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in Superconducting YBa₂Cu₃O₇ Observation of the Quasiparticle Hall Effect

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Observation of the quasiparticle Hall effect in superconducting $YBa₂Cu₃O₇$

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Abstract

Hall effect of quasiparticles in the superconducting state. ing their normal state values by more than a factor of 10. This behavior is ascribed to the analysis. Both the real and imaginary parts of σ_{xy} were found to peak near 40 K, exceedsurements in the frequency range 150-800 GHz without the need for Kramers-Kronig 10 to 200 K. The complex conductivity tensor was determined from transmission mea tensor of several YBa₂Cu₃O₇ thin films in magnetic fields up to 6 T at temperatures from Coherent time-domain spectroscopy was used to measure the complex transmission

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particles. $t_{xy} = -t_{yx} = \frac{-i}{2}(t_+ - t_-).$ the Hall, or damped cyclotron, motion of quasi-
as off-diagonal terms in the transmission tensor: ods. We ascribe the off-diagonal component to the polarization state. The nondegeneracy appears from 150 to 800 GHz, using time-domain meth-
basis, using wire grid linear polarizers to control $YBa_2Cu_3O_7$ thin films in the frequency range larly polarized radiation we work in a cartesian ments of the complex conductivity tensor in ago [4]. In this work we report the first measure-
required to characterize the conductivity tensor. mentally, although predicted theoretically long transmission measurements, rather than one, are ticles below T_c has never been reported experi-
ized radiation is lifted (the Faraday effect). Two ful. However, the Hall conductivity of quasipar-
generacy between left and right circularly polarsor in the presence of a magnetic field to be use-
ented perpendicular to the sample plane the dewe expect measurements of the *conductivity ten* ln the presence of a magnetic field ori-By analogy with other low-dimensional systems, dissipative response of the condensate. [1,2], and evidence for states at low energy [3]. state, which is dominated by the imaginary, nonthe rapid decrease in scattering rate just below T_c derstanding the dynamics of the superconducting the spectrum have led to important discoveries: Kronig transformation is particularly useful in un tivity in the gigahertz through terahertz region of determine the complex response without Kramersperature. Recent studies of quasiparticle conduc-
tained simply by inverting Eq. (1). The ability to tions which make use of the high transition tem-
imaginary parts of the conductivity, $\sigma_{\pm}(\omega)$ are obmechanism and are relevant to potential applica-
gation through the substrate. Both real and their conductivity, provide clues to the pairing the frequency dependent phase shift due to propanamics, as revealed through measurements of film thickness and n is the substrate index. Φ is anisotropic electronic system. Quasiparticle dy - where Z_0 is the impedance of free space, d is the sional, high-mobility and possibly highly perconductors constitute a unique low-dimen-Below T_c , quasiparticles in the cuprate su-

conductivity tensor by $pulse$. The time dependence of the terahertz pulse tion, $t_{+}(\omega)$, are degenerate and are related to the radiation at the moment it is struck by an optical cients of left and right circularly polarized radia-
to a current amplifier detects the field of incoming sence of a magnetic field the transmission coeffi-
Ti:sapphire laser. Similarly, an antenna connected termined over a broad spectral range. In the ab-
excited by an optical pulse from a mode-locked plex transmission coefficient $t(\omega)$ could be de-
of terahertz radiation when the photoconductor is transmitted through a thin film sample, the com-
A voltage-biased antenna generates a short pulse the time-dependence of an electromagnetic pulse are microfabricated photoconductive antennas [5]. superconductors. They showed that by resolving of the high-frequency conductivity of the cuprate presence of a static magnetic field is shown in Fig. of coherent time domain spectroscopy as a probe

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$$
t_{\pm}(\omega) = \frac{1}{Z_0 d \sigma_{\pm}(\omega) + n + 1} \cdot \frac{4n e^{i\Phi(\omega)}}{n + 1},
$$
 (1)

Because it is difficult to obtain broadband circu

between the arrival of optical pulses at the genera is recorded by continuously varying the interval 1. The sources and detectors of terahertz radiation measurement of the transmission tensor in the Nuss *et al.* [1] first demonstrated the utility The spectrometer we have designed for

paraboloidal mirrors. temperature and magnetic field, both $t_{xy}(\omega)$ and lenses and guided through the spectrometer with t_{xy} due to Faraday rotation is as follows: At each is coupled into space with hemispherical silicon The experimental procedure for measuring quency components out to 1 THz. The radiation tions typically yields an extinction ratio of 25. on the antenna geometry) and can contain fre-
film is remains linear. Taking the above precauradiation is centered at 200-500 GHz (depending thus ensuring that the polarization incident on the

a more pure and controllable polarization state. Sample B ($T_c = 88$ K) is a 40 nm film grown by and third wire grid polarizers were added to create on a sapphire substrate with a $CeO₂$ buffer layer. detector are highly polarized themselves, the first $84 K$) is a 70 nm film grown by off-axis sputtering the analyzing polarizer. Although the source and three in situ YBa₂Cu₃O₇ films. Sample A (T_c = the detector is not affected by the orientation of The measurements were performed on the polarization state of the pulse finally reaching radians for a 0.5 mm thick LaAlO₃ substrate. to the first. The third polarizer at 45° ensures that for a 3 mm thick sapphire substrate and $8x10^{-4}$ polarizer either parallel (t_x) or perpendicular (t_x) sample. The rotation was less than 2x10⁻⁴ radians sion tensor are selected by orienting the second checked using a bare substrate in place of the then the film. The components of the transmis-
measured at negative field. This procedure was ner windows of the cryostat, the substrate, and positive field can be removed by subtracting that passes through a linear polarizer, the outer and in-
The spurious component of t_x/t_{xx} measured at shown in Fig. 1, the pulse incident on the cryostat field while the signal due to birefringence is even. along the wavevector of the incident radiation. As Faraday effect is an odd function of the magnetic superconducting magnet, with the field oriented neto-optic effects we seek to measure. The The sample is placed at the center of a split-coil tage of the unique field dependence of the mag trometer to measure the complex Faraday effect. the effect of linear birefringence we take advansign of Nuss *et al.* are incorporated in the spec- $\Phi(\omega)$ is canceled in the ratio t_x/t_{xx} . To eliminate

substrate is largely eliminated by aligning the in-
relation ented sapphire. The waveplate-like effect of the normal state Hall effect, as can be seen from the ence in the terahertz range is 0.3 for $(1\overline{1}02)$ ori- $1/T^2$. This behavior is nothing more than the a more serious problem; the in-plane index differ- T_c , t_x/t_x is real and varies approximately as fused quartz 77 K inner windows. Substrates pose induced ellipticity in the transmitted light. Above birefringence – 0.1 mm Mylar outer windows and nary part represents the amount of magnetically dows is mitigated by using materials with low tion of the plane of polarization, and the imagi strate limit the extinction ratio. The effect of win-
in Figure 2. The real part corresponds to the rotabirefringence in the cryostat windows and sub- A at 150 GHz, in a magnetic field of 6 T, is shown in intensity). In the experimental configuration, The complex Faraday effect t_x/t_{xx} in film of two wire grid polarizers alone is about $100 (10⁴$ these films were qualitatively the same. ers be minimized. The amplitude extinction ratio lation on $LaAlO₃$. Results obtained for each of requires that the throughput with crossed polariz- $86 K$) is 80 nm thick, and was grown by laser ab-

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tor and detector. The amplitude spectrum of the cident polarization to within 1° of its optical axis,

Several features beyond the original de- $t_{xx}(\omega)$ are measured and the substrate phase shift

Achieving the optimum sensitivity to $t_{\rm rv}$ the same method, but on LaAlO₃. Sample C ($T_c =$

$$
\frac{t_{xy}}{t_{xx}} = \frac{Z_0 d \sigma_{xy}}{Z_0 d \sigma_{xx} + n + 1} \approx \frac{\sigma_{xy}}{\sigma_{xx}} = \tan(\theta_H), \quad (2) \qquad \frac{\sigma_{xy}(\omega)}{\sigma_0} = \frac{n_q}{n_0} \cdot \frac{\Gamma_0}{\Gamma}
$$

state [6] because 150 GHz is far below the normal cyclotron frequency ω_c , as a function of the tem-(377 Ω). The Hall angle at high frequency agrees Calculating $\sigma_{xy}(\omega)$ requires knowledge of cause the film impedance is much less than Z_0 and $\omega_c = eB/m^*c$. which is not made in later analysis, is accurate be-
rier density and scattering rate in the normal state, where θ_H is the Hall angle. The approximation, where σ_0 , n_0 , and Γ_0 are the dc conductivity, car-

cyclotron motion of quasiparticles. takes place in spite of a diminishing n_q , is caused sponse may be dominated instead by the damped accepted, the rise in σ_1 from T_c to 40 K, which peak at 40 K implies that the magneto-optic re-
constant at low temperature. As is now widely over a broad range of temperature below T_c . The monotonically with decreasing T, approaching a Faraday rotation is unrelated to vortex dynamics constraints that $n_q(0) = 0$ and that Γ decrease approaches zero. This contrast suggests that the and $\Gamma(T)$ were inferred from the data subject to the tive maximum near 40 K, and tends to zero as T conductivity σ_1 at 250 GHz. The functions $n_q(T)$ $Re(t_n/t_n)$ reverses sign again, increases to a posi-
shown in the inset, to the measured real part of the freezing out of vortex motion, while at 150 GHz compares a plot of Eq. (5), using $n_q(T)$ and $\Gamma(T)$ voltage become unmeasurably small due to where $n_q + n_s = n_0$. The main panel of Fig. 3 below T_c the low-frequency dissipation and Hall on dc measurements. At temperatures $10-15$ K lower T our results differ from expectations based tributed to vortex motion [8]. However, at still $\frac{1}{10}$ ics [10], we find a normalized conductivity frequency measurements [7] and has been at-
using a Drude model for the quasiparticle dynamchange of $\text{Re}(t_x/t_{xx})$, is also consistent with low measured on the same series of samples. Again

condensate and quasiparticle contributions [9]: drop in σ_1 at 100 GHz occurs at the same T, indiconductivity tensor may be written as a sum of scattering rate Γ_{el} . In our thin film samples the ing that in the presence of a magnetic field the has reached a temperature independent elastic extend the conventional two-fluid model, assum-
swept below the measurement frequency or that it Faraday effect due to quasiparticle motion. We 250 GHz below 40 K indicates either that Γ has

$$
\bar{\sigma} = \bar{\sigma}_{s} + \bar{\sigma}_{n} \approx \begin{bmatrix} \sigma_{s} & 0 \\ 0 & \sigma_{s} \end{bmatrix} + \begin{bmatrix} \sigma_{n} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{n} \end{bmatrix}.
$$
 (3)

rate, leads to the following off-diagonal compo- next use the same $n_q(T)$ and $\Gamma(T)$ as input to Eq. inomentum and energy independent scattering entire frequency range from 100-800 GHz. We Applying conventional transport theory, with a field conductivity for all three samples over our

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$$
\frac{\sigma_{xy}(\omega)}{\sigma_0} = \frac{n_q}{n_0} \cdot \frac{\Gamma_0}{\Gamma} \cdot \frac{\omega_c}{(1 - i \omega/\Gamma)^2 + (\omega_c/\Gamma)^2}, \quad (4)
$$

The structure near T_c , including the sign analyzing the complex zero-field conductivity state scattering rate.
 perature. We estimate the first two parameters by quantitatively with the dc Hall effect in the normal the quasiparticle density n_q , scattering rate Γ , and

$$
\frac{\sigma}{\sigma_0} = \frac{\sigma_s + \sigma_q}{\sigma_0} = \frac{i n_s \Gamma_0}{n_0 \omega} + \frac{n_q}{n_0} \frac{\Gamma_0}{(\Gamma - i \omega)},
$$
(5)

cating that Γ_{el} is reached at ~40 K. To test this hypothesis we focus on the by a rapid decrease in Γ . The drop in σ_1 at

 $\sigma_{xy}(\omega)$. The cyclotron frequency was chosen to be ment: (4) to estimate the quasiparticle contribution to rate shown in Fig. 3 reproduce the trends in zero The quasiparticle density and scattering 40 GHz at 6 T, corresponding to a temperature independent effective mass of $4m_e$.

Figure 4 compares the predictions of Eq. (4) with the real and imaginary parts of σ_{xy} as calculated from the transmission tensor. The data for sample B are shown because they cover the widest frequency range; the other two samples showed similar behavior. In each panel we plot σ_{xy} vs. T for several frequencies between 150 and 800 GHz. Referring first to the 150 GHz curves, one immediately notices that the model predicts a peak at 40 K in both the real and imaginary parts of σ_{xy} which is mirrored in the corresponding data. Most striking is the agreement in magnitude. The model and the data, which are both normalized to σ_0 at 100 K, agree to within about 50% at the peaks; both the real and imaginary parts appear to exceed the prediction by a factor of $~1.5$.

The model curves change dramatically as ω increases from 150 to 800 GHz because $\Gamma(T)$ inferred from the zero-field conductivity sweeps through this range. In fact Re $\sigma_{xy}(T)$ is much more sensitive to Γ than $\sigma_1(T)$; its shape evolves from a peak to a zero-crossing at 40 K as frequency increases. Again, the experimental results display the same trends as the model, implicating damped cyclotron motion of quasiparticles as the origin of the Hall effect over a broad range of temperature in the superconducting state.

Significant deviations between the model and experiment do appear, however, at the lowest temperatures at which measurements were performed. Of course the model predicts that $\sigma_{\rm rv}(T)$ tends to zero as we have assumed the quasiparticle density vanishes at $T = 0$. In contrast, both real and imaginary parts of the measured $\sigma_{xy}(T)$ appear to approach a non-zero asymptote as T tends to One possible explanation is that as zero. quasiparticles disappear into the condensate, the vortex contribution, masked at higher tempera-

m.

tures, begins to show through. Indeed, Choi et al. [11] have reported magneto-optic activity at 2 K in thin films of $YBa_2Cu_3O_7$ and interpreted the Faraday rotation in terms of vortex dynamics. Recent theoretical studies [12] of quasiparticle conductivity in the presence of disorder suggest another possibility. If a perfect $YBa₂Cu₃O₇$ crystal retains lines of the Fermi surface in the superconducting state, then disorder, clearly present in thin films, may induce finite areas of Fermi surface. In this picture the density of quasiparticles remains non-zero at $T=0$, although their contribution to the Hall effect has yet to be calculated.

In addition to understanding the Faraday effect better at the lowest temperatures, other important questions remain to be addressed. For example, will the Faraday rotation be significantly greater in samples with smaller Γ_{el} , as our model would predict? Second, the temperature dependence of the dc Hall angle [6] implies that the scattering rate for cyclotron motion differs from the transport scattering rate. Does this distinction extend to the superconducting state? The agreement in magnitude between model and experiment discussed earlier suggests that conventional dynamics describes the cyclotron motion in the superconducting state. However, this reasoning is based on the untested assumption that Γ is not strongly affected by a 6 T field. Future work will address these issues, focusing on the conductivity tensor as an additional probe of quasiparticle dynamics in the superconducting state.

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[12] P.J. Hirschfeld et al., Phys. Rev. Lett. 71, 3705 (1993); P.A. Lee, Phys. Rev. Lett. 71, 1887 (1993).

Figure Captions

Fig. 1: Time-domain spectrometer for measurement of the complex transmission tensor in the presence of a magnetic field. Diagonal t_{xx} and off-diagonal t_{xy} components are selected by rotating the analyzing polarizer through 90°. The effect of linear birefringence is removed by reversing the field direction.

Fig. 2: Real and imaginary parts of the ratio of the off-diagonal to diagonal transmission, t_x/t_{xx} , a quantity which is related to the Hall angle, measured at 150 GHz. Re($t_{\rm r}/t_{\rm rr}$) follows the dc Hall effect above and near T_c but rises to maximum near 40 K, at which temperature the dc Hall effect has become unmeasurably small.

Fig. 3: Real part of the conductivity σ_1 at 250 GHz measured in zero magnetic field, as a function of the temperature. The inset shows the quasiparticle density $n_q(T)$ and momentum relaxation rate $\Gamma(T)$ inferred from the complex $\sigma(T)$ at 250 GHz by assuming a Drude model for the quasiparticle conductivity. $\sigma_1(T)$ corresponding to these parameters is plotted as a solid line in the main panel.

Fig. 4: (a) Conventional transport model and (b) experimentally determined real and imaginary parts of σ_{xy} as a function of temperature for several frequencies between 150 and 800 GHz. The input parameters to the model are the quasiparticle density and momentum relaxation rate consistent with the zero-field conductivity (see Fig. 3 inset). The cyclotron frequency was chosen to be 40 GHz, corresponding to an effective mass of $4m_e$ and a field of 6 T.

Figure 3

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 $\label{eq:2.1} \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\sqrt{2\pi}}\frac{d\mu}{\$ $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{\mathbb{R}^3}\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$ $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$ $\label{eq:2.1} \frac{1}{\sqrt{2}}\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2\pi}}\right)^{2}d\mu\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{1}{\sqrt{2\pi}}\right)^{2}d\mu\int_{0}^{\infty}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{$ $\label{eq:2.1} \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{2} \left(\frac{1}{\sqrt{2}}\right)^{2} \left(\$

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