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Quasi Two Body Decays of Charm Mesons

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ABSTRACT

Quasi two body decays of charm mesons are studied by using a hard meson approximation in which the amplitude M is approximated by $M \simeq M_{ETC} + M_S$. The surface term M_S is given by a sum of all the possible pole amplitudes involving contributions of four-quark and hybrid mesons in addition to ground-state $\{Q\bar{Q}\}_0$ mesons to the intermediate states. Contributions of multi-hadron intermediate states are taken in the form of M_{ETC} . In the annihilation type of decays, hybrid mesons play an important role. The spectator type of decays can have significant contributions of four-quark mesons. The mixed type of decays are a little complicated since they can have all the contributions of possible hybrid and four-quark mesons in addition to the ground-state $\{Q\bar{Q}\}_0$.

The calculated branching ratios, $B(D^+ \rightarrow \bar{K}^0 \rho^+)$ and $B(D^0 \rightarrow K^- \rho^+)$, are of ordinary size in contrast with the factorization which predicts very large rates for these decays. The predicted $B(D_s^+ \rightarrow \pi^+ \omega)$ is very small. The $D_s^+ \rightarrow \pi^+ \rho^0$ decay can be suppressed in consistency with the other decays.

1. Introduction

Although nonleptonic weak decays of charm mesons have been studied from various theoretical approaches, *i.e.*, factorization[1], QCD sum rule[2], quark-line argument[3], ACD[4], \dots , no consensus has been reached. In these approaches, however, dynamical contributions of hadrons have not sufficiently been taken into account. Therefore, another perspective to nonleptonic weak processes, in which contributions of hadronic intermediate states are explicitly considered, will be needed. From this perspective[5],[6], we have already investigated two body decays of K and charm mesons and obtained good results, for example, a small violation of the $|\Delta I| = 1/2$ rule and a large violation of its charm counterpart can be understood in terms of contributions of exotic four-quark $(QQ)(\bar{Q}\bar{Q})$ mesons, a large violation of flavor $SU_f(3)$ symmetry in the $D \rightarrow K\bar{K}$ and $\pi\pi$ decays can be explained in terms of the large difference between the mass differences, $m_D - m_{\hat{\sigma}^*}$ and $m_D - m_{\hat{\sigma}}$, where $\hat{\sigma}^*$ and $\hat{\sigma}$ are $I = 0$ members of the scalar $[QQ][\bar{Q}\bar{Q}]$ mesons with $Q = u, d, s, \dots$ and the superscript s of $\hat{\sigma}^*$ implies that it involves an $(s\bar{s})$ pair. Therefore, it is meaningful to analyze quasi two body decays, $P_1 \rightarrow VP_2$, of charm mesons from the same perspective.

We start from the following approximate amplitude for $P_1(p_1) \rightarrow V(p_2)P_2(q)$,

$$M(P_1 \rightarrow VP_2) \simeq M_{ETC}(P_1 \rightarrow VP_2) + M_S(P_1 \rightarrow VP_2). \quad (1.1)$$

Equation (1.1) can be obtained[7] by extrapolating $q \rightarrow 0$ in the infinite momentum frame of the parent particle (IMF, *i.e.*, $p_1 \rightarrow \infty$). Here P_i and V denote pseudoscalar (PS) and vector mesons, respectively. M_{ETC} and M_S are given by,

$$M_{ETC}(P_1 \rightarrow VP_2) = -i(\sqrt{2}f_{P_2})^{-1} \langle V|[V_{P_2}, H_w]|P_1 \rangle - (P_1 \leftrightarrow \bar{P}_2), \quad (1.2)$$

$$M_S(P_1 \rightarrow VP_2) = -\frac{i}{\sqrt{2}f_{P_2}} \left\{ \sum_n \left(\frac{m_V^2 - m_1^2}{m_n^2 - m_1^2} \right) \langle V|A_{P_2}|n \rangle \langle n|H_w|P_1 \rangle \right. \\ \left. + \sum_l \left(\frac{m_V^2 - m_1^2}{m_l^2 - m_V^2} \right) \langle V|H_w|l \rangle \langle l|A_{P_2}|P_1 \rangle \right\} - (P_1 \leftrightarrow \bar{P}_2). \quad (1.3)$$

Notations are presented in refs. [5]–[8]. Dynamical contribution of various hadrons is manifest. Not only the ordinary $\{Q\bar{Q}\}_L$ (labeled by the level L in the sense of quark model) but also the glue-ball, hybrid and multi-quark mesons can contribute to the intermediate states of M_S if they exist. Thus exotic hadrons can play an important role if their masses happen to be close to those of the external hadrons. However, in the quasi two body decays of charm mesons under consideration, the last term in eq. (1.3) is small because of $m_n^2, m_l^2 \gg m_V^2, m_S^2$ and can be safely neglected. Therefore we do not need to worry about *asymptotic matrix elements* (matrix elements taken between single hadron states with infinite momentum) of axial charges, A_D , ($D = D^{\pm,0}, D_s^+$), which have not been measured yet. Contributions of orbitally excited $\{Q\bar{Q}\}_{L \neq 0}$ mesons also will be neglected since they are expected to be small[7].

The approximate expression of the amplitude, eq. (1.1) with eqs. (1.2) and (1.3), can be regarded as its decomposition[9] into (*continuum contribution*) + (*Born term*). This general structure is natural for the description of dynamical hadronic processes. The *continuum contribution* will, in general, develop a phase relative to the *Born term* which is usually treated to be real in the narrow width limit, *i.e.*, M_{ETC} can have a phase relative to M_S .

It is important to observe that the amplitude is thus governed by asymptotic matrix elements of charges, V_α and A_α , and the effective weak hamiltonian H_w . Therefore, the main task is now to estimate the asymptotic matrix elements of V_α , A_α and H_w .

In the next section, we will parametrize the asymptotic matrix elements of charges, V_α and A_α , in the framework of asymptotic flavor symmetry and asymptotic matrix elements of H_w using simple quark-line arguments. In the section 3, approximate amplitudes for quasi two-body decays will be given and resulting branching ratios will be compared with experiments. A brief summary will be given in the final section.

2. Asymptotic matrix elements of charges and the effective weak hamiltonian

In order to parametrize asymptotic matrix elements of charges, we use *asymptotic flavor symmetry* which is a useful prescription to treat broken flavor symmetry[10]. The asymptotic $SU_f(N)$ symmetry implies that a flavor charge V_α transforms an annihilation operator $a_\beta(k)$ of physical hadron β like π, K, η, \dots by

$$[V_\alpha, a_\beta(k)] = i \sum_\gamma f_{\alpha\beta\gamma} a_\gamma(k) + \delta f_{\alpha\beta}(k), \quad \delta f_{\alpha\beta}(k) \rightarrow 0 \quad \text{as } k \rightarrow \infty, \quad (2.1)$$

where $a_\gamma(k)$ should be taken over all possible particles γ with the same $J^{P(C)}$ as that of the particle β . Therefore, in the theory of asymptotic flavor symmetry, mixings among members of different multiplets (leakages to different multiplets) due to the flavor symmetry breaking can be taken into account. The well-known 1-8 mixing is an example of such effects in the framework of broken $SU_f(3)$. [In the exact $SU_f(N)$ symmetry, β and γ belong to the same $SU_f(N)$ multiplet and $\delta f_{\alpha\beta}(k)$ vanishes for any value of k .] The size of the leakage due to the flavor $SU_f(N)$ symmetry breaking is given approximately by the value of the form factor $f_+(0)$ of relevant vector current at zero momentum transfer squared. The estimated values $f_+^{\pi K}(0) \simeq 1$ and $f_+^{K^* D}(0) \simeq 0.7$ [11] suggest that in the world of the $\{Q\bar{Q}\}$ mesons, the leakage due to the $SU_f(3)$ symmetry breaking is negligibly small (except for the 1-8 mixings) while the $SU_f(4)$ symmetry breaking may make about 30 per cent leakages. Therefore, we parametrize the asymptotic matrix elements of the flavor charges as follows. The asymptotic matrix elements of V_π, V_K, A_π and A_K are very close to those in the symmetry *plus* intra-level mixings like the 1-8 mixing[12]. However asymptotic $SU_f(4)$ rotations through V_D can make leakages to different levels which can be described, for example, by

$$V_{D^0} |\pi^+(p)\rangle \simeq \alpha_D |D^+(p)\rangle + \alpha_{D'} |D'^+(p)\rangle + \dots, \quad (p \rightarrow \infty), \quad (2.2)$$

where α_D is the leakage factor discussed above and D' denotes the first radially excited state of D . Then the asymptotic ground-state-meson matrix elements of V_D

can be approximately parametrized by multiplying α_D ($\simeq \int_+^{K_D}(0) \simeq 0.7$ describing the leakage) to those in the symmetry limit. If we consider leakages only to the first radially excited state (in addition to the intra-level mixings), we can obtain $\alpha_{D'} \simeq \sqrt{1 - \alpha_D^2}$ inserting the commutation relation, $[V_{D^+}, V_{D^-}] = 2V_{\pi^0}$, between $\langle D^+ |$ and $|D^+ \rangle$ with infinite momentum. Its value $\alpha_{D'} \simeq 0.7$ can be obtained by using $\alpha_D \simeq 0.7$ estimated before. Since the commutation relations, $[V_{D^+}, A_{\pi^-}] = A_{D^0}$, *etc.*, relates the asymptotic matrix elements of A_D to those of A_{π} , we can again obtain approximately the values of them by multiplying the leakage factor α_D to those in the $SU_f(4)$ symmetry plus intra-level mixings, although we do not need to worry about the matrix elements of A_D in this note as was discussed before.

Asymptotic matrix elements of H_w can be parametrized by using an intuitive quark-line argument[6]. We review briefly it below. The effective nonleptonic weak hamiltonian H_w is approximately given by[13]

$$H_w \simeq c_- O_- + c_+ O_+ + h.c., \quad (2.3)$$

where O_{\pm} are the *normal ordered* four-quark operators. The operators, O_{\pm} , can be expanded into a sum of products of (a) two annihilation and two creation operators, (b) one annihilation and three creation operators, (c) one creation and three annihilation operators and (d) four annihilation or four creation operators of quarks and antiquarks. We associate these products of annihilation and creation operators with different types of weak vertices by requiring the usual connectedness of the quark-lines. For (a), we utilize the two annihilation and the two creation operators to annihilate and create, respectively, the quark and the anti-quark belonging to the ordinary meson states $|\{Q\bar{Q}\}\rangle$ and $\langle\{Q\bar{Q}\}|$ (or a hybrid $\{Q\bar{Q}g\}$ in place of the $\{Q\bar{Q}\}$) which sandwich O_{\pm} . However, in the case (b) and (c), we now have to add a spectator quark or antiquark to reach the physical processes $\langle\{QQ\bar{Q}\bar{Q}\}|O_{\pm}|\{Q\bar{Q}\}\rangle$ and $\langle\{Q\bar{Q}\}|O_{\pm}|\{QQ\bar{Q}\bar{Q}\}\rangle$. In this procedure, we have to be careful with the order of the quark(s) and anti-quark(s).

Noting that the wavefunction of ground-state $\{Q\bar{Q}\}_0$ meson should be *antisymmetric*[14] under the exchange of the quark and anti-quark constructing the

$\{Q\bar{Q}\}_0$ meson, we then obtain[5],[6],

$$\langle\{Q\bar{Q}\}_0|O_+|\{Q\bar{Q}\}_0\rangle = 0, \quad (2.4)$$

which implies that the *asymptotic* ground-state meson matrix elements of H_w with $|\Delta S| = 1$ and $|\Delta C| = 0$ satisfy the strict $|\Delta I| = 1/2$ rule and those of the charm changing H_w do its charm counterpart. If the masses of hybrid mesons are close to those of the parent charm mesons[15],[16], they can play a role in charm meson decays[7],[17]. In the flux tube model[16], for example, masses of hybrid mesons with $J^{PC} = 0^{-}(+)$ have been predicted to be around 2 GeV. Therefore we consider contributions of the hybrid mesons. Constraints on asymptotic matrix elements of H_w taken between the ground-state $\{Q\bar{Q}\}_0$ and the hybrid $\{Q\bar{Q}g\}$ will be analogous to eq. (2.4), *i.e.*,

$$\langle\{Q\bar{Q}g\}|O_+|\{Q\bar{Q}\}_0\rangle = 0, \quad (2.5)$$

if the matrix element is described by the quark-line diagram in Fig. 1.

Four-quark $\{QQ\bar{Q}\bar{Q}\}$ mesons are classified[18] into the following four types, $\{QQ\bar{Q}\bar{Q}\} = [Q\bar{Q}][\bar{Q}Q] \oplus (Q\bar{Q})(\bar{Q}Q) \oplus \{[Q\bar{Q}](\bar{Q}Q) \pm (Q\bar{Q})[\bar{Q}Q]\}$, where $()$ and $[]$

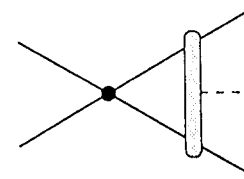


Fig.1 Quark-line diagram describing the matrix elements $\langle\{Q\bar{Q}g\}|H_w|\{Q\bar{Q}\}_0\rangle$. The solid lines represent quarks and anti-quarks, the dashed line a constituent gluon, the solid circle the weak vertex in the $m_W \rightarrow \infty$ limit and the gray-box the strong interactions.

denote symmetry and antisymmetry, respectively, with respect to the exchange of flavors between them. All these four types of four-quark mesons can have $J^P = 1^+$ and can contribute to quasi two body decays under consideration in contrast with the case of two body decays in which $\{(QQ)(\bar{Q}\bar{Q}) \pm (Q\bar{Q})(\bar{Q}Q)\}$ mesons could not participate because they have $J^P = 1^+$. The same procedure as the above leads us to

$$\langle [QQ][\bar{Q}\bar{Q}]|O_+|\{Q\bar{Q}\}_0 \rangle = 0 \quad (2.6)$$

and

$$\langle (QQ)(\bar{Q}\bar{Q})|O_-|\{Q\bar{Q}\}_0 \rangle = 0, \quad (2.7)$$

which are quite reasonable from the symmetry property of the wavefunctions of $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons. The nonvanishing $(QQ)(\bar{Q}\bar{Q})$ contributions, $\langle (QQ)(\bar{Q}\bar{Q})|O_+|\{Q\bar{Q}\}_0 \rangle$ and $\langle [Q\bar{Q}]_0|O_+|(Q\bar{Q})(\bar{Q}Q) \rangle$, can give a natural origin of the small violation of the $|\Delta I| = 1/2$ rule in the $K \rightarrow \pi\pi$ decays[6] and the large violation of its charm counterpart in the charm meson decays[5]. Constraints on the matrix elements of H_w taken between $[QQ](\bar{Q}\bar{Q}) \pm (Q\bar{Q})(\bar{Q}Q)$ and $\{Q\bar{Q}\}_0$ mesons also can be obtained through the same procedure,

$$\langle [QQ](\bar{Q}\bar{Q}) \pm (Q\bar{Q})(\bar{Q}Q)|O_{\mp}|\{Q\bar{Q}\}_0 \rangle = 0. \quad (2.8)$$

Explicit parametrization of the *asymptotic* matrix elements of H_w is listed in Appendix A.

The asymptotic matrix elements, $\langle P|H_w|V \rangle$ and $\langle V|H_w|P \rangle$, can be related to the asymptotic matrix elements $\langle P|H_w|P' \rangle$, for example, by

$$\langle \bar{K}^{*0}|H_w|D^0 \rangle = \pm \langle \bar{K}^0|H_w|D^0 \rangle, \quad (2.9)$$

which have already been obtained[19] from the realization of the commutation relations, $[A_\alpha, H_w^{(i)}] = [V_\alpha, H_w^{(i)}]$, with $\alpha = \pi^{\pm,0}$. This algebraic approach is complementary to the present quark-line argument. We choose the positive sign in eq. (2.9) hereafter.

3. Branching ratios for quasi two body decays

Before providing explicit expressions of decay amplitudes, we need a little more preparations. *METC* includes asymptotic matrix elements of V_D . As we have discussed in the previous section, an operation of V_D to a single hadron state $|\beta\rangle$ with infinite momentum can induce leakages to different multiplets (for example, radially excited states, *etc.*) in the framework of asymptotic flavor symmetry. We here consider leakages only to the first radially excited state, for simplicity. Then we obtain, for example,

$$\langle \rho^0|V_{D^0}H_w|K^0 \rangle \simeq \sqrt{\frac{1}{2}} \langle \bar{D}^{*0}|H_w|K^0 \rangle R,$$

where

$$R = \alpha_D + \alpha_{D'} \frac{\langle \bar{D}'^{*0}|H_w|K^0 \rangle}{\langle \bar{D}^{*0}|H_w|K^0 \rangle}. \quad (3.1)$$

D'^{*0} denotes the first radially excited state of D^{*0} . $R = 1$ if no leakage ($\alpha_D = 1$ and $\alpha_{D'} = 0$). α_D and $\alpha_{D'}$ have been estimated to be $\alpha_D \simeq \alpha_{D'} \simeq 0.7$ in the previous section. The matrix elements of H_w taken between two meson states will be proportional to the values of wavefunctions of these mesons at the origin[7]. Then the ratio, $|\langle \bar{D}'^{*0}|H_w|K^0 \rangle / \langle \bar{D}^{*0}|H_w|K^0 \rangle|$, is estimated to be

$$\left| \frac{\langle \bar{D}'^{*0}|H_w|K^0 \rangle}{\langle \bar{D}^{*0}|H_w|K^0 \rangle} \right| \simeq \left| \frac{\Psi_{D'^{*0}}(0)}{\Psi_{D^{*0}}(0)} \right| \simeq \left| \frac{\Psi_{\Psi'}(0)}{\Psi_{J/\Psi}(0)} \right| \simeq 0.79,$$

where $\Psi_\alpha(0)$ denotes the value of the wavefunction of the particle α at the origin. The second approximate equality comes from the fact that Ψ' and D'^{*0} belong to the same first radially excited states of J/Ψ and D^* , respectively, and the last one can be obtained from the observed ratio[20], $\Gamma(\Psi' \rightarrow e^+e^-)/\Gamma(J/\Psi \rightarrow e^+e^-) \simeq 0.44 \pm 0.11$. In this way we estimate $R \simeq 1.2$.

Substituting the constraints on matrix elements obtained in the previous section into the general form of the decay amplitude, eq. (1.1) with eqs. (1.2) and (1.3), we can write down explicitly the decay amplitudes. From these amplitudes,

we can see the following[7],[8], although they still include unknown parameters.

(i) The decays, $D^0 \rightarrow \bar{K}^0 \phi$, $D_s^+ \rightarrow \pi^+ \rho^0$ ($\pi^0 \rho^+$) and $D_s^+ \rightarrow \pi^+ \omega$, are described by the annihilation type quark-line diagrams (in the conventional sense) in the $m_V \rightarrow \infty$ limit. Therefore these decays are predicted to be strongly suppressed from a perspective of short distance physics[21], although the observed branching ratio $B(D^0 \rightarrow \bar{K}^0 \phi)$ is not very small[20]. In the present perspective, however, it implies that the intermediate state of M_S in the s -channel contains only one pair of valence quark Q and antiquark \bar{Q} [and possibly gluon(s)] if connectedness of the quark-lines describing the matrix elements of H_w is insisted. Therefore only the ordinary $\{Q\bar{Q}\}_L$ and hybrid $\{Q\bar{Q}g\}$ mesons can contribute to the single meson intermediate state $|n\rangle$ in the first term of $\{ \}$ on the right-hand-side (r.h.s.) of eq. (1.3). However, the matrix elements of H_w involving the orbitally excited $\{Q\bar{Q}\}_{L \neq 0}$ meson will be small[7] since the value of the wavefunction of $\{Q\bar{Q}\}_{L \neq 0}$ at the origin is expected to be small ($\Psi_{L \neq 0}(0) = 0$ in the nonrelativistic limit). The second term of $\{ \}$ on r.h.s. of eq. (1.3) has not $\{Q\bar{Q}\}$ but four-quark $\{QQ\bar{Q}\bar{Q}\}$ meson contributions in this type of decays. However, because of $m_\ell^2 \gtrsim m_1^2 = m_D^2, m_B^2 \gg m_V^2 = m_\phi^2, m_\rho^2, m_\omega^2$ for $\ell = \{cQ\bar{Q}\bar{Q}\}$ and also of the small overlapping of the wavefunctions between the ground-state $\{Q\bar{Q}\}_0$ and the $\{cQ\bar{Q}\bar{Q}\}$ mesons, the four-quark meson contributions will be small and therefore we neglect the second line in eq. (1.3) for these decays. In this way we see that the amplitude for the annihilation type of decay is described in terms of contributions of the ground-state $\{Q\bar{Q}\}_0$ and the hybrid $\{Q\bar{Q}g\}$ mesons to M_S in addition to M_{ETC} . One of the points is that in the present perspective, $M(D_s^+ \rightarrow \pi^+ \rho^0)$ is not automatically suppressed even in the $SU_I(2)$ symmetry limit in contrast with the conventional quark-line diagram approach[3]. In the latter approach, the $D_s^+ \rightarrow \pi^+ \omega$ and $D_s^+ \rightarrow \pi^+ \rho^0$ decays were described in terms of the same type of two annihilation diagrams and their amplitudes were given by a sum and a difference of the corresponding amplitudes, respectively. Then $SU_I(2)$ symmetry lead to the result that the $D_s^+ \rightarrow \pi^+ \rho^0$ decay amplitude would vanish while the $D_s^+ \rightarrow \pi^+ \omega$ amplitude could survive. Therefore, to obtain a suppression of the $D_s^+ \rightarrow \pi^+ \omega$ decay, some particular assumption would be needed[3]. However, the amplitude for the

quasi two-body decay $D \rightarrow VP$ should be anti-symmetrized with respect to the exchange of D and \bar{P} in the crossed channel[22] so that it may be continued smoothly to its $SU_I(4)$ symmetry limit. Under this anti-symmetrization, the $D_s^+ \rightarrow \pi^+ \omega$ amplitude vanishes in the $SU_I(2)$ symmetry limit while the $D_s^+ \rightarrow \pi^+ \rho^0$ amplitude is not necessarily vanishing. Therefore, to reproduce the observed suppression of the $D_s^+ \rightarrow \pi^+ \rho^0$ decay, something new has to happen. In the present perspective, this will be explained[7] by a magical cancellation among $M_{ETC}(D_s^+ \rightarrow \pi^+ \rho^0)$, $M_S^{(L=0)}(D_s^+ \rightarrow \pi^+ \rho^0)$ and $M_S^{(hybrid)}(D_s^+ \rightarrow \pi^+ \rho^0)$ later. A distinct point of the present perspective is that $M_{ETC}(D_s^+ \rightarrow \pi^+ \omega) = M_S^{(L=0)}(D_s^+ \rightarrow \pi^+ \omega) = 0$ and hybrid mesons cannot contribute to M_S , *i.e.*, $M_S^{(hybrid)}(D_s^+ \rightarrow \pi^+ \omega) = 0$. Within the present approximation in which contributions of excited states through the crossed channels are neglected, therefore, the $D_s^+ \rightarrow \pi^+ \omega$ amplitude is vanishing as was expected from the above quark-line argument.

(ii) In the spectator decays, $D^+ \rightarrow \bar{K}^0 \rho^+$, $\pi^+ \bar{K}^{*0}$, $D_s^+ \rightarrow \pi^+ \phi$, only the $\{QQ\bar{Q}\bar{Q}\}$ mesons (because of the connectedness of quark-lines at the weak vertex under consideration) can contribute to the s -channel intermediate states of M_S . Among the $\{QQ\bar{Q}\bar{Q}\}$ mesons, some of $\{QQ\}(\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]$ mesons are expected to give the most important contribution in M_S since their calculated masses[18] are very close to those of the parent charm mesons.

(iii) The mixed type of decays, which are described not only by the annihilation type of diagrams but also by the spectator diagrams (in the conventional sense), are a little more complicated since M_S can contain all the contributions from the ground-state $\{Q\bar{Q}\}_0$, the hybrid and the four-quark mesons.

For more detailed numerical discussion, we need to know values of parameters involved in the decay amplitudes, *i.e.*, the masses and widths of axial-vector four-quark and PS hybrid mesons, the phases δ_{2I} (I is the isospin of the final state) arising from M_{ETC} relatively to M_S , the parameters, k_0 , k_a^* , k_s^* , k_-^* , k_+^* and k_H which describe contributions of the $\{Q\bar{Q}\}_0$, $[QQ][\bar{Q}\bar{Q}]$, $(QQ)(\bar{Q}\bar{Q})$, $\{\{QQ\}(\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]\}$ and $\{Q\bar{Q}g\}$, respectively, to M_S and are defined, except for k_0 , in Appendix A, although these parameters have not been measured yet. Only the $k_0 = \langle K^{*+} | A_{\pi^+} | K^0 \rangle$ can be estimated to be $|k_0| \simeq 0.7$ [23] from the observed decay

rate[20], $\Gamma(K^* \rightarrow K\pi) \simeq 50$ MeV.

Evidences for existence of exotic mesons are now increasing[15],[24] in the region of their mass values higher than 1.5 GeV. We here use the calculated four-quark meson masses in ref. [18] and take the width of four-quark mesons to be $\Gamma_{exotic} \simeq 0.2$ GeV for simplicity. Hybrid meson masses have been calculated by using some different theoretical models[15],[16]. However the results are still in controversy. Therefore we estimate the hybrid meson mass (in particular, m_{π_H}) which reproduces the observed suppression of the $D_s^+ \rightarrow \pi^+\rho^0$ decay as follows. As discussed in (i), in the decays described in terms of only the annihilation diagrams, main terms of the amplitude are given by the ETC term, M_{ETC} , and the $\{Q\bar{Q}\}_0$ and $\{Q\bar{Q}g\}$ pole amplitudes, $M_S^{(L=0)}$ and $M_S^{(hybrid)}$, in the present approximation. For example, the amplitude for the $D_s^+ \rightarrow \pi^+\rho^0$ decay, in which the isospin of the final $\pi\rho$ state is unity (in the Cabibbo-angle favored decays of D_s^+ meson, the isospin of the final states is always unity because H_u is of $|\Delta I| = 1$), is given by

$$M(D_s^+ \rightarrow \pi^+\rho^0) \simeq -\frac{i}{f_\pi} \langle \bar{K}^{*0} | H_u | D^0 \rangle \left\{ e^{i\delta_2} + \left(\frac{m_{D_s}^2 - m_\rho^2}{m_{D_s}^2 - m_\pi^2} \right) k_0 - \left(\frac{m_{D_s}^2 - m_\rho^2}{m_{\pi_H}^2 - m_{D_s}^2} \right) k_H \right\}, \quad (3.2)$$

where the first, the second and the third terms on r.h.s. are arising from M_{ETC} , $M_S^{(L=0)}$ and $M_S^{(hybrid)}$, respectively. $\delta_{2I=2}$ is the phase of *non-resonant* $\pi\rho$ final state interactions with $I = 1$ and its size is expected to be $< 90^\circ$. If the mass of π_H is very close to m_{D_s} , the amplitude will be sensitive to the width as well as the mass of π_H , although the width was not explicitly shown in eq. (3.2). The parameter k_H is related to the overlapping between the wavefunctions of the $\{Q\bar{Q}g\}$ and $\{Q\bar{Q}\}_0$ mesons which is expected to be much smaller than that of two $\{Q\bar{Q}\}_0$ mesons. Therefore the value of k_H will be much smaller than k_0 which was estimated to be $\simeq 0.7$ before. Then, in order that M_{ETC} and $M_S^{(L=0)}$ may be canceled by $M_S^{(hybrid)}$ for $|k_H| \lesssim 0.1$ and $|\delta_2| < 90^\circ$, the width Γ_{π_H} cannot be very broad ($\lesssim 100$ MeV). If we take $|k_H| \simeq 0.1$ and $|\delta_2| \simeq 80^\circ$, then the mass and the width of the π_H should be $m_{\pi_H} \simeq 2.0$ GeV and $\Gamma_{\pi_H} \simeq 100$ MeV, respectively. The above value of m_{π_H} is close to its predicted one from the flux

tube model[16]. The mass of K_H which belongs to the same multiplet as π_H is estimated approximately from the above value of m_{π_H} by using the quark counting, i.e., $m_{K_H} \simeq m_{\pi_H} + (m_{D_s} - m_D) \simeq 2.1$ GeV. Our result is not very sensitive to the value of m_{K_H} as long as $m_{K_H} \gtrsim 2$ GeV.

The decays described in terms of the spectator diagrams can have four-quark meson contributions to M_S as stated in (ii). For example, the $D_s^+ \rightarrow \pi^+\phi$ decay amplitude in the present approximation is given by

$$M(D_s^+ \rightarrow \pi^+\phi) \simeq \frac{i}{\sqrt{2}f_\pi} \langle \bar{K}^{*0} | H_u | D^0 \rangle \left\{ \frac{f_\pi}{f_{D_s}} R e^{i\delta_2} - \left(\frac{m_{D_s}^2 - m_\phi^2}{m_{D_s}^2 - m_{\pi_A^{(*)}}^2} \right) k_u^* + \left(\frac{m_{D_s}^2 - m_\phi^2}{m_{D_s}^2 - m_{C_{\pi_A^{(*)}}^2}} \right) k_s^* + 2 \left(\frac{m_{D_s}^2 - m_\phi^2}{m_{D_s}^2 - m_{C_{\pi^{(*)}(+)}^2}} \right) (k_-^* + k_+^*) \right\}, \quad (3.3)$$

where $\pi_A^{(*)}$, $C_{\pi_A^{(*)}}$ and $C_{\pi^{(*)}(+)}$ denote the $[QQ][\bar{Q}\bar{Q}]$, $(QQ)(\bar{Q}\bar{Q})$ and $[QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]$ mesons, respectively, with $I = 1$ and $J^P = 1^+$. Their superscript s implies again that they include an $(s\bar{s})$ pair. The $[QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]$ mesons mix with each other to diagonalize G -parity. The argument of the $C_{\pi^{(*)}(+)}$ describes its G -parity. We here take $R = 1.25$ as was estimated before. The parameters, k_u^* and k_s^* , providing the contributions of $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons with $J^P = 1^+$, respectively, correspond to f_u^* and f_s^* describing the the scalar $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ meson contributions to two body decays of charm mesons in ref. [5]. These parameters are related to overlappings between the wavefunctions of the $\{Q\bar{Q}\}_0$ and the four-quark mesons. If possible difference between the spatial wavefunctions of the $[QQ][\bar{Q}\bar{Q}]$ and $(QQ)(\bar{Q}\bar{Q})$ mesons is neglected, then $k_s^*/k_u^* \simeq c_+/c_- = r$ will be obtained, where c_\pm are the Wilson coefficients given in eq. (2.3). We here put $k_u^* = 0.05$ and $k_s^*/k_u^* = r$, where r is expected to be not very far from unity. ($f_u^* = f_s^* = 0.05$ has reproduced well the observed branching ratios for two body decays of charm mesons in ref. [5] and [25].) The remaining parameters in eq. (3.3) are k_\pm^* describing the $[QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]$ meson contributions.

The amplitudes for mixed type of decays can contain further unknown parameters, the phases $\delta_{2I=1}$ and $\delta_{2I=3}$ of the M_{ETC} relative to M_S . As an example, the $D^0 \rightarrow K^- \rho^+$ decay amplitude is given by

$$\begin{aligned}
M(D^0 \rightarrow K^- \rho^+) & \simeq \frac{i}{\sqrt{2}f_K} \langle \bar{K}^{*0} | H_w | D^0 \rangle \left\{ \frac{2}{3} \mathcal{A}_{K\rho}^{(1/2)} e^{i\delta_1} + \frac{1}{3} \mathcal{A}_{K\rho}^{(3/2)} e^{i\delta_3} \right. \\
& - \left[\left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_K^2} \right) - \left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_{K^*}^2} \right) \right] k_0 \\
& - \left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_{K^*}^2} \right) k_a^* + \left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_{E_{K^*}^{\rho^*}}^2} \right) k_s^* \\
& \left. - 2 \left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_{C_{K^*}^{(\bar{18})}}^2} \right) (k_-^* + k_+^*) - \left(\frac{m_D^2 - m_\rho^2}{m_D^2 - m_{K_H}^2} \right) k_H \right\}, \quad (3.4)
\end{aligned}$$

in the present approximation, where

$$\mathcal{A}_{K\rho}^{(1/2)} = -\frac{1}{2} + \frac{3}{2} \left(\frac{f_K}{f_D} \right) R \quad \text{and} \quad \mathcal{A}_{K\rho}^{(3/2)} = 1$$

are arising from M_{ETC} with the isospin $I = 1/2$ and $3/2$ $\bar{K} \rho$ final states, respectively. $C_K^{(*)}(\bar{18})$ is a component of the $SU_f(3)$ $\bar{18}$ -plet of the $[QQ][(\bar{Q}\bar{Q})\pm(QQ)[\bar{Q}\bar{Q}]$ mesons. The $D \rightarrow \pi \bar{K}^*$ amplitudes can be written down in a similar way although $\mathcal{A}_{K\rho}^{(I)}$, etc. should be replaced by their $\pi \bar{K}^*$ analogues.

If spatial wavefunctions of the $[QQ][(\bar{Q}\bar{Q})\pm(QQ)[\bar{Q}\bar{Q}]$ mesons are assumed to be the same, k_+^*/k_-^* also can be related to c_+/c_- , i.e., $k_+^*/k_-^* \simeq c_+/c_- = r$. We here consider the following three cases, (a) $r \simeq 0.46$ in which only the perturbative QCD corrections to H_w [13] are take into account, (b) $r \simeq 1.0$ in which the perturbative QCD corrections to H_w are canceled and (c) $r \simeq 1.5$ chosen tentatively (dominance of non-perturbative QCD effects). Then, changing the values of the phases, δ_{2I} , ($I = 1/2$ and $3/2$), and the parameter k_-^* (although they are constrained to be

Table 1. Branching ratios (%) for quasi two body decays of charm mesons. The data values are the world average given in Particle Data Table in ref. [20]. The values with (*) and †) are given by ARGUS in ref. [26] and CLEO II in ref. [27], respectively. BSW is a theoretical prediction based on the factorization[1]. (a)–(c) are the calculated branching ratios in typical three cases: (a) $r = 0.46$, (b) $r = 1.0$ and (c) $r = 1.5$. The values with (*) are used as the input data.

Decays	BSW	(a) $r = 0.46$	(b) $r = 1.0$	(c) $r = 1.5$	Experiments
$D^+ \rightarrow \pi^+ \bar{K}^{*0}$	0.3	2.4	2.5	2.3	1.9 ± 0.7
$D^+ \rightarrow \bar{K}^0 \rho^+$	15.3	1.9	2.0	2.6	6.6 ± 1.7
$D_s^+ \rightarrow \pi^+ \phi$	2.8	0.7	2.5	3.1	2.8 ± 0.5
$D^0 \rightarrow \pi^+ K^{*-}$	9.1	5.1	5.3	6.6	4.5 ± 0.6 5.8 ± 0.7 (†)
$D^0 \rightarrow \pi^0 \bar{K}^{*0}$	3.9	4.6	4.1	4.6	2.1 ± 1.0 4.6 ± 2.2 (*)
$D^0 \rightarrow K^- \rho^+$	13.8	2.8	4.4	4.2	7.3 ± 1.1
$D^0 \rightarrow \bar{K}^0 \rho^0$	1.1	0.5	1.2	1.6	0.6 ± 0.3 1.2 ± 0.2 (*)
$D^0 \rightarrow \bar{K}^0 \omega$	2.7	1.7	2.2	2.4	2.5 ± 0.5
$D_s^+ \rightarrow \bar{K}^0 K^{*+}$	0.3	0.5	1.7	2.9	3.3 ± 0.9
$D_s^+ \rightarrow K^+ \bar{K}^{*0}$	2.4	5.9	3.7	2.1	2.6 ± 0.5
$D_s^+ \rightarrow \pi^+ \omega$		0.0	0.0	0.0	< 1.4
$D_s^+ \rightarrow \pi^+ \rho^0$	0.5	0.1	0.1	0.1	< 0.22
$D^0 \rightarrow \bar{K}^0 \phi$	1.1(*)	0.88(*)	0.88(*)	0.88(*)	0.88 ± 0.12

$|k_-^*| \ll 1$ and $|\delta_{2I}| < 90^\circ$), we search for reasonable fits in the above three cases. Typical results in the three cases are shown in Table 1 in which we put the values

of the decay constants as $f_D = 180$ MeV and $f_{D_s} = 200$ MeV. The values of k_a^* , k_H , $\delta_{2I=2}$, Γ_{exotic} and Γ_{hybrid} are chosen to be the same in the three cases, *i.e.*, $k_a^* = 0.05$, $k_H = 0.07$, $\delta_{2I=2} = -85^\circ$, $\Gamma_{exotic} = 200$ MeV and $\Gamma_{hybrid} = 100$ MeV. However the values of k_-^* and δ_{2I} , ($2I = 1$ and 3), to give the best fits are a little different in the above three cases; (a) $k_-^* = 0.07$, $\delta_1 = -75^\circ$, $\delta_3 = 45^\circ$. (b) $k_-^* = 0.07$, $\delta_1 = -75^\circ$, $\delta_3 = 45^\circ$, (c) $k_-^* = 0.06$, $\delta_1 = -65^\circ$, $\delta_3 = 30^\circ$.

From Table 1, we see that the cases (b) $r = 1.0$ and (c) $r = 1.5$ reproduce fairly well the observed branching ratios while the case (a) $r = 0.46$ seems to be far from the observation. In particular, it seems to be hard to reproduce sizable $\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 K^{*+})_{exp}$ and $\mathcal{B}(D_s^+ \rightarrow \pi^+ \phi)_{exp}$ in the case (a). Our values of $\mathcal{B}(D^+ \rightarrow \bar{K}^0 \rho^+)$ and $\mathcal{B}(D^0 \rightarrow K^- \rho^+)$ are of ordinary size in contrast with the prediction by BSW[1] (based on the factorization) which is much larger than the observed ones.

4. Summary

In summary, we have studied the quasi two body decays of charm mesons using a hard meson approximation and demonstrated that hybrid mesons can play an important role in the decays described in terms of the annihilation diagrams. For example, in the $D_s^+ \rightarrow \pi^+ \rho^0$ decay, the pole contribution of the hybrid meson, π_H , with $I = 1$ and $J^{P(C)} = 0^{-(+)}$ can cancel M_{ETC} and $M_S^{(I=0)}$ in consistency with the other decays, in particular, the $D^0 \rightarrow \bar{K}^0 \phi$ which also is described in terms of the same annihilation diagrams in the $m_H \rightarrow \infty$ limit but has a substantial decay rate. The $D_s^+ \rightarrow \pi^+ \omega$ decay which is again of the annihilation type is suppressed in the present approximation since $M_{ETC} = 0$ and no $\{Q\bar{Q}\}_0$ and no hybrid mesons can give significant contributions to this decay. The four-quark mesons can take part in this process only through the crossed channel and give a negligibly small contribution. In the spectator decays, M_{ETC} and four-quark meson poles provide main terms of their amplitudes. The predicted $\mathcal{B}(D^+ \rightarrow \pi^+ \bar{K}^{*0})$, $\mathcal{B}(D^+ \rightarrow \bar{K}^0 \rho^+)$ and $\mathcal{B}(D_s^+ \rightarrow \pi^+ \phi)$ are of ordinary size in the cases (b) $r = 1.0$ and (c) $r = 1.5$ while in the case (a) $r = 0.46$, the calculated $\mathcal{B}(D_s^+ \rightarrow \pi^+ \phi)_{r=0.46}$ and $\mathcal{B}(D_s^+ \rightarrow \bar{K}^0 K^{*+})_{r=0.46}$ are much smaller than the observed one. Therefore

the perturbative QCD corrections to H_H seem to be insufficient in nonleptonic weak decays of charm mesons.

In the above, we demonstrated that the $0^{-(+)}$ hybrid mesons could play an important role in quasi two body decays of charm mesons. However, some models on hybrid mesons, for example, the flux tube model[16], have predicted that many hybrid states with different $J^{P(C)}$ also can be around the charm meson masses. If it is true, some of these mesons could contribute to the decays of charm mesons. For more precise discussions, we may need to take into account these contributions.

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APPENDIX A. Asymptotic matrix elements of the effective weak hamiltonian

The *asymptotic* matrix elements of H_w , (a) $\langle \{Q\bar{Q}\}_0 | H_w | \{Q\bar{Q}\}_0 \rangle$, (b) $\langle \{Q\bar{Q}g\} | H_w | \{Q\bar{Q}\}_0 \rangle$, (c) $\langle [QQ][\bar{Q}\bar{Q}] | H_w | \{Q\bar{Q}\}_0 \rangle$, (d) $\langle (QQ)(\bar{Q}\bar{Q}) | H_w | \{Q\bar{Q}\}_0 \rangle$ and (e) $\langle [QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}] | H_w | Q\bar{Q} \rangle$, which satisfy eqs. (2.4)–(2.8) in the text, are parametrized. The notations of four-quark mesons with $J^P = 1^+$ are given partly in the text and in analogy with those in ref. [5] in which four-quark mesons were scalar. Non-strange components of the $[QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]$ mesons mix to diagonalize G -parity[18].

$$(a) \langle \{Q\bar{Q}\}_0 | H_w | \{Q\bar{Q}\}_0 \rangle : \quad \langle \bar{K}^0 | H_w | D^0 \rangle + \langle \pi^+ | H_w | D_s^+ \rangle = 0, \quad (A.1)$$

$$\langle \bar{K}^0 | H_w | D^{*0}(\lambda = 0) \rangle + \langle \pi^+ | H_w | D_s^{*+}(\lambda = 0) \rangle = 0, \quad etc., \quad (A.2)$$

$$(b) \langle [QQ][\bar{Q}\bar{Q}]; J^P = 1^+ | H_w | \{Q\bar{Q}\}_0 \rangle :$$

$$\langle \bar{K}_A^{(*)0} | H_w | D^0 \rangle = - \langle \pi_A^{s(*)+} | H_w | D_s^+ \rangle = - \frac{k_A^*}{2g_A^*} \langle \bar{K}^0 | H_w | D^0 \rangle, \quad (A.3)$$

(c) $\langle (QQ)(\bar{Q}\bar{Q}); J^P = 1^+ | H_w | \{Q\bar{Q}\}_0 \rangle$:

$$\begin{aligned} \sqrt{\frac{3}{2}} \langle E_{\pi\bar{K},A}^{(*)+} | H_w | D^+ \rangle &= \frac{3}{\sqrt{2}} \langle E_{\pi\bar{K},A}^{(*)0} | H_w | D^0 \rangle = 3 \langle C_{\bar{K},A}^{(*)0} | H_w | D^0 \rangle \\ &= \sqrt{3} \langle C_{\pi,A}^{s(*)+} | H_w | D_s^+ \rangle = - \frac{k_s^*}{g_s^*} \langle \bar{K}^0 | H_w | D^0 \rangle, \end{aligned} \quad (A.4)$$

(d) $\langle [QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}] | H_w | Q\bar{Q} \rangle_0$:

$$\begin{aligned} \sqrt{3} \langle C_{\bar{K}}^{(*)0}(\bar{1}\bar{8}) | H_w | D^0 \rangle &= - \sqrt{\frac{1}{2}} \langle E_{\pi\bar{K}}^{(*)+}(\bar{1}\bar{8}) | H_w | D^+ \rangle \\ &= - \sqrt{\frac{3}{2}} \langle E_{\pi\bar{K}}^{(*)0}(\bar{1}\bar{8}) | H_w | D^0 \rangle = \frac{k^*}{t^*} \langle \bar{K}^{*0} | H_w | D^0 \rangle, \end{aligned} \quad (A.5)$$

$$\langle C_{\bar{K}}^{(*)0}(1\bar{8}) | H_w | D^0 \rangle = \frac{k_{\pm}^*}{t^*} \langle \bar{K}^{*0} | H_w | D^0 \rangle, \quad (A.6)$$

$$- \sqrt{2} \langle C_{\pi}^{s(*)+}(+) | H_w | D_s^+ \rangle = \frac{(k_{-}^* + k_{+}^*)}{t^*} \langle \bar{K}^{*0} | H_w | D^0 \rangle, \quad (A.7)$$

$$- \sqrt{2} \langle C_{\pi}^{s(*)+}(-) | H_w | D_s^+ \rangle = \frac{(k_{-}^* - k_{+}^*)}{t^*} \langle \bar{K}^{*0} | H_w | D^0 \rangle, \quad (A.8)$$

(e) $\langle \{Q\bar{Q}g\} | H_w | \{Q\bar{Q}\}_0 \rangle$:

$$\langle \bar{K}_H^0 | H_w | D^0 \rangle = - \langle \pi_H^+ | H_w | D_s^+ \rangle = \frac{\sqrt{2}k_H}{h_H}, \text{ etc.}, \quad (A.9)$$

in IMF, where g_A^* , g_s^* , t^* and h_H are the asymptotic invariant matrix element of the axial-vector charge A_α taken between the $\{Q\bar{Q}\}_0$ and the exotic (the axial-vector $[QQ][\bar{Q}\bar{Q}]$, $(QQ)(\bar{Q}\bar{Q})$, $[QQ](\bar{Q}\bar{Q}) \pm (QQ)[\bar{Q}\bar{Q}]$ or the PS hybrid $\{Q\bar{Q}g\}$, respectively) meson states and are given by $g_A^* = \langle K_A^{(*)+} | A_{\pi^+} | K^0 \rangle$, $g_s^* = \langle C_{\bar{K},A}^{(*)+} | A_{\pi^+} | K^0 \rangle$, $t^* = \langle C_{\bar{K}}^{(*)+}(1\bar{8}) | A_{\pi^0} | K^{*+} \rangle$ and $h_H = \langle \rho^0 | A_{\pi^+} | \pi_H^- \rangle$. k_A^* , k_s^* , k_{\pm}^* and k_H are parameters introduced. They are expected to be much smaller than unity.