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Excited Λ_Q Baryons in the Large N_c Limit*

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Abstract

The spectrum of excited Λ_Q-type heavy baryons is considered in the large N_c limit. The universal form factors for Λ_b semileptonic decay to excited charmed baryons are calculated in the large N_c limit. We find that the Bjorken sum rule (for the slope of the Isgur–Wise function) and Voloshin sum rule (for the mass of the light degrees of freedom) are saturated by the first doublet of excited Λ_Q states.

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Experimental evidence for a doublet of excited charm baryons has recently been obtained [1]. They have masses 340 MeV and 308 MeV above the Λ_c. It is natural to interpret these states as the spin 3/2 and 1/2 isospin zero members of a doublet that has spin parity of the light degrees of freedom, $s_\ell^{\pi_\ell} = 1^-$.

Properties of these excited Λ_c baryons can be estimated using the nonrelativistic constituent quark model [2]. In this phenomenological model the observed excited Λ_c baryons have quark content *cud* with the *ud* pair in an isospin zero and spin zero state like the ground state Λ_c. However, unlike the ground state, in these excited Λ_c baryons the *ud* pair has a unit of orbital angular momentum about the charm quark.

In this paper we use the large N_c limit [3] to derive properties of excited Λ_Q baryons. The predictive power of this limit arises because the number of the light quarks in these baryons, $N_c - 1$, becomes large [4] as $N_c \to \infty$. In the physical three color case there are only two light quarks in a baryon with a single heavy quark and so we expect the large N_c limit to have only qualitative relevance. Nevertheless, unlike the nonrelativistic constituent quark model, the large N_c limit is the leading term in a systematic expansion of QCD and because of this we find its consequences interesting.

In the large N_c limit the light baryons, n, p, Δ, *etc.*, can be viewed as solitons in the chiral Lagrangian for pion self interactions [5]. The baryons containing a single heavy charm (or bottom) quark are bound states of these solitons with D and D^* (or B and B^*) mesons [6-12]. In this paper we use the bound state soliton picture to derive properties of the excited baryons that contain a heavy quark. However, since the results we derive do not depend on the couplings in the chiral Lagrangian for pion self interactions and pion-heavy meson interactions, they are interpreted as predictions of the large N_c limit. It should be possible to derive these results in other ways [13], however, the bound state soliton picture of Callan and Klebanov provides a convenient way to explore the consequences of the $N_c \to \infty$ limit for baryons containing a single heavy quark.

For baryons containing a single heavy quark the nucleon mass M_B plays a special role. In the large N_c limit the mass of the light degrees of freedom, $\bar{\Lambda}$, is equal to M_B. The equality, $\bar{\Lambda} \equiv M_{\Lambda_Q} - m_Q = M_B$, has a simple physical origin. In the nucleon a light quark responds to the mean color field created by $N_c - 1$ other quarks. With N_c large, replacing one of these other light quarks with a heavy quark has a negligible effect on this mean color field. Consequently, for large N_c, the mass of the light degrees of freedom in a baryon containing a single heavy quark is equal to M_B, with corrections to this relationship of order N_c^0.

In the bound state soliton picture Λ_Q-type bound states arise when the spin of the light degrees of freedom of the heavy meson and the spin of the nucleon are combined into a spin zero configuration, and the isospin of the heavy meson and the nucleon are combined into an isospin zero state. Other baryons (e.g., the Δ) only contribute to bound states with higher isospin. The spatial wavefunctions for Λ_Q-type bound states are controlled by the potential [11]

$$V_\Lambda(\vec{x}) = V_0 + \frac{1}{2}\kappa\vec{x}^2. \tag{1}$$

Higher powers of $\vec{x}$ are unimportant for large N_c. Both V_0 and κ are order N_c^0 and their values depend on nonperturbative strong interaction dynamics. When the orbital angular momentum of the bound state is non-zero, the Λ_Q-type baryons occur (in the $m_Q \to \infty$ limit) in degenerate doublets that arise from combining the orbital angular momentum of the bound state with the heavy quark spin. The harmonic oscillator potential in eq. (1) gives rise to an infinite tower of Λ_Q-type baryons with excitation energies

$$\Delta E^{(Q)}_{(n_1,n_2,n_3)} = (n_1 + n_2 + n_3)\sqrt{\kappa/\mu_Q}, \tag{2}$$

where μ_Q is the reduced mass

$$\frac{1}{\mu_Q} = \frac{1}{m_Q} + \frac{1}{M_B}. \tag{3}$$

In eq. (2) (n_1, n_2, n_3) are the quantum numbers that specify the bound states when the Schrödinger equation is solved by separating variables in cartesian coordinates.

For states with the same quantum numbers (n_1, n_2, n_3), but different heavy quarks, eq. (2) gives

$$\Delta E^{(c)}/\Delta E^{(b)} = \left(\frac{1+M_B/m_c}{1+M_B/m_b}\right)^{\frac{1}{2}} \simeq 1 + \frac{1}{2}\left(\frac{M_B}{m_c} - \frac{M_B}{m_b}\right) + \ldots \quad . \tag{4}$$

Eq. (2) was obtained by solving the Schrödinger equation including the kinetic energy of the heavy meson. This corresponds to taking simultaneously the limits $m_Q \to \infty$ and $N_c \to \infty$ with the ratio M_B/m_Q held fixed (recall M_B is of order N_c). If m_Q was taken to infinity first, then effects of order M_B/m_Q are neglected, and heavy quark flavor symmetry determines the ratio of excitation energies in eq. (4) to be unity. In the large N_c limit the leading corrections to heavy quark symmetry [14] arise from including the kinetic energy of the heavy meson in the Schrödinger equation for the soliton-heavy meson bound state [11]. This violates heavy quark flavor symmetry but leaves heavy quark spin symmetry intact. Despite the fact that M_B/m_c is not particularly small the ratio of excitation energies in eq. (4) differs from unity by less than 20%.

The excitation energies given in eq. (2) are of order $N_c^{-1/2}$. The first excited states have quantum numbers $(1,0,0)$, $(0,1,0)$, $(0,0,1)$. There is another basis of quantum numbers $[N, \ell, m]$, $N = n_1 + n_2 + n_3$, that is also useful. Here ℓ is the orbital angular momentum of the bound state and m is the component of the orbital angular momentum along the third (spin-quantization) axis. In this basis $N \geq \ell$ and even values of ℓ occur for N even while odd values of ℓ occur for N odd. The first excited states have $N = 1$, $\ell = 1$, and $m = 0, +1, -1$ giving $s_\ell^{\pi_\ell} = 1^-$ for the spin parity of the light degrees of freedom. Combining this with the spin of the heavy quark gives a doublet of negative parity states with total spins 3/2 an 1/2. For $Q = c$ these states correspond to the observed doublet of excited Λ_c states. Comparing eq. (2) with the experimental value of the excitation energy ($\simeq 340\ MeV$) gives $\kappa \simeq (440\ MeV)^3$.

In general, Λ_Q-type states have total spins $s = \ell \pm 1/2$ formed by combining the spin of the heavy quark with the orbital angular momentum ℓ. We label them by the quantum numbers $\{N, \ell; s, m\}$, where now m is the component of the total spin along the third (i.e., spin-quantization) axis. In this notation the ground state Λ_Q baryon has quantum numbers $\{0,0;1/2,m\}$ and the first excited Λ_Q doublet contains the states $\{1,1;1/2,m\}$ and $\{1,1;3/2,m\}$.

Ref. [11] considered the weak semileptonic decay $\Lambda_b \to \Lambda_c e \bar{\nu}_e$ in the large N_c limit. In this paper we discuss weak semileptonic decays of the Λ_b to excited Λ_c baryons. These decay amplitudes are determined by matrix elements of the vector and axial vector heavy quark currents. These matrix elements can be calculated using the bound state soliton picture we have outlined above. We find in the rest frame of the initial state, $v = (1, \vec{0})$, that

$$\langle \Lambda_c^{\{N,\ell;s',m'\}}(v')|\ \bar{h}_{v'}^{(c)} \gamma^\mu h_v^{(b)}\ |\Lambda_b^{\{0,0;1/2,m\}}(v)\rangle$$

$$= \delta^{\mu,0} (\ell, m'-m; 1/2, m | s', m') \mathcal{F}^{[N,\ell,m'-m]} \qquad (5a)$$

and

$$\langle \Lambda_c^{\{N,\ell;s',m'\}}(v')|\ \bar{h}_{v'}^{(c)} \gamma^\mu \gamma_5 h_v^{(b)}\ |\Lambda_b^{\{0,0;1/2,m\}}(v)\rangle$$

$$= \delta^{\mu,j} \sum_{m''} (\ell, m'-m''; 1/2, m'' | s', m') [\chi^\dagger(m'') \sigma^j \chi(m)] \mathcal{F}^{[N,\ell,m'-m'']}, \qquad (5b)$$

where $\mathcal{F}^{[N,\ell,m]}$ is an overlap of momentum space harmonic oscillator wave functions

$$\mathcal{F}^{[N,\ell,m]} = \int d^3q \phi_c^{*[N,\ell,m]}(\vec{q}) \phi_b^{[0,0,0]}(\vec{q} - M_B \vec{v}'). \qquad (6)$$

In eq. (5b) χ is a two-component Pauli spinor. The sum over m'' in eq. (5b) collapses to a single term since $\chi^\dagger(m'')\sigma^3\chi(m)$ vanishes for $m'' = -m$ and $\chi^\dagger(m'')\sigma^{1,2}\chi(m)$ vanishes for $m'' = m$. In eq. (6) $\phi_Q^{[N,\ell,m]}(\vec{q})$ denotes the normalized momentum space harmonic oscillator wave function. Its dependence on the type of heavy quark arises from the dependence of the reduced mass μ_Q on the heavy quark mass.

Eqs. (5) and (6) are valid in the kinematic region $|\vec{v}'| \lesssim \mathcal{O}(N_c^{-3/4})$. For recoil velocities greater than this the overlap $\mathcal{F}^{[N,\ell,m]}$ is very small and terms subdominant in N_c that we have neglected may be important [11]. When $v \cdot v' \neq 1$ the operator $\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}$ requires renormalization [15]. However, in the kinematic regime very near zero recoil where eqs. (5) and (6) apply, the subtraction point dependence of $\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}$ is negligible.

It is easiest to evaluate $\mathcal{F}^{[N,\ell,m]}$ in the case where $\vec{v}'$ is directed along the 3rd (i.e., spin-quantization) axis. The expression is particularly simple when the limit $m_Q \to \infty$ is taken first so that $\mu_Q = M_B$ independent of heavy quark type. Then

$$\mathcal{F}^{[N,\ell,m]} = \delta^{m,0}\frac{C^{N\ell}}{\sqrt{N!}}[M_B^3/\kappa]^{N/4}(v \cdot v' - 1)^{N/2}\exp\left(-\frac{1}{2}[M_B^3/\kappa]^{1/2}(v \cdot v' - 1)\right) \quad (7)$$

where

$$C^{N\ell} = \int d^3q\,\phi^{*[N,\ell,0]}(\vec{q})\phi^{(0,0,N)}(\vec{q}). \quad (8)$$

In eq. (7) we used $|\vec{v}'|^2 = 2(v \cdot v' - 1)$ which is appropriate for the region near zero recoil, $(v \cdot v' - 1) \lesssim \mathcal{O}(N_c^{-3/2})$, that we are considering.

The ground state $\Lambda_b \to \Lambda_c$ transition corresponds to the case $N = 0$, $\ell = 0$. Heavy quark spin symmetry implies that in this case the matrix elements of heavy quark bilinears have the form [16]

$$\langle\Lambda_c^{\{0,0;1/2,m'\}}(v')|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|\Lambda_b^{\{0,0;1/2,m\}}(v)\rangle$$

$$= \eta(v \cdot v')\bar{u}(v',s')\Gamma u(v,s). \quad (9)$$

Comparing with eqs. (5) and using eqs. (7) and (8) gives [11]

$$\eta(v \cdot v') = \exp\left[-\frac{1}{2}[M_B^3/\kappa]^{1/2}(v \cdot v' - 1)\right]. \quad (10)$$

Expanding η about zero recoil

$$\eta(v \cdot v') = 1 - \rho^2(v \cdot v' - 1) + \ldots , \tag{11}$$

and comparing with eq. (10) we see that

$$\rho^2 = \frac{1}{2}[M_B^3/\kappa]^{1/2}. \tag{12}$$

Transition matrix elements to excited Λ_c states with $\ell = 1$ are constrained by heavy quark symmetry to have the form [17]

$$\langle \Lambda^{\{N,1;1/2,m'\}}(v')|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|\Lambda_b^{\{0,0;1/2,m\}}(v)\rangle$$

$$= \frac{\sigma^{(N)}(v \cdot v')}{\sqrt{3}}\bar{u}(v', s')\gamma_5(\not{v} + v \cdot v')\Gamma u(v, s), \tag{13a}$$

$$\langle \Lambda_c^{\{N,1;3/2,m'\}}(v')|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|\Lambda_b^{\{0,0;1/2,m\}}(v)\rangle$$

$$= \sigma^{(N)}(v \cdot v')\bar{u}_\mu(v', s')v^\mu\Gamma u(v, s). \tag{13b}$$

Comparing these expressions with eqs. (5), (6) and (7) gives

$$\sigma^{(N)}(v \cdot v') = \frac{C^{N1}}{\sqrt{2(N!)}}[M_B^3/\kappa]^{N/4}(v \cdot v' - 1)^{(N-1)/2}$$

$$\cdot \exp\left[-\frac{1}{2}[M_B^3/\kappa]^{1/2}(v \cdot v' - 1)\right]. \tag{14}$$

Note that fractional powers of $(v \cdot v' - 1)$ do not occur in eq. (14) because N must be odd. At zero recoil $\sigma^N(1)$ is zero for $N > 1$ while for the first excited state

$$\sigma^{(1)}(1) = [M_B^3/4\kappa]^{1/4}, \tag{15}$$

using $C^{11} = 1$. Equations (14) and (15) are the main results of this paper. For simplicity we derived our expressions for the Isgur–Wise functions $\sigma^{(N)}(v \cdot v')$ by taking the limit $m_Q \to \infty$ followed by the limit $N_c \to \infty$. However, eqs. (5) and (6) can be used to include corrections to the heavy quark limit to all orders in M_B/m_Q. As we have noted, these corrections do not violate heavy quark spin symmetry. Therefore the form of the matrix elements given in eqs. (13) still holds, but the functions $\sigma^{(N)}(v \cdot v')$ become dependent on the heavy quark masses.

The fact that $\sigma^{(N)}(1)$ is zero for $N > 1$ means that in the large N_c limit the Bjorken sum rule [17,18] for the slope ρ^2 of the Isgur–Wise function $\eta(v \cdot v')$ and Voloshin sum rule [19] for the mass of the light degrees of freedom are saturated by the first doublet of excited Λ_c states. The Bjorken sum rule for the slope of the Isgur–Wise function $\eta(v \cdot v')$ is [17]

$$\rho^2 = \sum_N |\sigma^{(N)}(1)|^2, \tag{16}$$

while the Voloshin sum rule for the mass of the light degrees of freedom, $\bar{\Lambda} = M_{\Lambda_Q} - m_Q$, reads

$$\bar{\Lambda} = \sum_N 2\Delta E_N |\sigma^{(N)}(1)|^2. \tag{17}$$

(Eq. (17) is a generalization of the heavy meson result derived in Ref. [19] to the case of heavy baryons.) Using eq. (15) and our explicit expressions for ΔE_N, ρ^2 and $\bar{\Lambda} = M_B$, it is straightforward to verify that in the large N_c limit these two sum rules hold.

There are other properties of excited heavy baryons that can be examined in the large N_c limit. For example, at the leading order in chiral perturbation theory [20], the strong couplings of the ground state Λ_Q to $\Sigma_Q\pi$ or $\Sigma_Q^*\pi$ are of order $N_c^{1/2}$ and can be related to the pion-nucleon coupling [9]. However, because of the orthogonality of the harmonic oscillator wave-functions the analogous couplings for excited Λ_Q states [21] are only of order $N_c^{-1/2}$.

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