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Exclusive heavy meson pair production at large recoil

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Abstract

The Brodsky-Lepage formalism (BL) in perturbative QCD is applied to estimate the validity of the heavy quark effective theory and the high energy scaling laws to the exclusive heavy meson pair production in two-photon collision as well as e^+e^- annihilation. In addition to the ordinary heavy mesons such as B and D, the heavy mesons of which both quark masses are larger than the typical strong interaction scale Λ_{QCD} are considered. Explicit forms of transition amplitudes are presented for the heavy mesons with two heavy quarks in the peaking approximation and the modifications for the ordinary heavy meson production are described. The result in the heavy quark limit is consistent with the heavy quark effective theory only in a certain range of center of mass energy. In the high energy limit, the transition amplitudes turn out to comply with the high energy scaling laws. The common features and different points of the e^+e^- and $\gamma\gamma$ cases are described in detail. The limitaion of the BL formalism and the heavy quark effective theory is discussed.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is widely believed to be a true theory of strong interactions. The asymptotic freedom [1] of QCD enables us to perturbatively treat hard processes, because the strong coupling constant α_s becomes small at short distances. On the other hand, quark interaction within hadrons is strong enough to bind quarks into unseparable pairs. At present there is no quantitative framework within QCD to deal with the strong interaction and such a fundamental problem as evaluation of the hadron spectrum is beyond reach of the theory.

For these reasons, the hadron physics in the strong coupling limit has been approached on phenomenological grounds with some simple ansatz to be justified by the underlying theory. Until now, every concrete and model-independent results have relied on the existence of small parameters as well. Among them are the large-N expansion [2] and the chiral perturbation theory [3] for the dynamics of light mesons.

Recently, the same philosophy has been applied to heavy hadron systems with a single heavy quark such as b or c quarks and it led to an effective theory called heavy quark effective theory (HQET) [4–6]. The HQET of such heavy hadrons in the infinite heavy quark mass limit remains well defined and reveals a new SU($2N_Q$) spin-flavor symmetry for N_Q heavy-quark species. This symmetry gives strong relationships to spectroscopy and to both elastic and transition form factors of heavy quark operators between different heavy quark states.

Essentially, the heavy quark symmetry is due to a big separation between the heavy quark mass scale m_Q and all other dynamical scales in the problem, especially the strong interaction scale Λ_{QCD}. Consider, for example, a heavy meson made up of a heavy quark and a so-called light degree of freedom (i.e., a compound of light quarks, light anti-quarks, and gluons) with an effective mass Λ. It is assumed that at least one heavy quark inside a heavy meson has a mass much larger than all other scales in the problem. Then, the momentum transfer between the heavy quark and the remainder of the heavy meson is too small to change the heavy quark velocity [7]. On the other hand, since the gluon magnetic coupling to a heavy quark goes as $1/m_Q$, the strong interaction of the heavy quark is independent of its spin. Therefore, the heavy-hadron dynamics is completely determined by only the heavy-quark velocity and color.

The heavy quark spin-flavor symmetry has been applied mainly to matrix elements with *space-like* momentum transfers in heavy hadron decay processes where the heavy quark serves as a natural upper bound on the momentum transfer in a heavy hadron decay process. This natural bound justifies the application of heavy quark symmetry to heavy hadron decay. Recently, Körner and Kroll [8] analyzed the large recoil behavior of transition form factors in the heavy hadron decay process and found that, at the leading order of the heavy mass scale, the large recoil form factors exhibit a new type of heavy quark symmetry. The new symmetry also is due to the fact that the momentum transfer is still much less than the heavy quark mass.

However, the momentum transfer can be very large in processes involving more than one heavy quark such as bound states with two or more heavy quarks or scattering states of two or more heavy mesons. In this case, the application of HQET will be confined to a particular regime of momentum transfer. It is, therefore, interesting to consider the heavy meson production through non-QCD interactions such as e^+e^- annihilation and $\gamma\gamma$ collision to estimate the validity of the HQET.

The nontrivial consequences of heavy quark symmetry for $e^+e^- \to M\bar{M}$ and for $\gamma\gamma \to M\bar{M}$ have been considered previously in Ref. [9] and in Ref. [10], respectively. They used analyticity and crossing symmetry, and pointed out that their results can not be valid for the whole range of c.m. energy. The processes can be viewed to proceed through two steps: the formation of a $h\bar{h}$ pair at the electromagnetic vertex and its subsequent hadronization into two heavy mesons. As the state evolves from the two heavy quarks into the two heavy mesons, the dynamics involves light degrees of freedom. However, since the heavy quarks are much heavier than any of the other scales in the problem, the recoil of heavy quarks is negligible during the formation of the two mesons and the dynamics of the light degrees of freedom depend only on the scalar product of heavy meson velocities. As a result, the

amplitude of the process is written as the amplitude for creation of a point particle of mass $m_Q \approx M$ times a form factor depending only on the final meson velocities.

It should be, however, emphasized that the argument relies on one crucial assumption: the momentum transfer is much less than the heavy quark mass. There are two main sources to break the heavy quark symmetry down very badly: threshold effects [11] and high-momentum scaling [12]. The heavy quark symmetry has broken down near the threshold region because of the $Q\bar{Q}$ resonance formation where the natural scale of momentum transfer is order of m_Q. On the other hand, there is a general result [12] of perturbative QCD (PQCD) that the form factor of all mesons should fall like $1/Q^2$ asymptotically when $Q^2 \gg M^2$ and Λ^2, and the scaling law is not compatible with the property of the Isgur-Wise function in the heavy quark limit.

Recently, the BL formalism [13] in PQCD has been employed by Cohen and Milana [14] to estimate the HQET results for the production process of a pair of pseudoscalar mesons in e^+e^- annihilation. They concluded that, for sufficiently heavy mesons, the HQET result is valid but the $1/m_Q$ correction is large for realistic quark masses.

In this paper, we investigate the physical implications of the heavy quark limit and high enery limit in more detail by extending the work by Cohen and Milana [14] to the processes involving heavy vector mesons and by applying the BL formalism to the heavy meson pair production by *two-photon* collision as well as e^+e^- annihilation. In addition, there is another crucial difference between Ref. [14] and our present work. In addition to the ordinary heavy mesons, the heavy mesons of which two quarks have mass larger than the strong interaction scale Λ_{QCD} are considered. Every actual calculation has been done for the latter heavy meson systems and the modifications for the ordinary heavy meson production are described in detail.

The paper is organized as follows. Section II is devoted to the introduction of the BL formalism in PQCD. In Section III, the general features of heavy meson pair production by e^+e^- or $\gamma\gamma$ are explained and the explicit forms of the production amplitudes both for two-heavy-quark (HH) meson systems and for heavy-light-quark (HL) meson systems are presented in the center of mass frame. We can see that if one heavy quark mass is much larger than any other scale such as the strong interaction scale Λ_{QCD} or/and the other quark mass m_q, the amplitudes exhibit heavy quark symmetries. Two limits - heavy quark and high energy are intensively discussed in Section IV. We have found that both e^+e^- and $\gamma\gamma$ processes have the same Isgur-Wise function in the heavy quark limit. Finally, the limitation of the results and several aspects for more quantitative analysis are pointed out and the conclusion is made.

II. BRODSKY-LEPAGE FORMALISM IN PERTURBATIVE QCD

The main idea of the BL formalism is that amplitudes for large-recoil exclusive processes factor into two parts at high energies: (1) a parton distribution amplitude $\phi(x_i, Q^2)$ for each hadron - the probability amplitude for finding valence partons in the hadron, each carrying some fraction x_i of the hadron's momentum, and all collinear up to Q, the typical momentum transfer in the process; and (2) a hard scattering amplitude T_H - the scattering amplitude of collinear valence partons for each hadron. The production amplitude for meson pair production is given by the factorized form

$$\mathcal{M}_{M(M^*),\bar{M}(\bar{M}^*)} = \frac{[Mf_M]^2}{6}\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \;\; \mathrm{Tr}\Big[\tilde{\phi}_{M(M^*)}(x,Q^2)T_H(x,y;Q^2)\phi_{\bar{M}(\bar{M}^*)}(y,Q^2)\Big], \quad (1)$$

where $\tilde{\phi} = \gamma^0\phi^+\gamma^0$, M is the heavy meson mass, and the dimensionless variables x and y are the longitudinal momentum fractions of effective lighter quarks. The normalization is the same as that in Ref. [14,15]. The scale Q^2 is set from higher order calculations, but it reflects the minimum momentum transfer in the process. The wave function form factor f_M in Eq. (1) is the usual heavy meson decay constant determined by its respective wave function at the origin. The main dynamical dependence of the form factor is controlled by the hard scattering amplitude T_H, computed by replacing each hadron by collinear constituents $p_i^\mu = x_i P_M^\mu$. The collinear divergences are summed in $\phi_{M(M^*)}$ and $\phi_{\bar{M}(\bar{M}^*)}$, and $T_H(x,y;Q^2)$ are systematically computed as a perturbation expansion in $\alpha_s(Q^2)$. The spin wave functions are given according to their spin structure by

$$\phi_M(x,Q^2) = f(x,Q^2)\frac{1+\not{v}}{2}\gamma_5, \quad \phi_{\bar{M}}(x,Q^2) = f^*(x,Q^2)\frac{1-\not{v}'}{2}\gamma_5,$$
$$\phi_{M^*}(x,Q^2) = f(x,Q^2)\frac{1+\not{v}}{2}\not{\epsilon}, \quad \phi_{\bar{M}^*}(x,Q^2) = f^*(x,Q^2)\frac{1-\not{v}'}{2}\not{\epsilon}, \quad (2)$$

where v and v' are the heavy meson and anti-meson velocities, respectively.

To leading order in the strong coupling constant α_s, the hard scattering amplitude is governed by one-gluon exchange. On the other hand, the distribution amplitudes $f(x,Q^2)$ and $f^*(x,Q^2)$ are known [8,16] to have a peak at $x = \Lambda/M$ with a width which shrinks with increasing quark (or hadron) mass. This implies that, as a good approximation, the quarks and the heavy hadron composed of them have the same velocity, and $f(x,Q^2)$ and $f^*(x,Q^2)$ can be taken as follows

$$f(x,Q^2) = f^*(x,Q^2) \approx \delta(x - \frac{\Lambda(m_q, < Q^2 >)}{M}), \quad (3)$$

where m_q is the lighter quark current mass and $< Q^2 >$ the average square of transverse momentum. Approximately the Λ which is the difference of the heavy meson mass M and the heavier quark mass m_Q is given in terms of m_q and $< Q^2 >$ [14,16] by

$$\Lambda \equiv M - m_Q \approx [m_q^2 + < Q^2 >]^{1/2}. \quad (4)$$

The so-called peaking approximation is much more reliable as the lighter quark current mass m_q is larger than $< Q^2 >$ of $O(\Lambda_{QCD})$, while the approximation is not so good for u and d quarks.

III. HEAVY MESON PAIR PRODUCTION

A clean production mechanism for exclusive heavy mesons can be provided by the e^+e^- or $\gamma\gamma$ [17] annihilation because they do not strongly interact. In fact, much effort has recently been devoted to the study of exclusive processes at large recoil with the e^+e^- and $\gamma\gamma$ [17,18] beams in the context of PQCD.

Let us list a few properties of the two production mechanisms.

- The pointlike structure of the electron and photon results in substantial simplifications of the analysis of these exclusive production amplitudes.

- Any odd-C hadronic state is produced in e^+e^- annihilation, while an even-C hadronic state can be created only in two-photon annihilation with minimum complexity. This difference provides a clean environment for independent identification of the C-odd or even color-singlet composites of quarks and gluons.

- While it is not easy to control the incident electron polarization, the photon polarization can be easily varied, allowing detailed tests of theory.

- The e^+e^- center of mass frame is spatially fixed but the $\gamma\gamma$ center of mass frame is boosted along the beam direction. The boost property makes $\gamma\gamma$ experiments are more difficult than e^+e^- experiments and requires more sophisticated data analyses.

To leading order in α_s, four Feynman diagrams contribute to the heavy meson pair production in e^+e^- annihilation as shown in Fig. 1. They can be classified into two parts. The first part (Fig. 1(a)) is for the heavier quark pair production preceding the hadronization into heavy mesons via one virtual gluon exchange. In the second part, the lighter quarks are first produced and then they are hadronized into two heavy mesons through one virtual gluon emission.

When the initial lepton masses are neglected, the transition amplitudes are given by

$$\mathcal{M}^{ee}_{PP} = F^{ee}L^{ee}\left[e_Q\frac{2(1-x)-\hat{s}x}{x^3} - e_q\frac{2x-\hat{s}(1-x)}{(1-x)^3}\right], \tag{5}$$

$$\begin{aligned}
\mathcal{M}^{ee}_{PV}(\pm) &= \mathcal{M}^{ee}_{VP}(\mp) = \pm\frac{N}{\sqrt{2}}F^{ee}L^{ee}\left[\frac{e_Q}{x^3}+\frac{e_q}{(1-x)^3}\right]\hat{s}z,\\
\mathcal{M}^{ee}_{PV}(0) &= \mathcal{M}^{ee}_{VP}(0) = 0,
\end{aligned} \tag{6}$$

$$\begin{aligned}
\mathcal{M}^{ee}_{VV}(+,+) &= \mathcal{M}^{ee}_{VV}(-,-) = 2F^{ee}L^{ee}\left[e_Q\frac{1-x}{x^3} - e_q\frac{x}{(1-x)^3}\right],\\
\mathcal{M}^{ee}_{VV}(+,-) &= \mathcal{M}^{ee}_{VV}(-,+) = 0,\\
\mathcal{M}^{ee}_{VV}(\pm,0) &= -\mathcal{M}^{ee}_{VV}(0,\pm) = \frac{N}{\sqrt{2}}F^{ee}L^{ee}\left[\frac{e_Q}{x^3} - \frac{e_q}{(1-x)^3}\right]\hat{s}z,
\end{aligned}$$

$$\mathcal{M}^{ee}_{VV}(0,0) = F^{ee}L^{ee}\left[e_Q\frac{2(1-x)+\hat{s}x}{x^3} - e_q\frac{2x+\hat{s}(1-x)}{(1-x)^3}\right], \tag{7}$$

where

$$\hat{s} = s/M^2,\quad \beta=\sqrt{1-4/\hat{s}},\quad z=\beta\cos\theta,\quad N = 1/\sqrt{\hat{s}(\beta^2-z^2)}, \tag{8}$$

$$F^{ee} = -\frac{16\pi^2\alpha_e\alpha_s C_F}{3M\hat{s}^3}\left[\frac{f_M}{M}\right]^2,\quad L^{ee} = \bar{v}(p_2)(\not{p}_1 - \not{p}_2)u(p_1), \tag{9}$$

with the color factor $C_F = 4/3$ for the $\mathrm{SU(3)_C}$ group. The θ is the scattering angle between a photon and a heavy meson, and the subscripts P and V stand for a pseudoscalar and a vector, respectively.

It is noted that every expression is symmetric under the role exchange of two constituent quarks, i.e. $x \leftrightarrow (1-x)$ and $e_Q \leftrightarrow e_q$ and Eq. (5) is the same as Eq. (10) in Ref. [19]. On the other hand, x can be large as much as 1/2. Certainly, the presented results can not be directly applied for HL meson systems. It is, however, not so difficult to get their production amplitudes from the above expressions by noting that, for HL meson systems, both transverse momentum and light quark mass effects are order of x^2 and so can be neglected [15] because they are very small for a heavier quark mass m_Q. On the other hand, the leading s terms are not altered by transverse momentum or mass effects, since they are controlled by the longitudinal momentum of each particle but not by the small light quark mass and/or small transverse momentum. We can get the expressions for these HL meson cases from the above expressions by maintaining only terms which are leading and next-to-leading in x and also leading in s terms. In this case, x is approximately given by $\sqrt{< Q^2 >}/M$ and then Eq. (5) is found to be consistent with the result of Ref. [14].

One noteworthy feature of the QCD predictions found by Ji and Brodsky [19] is the existence of a zero in the form factor for pseudoscalar meson-pair production at the specific timelike value $\hat{s} = 2m_Q/\Lambda$ when m_Q is much larger than Λ, while Λ itself is larger than Λ_{QCD}.

In the two-photon case, twenty Feynman diagrams are involved even in the leading order of α_s as shown in Fig. 2. The diagrams can be classified into three parts. Six diagrams of

Fig. 2(a) represent the heavier quark pair production by two-photon collision followed by one virtual gluon emission to allow the produced heavier quarks to hadronize into heavy mesons. Fig. 2(b) are for diagrams obtained by exchanging heavier quark lines with lighter anti-quark line in Fig. 2(a). The last part (Fig. 2(c)) consists of eight diagrams where one photon produces a pair of heavier quarks and the other photon produces a pair of lighter quarks.

After a lengthy calculation, the resulting pseudoscalar-pseudoscalar (PP) production amplitude $\mathcal{M}^{\gamma\gamma}_{PP}(\lambda,\lambda')$ for the initial photon helicities (λ,λ') is written as

$$\mathcal{M}^{\gamma\gamma}_{PP}(\lambda,\lambda') = 2\frac{F^{\gamma\gamma}}{(1-z^2)^2}\Big\{(1+z^2)\big[e_Q^2F_{PP}(\lambda,\lambda')+e_q^2\tilde{F}_{PP}(\lambda,\lambda')\big]-2e_Qe_qG_{PP}(\lambda,\lambda') \\ -(1-z^2)\Big[\frac{1-x}{x}e_Q^2H_{PP}(\lambda,\lambda')+\frac{x}{1-x}e_q^2\tilde{H}_{PP}(\lambda,\lambda')\Big]\Big\}, \quad (10)$$

where

$$F^{\gamma\gamma} = -\frac{16\pi^2\alpha_e\alpha_sC_F}{3\hat{s}^2x^2(1-x)^2}\left[\frac{f_M}{M}\right]^2, \quad (11)$$

$$F_{PP}(\lambda,\lambda') = \left[(1-x)[2-x(\hat{s}+2)]+\frac{\hat{s}}{2}\delta_{\lambda,-\lambda'}\right](\beta^2-z^2),$$

$$\tilde{F}_{PP}(\lambda,\lambda') = \left[x[2-(1-x)(\hat{s}+2)]+\frac{\hat{s}}{2}\delta_{\lambda,-\lambda'}\right](\beta^2-z^2), \quad (12)$$

$$G_{PP}(\lambda,\lambda') = \left[\frac{\hat{s}}{2}[1+z^2+(1-z^2)\delta_{\lambda,\lambda'}]-2x(1-x)(2+\hat{s})\right](\beta^2-z^2) \\ -\big[\hat{s}(1-z^2)-2(3-z^2)\big]\delta_{\lambda,\lambda'}, \quad (13)$$

$$H_{PP}(\lambda,\lambda') = \big[2-x(\hat{s}+2)\big]\big[(1-x)(\beta^2-z^2)+z^2\delta_{\lambda,\lambda'}\big]+x(\hat{s}+2)\delta_{\lambda,\lambda'},$$

$$\tilde{H}_{PP}(\lambda,\lambda') = \big[2-(1-x)(\hat{s}+2)\big]\big[x(\beta^2-z^2)+z^2\delta_{\lambda,\lambda'}\big]+(1-x)(\hat{s}+2)\delta_{\lambda,\lambda'}, \quad (14)$$

And the pseudoscalar-vector (PV) or vector-pseudoscalar (VP) production amplitude $\mathcal{M}^{\gamma\gamma}_{PV(VP)}(\lambda,\lambda')$ for initial photon helicities (λ,λ') and final vector meson helicity σ is

$$\mathcal{M}^{\gamma\gamma}_{PV(VP)}(\lambda,\lambda';\sigma) = -\frac{4F^{\gamma\gamma}}{(1-z^2)^2}A_{PV(VP)}(\lambda,\lambda';\sigma), \quad (15)$$

where

$$A_{PV}(+,+;\pm) = -A_{PV}(-,-;\mp) = -A_{VP}(+,+;\pm) = A_{VP}(-,-;\mp) \\ = \sqrt{\frac{\hat{s}}{2}}\big[(1-x)e_Q^2-xe_q^2-(1-2x)e_Qe_q\big](\frac{1}{\beta}\pm1)z\sqrt{\beta^2-z^2},$$

$$A_{PV}(+,-;\pm) = -A_{PV}(-,+;\mp) = A_{VP}(+,-;\mp) = -A_{VP}(-,+;\pm) \\ = \sqrt{\frac{\hat{s}}{2}}\Big[(e_Q-e_q)[(1-x)e_Q-xe_q](\beta\pm z) \\ -(e_Q-e_q)\big[(1-2x)(e_Q-e_q)\frac{z}{\beta}\pm(e_Q+e_q)\big]z \\ -\frac{1-z^2}{2}\left(\frac{1-x}{x}e_Q^2-\frac{x}{1-x}e_q^2\right)(\beta\mp z)\Big]\sqrt{\beta^2-z^2},$$

$$A_{PV}(+,+;0) = -A_{PV}(-,-;0) = A_{VP}(+,+;0) = -A_{VP}(-,-;0) \\ = -\beta\Big[2(e_Q-e_q)[(1-x)e_Q+xe_q](1-\frac{z^2}{\beta^2}) \\ -(1-z^2)\left(\frac{1-x}{x}e_Q^2-\frac{x}{1-x}e_q^2\right)\Big],$$

$$A_{PV}(+,-;0) = -A_{PV}(-,+;0) = -A_{VP}(+,-;0) = A_{VP}(-,+;0) \\ = \frac{\hat{s}}{\beta}(\frac{1}{2}-x)(e_Q-e_q)^2(\beta^2-z^2)z. \quad (16)$$

Finally, the vector-vector (VV) production amplitude $\mathcal{M}^{\gamma\gamma}_{VV}(\lambda,\lambda';\sigma,\sigma')$ with initial photon helicities (λ,λ') and final vector meson helicities (σ,σ') is

$$\mathcal{M}^{\gamma\gamma}_{VV}(\lambda,\lambda';\sigma,\sigma') = 2F^{\gamma\gamma}\frac{\beta^2-z^2}{(1-z^2)^2}\Big\{(1+z^2)\big[e_Q{}^2F_{VV}(\lambda,\lambda';\sigma,\sigma')+e_q{}^2\tilde{F}_{VV}(\lambda,\lambda';\sigma,\sigma')\big] \\ -2e_Qe_qG_{VV}(\lambda,\lambda';\sigma,\sigma') \\ -(1-z^2)\Big[\frac{1-x}{x}e_Q^2H_{VV}(\lambda,\lambda';\sigma,\sigma')+\frac{x}{1-x}e_q^2\tilde{H}_{VV}(\lambda,\lambda';\sigma,\sigma')\Big]\Big\}, \quad (17)$$

where

$$F_{VV}(+,+;+,+) = 2(1-x)\left[1-x+\frac{1}{\beta}\right],$$

$$F_{VV}(+,+;+,-) = 0, \quad F_{VV}(+,-;+,+) = 2(1-x)^2-\frac{2}{\beta^2},$$

$$F_{VV}(+,-;+,-) = \frac{\hat{s}}{2}\left(\frac{\beta+z}{\beta-z}\right)\frac{1+z^2-2\beta z}{\beta^2(1+z^2)},$$

$$F_{VV}(+,+;+,0) = -F_{VV}(+,+;0,+) = \frac{N}{\sqrt{2}}\hat{s}\left(1+\frac{1}{\beta}\right)(1-x)z,$$

$$F_{VV}(+,-;+,0) = -F_{VV}(+,-;0,-) = \frac{N}{\sqrt{2}}\hat{s}\left(1+\frac{z}{\beta}\right)\left[\frac{2z}{1+z^2}-\frac{2}{\beta}+\beta(1-x)\right],$$

$$F_{VV}(+,+;0,0) = (1-x)[2+x(\hat{s}-2)],$$

$$F_{VV}(+,-;0,0) = (1-x)[2+x(\hat{s}-2)] + \frac{\hat{s}}{2} - \frac{4}{\beta^2}, \tag{18}$$

$$\begin{aligned}
G_{VV}(+,+;+,+) &= -4x(1-x) - \frac{2}{\beta} - \frac{2}{\beta^2-z^2}\left[1+z^2-2\frac{z^2}{\beta}\right],\\
G_{VV}(+,+;+,-) &= 0, \quad G_{VV}(+,-;+,+) = -2\left[2x(1-x)+\frac{z^2}{\beta^2}\right],\\
G_{VV}(+,-;+,-) &= \frac{\beta^2+z^2}{\beta^2-z^2}\left[\frac{2z^2}{\beta^2}+\hat{s}\frac{1+z^2}{2}-\frac{2\hat{s}z^2}{\beta^2+z^2}\right],\\
G_{VV}(+,+;+,0) &= -G_{VV}(+,+;0,+) = \frac{N}{\sqrt{2}}\hat{s}\left(1+\frac{1}{\beta}\right)z,\\
G_{VV}(+,-;+,0) &= -G_{VV}(+-;0-) = -\frac{N}{\sqrt{2}}\hat{s}\left(1+\frac{z}{\beta}\right)\left(\beta-2z+2\frac{z^2}{\beta}\right),\\
G_{VV}(+,+;0,0) &= 2x(1-x)(\hat{s}-2) + 2\frac{1+z^2}{\beta^2-z^2},\\
G_{VV}(+,-;0,0) &= 2x(1-x)(\hat{s}-2) + \frac{4}{\beta^2} - \frac{\hat{s}+4}{2\beta^2}(1+z^2),
\end{aligned} \tag{19}$$

$$\begin{aligned}
H_{VV}(+,+;+,+) &= 2(1-x)^2 + 2\frac{\beta+z^2}{\beta^2-z^2}\left[1-x+\frac{x}{\beta}\right],\\
H_{VV}(+,+;+,-) &= 0, \quad H_{VV}(+,-;+,+) = 2(1-x)^2, \quad H_{VV}(+,-;+,-) = 0,\\
H_{VV}(+,+;+,0) &= -H_{VV}(+,+;0,+) = -\frac{N}{\sqrt{2}}\hat{s}\left(1+\frac{1}{\beta}\right)xz,\\
H_{VV}(+,-;+,0) &= -H_{VV}(+,-;0,-) = \frac{N}{\sqrt{2}}\hat{s}(1-x)(\beta+z),\\
H_{VV}(+,+;0,0) &= \left[1-x+\frac{z^2}{\beta^2-z^2}\right][2+x(\hat{s}-2)] - x\frac{\hat{s}-2}{\beta^2-z^2},\\
H_{VV}(+,-;0,0) &= (1-x)[2+x(\hat{s}-2)].
\end{aligned} \tag{20}$$

and the tilded functions $\tilde{F}_{VV}(\lambda,\lambda';\sigma,\sigma')$ and $\tilde{H}_{VV}(\lambda,\lambda';\sigma,\sigma')$ are obtained with x replaced by $(1-x)$ in the corresponding untilded functions $F_{VV}(\lambda,\lambda';\sigma,\sigma')$ and $H_{VV}(\lambda,\lambda';\sigma,\sigma')$, respectively. On the other hand, other terms can be determined by parity and charge conjugation symmetries. The amplitude $\mathcal{M}_{VV}(-\lambda,-\lambda';-\sigma,-\sigma')$ is the same as $\mathcal{M}_{VV}(\lambda,\lambda',\sigma,\sigma')$ and $\mathcal{M}_{VV}(-\lambda,-\lambda';\sigma,\sigma')$ can be obtained by changing the sign of every β in $\mathcal{M}_{VV}(\lambda,\lambda',\sigma,\sigma')$.

The $\gamma\gamma$ transition amplitudes for the HL mesons with light u and d quarks, can be obtained by maintaining only the leading and next-to-leading x and leading s terms in the above expressions as in the e^+e^- case.

Certainly the results are valid only if every gluon momentum transfer is sufficiently large compared to the strong interaction scale Λ_{QCD}. To satisfy this condition, the c.m. energy should be high enough. The square of minimal momentum transfer in Figs. 2(a) is $\Lambda^2\hat{s}$. Even for $\Lambda = \Lambda_{QCD}$, it is at least larger than $4\Lambda_{QCD}^2$ so that the perturbative results are rather reliable.

IV. TWO LIMITS

A. Heavy quark limit

By definition, the heavy quark limit is the case where the center of mass energy $\sqrt{s}$ is not so high and one quark is much heavier than the other one. In this limit, all x and $x\hat{s}$ terms can be neglected in Eqs. (5-20) except for overall factors and therefore the transition amplitudes $\mathcal{M}^{ee}_{XY}$ and $\mathcal{M}^{\gamma\gamma}_{XY}$ are reduced to

$$\mathcal{M}^{ee}_{XY} = F^{ee}L^{ee}\left[\frac{M}{\Lambda}\right]^3[e_Q]B_{XY}, \tag{21}$$

$$\mathcal{M}^{\gamma\gamma}_{XY} = -\frac{F^{\gamma\gamma}}{1-z^2}\left[\frac{M}{\Lambda}\right][e_Q^2]C_{XY}. \tag{22}$$

where XY is a generic notation for (PP, PV, VV) and the nonvanishing B_{XY} and C_{XY} are given by

$$\begin{aligned}
B_{PP} &= 2,\\
B_{PV}(\pm) &= B_{VP}(\mp) = \pm\frac{N}{\sqrt{2}},\\
B_{VV}(+,+) &= B_{VV}(-,-) = 2,\\
B_{VV}(\pm,0) &= -B_{VV}(0,\pm) = \frac{N}{\sqrt{2}},\\
B_{VV}(0,0) &= 2,
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
C_{PP}(\lambda,\lambda') &= 4[\beta^2 - z^2\delta_{\lambda,-\lambda'}],\\
C_{PV}(+,-;\pm) &= -C_{PV}(-,+;\mp) = -(\beta\mp z)\sqrt{2\hat{s}(\beta^2-z^2)},
\end{aligned}$$

$$C_{PV}(+,+;0) = -C_{PV}(-,-;0) = 4\beta,$$
$$C_{VV}(+,+;+,+) = 4\beta(1+\beta), \quad C_{VV}(+,-;+,+) = 4(\beta^2 - z^2),$$
$$C_{VV}(+,-;+,0) = -C_{VV}(+,-;0,-) = (\beta+z)\sqrt{2\hat{s}(\beta^2-z^2)},$$
$$C_{VV}(+,+;0,0) = 4\beta^2, \quad C_{VV}(+,-;0,0) = 4(\beta^2-z^2). \tag{24}$$

The amplitudes $C_{VV}(-\lambda,-\lambda';-\sigma,-\sigma')$ are equal to $C_{VV}(\lambda,\lambda',\sigma,\sigma')$ and $C_{VV}(-\lambda,-\lambda';\sigma,\sigma')$ can be obtained by changing the sign of every β in $C_{VV}(\lambda,\lambda',\sigma,\sigma')$.

Note that only the terms with the heavy quark charge e_Q survive. The property can be understood in the context of the HQET. First, since the heavy quark pair production through the QCD interaction is not allowed, the diagrams in Fig. 1(b) as well as Fig. 2(b) can not contribute. Second, the diagrams in Fig. 2(c) are suppressed because of the velocity superselection rule (VSR) [7]. Let us explain this point in more detail. Two heavy quarks are produced from one photon and combined with light quarks produced from the other photon. In the c.m. frame, the produced heavy quarks are moving in the hemisphere along an original photon beam direction and one hard-gluon exchange is required to force one heavy quark to hadronize into a heavy meson flying into the hemisphere opposite to the original photon direction. In the course of this process, the heavy quark velocity should be changed very much. But this big velocity change is forbidden in the heavy quark limit.

On the other hand, one can extract the PQCD form factors corresponding to the so-called Isgur-Wise function from Eqs. (21) and (22) :

$$\xi^{ee}_{BL}(-v\cdot v') = \xi^{\gamma\gamma}_{BL}(-v\cdot v') = \frac{2\pi\alpha_s C_F}{3(1+v\cdot v')^2\Lambda^3}\left[M f_M^2\right]. \tag{25}$$

Two factors are equal and have one common factor $[Mf_M^2]$ which is independent of the heavy quark mass m_Q because

$$f_M \propto \frac{1}{\sqrt{M}}, \tag{26}$$

is satisfied in the context of the HQET and the heavy meson mass M is equal to the heavy quark mass m_Q in the heavy quark limit. As a result, all transition amplitudes are independent of m_Q. This property exists even if the light quark mass is larger than the strong interaction scale Λ_{QCD}. This is nothing but a manifestation of the heavy quark flavor symmetry.

B. High energy limit

In the high energy limit, only terms with $\hat{s}$ survive and therefore the nonvanishing transition amplitudes $\mathcal{M}^{ee}$ and $\mathcal{M}^{\gamma\gamma}$ are given by

$$\mathcal{M}^{ee}_{PP} = -\mathcal{M}^{ee}_{VV}(0,0) = -F^{ee}L^{ee}\hat{s}\left[\frac{e_Q}{x^2} - \frac{e_q}{(1-x)^2}\right], \tag{27}$$

and

$$\mathcal{M}^{\gamma\gamma}_{XY} = 2F^{\gamma\gamma}\frac{\hat{s}}{1-z^2}D_{XY} \tag{28}$$

where D_{XY} is given by

$$\begin{aligned}
D_{PP}(\lambda,\lambda') &= -2x(1-x)(e_Q-e_q)^2 \\
&\quad + \delta_{\lambda,-\lambda'}\left\{\frac{1+z^2}{2}(e_Q^2+e_q^2) + (1-z^2)\left[(1-x)e_Q - xe_q\right]\left[e_Q - e_q\right]\right\}, \\
D_{PV(VP)}(\pm,\mp;0) &= \pm(e_Q-e_q)^2(1-2x)z, \\
D_{VV}(\pm,\mp;\pm,\mp) &= (1-z^2)(e_Q-e_q)^2, \\
D_{VV}(\lambda,\lambda';0,0) &= 2x(1-x)(e_Q-e_q)^2 \\
&\quad + \delta_{\lambda,-\lambda'}\left\{\frac{1+z^2}{2}(e_Q+e_q)^2 - (1-z^2)\left[(1-x)e_Q^2 + xe_q^2\right]\right\}.
\end{aligned} \tag{29}$$

Let us now consider the case that x is also very small in the high energy limit. Then the PP production amplitudes $\mathcal{M}^{ee}_{PP}$ and $\mathcal{M}^{\gamma\gamma}_{PP}(\lambda,\lambda')$ are reduced to

$$\mathcal{M}^{ee}_{PP} = \frac{16\pi^2\alpha_e\alpha_s C_F}{3M\Lambda^2\hat{s}}[e_Q]\,[f_M]^2\,\frac{L^{ee}}{\hat{s}}, \tag{30}$$
$$\mathcal{M}^{\gamma\gamma}_{PP}(\lambda,\lambda') = -\frac{16\pi^2\alpha_e\alpha_s C_F}{3\Lambda^2\hat{s}}[f_M]^2\left[(e_Q-e_q)^2\frac{1+z^2}{1-z^2} + 2e_Q^2\right]\delta_{\lambda,-\lambda'}, \tag{31}$$

where one $1/\hat{s}$ factor in $\mathcal{M}^{ee}_{PP}$ comes from the s-channel photon propagator. First, one interesting point is that the amplitude $\mathcal{M}^{\gamma\gamma}_{PP}(\lambda,\lambda')$ depends not only on the heavy quark charge e_Q but also on the light quark charge e_q. This aspect causes the charged meson production rate to be larger than the neutral meson production and to exhibit an angular dependence different from the neutral meson production. The difference can be, therefore, utilized to clearly distinguish the charged heavy meson production from the neutral heavy meson production in the $\gamma\gamma$ collision. Second, two overall factors in Eqs. (30) and (31) are noted to be proportional to $1/\hat{s}$. Since every momentum transfer in the heavy meson pair production processes is proportional to $\hat{s}$, the scaling is consistent with a general property [12] of PQCD that, at large momentum transfer, the meson form factors should fall as $1/Q^2$.

V. DISCUSSION AND CONCLUSION

The BL formalism in PQCD was employed to estimate the validity of HQET in the heavy quark limit and the large-momentum scaling law in the high energy limit. In addition to the ordinary heavy mesons made up of a heavy quark and a light quark, the heavy mesons of which two quark masses are larger than Λ_{QCD} were also considered. In the two cases, the HQET turns out to be applicable within a constrained kinematic regime given explicitly below and the high energy scaling laws are shown to be satisfied at very high energies.

Above all, it is clear from the expression of Eqs. (5) and (7) that the following condition in the e^+e^- case should be satisfied

$$\hat{s} \ll 2/x - 2, \tag{32}$$

where, for $\Lambda = 200$ MeV, x is approximately 1/8 and 1/25 for the c quark and the b quark, respectively. The condition severely constrains the validity regime of the HQET for realistic heavy quarks such as b and c quarks. In particular, the validity region for a charm-quark hadron is very restricted and very large $1/m_c$ corrections are expected. For a bottom-quark hadron system, the upper bound on $\hat{s}$ is three times larger than a charm-quark hadron system and so the heavy quark symmetry may be employed for qualitative and even quantitative physical analyses. The upper bound on $\hat{s}$ in the $\gamma\gamma$ case is a little complicated, because many

terms are angle-dependent, but the bound is essentially the same as that in the e^+e^- case while the collision angle θ is large enough. Furthermore, from a more detailed analysis, the condition (32) turns out to be relatively strong. For some helicity amplitudes, there is no specific bound on $\hat{s}$ for the application of HQET. On the other hand, the gluon momentum transfer should be sufficiently large enough to enable the BL formalism to be applied. The minimal momentum transfer in the processes under consideration is

$$Q^2_{min} = \hat{s}\Lambda^2 > 4\Lambda^2. \tag{33}$$

Therefore, the process is in the perturbative regime so long as it is well above threshold.

Two crucial assumptions were made in our calculation. One assumption is that, on the average, the heavy quark and the light anti-quark share the heavy meson momentum in proportion to their masses and the spread of the momentum fraction is very small [8,14,16]. The other assumption is that one-gluon exchange dominates. Therefore, for more detailed analysis, one must use more exact meson wave functions and include radiative corrections and higher-twist terms for the HL meson systems. It is, however, expected that the gross features of the results in this paper are maintained even for such modifications.

The present results lack in exactness and so further studies are required. First of all, the validity of the peaking approximation should be closely checked. In the case that one considers a heavy meson consisting of two heavy quarks, the bound wave functions can be described very well with a nonrelativistic potential, i.e. Coulomb potential [19]. As the quark mass increases, the wave function gets peaked and takes a delta function form. If we consider a meson made up of a heavy quark and *one effective light quark*, the light quark is moving very fast and thus the relativistic effects are considerable. The peaking approximation might be very crude. To attack the problem, one might have to employ nonperturbative frameworks such as lattice techniques or light-front Tamm-Dancoff method. On the other hand, QCD radiative corrections [20] can be treated perturbatively and systematically in the perturbative regime.

In conclusion, the HQET can be applied to discuss the exclusive heavy meson pair production through non-QCD interactions only for a certain confined range of the center of mass energy. The larger the c.m. energy is, the more reliable the perturbative result is. There should be some sort of compromise between PQCD and HQET in order that two frameworks be justified. As the c.m. energy increases to a very large value, all particle masses can be neglected and the meson form factors are forced to comply with the power counting laws [12]. In addition, there are two interesting features : the existence of a zero form factor in the e^+e^- case [19] is reconfirmed in the heavy quark limit, and the meson form factors in the $\gamma\gamma$ case are shown to be dependent on the light quark electric charge as well as the heavy quark electric charge when the high energy and the heavy quark limit are considered at the same time.

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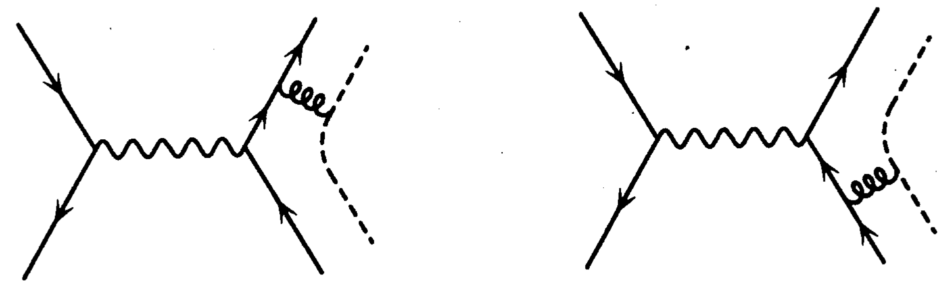
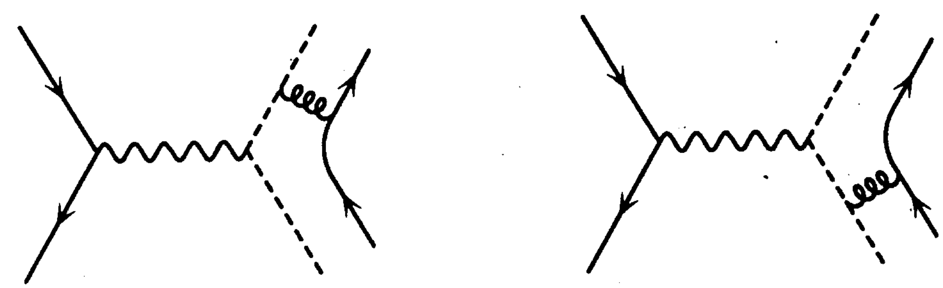
FIGURES

FIG. 1. Heavy meson pair production in e^+e^- annihilation. The left solid line is for the incident electron and the right solid line is for the heavier quark. The wiggly(curly) line is for a photon(gluon), while the dotted one is for the lighter quark.

FIG. 2. Heavy meson pair production in photon-photon collision. The wiggly(curly) line is for a photon(gluon), while the solid(dotted) one is for the heavier(lighter) (anti-)quark.



(a)



(b)

Fig. 1

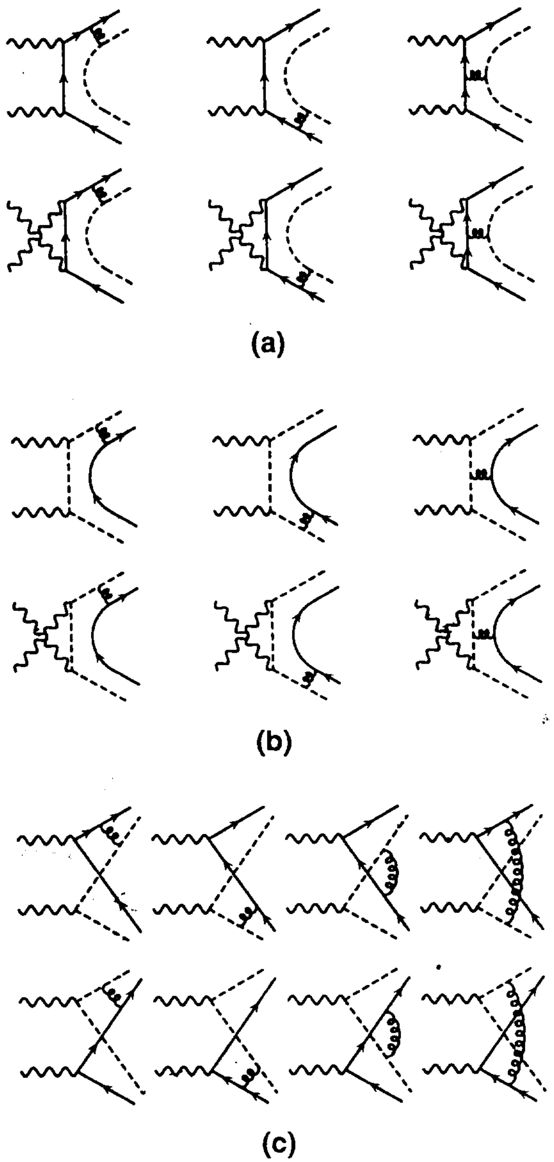


Fig. 2