 1 \\ -\frac{\sqrt{2}}{3} \quad c_1 = c_2 = 8 \end{array} \right\}. \end{aligned} \quad (15)$$

With the definitions and results of eqs. (7)-(15) and using the dynamical coefficients C_{ab}^σ of Table 1, the transition amplitude is then given by

$$\begin{aligned} \langle N'\sigma | T | N \rangle &= \delta^3(0) \frac{\lambda^2}{\Delta E} \frac{\sqrt{3}}{\pi^{7/4}} \frac{R_N^6 R_\sigma^9 / 2 (5R_N^2 + R_\sigma^2)}{(3R_N^2 + R_\sigma^2)^{5/2} (R_N^2 + R_\sigma^2)^4} 16.8 \\ &\cdot \exp \left\{ -\frac{R_N^2 (3R_N^2 + 5R_\sigma^2)}{24 (3R_N^2 + R_\sigma^2)} q^2 \right\}. \end{aligned} \quad (16)$$

We now can identify the T-matrix element (16) with that obtained from the phenomenological Lagrangian

$$\mathcal{L}_{NN\sigma} = g_{NN\sigma} \bar{\psi}_N \psi_N \phi_\sigma. \quad (17)$$

Making the identification in the center-of-mass frame, we obtain for the $NN\sigma$ vertex form factor

$$\begin{aligned} g_{NN\sigma}(q^2) &= \frac{\lambda^2}{\Delta E} 4\sqrt{3} \pi^{-1/4} m_\sigma^{1/2} \frac{R_N^6 R_\sigma^9 / 2 (5R_N^2 + R_\sigma^2)}{(3R_N^2 + R_\sigma^2)^{5/2} (R_N^2 + R_\sigma^2)^4} 16.8 \\ &\cdot \exp \left\{ -\frac{R_N^2 (3R_N^2 + 5R_\sigma^2)}{24 (3R_N^2 + R_\sigma^2)} q^2 \right\}, \end{aligned} \quad (18)$$

where m_σ is the mass of the $q^2\bar{q}^2$ σ -meson.

We now turn to the numerical results. In the framework of the bag model the rms radius of a hadron of mass m_H is given by $\langle r^2 \rangle_H = 3.64 \text{ GeV}^{-1} (m_H/m_p)^{1/3}$. We thus obtain

$R_N = 3.64 \text{ GeV}^{-1}$, $R_\pi = 3.14 \text{ GeV}^{-1}$ and $R_\sigma = 3.07 \text{ GeV}^{-1}$. Fitting the $g_{NN\pi}$ coupling constant to $g_{NN\pi}^2/4\pi = 14$ yields for the strength parameter of the 3P_0 -vertex a value of $\lambda = 2.84$. This fitted 3P_0 strength is comparable to the one obtained by Weber [15] in terms of quark and gluon condensates, where he effectively obtains a value of $\lambda = 2.7$. For the sigma-nucleon coupling constant we obtain $g_{NN\sigma}^2/4\pi = 8.79$ for $\Delta E = m_\sigma/2$, or $g_{NN\sigma}^2/4\pi = 2.20$ for $\Delta E = m_\sigma$. This range includes the values 2.5 [9] to 7.1 [4] considered in models with σ -exchange. The corresponding mean square σN interaction radius of our model is $\langle r^2 \rangle_{\sigma N}^{1/2} = 0.48 \text{ fm}$, which is an intermediate value compared to the ones used in the literature. Additive quark models prefer a larger interaction radius of $\langle r^2 \rangle_{\sigma N}^{1/2} \approx 0.7 \text{ fm}$ [7,9], whereas one-boson-exchange models typically use $\langle r^2 \rangle_{\sigma N}^{1/2} \approx 0.25 \text{ fm}$ [4].

In summary, we have shown that the lowest scalar-isoscalar $q^2\bar{q}^2$ state can indeed be interpreted as the σ -meson in one-boson-exchange models, needed to explain the intermediate range attraction of the nucleon-nucleon system, as mass, quantum numbers, coupling constant and the slope of the form factor agree with the expectations.

This work was supported by a grant of the German Federal Ministry of Research and Technology (BMFT) under contract No. 06 Tü 714.

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Table 1: Coefficients of fractional parentage for the interacting $q^2\bar{q}^2$ system with the quantum numbers $\sigma \equiv I^G(J^{PC}) = 0^+(0^{++})$ at a mass of $m=668 \text{ MeV}/c^2$. Here, the index 1,8 of the $q\bar{q}$ pairs denote the dimensionality of the color representation.

$ (q\bar{q})_a \otimes (q\bar{q})_b \rangle_\sigma$	C_{ab}^σ
$ \eta^1; \eta^1 \rangle$	0.4514
$ \pi^1; \pi^1 \rangle$	0.5940
$ \omega^1; \omega^1 \rangle$	0.0010
$ \rho^1; \rho^1 \rangle$	-0.0699
$ \eta^8; \eta^8 \rangle$	0.0496
$ \pi^8; \pi^8 \rangle$	0.1862
$ \omega^8; \omega^8 \rangle$	-0.2332
$ \rho^8; \rho^8 \rangle$	-0.5891

Table 2: Values of the reduced matrix element $F(s_1, t_1; s_2, t_2)$

$(s_1, t_1)(s_2, t_2)$	$F(s_1, t_1; s_2, t_2) \cdot 12$
$\pi\pi$	$5\sqrt{3}$
$\eta\eta$	3
$\rho\rho$	13
$\omega\omega$	$5\sqrt{3}$

Fig. 1: Quark line diagrams for the nucleon-nucleon- $q\bar{q}$ (a) and nucleon-nucleon- $q^2\bar{q}^2$ (b) vertices. The dots indicate the $q\bar{q} \ ^3P_0$ pair-creation vertex.