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INSTITUTE OF THEORETICAL PHYSICS ACADEMIA SINICA

AS-ITP-94-14
May 1994

sw9421

BLOCKING EFFECT AND ODD–EVEN DIFFERENCES IN THE MOMENTS OF INERTIA OF RARE–EARTH NUCLEI

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Blocking effect and odd-even differences in the moments of inertia of rare-earth nuclei

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Abstract

A phenomenological analysis of the experimental odd-even differences in the moments of inertia, $\delta J/J$, of well-deformed rare-earth nuclei is reviewed, which reveals that there exist large fluctuations in $\delta J/J$ with the blocked levels in odd-A nuclei. Calculation using the particle-number conserving treatment shows that the odd-even difference in the moments of inertia is a pure quantum mechanical interference effect and the experimental strong fluctuations in $\delta J/J$ with the blocked level can be reproduced satisfactorily. The calculated value of $\delta J/J$ depends sensitively on the energetic location and Coriolis response of the blocked level and the underlying physics is discussed. Particularly, $\delta J/J$ is especially large if the blocked orbital is a high-j intruder orbital near the Fermi surface. In contrast, if the blocked orbital is of normal parity with low j and high Ω (e.g., proton [404]7/2, [402]5/2), $\delta J/J$ almost vanishes.

PACS numbers: 21.10Re, 27.70+q, 21.60-n.

I. INTRODUCTION

One of the most striking discoveries in high-spin nuclear physics was the finding of almost identical superdeformed (SD) bands in some neighboring nuclei [1-3]. Several explanations [3-5] were put forward assuming the occurrence of such identical bands to be a specific properties of the superdeformed rotational bands. All these explanations assume [6] that the main contributing factor to the odd-even difference in the moments of inertia, namely, the pairing interaction, is substantially weakened for high-spin superdeformed states. Shortly afterwards, it was recognized that identical bands are also present in normally deformed pairs of even- and odd-mass nuclei at low spin [6, 7] and in normally deformed pairs of even-mass nuclei [8, 9], i.e., the occurrence of identical bands is not necessarily related to the phenomenon of superdeformation or excitation of very high-spin states in nuclei. It is well-known that the pairing interaction plays a substantial role in the description of collective motion of normally deformed nuclei at low-lying excited states [10], e.g., the pairing interaction may be responsible for the observed reduction of nuclear moments of inertia compared to that of a rigid rotor [11-15]. However, according to the conventional BCS approximation for treating the pairing interaction the moments of inertia associated with one quasiparticle states in odd-A nuclei should be larger than those of the ground state configuration of adjacent even-even nuclei by a factor of $\sim$ 15% [10]. Therefore, it was asserted [6, 7] that the occurrence of identical bands in normally deformed pairs of even- and odd-mass nuclei at low spin presents a serious challenge to the mean-field (BCS) approximation.

General considerations show that the BCS theory is very suitable for a system of large number of particles. The question is, however, how reliable is the BCS approximation for treating the eigenvalue problem of the cranked shell model (CSM) Hamiltonian [16, 17]. One of the crucial problems is the number of nucleons in a nucleus ($\sim 10^2$), particularly the number of valence nucleons ($\sim$ 10) which dominate the behavior of low-lying excited states, is very limited. Therefore, the serious defects (particle-number non-conservation, spurious states, etc.) should be considered seriously, and the conclusions drawn from the BCS approximation, particularly the statement concerning the nuclear features which depend sensitively on the particle number, need careful re-examination. To overcome the defect of particle number nonconservation in the BCS approximation, there have been developed various methods, including the various types of particle number projection method [18–27] and the generator coordinate method [28, 29], and improved agreement with experiment compared to the simple BCS approximation was obtained. Another crucial problem is the blocking effect, which is responsible for various odd-even differences in nuclear properties and is especially important for low-lying excited states. The blocking effects on the moments of inertia were addressed in the BCS formalism in many papers, e.g. refs. 10, 13, 17, 30, 31. However, while the defect of number non-conservation may be partly remedied by various types of number-projection, the most serious defect of the BCS treatment is that it is not able to treat the blocking effect properly [17]. Just as Rowe has emphasized [17], *while the blocking effects are straightforward, it is very difficult to treat them in the BCS formalism because they introduce different quasiparticle bases for different blocked levels* (see pp. 194-195 of ref. 17), which seems worth of much attention. The odd-even differences in the moments of inertia $\delta J/J \sim 15\%$ is only a rough estimate based on the BCS approximation. In fact, the observed odd-even differences in nuclear moments of inertia show large fluctuation [10], including the identical bands observed in normally deformed pairs of even- and odd-mass nuclei at low spin, for which the conventional BCS treatment offers no satisfactory explanation. Therefore, in this paper we prefer addressing this problem using a particle-number conserving (PNC) method in which the blocking effects are taken into account exactly [32, 33].

The experimental odd-even differences in the moments of inertia of the rare-earth nuclei have been analyzed in detail in refs. 6 and 7. In Sec. II we will give an additional analysis of the variation of odd-even differences in the moments of inertia with the blocked level (see Figs. 1–3). Two kinds of odd-even differences in the moments of inertia (see eqs. (2) and (3)) are compared and some distinctions are found between the odd-even differences for the odd-N and odd-Z well-deformed rare-earth nuclei. In Sec. III the odd-even differences in the moments of inertia of the rare-earth nuclei are calculated using the PNC treatment [32, 33] for the eigenvalue problem of the CSM Hamiltonian

$$H_{CSM} = H_0 + H_P = H_{Nil} + H_C + H_P, \qquad (1)$$

where H_{Nil} is the Nilsson Hamiltonian, $H_C = -\omega J_x$ is the Coriolis interaction and H_P the pairing interaction. In the PNC approach the particle number is conserved *from beginning to end* (unlike the number-projection technique). The moments of inertia of a lot of well-deformed even-even rare-earth nuclei have been calculated in a previous paper [33] and the agreement between the calculated and experimental results is satisfactory. In this paper we will show that the odd-even difference in the moments of inertia is a pure quantum mechanical effect and the experimental large fluctuations of the odd-even differences in moments of inertia with the

blocked level can be reproduced satisfactorily by the PNC calculation. The underlying physics is discussed in detail. A brief summary is given in Sec. IV.

II. PHENOMENOLOGICAL ANALYSIS

It has been known that the experimental moments of inertia of odd-A nuclei exceed those of the ground bands of adjacent even-even nuclei by amounts that are typically of order 20%, but show large fluctuations [10]. Particularly, the moment of inertia of an odd-A nucleus whose unpaired nucleon occupying a high-j intruder orbit is systematically much larger than those of the ground band moments of inertia of neighboring even-even nuclei [10]. For example, the bandhead moment of inertia of the ground band [642]5/2 of ^{161}Dy is $2J = 159.4\ \hbar^2\mathrm{MeV}^{-1}$ (determined by the two lowest observed energy levels), which is over twice as large as that of ^{160}Dy ($2J = 69.1\ \hbar^2\mathrm{MeV}^{-1}$) and ^{162}Dy ($2J = 74.4\ \hbar^2\mathrm{MeV}^{-1}$). In sharply contrast to this, it was recognized recently [6, 7] that the moments of inertia of some odd-Z nuclei are almost identical to that of the seniority-zero configuration of the neighboring even-even nucleus with one proton less. For example, the bandhead moment of inertia of the [404]5/2 band in ^{171}Lu ($2J = 73.8\ \hbar^2\mathrm{MeV}^{-1}$) is almost identical to that of the ground band in ^{170}Yb ($2J = 71.2\ \hbar^2\mathrm{MeV}^{-1}$), but moderately larger than that of ^{172}Hf ($2J = 63.0\ \hbar^2\mathrm{MeV}^{-1}$). Therefore, it seems worthwhile to make a systematic review of the variation of the odd-even differences in the moments of inertia with the blocked level.

Like the usual definition of the odd-even mass difference, the relative odd-even difference in the moments of inertia may be defined as

$$\left.\frac{\delta J}{J}\right|_{av} = \frac{J(A) - \frac{1}{2}[J_0(A+1) + J_0(A-1)]}{\frac{1}{2}[J_0(A+1) + J_0(A-1)]}, \qquad (A\ \text{odd}) \qquad (2)$$

where $[J_0(A+1) + J_0(A-1)]/2$, as a reference, is the average of the ground band moments of inertia of neighboring even-even nuclei. As has been noted in ref. 6, the situation may be different if one compare the moment of inertia of an odd-A nucleus with its neighboring even-even nucleus having one less nucleon; i.e., we may define

$$\frac{\delta J}{J} = \frac{J(A) - J_0(A-1)}{J_0(A-1)}. \qquad (A\ \text{odd}) \qquad (3)$$

The isotonic variations of $\delta J/J|_{av}$ and $\delta J/J$ for the ground state bands of odd-N rare-earth nuclei are displayed in Fig. 1. The isotopic variations of $\delta J/J|_{av}$ and $\delta J/J$ for the rotational bands of odd-Z nuclei are displayed in Figs. 2 and 3. From Figs. 1–3 several observations can be made:

(a). For the rotational band whose unpaired nucleon occupying the high-j intruder orbital (neutron $N = 6$, $i_{13/2}$; proton, $N = 5$, $h_{11/2}$), the odd-even differences in the moments of inertia are unusually large; e.g., for the ground band of the odd-N nuclei,

	^{161}Dy [642]5/2,	^{167}Er [633]7/2,	^{179}Hf [624]9/2
$\delta J/J\|_{av}$	1.23,	0.51,	0.39

Similarly, for the rotational bands in the odd-Z nuclei,

	^{161}Ho [523]7/2,	^{171}Lu [514]9/2,	^{171}Lu* [541]1/2
$\delta J/J\|_{av}$	0.42,	0.32,	0.42

In contrast, for the rotational band whose unpaired particle occupying low-j and high-Ω (strongly deformation aligned) orbital , e.g., proton [404]7/2 ($g_{7/2}$, $\Omega = j = 7/2$), [402]5/2 ($d_{5/2}$, $\Omega = j = 5/2$) etc., the value of $\delta J/J|_{av}$ is especially small (~ 0.10), which is displayed in Fig. 3. The underlying physics will be illustrated in Sec. III.

(b). For the odd-Z rare-earth nuclei, $\delta J/J$ is, in general, *smaller* than the corresponding value of $\delta J/J|_{av}$ (Figs. 2 and 3). As has been pointed out [6] that such systematics are counter to the expectations of a paired system, which would require comparison with the average of neighboring isotones. In particular, for the rotational bands building on the proton orbital [404]7/2 or [402]5/2 (Fig. 3), the value of $\delta J/J$ ($< \delta J/J|_{av} \sim 0.1$) is nearly zero; i.e., the moment of inertia of an odd-Z nuclei is almost equal to that of the neighboring even-even nucleus having one less proton. In this case, identical bands in normally deformed pairs of even- and odd-mass nuclei may emerge [6, 7].

However, the situation is different for the odd-N rare-earth nuclei. The value of $\delta J/J$ is usually a little *larger* than the corresponding value of $\delta J/J|_{av}$ (except for a few cases). In fact, for almost all the odd-N rare-earth well-deformed nuclei, the values of both $\delta J/J|_{av}$ and $\delta J/J$ are larger than 0.10; i.e., it is rarely found that the ground state band moment of inertia of an odd-N nucleus is almost identical to those of the neighboring even-even nuclei.

The relation between the magnitudes of $\delta J/J|_{av}$ and $\delta J/J$ mentioned above is partly connected with the change in deformation of well-deformed rare-earth nuclei with proton or neutron numbers. For examples, the variations in the quadruple deformation ϵ_2 for some well-deformed rare-earth nuclei [36] are as follows:

N=94	^{160}Dy 0.248,	^{162}Er 0.245,	^{164}Yb 0.239,	^{166}Hf 0.219
N=96	^{162}Dy 0.261,	^{164}Er 0.258,	^{166}Yb 0.246,	^{168}Hf 0.235
N=98	^{164}Dy 0.267,	^{166}Er 0.267,	^{168}Yb 0.255,	^{168}Hf 0.245
N=100		^{168}Er 0.273,	^{170}Yb 0.265,	^{172}Hf 0.254
N=102		^{170}Er 0.276,	^{172}Yb 0.269,	^{174}Hf 0.258
N=104			^{174}Yb 0.266,	^{176}Hf 0.256

It is seen that for these nuclei $\epsilon_2(Z,\ N) > \epsilon(Z+2,\ N)$, which may partly responsible for the fact that $J_0(Z,\ N) > J_0(Z+2,\ N)$, which implies $\delta J/J < \delta J/J|_{av}$, observed in odd-$Z$ nuclei. Similarly, $\epsilon_2(Z,\ N) > \epsilon(Z,\ N-2)$ for $N < 104$, which may partly account for the fact that $J_0(Z,\ N) > J_0(Z,\ N-2)$, which implies $\delta J/J > \delta J/J|_{av}$, observed in odd-$N$ nuclei. However, it should be emphasized that the odd-even difference in the moments of inertia is a pure quantum mechanical effect and depends intimately on the intrinsic configuration structure, which will be discussed in Sec. III.

(c). The values of $\delta J/J|_{av}$ and $\delta J/J$ vary in a rather wide range, but there exists no distinct line of demarcation between the "identical" and non-identical bands. The results for some typical (most β-stable) odd-A rare-earth nuclei are as follows:

rotational bands	[523]7/2 ($h_{11/2}$)	[514]9/2 ($h_{11/2}$)	[413]5/2 ($g_{7/2}$)	[411]3/2 ($d_{5/2}$)	[411]1/2 ($d_{3/2}$)	[404]7/2 ($g_{7/2}$)	[402]5/2 ($d_{5/2}$)
odd-Z nuclei	$^{161}_{67}$Ho	$^{171}_{71}$Lu*	$^{157}_{63}$Eu	$^{159}_{65}$Tb	$^{167}_{69}$Tm	$^{171}_{71}$Lu	$^{171}_{71}$Lu*
$\delta J/J\|_{av}$	0.42	0.32	0.19	0.20	0.13	0.10	0.056
$\delta J/J$	0.31	0.24	0.16	0.14	0.08	0.037	-0.008

rotational bands	[642]5/2 ($i_{13/2}$)	[633]7/2 ($i_{13/2}$)	[624]9/2 ($i_{13/2}$)	[523]5/2 ($f_{7/2}$)	[521]3/2 ($h_{9/2}$)	[514]7/2 ($f_{7/2}$)	[512]5/2 ($h_{9/2}$)	[521]1/2 ($p_{3/2}$)
odd-N nuclei	^{161}Dy$_{95}$	^{167}Er$_{99}$	^{170}Hf$_{107}$	^{165}Er$_{97}$	^{157}Gd$_{93}$	^{177}Hf$_{105}$	^{173}Yb$_{103}$	^{171}Yb$_{101}$
$\delta J/J\|_{av}$	1.23	0.51	0.39	0.30	0.29	0.21	0.15	0.13
$\delta J/J$	1.31	0.52	0.39	0.38	0.36	0.17	0.17	0.06

To display the variation in $\delta J/J|_{av}$ with the neutron numbers and the Nilsson orbital occupied by the odd nucleon, in Fig. 4a are shown the experimental $\delta J/J|_{av}$ by open circles for the ground state bands of some typical (most β-stable) odd-N rare-earth nuclei. Similar plot for odd-Z rare-earth nuclei is shown in Fig. 4b. We can see that strong fluctuations in $\delta J/J|_{av}$

exhibit clearly in Fig. 4. Particularly, in Fig. 4a there exist three peaks of $\delta J/J|_{av}$ corresponding to the blocked neutron orbitals [642]5/2, [633]7/2, and [624]9/2, respectively, which originate from the high-j intruder spherical orbital $i_{13/2}$ having stong Coriolis response. Similarly, the two peaks of $\delta J/J|_{av}$ in Fig. 4b correspond to the proton ortibals [523]7/2 and [514]9/2, which originate from the high-j intruder spherical orbital $h_{11/2}$. On the contrary, there exists a valley ($\delta J/J < 0.1$) in Fig. 4b near $Z \sim 71$, which is connected with the orbitals [404]7/2 and [402]5/2 having little Coriolis response. In fact, the majority of identical bands in normally deformed nuclei at low spin occur in this region. For comparison, the calculated $\delta J/J|_{av}$ using the PNC treatment (Sec. III) are also shown in Fig. 4 by solid circles. The general tendency of the experimental variation of $\delta J/J|_{av}$ with the blocked level is reproduced satisfactorily by the PNC calculation. Considering no free parameters involved in the PNC calculation, the results seem encouraging. The underlying physics will be discussed in Sec. III.

III. MICROSCOPIC CALCULATION AND DISCUSSIONS

A. Sketch of the PNC formalism

A particle-number-conserving (PNC) method for calculating the low-lying eigenstates of H_{CSM} was developed [32], in which the many-particle configuration (MPC) truncation is used instead of the usual single-particle level truncation and the blocking effects are taken into account exactly. To reveal clearly the influence of pairing interaction on the moment of inertia, an improved PNC approach was developed [33]; i.e., firstly, the one-body part of H_{CSM}, $H_0 = H_{Nil} - \omega J_x = \sum_i h_0(i)$ is diagonalized exactly to obtain the cranked Nilsson (CN) orbitals, and then $H_{CSM} = H_0 + H_P$ is diagonalized in a sufficiently large cranked many-particle configuration (CMPC) space to obtain the accurate solutions of the low-lying eigenstates of H_{CSM}. The moments of inertia of the ground bands in a series of well-deformed even-even rare-earth nuclei have been calculated [33] using this approach. It is well-known that the BCS theoretical moments of inertia of the ground bands in rare-earth and actinide even-even nuclei are systematically smaller than the experimental ones by a factor of 10–40%, i.e., systematic excessive reduction of the nuclear moments of inertia was found [10, 14]. Many efforts to reduce the discrepancy between theory and experiment have not got decisive success [27, 34]. This long-standing discrepancy disappears in the PNC calculation [33]. In this paper this PNC approach

is used to calculate the moments of inertia of odd-A rare-earth nuclei. The details of calculation has been presented in ref. 33. For convenience, a sketch of the PNC formalism is given below.

Usually the Nilsson orbitals [35] are characterized by π (parity) and Ω (eigenvalue of j_z) and are conventionally denoted by the asymptotic quantum numbers $[Nn_z\Lambda\Sigma]\Omega$. Each Nilsson level is two-fold degenerate ($\pm\Omega$). For the CN orbitals, j_z is no longer conservative and the degeneracy is removed. Each CN orbital is characterized by π and signature $r = e^{-i\pi\alpha} = \pm i(\sim \alpha = \mp 1/2)$, and denoted by $|\mu\alpha\rangle$, corresponding to the energy eigenvalue $\epsilon_{\mu\alpha}$. Hereafter, $|\mu\alpha\rangle$ is often briefly denoted by $|\mu\rangle$. The CMPC of an n-particle system can be expressed as $|\mu_1\mu_2\cdots\mu_n\rangle$, μ_1, μ_2, μ_n being the occupied CN orbitals. Each CMPC, simply labelled by $|i\rangle$, is characterized by $E_i(= \sum_{\mu_i} \epsilon_{\mu_i}$, configuration energy), parity and signature. When the pairing interaction is taken into account, we may diagonalize H_{CSM} in a sufficiently large CMPC space (i.e., all the CMPC's with energies $E_i - E_0 \le E_c$ are considered, E_0 being the energy of the lowest CMPC and E_c the truncation energy) to obtain the solutions of the yrast and low-lying excited states. Assume one low-lying excited state of H_{CSM} is expressed as $|\Psi\rangle = \sum_i C_i|i\rangle$, the angular momentum alignment is

$$\langle\Psi|J_x|\Psi\rangle = \sum_i |C_i|^2\langle i|J_x|i\rangle + 2\sum_{i<j} C_i^* C_j\langle i|J_x|j\rangle. \tag{4}$$

Considering J_x being a one-body operator, the matrix element $\langle i|J_x|j\rangle$ ($i \neq j$) is non-zero only when $|i\rangle$ and $|j\rangle$ differ by one particle occupation. After certain permutation of creation operators, $|i\rangle$ and $|j\rangle$ are brought into the form $|i\rangle = (-)^{M_{i\mu}}|\mu\cdots\rangle$, $|j\rangle = (-)^{N_{j\nu}}|\nu\cdots\rangle$, where the ellipses stand for the same particle occupation and $(-)^{M_{i\mu}} = \pm 1$, $(-)^{N_{j\nu}} = \pm 1$, according to the permutation is even or odd. Thus, the kinematic moment of inertia of the state $|\Psi\rangle$ can be expressed in terms of the single-particle picture as follows

$$\begin{aligned} J &= \frac{1}{\omega}\langle\Psi|J_x|\Psi\rangle = \sum_\mu J_{\mu\mu} + \sum_{\mu<\nu} J_{\mu\nu}, \\ \sum_\mu J_{\mu\mu} &= \frac{1}{\omega}\sum_\mu \langle\mu|j_x|\mu\rangle \sum_i |C_i|^2 P_{i\mu} = \frac{1}{\omega}\sum_\mu \langle\mu|j_x|\mu\rangle n_\mu, \\ J_{\mu\nu} &= \frac{2}{\omega}\langle\mu|j_x|\nu\rangle \sum_{i<j} (-)^{M_{i\mu}+N_{j\nu}} C_i^* C_j, \quad (\mu \neq \nu) \end{aligned} \tag{5}$$

where $n_\mu = \sum_i |C_i|^2 P_{i\mu}$ is the particle occupation probability of the CN orbital $|\mu\rangle$ in the state $|\Psi\rangle$ and $P_{i\mu} = 1$, if $|\mu\rangle$ is occupied in $|i\rangle$, and $P_{i\mu} = 0$ otherwise. If the pairing interaction is missing, only one CMPC appears in $|\Psi\rangle$ and all the interference terms $J_{\mu\nu}$ vanish. When the pairing interaction is taken into account, the diagonal part ($\sum_\mu J_{\mu\mu}$) changes only a little (see

Tables 2a, 3a, 4a, 5a), which can be understood from the slight change in particle occupation due to pairing correlation. The reduction of the moments of inertia originates mainly from the destructive interference ($\sum_{\mu<\nu} J_{\mu\nu} < 0$) due to the anti-alignment effect of pairing interaction. The off-diagonal part ($\sum_{\mu<\nu} J_{\mu\nu}$) depends sensitively on the features and distribution of the CN orbitals near the Fermi surface. Each $J_{\mu\nu}$ ($\mu \neq \nu$) depends on the energetic location of the CN orbital ϵ_μ and ϵ_ν and the magnitude of the matrix element $\langle\mu|j_x||\nu\rangle$, which is especially large for both μ and ν being the high-j intruder orbitals (the neutron $i_{13/2}$ orbitals and proton $h_{11/2}$ orbitals for rare-earth nuclei). If μ or ν is far away from the Fermi surface, $J_{\mu\nu}$ would be negligibly small. Therefore, only when both μ and ν are near the Fermi surface, $J_{\mu\nu}$ would be of importance (see Tables 2b, 3b, 4b, 5b). Also it should be noted that the contribution to the moments of inertia from a harmonic oscillator closed major shell is zero. Therefore, for the rare-earth nuclei, no contribution comes from $N \leq 3$ proton shells and $N \leq 4$ neutron shells, which are closed for the low-lying excited bands at low spin. Similarly, the contributions from the $N \geq 6$ proton shells and $N \geq 7$ neutron shells are very small, even when the pairing interaction is taken into account, because these shells are completely vacant in the lowest configuration of rare-earth nuclei. Therefore, almost all the contributions to the moments of inertia of rare-earth nuclei come from the $N = 4, 5$ proton and $N = 5, 6$ neutron shells (see Tables 2a, 3a, 4a, 5a).

It is seen that the transitions between adjacent high-j intruder orbitals ($\Delta\Omega = \pm1$) in the vicinity of the Fermi surface play a decisive role in the contributions to the moments of inertia; e.g., the neutron $i_{13/2}$ shell, [660]1/2 ↔ [651]3/2 ↔ [642]5/2 ↔ [633]7/2 ↔ [624]9/2 ↔ [615]11/2, (Tables 2b, 3b), and the proton $h_{11/2}$ shell, [532]5/2 ↔ [523]7/2 ↔ [514]9/2, etc., (Tables 4b, 5b).

For odd-A nuclei, if a single-particle level ν_0 is occupied by an odd nucleon, the pairing correlation is reduced (blocking effect). Calculation shows that $J_{\mu\nu_0}$ becomes positive, whose magnitude depends on the energetic location and Coriolis response of the blocked level ν_0, hence the calculated moments of inertia show large variation with the blocked level.

B. Calculated results and discussions

The moments of inertia of a series of well-deformed odd-A rare-earth nuclei were calculated and the comparison between the calculated and experimental odd-even differences in the moments of inertia, $\delta J/J|_{av}$, is displayed in Fig. 4. The experimental large fluctuations in $\delta J/J|_{av}$ are reproduced rather well by the PNC calculation.

As illustrative examples, the calculated results for the bandhead moments of inertia of four groups of typical rare-earth nuclei are presented in Tables 1–5. The results for the other rare-earth nuclei are similar. The comparison between the calculated and experimental moments of inertia are given in Table 1 and the detailed analyses of the contributions to the moments of inertia are shown in Tables 2–5. In the calculations the Nilsson parameters (ϵ_2, ϵ_4, κ, μ) are taken from the Lund systematics [35, 36] and no change is made to improve the calculated moments of inertia. The pairing interaction strength G_n and G_p are determined unambiguously [33] by the experimental odd-even differences in binding energies [37]

$$\begin{aligned} P_N &= \tfrac{1}{2}[B(Z,N) + B(Z,N+2)] - B(Z,N+1) \\ &= E_g(Z,N+1) - \tfrac{1}{2}[E_g(Z,N) + E_g(Z,N+2)] \\ P_P &= \tfrac{1}{2}[B(Z,N) + B(Z+2,N)] - B(Z+1,N) \\ &= E_g(Z+1,N) - \tfrac{1}{2}[E_g(Z,N) + E_g(Z+2,N)], \end{aligned} \tag{6}$$

where E_g is the ground state energy of nucleus at $\omega = 0$. In the PNC calculation of E_g of an odd-A nucleus the blocking effect has been taken into account exactly. The values of G_n and G_p thus obtained are listed in Table 1 of ref. 33. The CMPC truncation energy is chosen as $E_c = 0.85\hbar\omega_0$ (e.g., for ^{170}Yb, $\hbar\omega_{0n} = 7.837$ MeV, $\hbar\omega_{0p} = 6.966$ MeV) and the accuracy of the solutions of low-lying excited states has been discussed in ref. 33. From Table 1 it is seen that the agreement between the calculated and experimental moments of inertia is satisfactory. Now some discussions are given as follows.

From Table 1 it is seen that the calculated moments of inertia for even-even nuclei are greatly reduced due to strong pairing correlation (anti-alignment effect). This is a pure quantum mechanical effect. For example, the calculated $2J_0(^{160}\text{Dy})|_{G=0} = 187.6\ \hbar^2\text{MeV}^{-1}$ and $2J_0(^{162}\text{Dy})|_{G=0} = 160.6\ \hbar^2\text{MeV}^{-1}$ are reduced to 68.7 and 71.2 $\hbar^2\text{MeV}^{-1}$, respectively, which are very close to the experimental results. The PNC calculation shows that the contribution to the moments of inertia from the diagonal part ($\sum_\mu J_{\mu\mu}$) changes only a little due to pairing correlation (see Table 2a) and the vast majority of the reduction of the moments of inertia of ^{160,162}Dy comes from the negative off-diagonal part ($\sum_{\mu<\nu} J_{\mu\nu} < 0$), which vanishes for $G = 0$. Particularly, when both μ and ν are the high-j intruder orbitals in the vicinity of the Fermi surface (e.g., [651]3/2, [642]5/2, [633]7/2, etc.), the value of $J_{\mu\nu}$ is especially large (but negative! see Table 2b). These interference terms due to pairing play a decisive role in the reduction of the moments of inertia.

As for ^{164}Er and ^{166}Er, the experimental moments of inertia of ^{166}Er ($2J_0 = 74.5\ \hbar^2\text{MeV}^{-1}$)

is larger than that of ^{164}Er ($2J_0 = 65.7\ \hbar^2$MeV^{-1}) by a factor of 13.4%, which is reproduced rather well by the PNC calculation

$$\frac{J_{cal}(^{166}\mathrm{Er}) - J_{cal}(^{164}\mathrm{Er})}{J_{cal}(^{164}\mathrm{Er})} = 12.6\%$$

The reason is as follows. The calculation shows that the contributions to the moments of inertia from protons are nearly the same for both ^{164}Er and ^{166}Er (see Table 1), but the contributions from neutrons are rather different. For ^{166}Er ($N = 98$), there exists a large gap in the neutron Nilsson level scheme immediately above the Fermi surface, which leads to a significant pairing reduction, hence a larger moment of inertia of ^{166}Er compared to that of ^{164}Er.

In contrast, the experimental moments of inertia of ^{172}Hf ($2J_0 = 63.0\ \hbar^2$MeV^{-1}) is smaller than that of ^{170}Yb ($2J_0 = 71.2\ \hbar^2$MeV^{-1}) by a factor of 13%, which is also approximately reproduced by the PNC calculation

$$\frac{J_{cal}(^{170}\mathrm{Hf}) - J_{cal}(^{172}\mathrm{Hf})}{J_{cal}(^{172}\mathrm{Hf})} \sim 9\%.$$

This is intimately connected with the moderate gap in the proton Nilsson level scheme at $Z = 70$, which leads to a weaker pairing correlation in ^{170}Yb ($Z = 70$) than in ^{172}Hf ($Z = 72$), hence the calculated J_p for ^{170}Yb ($2J_p = 25.46\ \hbar^2$MeV^{-1} is larger than that for ^{172}Hf ($2J_p = 21.7\ \hbar^2$MeV^{-1}). The smaller difference in the calculated values of J_n for ^{170}Yb$_{100}$ and ^{172}Hf$_{100}$ may partly comes from the small change in deformation ($\epsilon_2 = 0.265$ for ^{170}Yb, and 0.254 for ^{172}Hf) [36].

Now let us consider the moments of inertia of odd-A nuclei and the odd-even differences.

(a). First, we address the first group (^{161}Dy [642]5/2 band, ^{160,162}Dy g.s. bands) and try to explain why the odd-even difference in the moments of inertia is so large. From Table 2b we see that, unlike the even-even nuclei ^{160,162}Dy, for the ^{161}Dy [642]5/2 band the value of $J_{\mu\nu}$ becomes positive for μ or ν = [642]5/2 ($\alpha = \pm 1/2$), due to the important blocking effect and the strong Coriolis response of the [642]5/2 level, hence the reduction of the moments of inertia due to pairing observed in ^{160,162}Dy disappears in the ^{161}Dy [642]5/2 band. Therefore, it is not surprising that the calculated neutron contribution to the moment of inertia for the ^{161}Dy [642]5/2 band is greatly increased ($2J_n = 117.9\ \hbar^2$MeV^{-1}) and close to the value for vanishing pairing interaction ($G_n = 0$, $2J_n = 113.5\ \hbar^2$MeV^{-1}). Similar argument may account for the observed large odd-even differences in the moments of inertia for the [633]7/2 and [624]9/2 bands in odd-N rare-earth nuclei.

(b) Second, we discuss the calculated moment of inertia for the ^{165}Er [523]5/2 band. The contribution to the moments of inertia from protons are almost the same for ^{164,166}Er and ^{165}Er [523]5/2 band, so the odd-even difference mainly comes from the off-diagonal part of neutron contribution (Tables 3a and 3b). However, the most important interference terms are those concerning the high-j intruder orbitals ([660]1/2, [651]3/2, [642]5/2, [633]7/2, [624]9/2 etc.) and the contribution to the moment of inertia from the normal parity orbitals ([523]5/2, [514]7/2, etc.) are of minor importance. Therefore, the blocking effect of the orbital [523]5/2 only leads to a moderate increase of moment of inertia of ^{165}Er [523]5/2 band compared to the neighboring even-even nuclei.

(c) Third, we investigate the ^{171}Lu [514]9/2 band. The observed moments of inertia of the ^{171}Lu [514]9/2 band ($2J = 88.3\ \hbar^2$MeV^{-1}) exceed those of ^{170}Yb and ^{172}Hf by a factor of about 30%, which is reproduced satisfactorily by the PNC calculation (Tables 1 and 4), i.e.

	experimental	calculated
$\frac{J(^{171}\mathrm{Lu}\ [514]9/2) - J_0(^{170}\mathrm{Yb})}{J_0(^{170}\mathrm{Yb})}$	24%	22%
$\frac{J(^{171}\mathrm{Lu}\ [514]9/2) - J_0(^{172}\mathrm{Hf})}{J_0(^{172}\mathrm{Hf})}$	40%	33%

The reason is that the high-j intruder proton orbital [514]9/2 plays an important role in the contribution to the moments of inertia (Table 4b). The value of $J_{\mu\nu}$ for μ or ν = [514]9/2 is rather large (but negative) for the even-even nuclei, but becomes positive for the ^{171}Lu [514]9/2 band due to the blocking effect, and then leads to a rather large odd-even difference in the moments of inertia.

(d) Finally, we consider the ground state [404]7/2 band of ^{171}Lu. Recently, it was recognized [6, 7] that the moment of inertia of the ^{171}Lu [404]7/2 band ($2J = 73.8\ \hbar^2$MeV^{-1}) is nearly equal to that of the neighboring even-even nucleus having one less proton, ^{170}Yb ($2J = 71.2\ \hbar^2$MeV^{-1}), which was considered as an experimental evidence for identical bands in normally deformed nuclei at low spin [6, 7]. However, it was also noted that [6] the difference in the moments of inertia between the ^{171}Lu [404]7/2 band and that of the neighboring even-even nucleus having one more proton, ^{172}Hf ($2J = 63.0\ \hbar^2$MeV^{-1}) is so large that it is hard to consider the ground band of ^{172}Hf and the ^{171}Lu [404]7/2 band as identical, which seems rather odd and hard to understand. It is interesting to note that such feature of the moments of inertia can be reproduced satisfactorily by the PNC calculation,

	experimental	calculated
$\frac{J(^{171}\text{Lu [404]7/2}) - J_0(^{170}\text{Yb})}{J_0(^{170}\text{Yb})}$	3.7%	3.7%
$\frac{J(^{171}\text{Lu [404]7/2}) - J_0(^{172}\text{Hf})}{J_0(^{172}\text{Hf})}$	13%	17%

The reason is that the [404]7/2 orbital with low j and high Ω ($g_{7/2}$, $\Omega = j = 7/2$) has a very small Coriolis response and the contribution to the moments of inertia from the [404]7/2 orbital is trifling (Table 5b), so the blocking effect of the [404]7/2 orbital is not worth mentioning. Therefore, it is not surprising that the moment of inertia of the ground state band [404]7/2 in ^{171}Lu is nearly equal to that of the ground band in ^{170}Yb. The reason why the moments of inertia of ^{172}Hf is much smaller than those of ^{170}Yb and ^{171}Lu [404] band has been discussed above, which is intimately connected with the gap in the proton Nilsson level scheme at $Z = 70$.

IV. SUMMARY

The variation of the odd-even differences in the moments of inertia of well-deformed rare-earth nuclei with the blocked level was addressed both phenomenologically and microscopically. The experimental large fluctuations in $\delta J/J$ can be reproduced satisfactorily by the PNC calculation. The underlying physics of such large variations in $\delta J/J$ is discussed in detail. It is noted that treating the blocking effects properly is essential to account for the experimental large fluctuations in $\delta J/J$. The calculated value of $\delta J/J$ is especially large if the blocked orbital is a high-j intruder near the Fermi surface. In contrast, if the blocked orbital is of low-j and high-Ω (e.g., proton [404]7/2 and [402]5/2), the calculated $\delta J/J$ almost vanishes. In this case, the occurrence of identical bands in pairs of even- and odd-mass nuclei at low spin seems understandable.

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Table Captions

Table 1 Comparison of the calculated and experimental bandhead moments of inertia of four groups of rare-earth nuclei. Columns 2, 3, and 4 are the calculated contribution to the moments of inertia for vanishing pairing ($G_n = G_p = 0$) from protons, neutrons, and their sum, respectively. When the pairing interaction is taken into account, the corresponding calculated results are given in columns 4, 5, and 6. The pairing strength (G_n and G_p), are determined by the experimental odd-even mass differences [37] and the values of G_n and G_p are taken from ref. 33. The experimental bandhead moments of inertia [38] extracted from the lowest two levels of each band are given in the final column.

Table 2 (a) Structure analysis of the neutron contributions from each major shell to the moments of inertia of the ground bands of ^{160,162}Dy and the [642]5/2 band in ^{161}Dy. No contribution comes from the neutron $N = 0, 1, 2, 3$ shells.

(b) The off-diagonal part of the contribution to the moments of inertia from neutrons.

Table 3 The same as Table 2, but for the ground bands of ^{164,166}Er and the neutron [523]7/2 band of ^{165}Er.

Table 4 The same as Table 2, but for the ground bands of ^{170}Yb, ^{172}Hf and the proton [514]9/2 band of ^{171}Lu.

Table 5 The same as Table 2, but for the ground bands of ^{170}Yb, ^{172}Hf and the proton [404]7/2 band of ^{171}Lu.

Figure Captions

Fig. 1 The relative odd-even differences in the moments of inertia for odd-N well-deformed rare-earth nuclei. $\delta J/J$ (see eq. (3)) and $\delta J/J|_{av}$ (see eq. (2)) are denoted by solid and open circles, respectively. The blocked neutron Nilsson level for each band is also indicated. The experimental data of the bandhead moments of inertia are taken from ref. 38.

Fig. 2 The same as Fig. 1, but for the odd-Z well-deformed rare-earth nuclei.

Fig. 3 The same as Fig. 2, but for the odd-Z rare-earth nuclei, whose odd proton occupies the [404]7/2, or [402]5/2 orbitals.

Fig. 4 The relative odd-even differences in the moments of inertia $\delta J/J|_{av}$ (see eq. (2)) of some typical (most β-stable) rare-earth nuclei versus the particle numbers and the the corresponding Nilsson levels blocked by the odd particles. The experimental and calculated $\delta J/J|_{av}$ are denoted by open and solid circles, respectively.

(a), odd-N nuclei. (b), odd-Z nuclei.

Table 1

Rotational band	$2J_{cal}$ ($\hbar^2$MeV^{-1}) G_p, $G_n = 0$			G_p, $G_n \neq 0$			$2J_{exp}$ ($\hbar^2$MeV^{-1})
	proton	neutron	total	proton	neutron	total	
^{160}Dy	61.26	126.32	187.58	28.98	39.70	68.68	69.1
^{161}Dy [642]5/2	60.18	113.54	173.82	28.66	117.92	146.58	159.4
^{162}Dy	59.46	101.14	160.60	29.66	41.56	71.22	74.4
^{164}Er	44.16	106.74	150.90	23.82	42.32	66.14	65.7
^{165}Er [523]5/2	42.98	103.20	146.18	24.86	60.62	85.48	90.6
^{166}Er	42.10	99.22	141.32	24.97	49.55	74.50	74.5
^{170}Yb	41.66	80.26	121.92	25.46	43.74	69.20	71.2
^{171}Lu [514]9/2	35.08	81.98	117.06	40.20	44.36	84.56	88.3
^{172}Hf	37.86	84.08	121.94	21.70	41.92	63.62	63.0
^{170}Yb	41.66	80.26	121.92	25.46	43.74	69.20	71.2
^{171}Lu [404]7/2	39.74	81.98	121.72	27.40	44.36	71.76	73.8
^{172}Hf	37.86	84.08	121.94	21.70	41.92	63.62	63.0

Table 2

(a)

		$G_n = 0$	$G_n \neq 0$		
		$2\sum_\mu J\mu\mu$	$2\sum_\mu J\mu\mu$	$2\sum_{\mu<\nu} J\mu\nu$	$2J_{\text{ncal}}$
${}^{160}_{66}Dy_{94}$	N=4		0.11	−0.02	0.09
	N=5	42.45	42.20	−20.67	21.54
	N=6	83.87	72.25	−54.17	18.07
	all shells	126.32	114.56	−74.86	39.70
${}^{161}_{66}Dy_{95}$	N=4		0.09	−0.01	0.08
[642]5/2	N=5	41.93	41.20	−18.32	22.87
	N=6	71.61	70.46	24.51	94.97
	all shells	113.54	111.74	6.18	117.92
${}^{162}_{66}Dy_{96}$	N=4		0.09	−0.05	0.04
	N=5	41.68	39.97	−18.24	21.72
	N=6	59.45	63.13	−43.34	19.79
	all shells	101.14	103.19	-61.63	41.56

(b)

	$2J_{\mu\nu}$ ($\hbar^2$ MeV $^{-1}$)					
neutron orbitals	^{160}Dy		^{161}Dy [642]5/2		^{162}Dy	
μ , ν	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$
[541] 1/2, [530] 1/2	−0.28	−0.22	−0.20	−0.16	−0.21	−0.17
[541] 1/2, [532] 3/2	−0.25	−0.28	−0.16	−0.19	−0.17	−0.19
[514] 9/2, [505]11/2	−0.37	−0.37	−0.26	−0.26	−0.28	−0.28
[530] 1/2, [532] 3/2	−0.05	−0.10				
[530] 1/2, [521] 3/2	−2.05	−2.12	−0.97	−0.98	−0.96	−0.97
[530] 1/2, [521] 1/2	−0.29	−0.24	−0.27	−0.23	−0.28	−0.23
[660] 1/2, [651] 3/2	−2.81	−4.40	−1.48	−2.46	−1.64	−2.47
[532] 3/2, [523] 5/2	−2.88	−2.88	−2.72	−2.72	−2.41	−2.41
[532] 3/2, [521] 1/2	−0.30	−0.34	−0.29	−0.33	−0.30	−0.34
[651] 3/2, [642] 5/2	-14.57	-14.52	5.83	7.81	-6.08	-6.06
[521] 3/2, [523] 5/2	−0.70	−0.70	−1.00	−1.00	−0.83	−0.83
[521] 3/2, [512] 5/2	−2.09	−2.09	−2.14	−2.14	−2.15	−2.15
[642] 5/2, [633] 7/2	-8.25	-8.25	9.31	6.79	-12.42	-12.42
[523] 5/2, [514] 7/2	−0.88	−0.88	−0.97	−0.97	−1.30	−1.30
[633] 7/2, [624] 9/2	−0.62	−0.62	−0.67	−0.67	−1.10	−1.10
[512] 5/2, [503] 7/2	−0.05	−0.05			−0.07	−0.07
[660] 1/2, [642] 5/2		−0.10				−0.04
[521] 1/2, [510] 1/2					−0.07	−0.06
[521] 1/2, [512] 3/2					−0.05	−0.07
Total:	-74.86		6.18		-61.63	

Table 3

(a)

		$G_n = 0$	$G_n \neq 0$		
		$2\sum_\mu J\mu\mu$	$2\sum_\mu J\mu\mu$	$2\sum_{\mu<\nu} J\mu\nu$	$2J_{\mathrm{ncal}}$
$^{164}_{68}\mathrm{Er}_{96}$	N=4		0.09	−0.01	0.08
	N=5	43.38	40.53	−18.99	21.54
	N=6	63.36	67.97	−47.29	20.68
	all shells	106.74	108.60	−66.28	42.32
$^{165}_{68}\mathrm{Er}_{97}$	N=4		0.08	−0.07	0.02
[523]7/2	N=5	39.25	39.96	−4.46	35.50
	N=6	63.94	63.04	−37.95	25.10
	all shells	103.20	103.09	−42.47	60.62
$^{166}_{68}\mathrm{Er}_{98}$	N=4		0.05	−0.01	0.04
	N=5	34.74	36.47	−12.38	24.09
	N=6	64.48	62.62	−37.20	25.42
	all shells	99.22	99.15	−49.59	49.55

(b)

	$2J_{\mu\nu}$ ($\hbar^2$ MeV $^{-1}$)					
neutron orbitals	^{164}Er		^{165}Er [523]5/2		^{166}Er	
μ , ν	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$
[541] 1/2, [530] 1/2	−0.09	−0.07				
[541] 1/2, [532] 3/2	−0.09	−0.10				
[514] 9/2, [505]11/2	−0.31	−0.31	−0.29	−0.29	−0.24	−0.24
[530] 1/2, [521] 3/2	−1.14	−1.14	−0.47	−0.47	−0.38	−0.38
[530] 1/2, [521] 1/2	−0.31	−0.25	−0.33	−0.26	−0.32	−0.25
[532] 3/2, [523] 5/2	−2.44	−2.44	0.45	0.60	−0.68	−0.67
[532] 3/2, [521] 1/2	−0.31	−0.37	−0.32	−0.38	−0.31	−0.37
[660] 1/2, [651] 3/2	−1.69	−3.15	−0.61	−1.09	−0.47	−1.03
[651] 3/2, [642] 5/2	−7.16	−7.11	−2.74	−2.72	−2.37	−2.36
[521] 3/2, [523] 5/2	−0.72	−0.72	0.47	0.31	−0.15	−0.15
[521] 3/2, [512] 5/2	−2.25	−2.25	−2.12	−2.12	−1.95	−1.95
[642] 5/2, [633] 7/2	−12.71	−12.70	−13.64	−13.64	−13.30	−13.30
[523] 5/2, [514] 7/2	−1.56	−1.56	1.02	0.52	−1.71	−1.71
[633] 7/2, [624] 9/2	−1.33	−1.33	−1.73	−1.73	−2.07	−2.07
[521] 1/2, [510] 1/2	−0.09	−0.07	−0.14	−0.12	−0.18	−0.15
[521] 1/2, [512] 3/2	−0.06	−0.07	−0.11	−0.13	−0.13	−0.16
[512] 5/2, [503] 7/2	−0.07	−0.07	−0.09	−0.09	−0.10	−0.10
[530] 1/2, [532] 3/2		−0.05				
[660] 1/2, [642] 5/2		−0.07				
[624] 9/2, [615]11/2					−0.08	−0.08
Total:	−66.28		−42.47		−49.59	

Table 4

(a)

		$G_p = 0$	$G_p \neq 0$		
		$2\sum_\mu J\mu\mu$	$2\sum_\mu J\mu\mu$	$2\sum_{\mu<\nu} J\mu\nu$	$2J_{\text{ncal}}$
$^{170}_{70}Yb_{100}$	N=4	15.39	14.44	-3.36	11.08
	N=5	26.27	27.55	-13.23	14.32
	N=6	0.00	3.12	-3.06	0.06
	all shells	41.66	45.11	-19.65	25.46
$^{171}_{71}Lu_{100}$	N=4	15.62	14.53	-3.42	11.11
[514]9/2	N=5	19.46	21.79	7.32	29.11
	N=6	0.00	2.75	-2.77	-0.02
	all shells	35.08	39.08	1.12	40.20
$^{172}_{72}Hf_{100}$	N=4	10.43	11.85	-3.05	8.80
	N=5	27.43	27.07	-13.83	13.24
	N=6	0.00	5.99	-6.33	-0.34
	all shells	37.86	44.91	-23.21	21.70

(b)

	$2J_{\mu\nu}$ ($\hbar^2$ MeV $^{-1}$)					
proton orbitals	^{170}Yb		^{171}Lu [514]9/2		^{172}Hf	
μ , ν	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$
[420] 1/2, [411] 3/2	-0.05	-0.05	-0.06	-0.06	-0.04	-0.04
[420] 1/2, [411] 1/2	-0.11	-0.10	-0.08	-0.08	-0.06	-0.06
[541] 3/2, [532] 5/2	-0.31	-0.31	-0.41	-0.41	-0.32	-0.32
[532] 5/2, [523] 7/2	-0.92	-0.92	-0.47	-0.47	-0.40	-0.40
[413] 5/2, [404] 7/2	-0.72	-0.72	-0.74	-0.74	-0.49	-0.49
[411] 3/2, [402] 5/2	-0.50	-0.50	-0.52	-0.52	-0.51	-0.51
[523] 7/2, [514] 9/2	-4.30	-4.30	3.59	6.17	-4.06	-4.06
[411] 1/2, [402] 3/2	-0.25	-0.29	-0.25	-0.30	-0.23	-0.27
[404] 7/2, [402] 5/2	-0.03	-0.03			-0.15	-0.15
[514] 9/2, [505]11/2	-0.30	-0.30	0.15	0.11	-0.82	-0.82
[541] 1/2, [532] 3/2	-0.40	-0.48	-0.37	-0.44	-0.73	-0.90
[541] 1/2, [530] 1/2	-0.04	-0.08		-0.07	-0.11	-0.13
[660] 1/2, [651] 3/2	-0.91	-1.61	-0.83	-1.48	-0.89	-2.69
[651] 3/2, [642] 5/2	-0.27	-0.27	-0.23	-0.23	-0.59	-0.57
[532] 3/2, [523] 5/2	-0.27	-0.27			-0.38	-0.38
[411] 1/2, [400] 1/2						-0.05
[642] 5/2, [633] 7/2					-0.80	-0.80
Total:	-19.65		1.12		-23.21	

Table 5

(a)

		$G_p = 0$	$G_p \neq 0$		
		$2\sum_\mu J\mu\mu$	$2\sum_\mu J\mu\mu$	$2\sum_{\mu<\nu} J\mu\nu$	$2J_{\text{ncal}}$
${}^{170}_{70}\text{Yb}_{100}$	N=4	15.39	14.44	-3.36	11.08
	N=5	26.27	27.55	-13.23	14.32
	N=6	0.00	3.12	-3.06	0.06
	all shells	41.66	45.11	-19.65	25.46
${}^{171}_{71}\text{Lu}_{100}$	N=4	12.96	12.67	-1.21	11.45
[404]7/2	N=5	26.78	27.50	-11.49	16.01
	N=6	0.00	2.75	-2.80	-0.06
	all shells	39.74	42.91	-15.51	27.40
${}^{172}_{72}\text{Hf}_{100}$	N=4	10.43	11.85	-3.05	8.80
	N=5	27.43	27.07	-13.83	13.24
	N=6	0.00	5.99	-6.33	-0.34
	all shells	37.86	44.91	-23.21	21.70

(b)

proton orbitals	$2J_{\mu\nu}$ ($\hbar^2$ MeV $^{-1}$)					
	^{170}Yb		^{171}Lu [404]7/2		^{172}Hf	
μ , ν	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$	$\alpha = \frac{1}{2}$	$\alpha = -\frac{1}{2}$
[420] 1/2, [411] 3/2	-0.05	-0.05			-0.04	-0.04
[420] 1/2, [411] 1/2	-0.11	-0.10	-0.06	-0.06	-0.06	-0.06
[541] 3/2, [532] 5/2	-0.31	-0.31	-0.31	-0.31	-0.32	-0.32
[532] 5/2, [523] 7/2	-0.92	-0.92	-0.33	-0.33	-0.40	-0.40
[413] 5/2, [404] 7/2	-0.72	-0.72	0.26	0.10	-0.49	-0.49
[411] 3/2, [402] 5/2	-0.50	-0.50	-0.52	-0.52	-0.51	-0.51
[523] 7/2, [514] 9/2	-4.30	-4.30	-4.34	-4.34	-4.06	-4.06
[411] 1/2, [402] 3/2	-0.25	-0.29	-0.25	-0.29	-0.23	-0.27
[404] 7/2, [402] 5/2	-0.03	-0.03	0.07	0.05	-0.15	-0.15
[514] 9/2, [505]11/2	-0.30	-0.30	-0.30	-0.30	-0.82	-0.82
[541] 1/2, [532] 3/2	-0.40	-0.48	-0.37	-0.45	-0.73	-0.90
[541] 1/2, [530] 1/2	-0.04	-0.08	-0.03	-0.07	-0.11	-0.13
[660] 1/2, [651] 3/2	-0.91	-1.61	-0.85	-1.51	-0.89	-2.69
[651] 3/2, [642] 5/2	-0.27	-0.27	-0.23	-0.22	-0.59	-0.57
[532] 3/2, [523] 5/2	-0.27	-0.27			-0.38	-0.38
[411] 1/2, [400] 1/2						-0.05
[642] 5/2, [633] 7/2					-0.80	-0.80
Total:	-19.65		-15.51		-23.21	

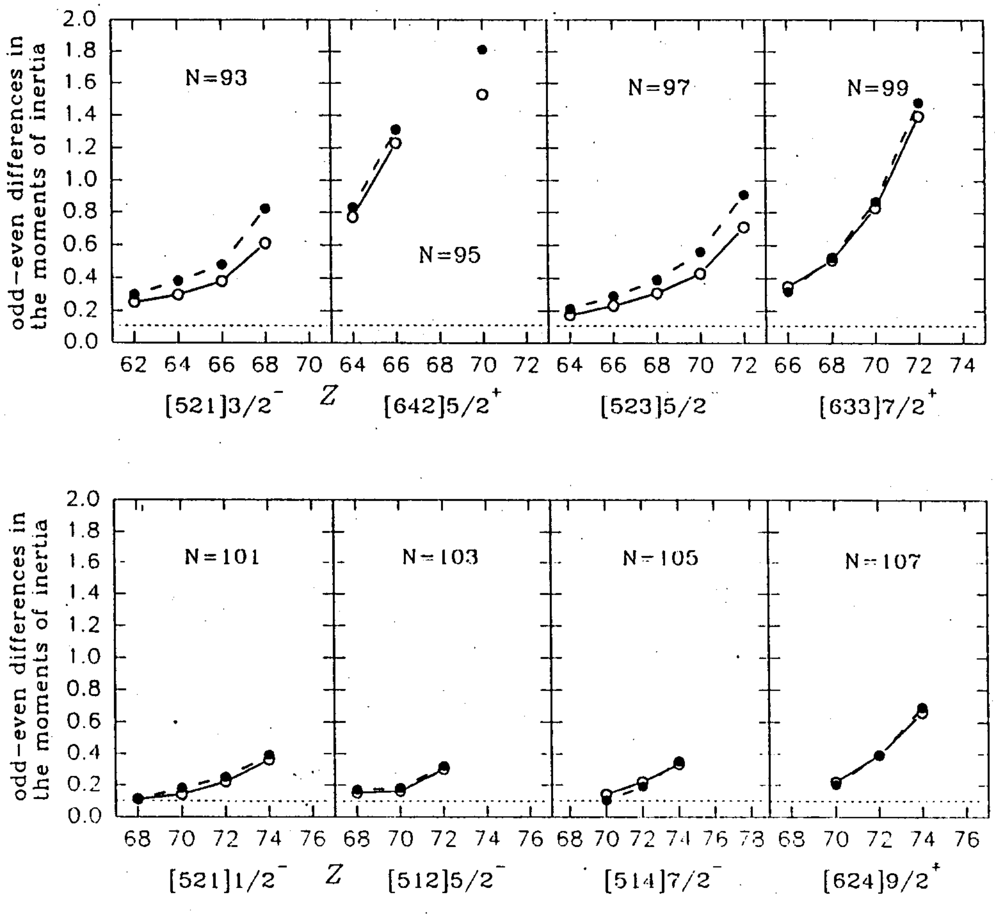
odd-even differences in the moments of inertia
N=93
N=95
N=97
N=99
[521]3/2⁻
[642]5/2⁺
[523]5/2
[633]7/2⁺
Z
N=101
N=103
N=105
N=107
[521]1/2⁻
[512]5/2⁻
[514]7/2⁻
[624]9/2⁺

Fig. 1

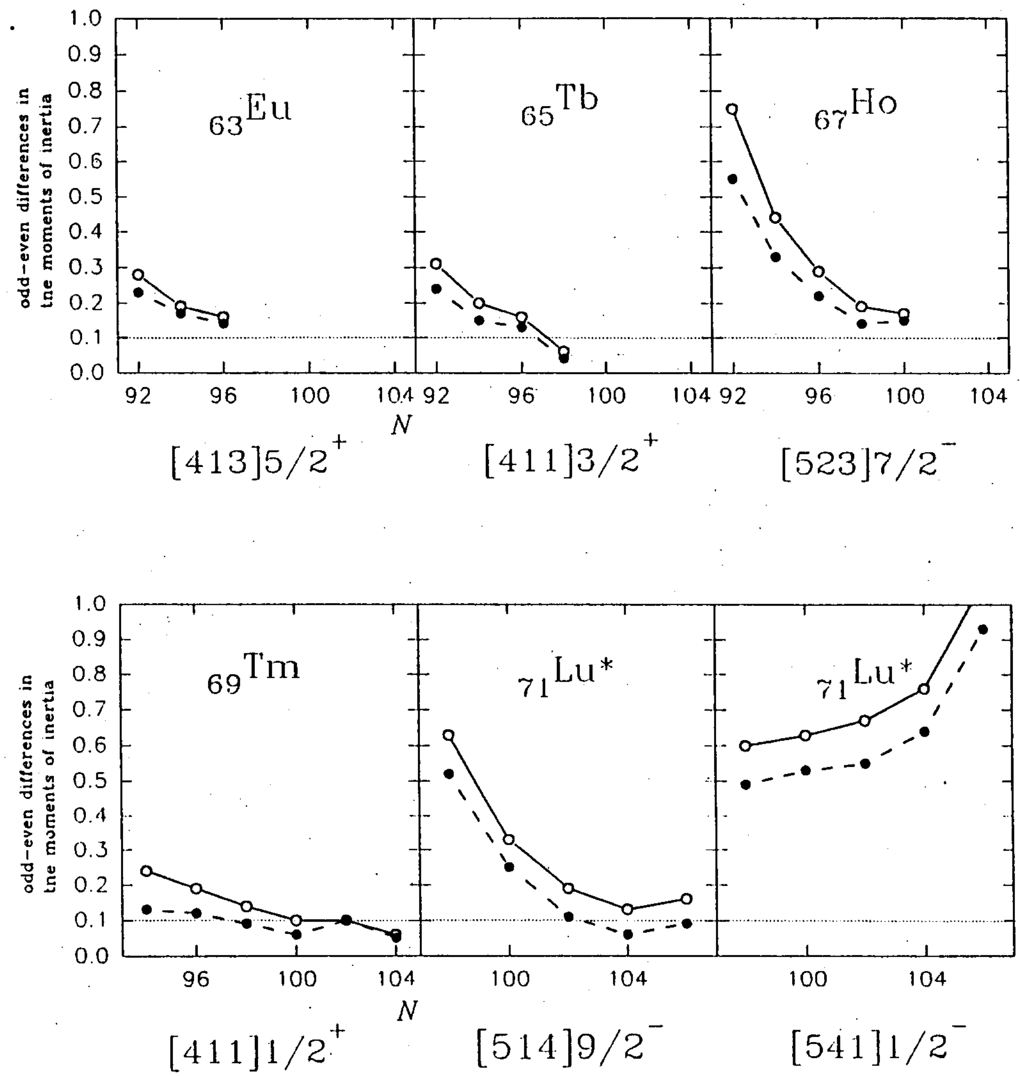
odd-even differences in the moments of inertia
63Eu
65Tb
67Ho
N
[413]5/2⁺
[411]3/2⁺
[523]7/2⁻
69Tm
71Lu*
71Lu*
[411]1/2⁺
[514]9/2⁻
[541]1/2⁻

Fig. 2

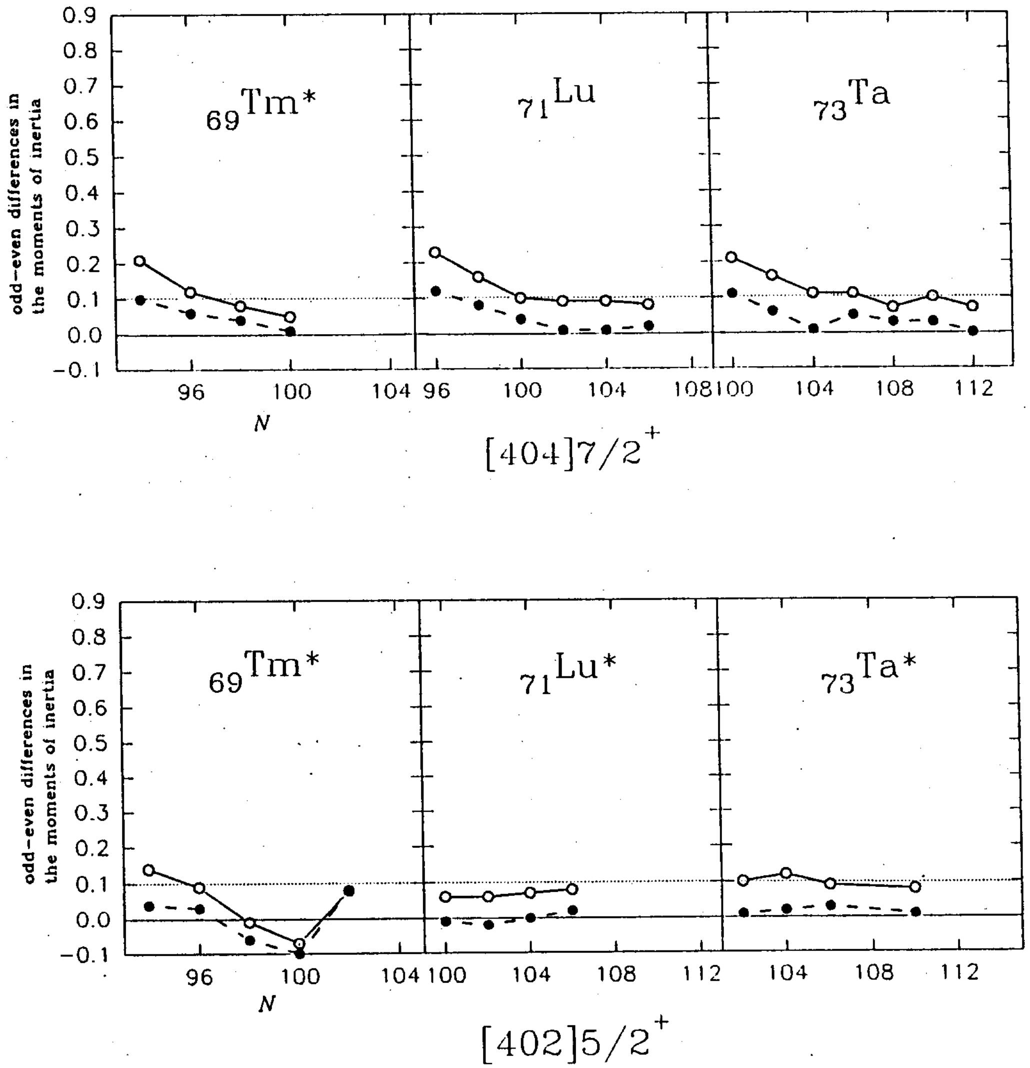
odd-even differences in the moments of inertia
$_{69}$Tm*
$_{71}$Lu
$_{73}$Ta
N
[404]7/2+
$_{69}$Tm*
$_{71}$Lu*
$_{73}$Ta*
N
[402]5/2+

Fig. 3

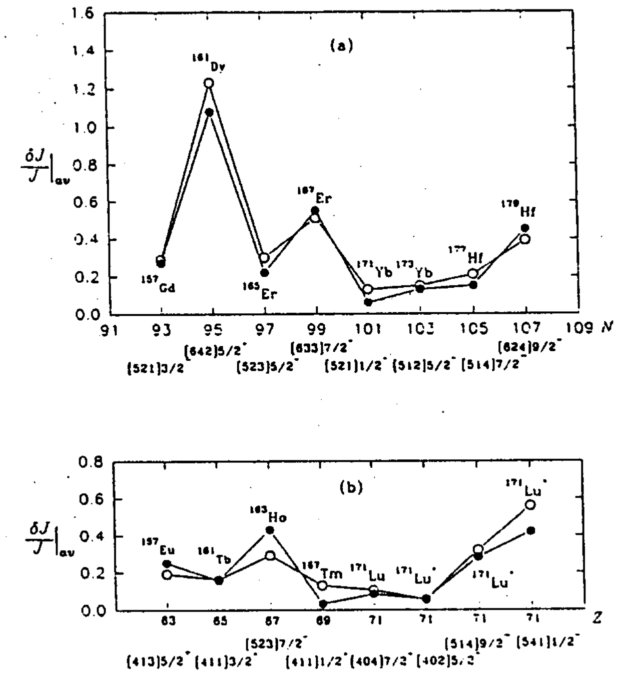
(a)
δJ/J|av
161Dy
167Er
157Gd
165Er
171Yb
173Yb
177Hf
179Hf
N
(b)
157Eu
161Tb
165Ho
167Tm
171Lu
171Lu*
Z

Fig. 4