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The Gaussian Approximation of The Abelian Higgs Model in The Functional Schrödinger Picture

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To establish the BRST invariant functional variational method for the gauge field theories in the covariant gauge, we have computed the Gaussian effective potential of the Abelian Higgs model in the functional Schrödinger picture. We obtain the same result as that of Ibañez-Meier, Stancu and Stevenson, but we have used the BRST invariant variational procedures.

1. Introduction

The functional Schrödinger picture formulation of quantum field theories has been shown to provide a practical device in obtaining non-perturbative informations through functional variational method[1]. It has also been proved to be especially convenient in dealing with the gauge degrees of freedom in quantization of gauge field theories[2][3]. However, the application of the functional variational method to the gauge field theories has not been succesful due to the difficulties in imposing the gauge fixing conditions properly[4].

Recently, Ibañez-Meier, Stancu and Stevenson have been able to compute the Gaussian effective potential of the Abelian Higgs model by using the particle picture[5]. However, they ignored the Faddeev-Popov ghost terms and had to make variation with respect to the gauge parameter ξ. As a result, their trial wave functional does not satisfy the BRST invariance condition, although their result is physically reasonable.

Although it is difficult to solve the BRST invariance condition for the state functional directly, the decomposition of the state into the ghost part and the other part as, $|\Psi>=|Ghost>|other>$, simplifies the invariance condition[6]. One way to achieve this is to choose the ghost part of the wave functional as a delta functional. This choice is, however,too strong condition for non-Abelian gauge theories in that the ghost and the ghost related terms must vanish. Another way is to make a transformation of field variables so that the matrix elements of operators can easily be computed[2]. The purpose of this paper is to estabilish the BRST invariant variational approximation method by using the latter method .

In sec.2, we seek the vacuum wave functional that satisfies the BRST constraint equation and then evaluate the effective potential by the variational method in the functional Schrödinger picture. In sec.3 we renormalize the effective potential, and in the last section the discussions on the relations with other results are given.

2. Calculation of the effective potential

We consider the Abelian Higgs model described by the Lagrangian density,

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}(\partial_\mu\phi_1 - eA_\mu\phi_2)^2 + \frac{1}{2}(\partial_\mu\phi_2 + eA_\mu\phi_1)^2 - \frac{1}{2}m_B^2\phi_a\phi_a \\ &- \lambda_B(\phi_a\phi_a)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + i\partial^\mu\bar{\eta}\partial_\mu\eta,\end{aligned} \tag{2.1}$$

where $\phi = (\phi_1, \phi_2)$ are two real components of the charged scalar field, and $(\eta, \bar{\eta})$ are Faddev-Popov ghosts. From the Lagrangian (2.1) one obtains the Hamiltonian density of the system,

$$\begin{aligned}\mathcal{H} = &\frac{1}{2}\pi_a\pi_a + \frac{1}{2}\partial_i\phi_a\partial_i\phi_a + \frac{1}{2}m_B^2\phi_a\phi_a + \lambda_B(\phi_a\phi_a)^2 \\ &+ \frac{1}{2}e_B^2A_iA_i\phi_a\phi_a + \frac{1}{2}\pi_{A_i}\pi_{A_i} + \frac{1}{2}B_iB_i - \pi_i\partial_iA_0 \\ &- \frac{1}{2}\xi\pi_{A_0}\pi_{A_0} + \pi_{A_0}\partial_iA_i - i\wp\bar{\wp} - i\nabla\bar{\eta}\nabla\eta,\end{aligned} \tag{2.2}$$

and the equal-time (anti)commutation relations,

$$\begin{aligned}[\pi_a(x), \phi_b(x')]_{x_0=x_0'} &= -\delta_{ab}\delta(\vec{x} - \vec{x}') \\ [\pi_{A_\mu}(x), A_\nu(x')]_{x_0=x_0'} &= -ig_{\mu\nu}\delta(\vec{x} - \vec{x}') \\ \{\eta(x), \wp(x')\}_{x_0=x_0'} &= -i\delta(\vec{x} - \vec{x}') \\ \{\bar{\eta}(x), \bar{\wp}(x')\}_{x_0=x_0'} &= -i\delta(\vec{x} - \vec{x}'),\end{aligned} \tag{2.3}$$

where $\wp$ and $\bar{\wp}$ are conjugate momenta to the ghost fields η and $\bar{\eta}$, respectively. The equal-time (anti)commutation relations(2.3) lead to the Schrödinger picture representations of the quantum operators:

$$\begin{aligned}\phi_a &\to \phi_a(x), & \pi_a &\to -i\frac{\delta}{\delta\phi} \\ A^\mu &\to A^\mu(x), & \pi_{A_\mu} &\to i\frac{\delta}{\delta A^\mu} \\ \eta &\to \eta(x), & \wp &\to -i\frac{\delta}{\delta\eta} \\ \bar{\eta} &\to \bar{\eta}(x), & \bar{\wp} &\to -i\frac{\delta}{\delta\bar{\eta}}.\end{aligned} \tag{2.4}$$

One can easily show that the Lagrangian (2.1) is invariant under the BRST transformation

$$\begin{aligned}\delta A_\mu &= (\partial_\mu\eta)\varepsilon, \\ \phi_a &= ie\epsilon_{ab}\phi_b\eta\varepsilon, \\ \delta\bar{\eta} &= \frac{1}{\xi}(\partial_\mu A^\mu)\varepsilon, \\ \delta\eta &= 0,\end{aligned} \tag{2.5}$$

where ε is a Grassmann constant which anticommute with η and $\bar{\eta}$. This invariance can be realized by imposing the BRST invariance condition on the physical states[7],

$$Q\Psi(\phi_a, A^\mu, \eta, \bar{\eta}; t) = 0 \tag{2.6}$$

where

$$Q = i\int d^3x[\eta(\partial_i\frac{\delta}{\delta A_i} - e\epsilon_{ab}\phi_a\frac{\delta}{\delta\phi_b}) - \frac{\delta}{\delta\bar{\eta}}\frac{\delta}{\delta A_0}]. \tag{2.7}$$

Writing the wave functional appearing in Eq.(2.6) in the form

$$\Psi(\phi_a, A^\mu, \eta, \bar{\eta}; t) = e^{\bar{\eta}(x)D(x-x';t)\eta(x')}\Phi(\phi_a, A^\mu; t), \tag{2.8}$$

the BRST invariance condition (2.6) can be written as the condition on Φ:

$$[\partial_i\frac{\delta}{\delta A_i} - e\frac{\delta}{\delta\theta} - \bar{D}\frac{\delta}{\delta A_0}]\Phi(\phi_a, A^\mu; t) = 0, \tag{2.9}$$

where $\bar{D}(x, x'; t) = D(x', x; t)$. By using the change of variables

$$\begin{aligned}r(x) &= \nabla\cdot\vec{A}(x) - \bar{D}^{-1}(x - x'; t)\nabla^2A^0(x'), \\ s(x) &= \nabla\cdot\vec{A}(x) - \frac{1}{e}\nabla^2\theta(x), \\ u(x) &= a\nabla\cdot\vec{A} - \frac{b}{e}\nabla^2\theta(x) - c\bar{D}^{-1}(x - x'; t)\nabla^2A^0(x'),\end{aligned} \tag{2.10}$$

where a, b and c are constant numbers such that $a - b - c \neq 0$, the BRST invariance condition (2.9) becomes

$$\frac{\delta}{\delta u}\Phi(\phi_a, A^\mu; t) = 0. \tag{2.11}$$

This means that physical states must be independent of the variable $u(x)$[2].

We now have to find the wave functional Ψ that satisfies both the functional Schrödinger equation and the BRST invariance condition (2.11). One way to realize this procedure is to solve the Schrödinger equation for general wave functional Ψ and then require the BRST invariance condition (2.11) when one computes the inner product,

$$< \Psi_1|\Psi_2 >= \int \mathcal{D}\phi\mathcal{D}A\mathcal{D}\eta\mathcal{D}\bar{\eta}\delta(u)\bar{\Psi}_1\Psi_2, \tag{2.12}$$

which guarantees that only the BRST invariant state functionals contribute to the physical amplitudes. Eqs (2.10) and (2.12) imply that there exist infinitely many equivalent ways to realize the covariant gauge quantization procedure. One can easily show, for example, that when $(a,b,c) = (0,0,1)$ it is equivalent to the Weyl gauge, and when $(a,b,c) = (1,0,0)$ to the Coulomb gauge. And we can choose the values of a,b and c conveniently for given problems[2].

If we choose nonzero b, we must use the polar coordinates for the scalar fields, which makes the problem complicated in the functional Schrödinger picture. For the gauge field theoris,therefore, the computations become greatly simplified if one chooses $(a,b,c) = (1,0,-1)$. Then, for the functional variational approximation, we can choose Gaussian trial wave functional,

$$\Psi = Ne^{\int d^3x d^3x'[\bar{\eta}(x)D(x,x')\eta(x') - \frac{1}{4}A^{\mu}(x)G^{-1}_{\mu\nu}(x,x')A^{\nu}(x') - \frac{1}{4}(\phi_a(x)-\varphi_a)F^{-1}_{ab}(x,x')(\phi_b(x)-\varphi_b)]} \tag{2.13}$$

where $G_{\mu\nu}$ and F_{ab} are 4×4 and 2×2 matrix functions, respectively. For this choice of (a,b,c) the inner product between two states $|\Psi_1>$ and $|\Psi_2>$ becomes

$$< \Psi_1|\Psi_2 >= \int \mathcal{D}\phi\mathcal{D}A_\mu\mathcal{D}\eta\mathcal{D}\bar{\eta}\delta(\nabla\cdot\vec{A} + \bar{D}^{-1}\nabla^2 A^0)\bar{\Psi}_1\Psi_2. \tag{2.14}$$

which guarantees the BRST invariance of the physical informations. Given the trial wave functional our problem is to minimize the energy expectation value,

$$E =< \mathcal{H} >= \int d^3x \int \mathcal{D}\phi_a\mathcal{D}A_\mu\mathcal{D}\eta\mathcal{D}\bar{\eta}\delta(u)\bar{\Psi}\mathcal{H}\Psi. \tag{2.15}$$

In the momentum space, Eq.(2.15) can be written as,

$$\begin{aligned}
< \mathcal{H} >=& \frac{1}{2}m_B^2\varphi_c^2 + \lambda_B\varphi_c^4 + \frac{1}{8}\int_p F_{aa}^{-1} + \frac{1}{2}\int_p (p^2+m_B^2)F_{aa}(p) \\
&+ \lambda_B[\int_p\int_p' (F_{aa}F_{bb} + 2F_{ab}F_{ab}) + \int_p (2\varphi_c^2 F_{aa} + 4\varphi_c^2\delta_{a1}\delta_{b1}F_{ab})] \\
&+ \frac{1}{2}e_B^2\int_p\int_p' \Lambda_{ii}(F_{aa}+\varphi_c^2) + \frac{1}{2}\int_p (p^2\Lambda_{ii} - p_ip_j\Lambda_{ij}) + \frac{1}{8}\int_p \Lambda_{ii}^{-1}(p) - \frac{\xi}{8}\int_p \Lambda_{00}^{-1}(p) \\
&- \frac{1}{2}\int_p p_i\Lambda_{\mu i}^{-1}(p)\Lambda_{\mu 0} + \frac{1}{2\alpha}\int_p p^2 p_i\Lambda_{i0} - \frac{i}{2\alpha}\int_p p^4 D^{-1}(p)\Lambda_{00}(p) \\
&- \frac{1}{2}\int_p p_i\Lambda_{0\mu}^{-1}(p)\Lambda_{i\mu} + \frac{1}{2\alpha}\int_p p_ip_jD^{-1}p^2\Lambda_{ij} + \frac{1}{2\alpha}\int_p p^4 D^{-2}(p)p_i\Lambda_{i0}(p) \\
&- \frac{1}{8\alpha^2}\int_p (p^2p_ip_j\Lambda_{ij} + 2ip^4\Lambda_{i0}D^{-1} - p^6D^{-2}\Lambda_{00}) \\
&+ \frac{\xi}{8\alpha^2}\int_p (-p^4p_ip_j\Lambda_{ij}D^{-2} - 2ip_ip^6D^{-3}\Lambda_{i0} + p^8\Lambda_{00}D^{-4}) \\
&- i\int_p [D(p) + p^2(D(p)+(D^+)^{-1})^{-1}],
\end{aligned} \tag{2.16}$$

where we have used the relations:

$$\begin{aligned}
&\varphi_c \equiv \varphi_1 \\
&\int_p \equiv \int \frac{d^3p}{(2\pi)^3}, \qquad \delta(u) \equiv \lim_{\alpha\to 0} e^{-\frac{1}{2\alpha}\int d^3x u^2} \\
&\Lambda_{\mu\nu}^{-1} \equiv G_{\mu\nu}^{-1} + \frac{1}{\alpha}[-\delta_{i\mu}\delta_{j\nu}\partial_i\partial_j + \delta_{\mu i}\delta_{\nu 0}\partial_i D^{-1}\nabla^2 + \delta_{\mu 0}\delta_{\nu i}\partial_i D^{-1}\nabla^2 \\
&\quad + \delta_{\mu 0}\delta_{\nu 0}\nabla^2 D^{-2}\nabla^2].
\end{aligned} \tag{2.17}$$

Minimization of $< \mathcal{H} >$ with respect to D^{-1}, Λ_{00} and Λ_{i0} leads to trivial results which are φ_c-independent and can be ignored. Note that $D(x,x';t)$ is the ghost field contribution as can be seen from Eq.(2.8). This means that the ghost fields for the Abelian gauge theory do not contribute to the effective potential. And if we choose $\vec{p} = (0,0,p)$, Λ^{-1} is diagonalized and $\Lambda_{\mu\nu}^{-1}$, for $\mu,\nu = 0,3$, is determined. So we have only to determine the diagonal part of $\Lambda_{ij}^{-1}(i,j=1,2)$ and F_{ab}^{-1}. One can see from eq.(2.16) that the off-diagonal terms of F_{ab}^{-1} are also φ_c independent. Thus the condition, $\frac{\delta}{\delta\Lambda_{ij}} < \mathcal{H} >= 0$, leads to

$$\frac{1}{2}e_B^2\delta_{ij}[\int_p F_{aa} + \varphi_c^2] - \frac{1}{8}\Lambda_{ij}^{-2} + \frac{1}{2}p^2\delta_{ij} = 0, \quad i,j = 1,2 \tag{2.18}$$

which can be written as

$$\Lambda_{ii}^{-2} = 4p^2 + 4e_B^2[\int_p F_{aa} + \varphi_c^2]. \tag{2.19}$$

The condition, $\frac{\delta}{\delta F_{ab}} < \mathcal{H} >= 0$, leads to

$$F_{ab}^{-2} = 4(p^2 + m_B^2 + e_B^2 \int_p \Lambda_{ii})\delta_{ab} + 8\lambda_B[\int_p (2F_{cc}\delta_{ab} + 4F_{ab}) + 2\varphi_c^2\delta_{ab} + 4\varphi_c^2\delta_{a1}\delta_{b1}]. \tag{2.20}$$

If we introduce Δ, Ω and ω defined by

$$\begin{aligned}\Delta^2 &= e_B^2(I_0(\Omega^2) + I_0(\omega^2) + \varphi_c^2)\\ \Omega^2 &= m_B^2 + 4\lambda_B[3I_0(\Omega^2) + I_0(\omega^2) + 3\varphi_c^2] + 2e_B^2 I_0(\Delta^2)\\ \omega^2 &= m_B^2 + 4\lambda_B[I_0(\Omega^2) + 3I_0(\omega^2) + \varphi_c^2] + 2e_B^2 I_0(\Delta^2),\end{aligned} \tag{2.21}$$

where I_0 and I_1 denote the integrals

$$I_0(\Omega^2) \equiv \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\sqrt{p^2+\Omega^2}}, \quad I_1(\Omega^2) \equiv \int \frac{d^3p}{(2\pi)^3} \frac{1}{2}\sqrt{p^2+\Omega^2}, \tag{2.22}$$

the matrix functions can be written as

$$\begin{aligned}\Lambda_{ii}^{-2} &\equiv 4(p^2 + \Delta^2)\\ F_{11}^{-2} &\equiv 4(p^2 + \Omega^2)\\ F_{22}^{-2} &\equiv 4(p^2 + \omega^2).\end{aligned} \tag{2.23}$$

By inserting (2.19) and (2.23) back into (2.18) and (2.20), we obtain the effective potential

$$\begin{aligned}V_{eff} &= J(\Omega^2) + J(\omega^2) + \frac{1}{2}m_B^2[\varphi_c^2 + I_0(\Omega^2) + I_0(\omega^2)]\\ &+ \lambda_B[3I_0^2(\Omega^2) + 3I_0^2(\omega^2) + 2I_0(\Omega^2)I_0(\omega^2) + 6\varphi_c^2 I_0(\Omega^2) + 2\varphi_c^2 I_0(\omega^2) + \varphi_c^4]\\ &+ e_B^2 I_0(\Delta^2)[I_0(\Omega^2) + I_0(\omega^2) + \varphi_c^2] + 2J(\Delta^2),\end{aligned} \tag{2.24}$$

where $J(\alpha) = I_1(\alpha) - \frac{1}{2}\alpha^2 I_0(\alpha)$.

3. Renormalization

We first introduce another divergent integral:

$$I_{-1} \equiv -2\frac{dI_0}{d\Omega^2} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega_p^3} = \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+\Omega^2)^2} \tag{3.1}$$

where $\omega_p = \sqrt{p^2+\Omega^2}$. Using this, we can obtain the following convenient relations between $I_{\pm 1}$ and I_0:

$$\begin{aligned}I_1(\Omega^2) &= I_1(0) + \frac{\Omega^2}{2}I_0(0) - \frac{\Omega^4}{8}I_{-1}(\mu^2) + f(\Omega^2)\\ I_0(\Omega^2) &= I_0(0) - \frac{\Omega^2}{2}I_{-1}(\mu^2) + 2f'(\Omega^2)\\ I_{-1}(\Omega^2) &= I_{-1}(\mu^2) - \frac{1}{8\pi^2}\ln\frac{\Omega^2}{\mu^2},\end{aligned} \tag{3.2}$$

where $f(\Omega^2) = \frac{\Omega^4}{64\pi^2}[\ln\frac{\Omega^2}{\mu^2} - \frac{3}{2}]$ and $f'(\Omega^2)$ is the derivative of f with respect to Ω^2. The theory contains two divergent integrals $I_{-1}(\mu^2)$ and $I_0(0)$. To absorb the divergences to the physical parameters, we impose the renormalization conditions,

$$\begin{aligned}\frac{dV_{eff}}{d\Omega^2}|_{\Omega_0} &= \frac{m_R^2}{24\lambda_R},\\ \frac{d^2V_{eff}}{d(\Omega^2)^2}|_{\Omega_0} &= \frac{1}{72\lambda_R},\end{aligned} \tag{3.3}$$

as in the case of scalar ϕ^4 theory.

From the first equation of (3.3), we obtain

$$\frac{m_B^2}{\lambda_B} + \frac{1}{\lambda_B}(16\lambda_B + 2e_B^2)I_0(0) = \frac{m_R^2}{\lambda_R}, \tag{3.4}$$

and the second equation of (3.3) can be written as

$$\frac{d^2V_{eff}}{d(\Omega^2)^2}|_{\Omega_0} = \frac{1}{\lambda}(\frac{1}{72} - \frac{\lambda}{12}I_{-1} - \lambda^2 I_{-1}^2) + finite\ terms. \tag{3.5}$$

For the right hand side of eq.(3.3) to be finite, λ_B must satisfy the condition

$$\frac{1}{72} - \frac{\lambda_b}{12}I_{-1} - \lambda_B^2 I_{-1}^2 = 0. \tag{3.6}$$

This equation has two solutions.

$$\lambda_B I_{-1} = -\frac{1}{6} \quad or \quad \frac{1}{12}. \tag{3.7}$$

For the solution $\lambda_B I_{-1} = -\frac{1}{6}$, the renormalization condition becomes

$$\frac{1}{\lambda_B} + 6I_1 = \frac{1}{\lambda_R}. \tag{3.8}$$

If $\lambda_B I_{-1} = \frac{1}{12}$, the renormalization condition becomes

$$\frac{1}{\lambda_B} - 12I_1 = \frac{1}{\lambda_R}. \tag{3.9}$$

If we rewrite the gap equations (2.21) by using these conditions, we can see that we need another renormalization condition for both case:

$$\begin{aligned} \lambda_B \phi_c^2 &= \lambda_R \Phi_c^2 \\ e_B^2 I_{-1}(\mu^2) &= e_R^2. \end{aligned} \tag{3.10}$$

For the positive λ_B, the second solution of (3.7), we obtain from the eqs.(3.4),(3.9)and (3.10):

$$\begin{aligned} \varphi_c^2 &= I_{-1}(\mu^2)\Phi_c^2, \quad \lambda_B = \lambda_R/I_{-1}(\mu^2), \\ e_B^2 &= e_R^2/I_{-1}(\mu^2), \qquad m_B^2 = m_0^2/I_{-1}(\mu^2), \end{aligned} \tag{3.11}$$

where λ_R, e_R^2 and m_0^2 are finite and μ^2 is a finite mass scale. This is the so-called 'autonomous' renormalizition condition of Ibañez-Meier et al [5]. Dimensional regularization can justify setting the scaleless integral $I_0(0)$ equal to zero. Setting $I_0(0) = 0$ and using the formula (3.2), the gap equation Eq.(2.21) can be written as

$$\begin{aligned} \Delta^2 &= e_R^2(\Phi_c^2 - \frac{1}{2}\Omega^2 - \frac{1}{2}\omega^2) + \varepsilon_{\Delta^2} \\ \Omega^2 &= 4\lambda_R(3\Phi_c^2 - \frac{3}{2}\Omega^2 - \frac{1}{2}\omega^2) - e_R^2\Delta^2 + \varepsilon_{\Omega^2} \\ \omega^2 &= 4\lambda_R(\Phi_c^2 - \frac{1}{2}\Omega^2 - \frac{3}{2}\omega^2) - e_R^2\Delta^2 + \varepsilon_{\omega^2}, \end{aligned} \tag{3.12}$$

where ε terms are infinitesimal, $O(1/I_{-1})$ terms. Ignoring those terms, Eq.(3.12) can be solved to yield

$$\begin{aligned} \Delta^2 &= \frac{e_R^2}{1 + 8\lambda_R - e_R^4}\Phi_c^2, \\ \Omega^2 &= \frac{4\lambda_R(3 + 16\lambda_R) - e_R^4(1 + 8\lambda_R)}{(1 + 4\lambda_R)(1 + 8\lambda_R - e_R^4)}\Phi_c^2, \\ \omega^2 &= \frac{4\lambda_R - e_R^4}{(1 + 4\lambda_R)(1 + 8\lambda_R - e_R^4)}\Phi_c^2. \end{aligned} \tag{3.13}$$

Total derivative of V_{eff} with respect to φ_c becomes

$$\begin{aligned} \frac{dV_{eff}}{d\varphi_c} &= \frac{\partial V_{eff}}{\partial \varphi_c} = \varphi_c[m_B^2 + 4\lambda_B(3I_0(\Omega) + I_0(\omega) + \varphi_c^2) + 2e_B^2 I_0(\Delta)] \\ &= \varphi_c(\Omega^2 - 8\lambda_B\varphi_c^2). \end{aligned} \tag{3.14}$$

In order for V_{eff} to be finite in terms of the rescaled field Φ_c, we must have a cancellation between the finite part of Ω^2 and $8\lambda_B\varphi_c^2 = 8\lambda_R\Phi_B^2$, which implies the constraint equation,

$$e_R^4 = 4\lambda_R\frac{1 - 8\lambda_R - 64\lambda_R^2}{1 - 32\lambda_R^2}. \tag{3.15}$$

Using this constraint equation, Eq.(3.13) becomes

$$\begin{aligned} \Delta^2 &= \frac{e_R^2(1 - 32\lambda_R^2)}{1 + 4\lambda_R}\Phi_c^2 + O(1/I_{-1}), \\ \Omega^2 &= 8\lambda_R\Phi_c^2 + O(1/I_{-1}), \\ \omega^2 &= \frac{32\lambda_R}{1 + 4\lambda_R}\Phi_c^2 + O(1/I_{-1}). \end{aligned} \tag{3.16}$$

The renormalized effective potential is obtained from Eq.(3.14). With the leading divergent terms cancelled in Eq.(3.14), one need to consider the infinitesimal parts, $O(1/I_{-1})$ of Ω^2, since they are multiplied by φ_c which is divergent, in order to obtain the finite part. Using explicit form of $\varepsilon_{\Delta^2}, \varepsilon_{\Omega^2}$ and ε_{ω^2}, one can solve for the $O(1/I_{-1})$ corrections to Ω^2 in Eq.(3.16). One thus obtains:

$$\frac{dV_{eff}}{d\Phi_c} = 2\Phi_c[2(\frac{d\Delta^2}{d\Phi_c^2})f'(\Delta^2) + (\frac{d\Omega^2}{d\Phi_c^2})f'(\Omega^2) + (\frac{d\omega^2}{d\Phi_c^2})f'(\omega^2)] + \Phi_c m^2. \tag{3.17}$$

Thus, by integrating Eq.(3.17) with respect to Φ_c, one obtains the renormalized effective potential

$$V_{eff} = 2f(\Delta^2) + f(\Omega^2) + f(\omega^2) + \frac{1}{2}m^2\Phi_c^2. \tag{3.18}$$

This can be conveniently reparametrized by vacuum value Φ_v, defined as the position of the minimum of V_{eff}, as

$$V_{eff} = K\Phi_c^4[\ln(\frac{\Phi_c^2}{\Phi_v^2}) - \frac{1}{2}] + \frac{1}{2}m^2\Phi_c^2(1 - \frac{1}{2}\frac{\Phi_c^2}{\Phi_v^2}), \tag{3.19}$$

where $K = \frac{\lambda_R(1+8\lambda_R)(1-8\lambda_R+32\lambda_R^2+256\lambda_R^3)}{8\pi^2(1+4\lambda_R)^2}$ and $m^2 = \frac{1-32\lambda_R^2}{1+4\lambda_R}m_0^2$. This result is the same as that of ref.[5].

For the case of negative λ_B, we see that V_{eff} cannot be finite in terms of the rescaled Φ_R from (3.14). Therefore, for the case λ_B is negative, we still need to find a consistent renormalization method.

4. Discussions

The main difficulty in establishing the consistent variational approximation method for the gauge field theories has been in finding the BRST invariant procedure for the variational calculations. We have established the BRST invariant variational method by transforming the field variables in such a way that the BRST invariance condition appears as a simple delta functional in the inner product of the Hilbert space elements. In this procedure it is natural to take the trial wave functional in a Gaussian form which includes the ghost fields contributions. This clearly exhibits how the ghost fields contribute to the effective potential in the functional Schrödinger picture. As has been shown, the ghosts and the ghost related terms contribute only to the infinite constant terms in the effective potential, and thus have no physical effects for the Abelian gauge theories. This is the reason why Ibañez-Meier, Stancu and Stevenson have obtained the correct result by ignoring the ghost contribution[5]. For the non-Abelian gauge theories, however, we have to include the ghost terms in a BRST invariant way to obtain the correct results.

As has been explained in the introduction, there exists another way to achieve the BRST invariance, i.e., by choosing the ghost part of the trial wave funtional as a delta functional[6]. This gives the same result as that of ref.[5] for the Abelian theories. This method, however, cannot be applied to the non-Abelian gauge theories since the ghost contributions cannot be ignored for the non-Abelian cases.

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