

The Jacobi Polynomials QCD analysis of the CCFR Data for xF_3 and the Q^2 -Dependence of the Gross–Llewellyn Smith Sum Rule

Andrei L. Kataev

Theory Division, CERN, CH-1211 Geneva 23, Switzerland;¹

Institute for Nuclear Research of the Russian Academy of Sciences, Moscow 117312,
Russia²

Aleksander V. Sidorov

Bogoliubov Theoretical Laboratory, Joint Institute for Nuclear Research, 141980 Dubna,
Russia³

Abstract

We present the results of our QCD analysis of the recent CCFR data for the structure function $xF_3(x, Q^2)$ of the deep-inelastic neutrino–nucleon scattering. The analysis is based on the Jacobi polynomials expansion of the structure functions. The concrete results for the parameter $\Lambda_{\overline{MS}}^{(4)}$ and the shape of quark distributions are determined. At the reference scale $|Q_0^2|=3 \text{ GeV}^2$ our results are in satisfactory agreement with the ones obtained by the CCFR group with the help of another method. The Q_0^2 -dependence of the experimental data for the Gross–Llewellyn Smith sum rule is extracted in the wide region of high-momentum transfer. Within systematical experimental uncertainties the results obtained are consistent with the perturbative QCD predictions. We reveal the effect of the discrepancy between our results and the analysed perturbative QCD predictions at the level of the statistical error bars. The importance of taking account, in our procedure, of a still unknown next-next-to-leading approximation of the moments of the structure function $xF_3(x, Q^2)$ is stressed.

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¹E-mail: KATAEV@CERNVM.BITNET

²On leave of absence at CERN from February 1994

³E-mail: SIDOROV@THEOR.JINRC.DUBNA.SU

1. Introduction

The deep-inelastic lepton–nucleon scattering is the source of important information about the nucleons structure. In the last years the accuracy of the obtained experimental data became large enough to study in detail the status of the comparison of the available data with the theoretical predictions of QCD in the different regions of momentum transfer [1].

The most precise data for the structure function (SF) $xF_3(x, Q^2)$ was recently obtained by the CCFR collaboration at the FERMILAB collider [2]–[4] (for a detailed description see Refs. [5]). The theoretical analysis of the obtained experimental data for the process of nucleon destruction by the charged currents was made by the members of the CCFR collaboration, with the help of the computer program developed in Ref. [6] and based on the direct integration of the Altarelli–Parisi equation [7]. The fits to the data [2]–[4] were only made where perturbative QCD is expected to be valid. The results of a next-to-leading order (NLO) fit of the non-singlet SF $xF_3(x, Q^2)$ were obtained for the different values of the Q^2 cut. In particular in the case of the cut $|Q^2| > 10 \text{ GeV}^2$ the CCFR collaboration got the following value of the QCD scale parameter $\Lambda_{\overline{MS}}^{(4)}$ (see [4]) :

$$\Lambda_{\overline{MS}}^{(4)} = 171 \pm 32(stat.) \pm 54(syst) \text{ MeV} , \quad (1)$$

which corresponds to $f = 4$ numbers of active flavours. Notice that the former cut allows one to neglect the effects of the high-twist (HT) contribution and the target mass (TM) corrections to $xF_3(x, Q^2)$.

Another important characteristic of the deep-inelastic neutrino–nucleon scattering is the Gross–Llewellyn Smith (GLS) sum rule [8]

$$GLS(Q^2) = \frac{1}{2} \int_0^1 \frac{xF_3^{\bar{\nu}p+\nu p}(x, Q^2)}{x} dx. \quad (2)$$

In the work of Ref. [3], the following result of the measurement of the GLS sum at the scale $|Q_0^2| = 3 \text{ GeV}^2$ was reported :

$$GLS(|Q_0^2| = 3 \text{ GeV}^2) = 2.50 \pm 0.018(stat) \pm 0.078(syst). \quad (3)$$

This result has been obtained by a special procedure of either interpolation or extrapolation of the $xF_3(x, Q^2)$ data with the help of the best fit of the Q^2 -dependence and taking into account the logarithmic Q^2 -dependence as predicted by QCD [3, 5]. Equation 3 was already used as the bases of the NLO QCD analysis [9, 10] by taking into account the NLO perturbative QCD corrections to the GLS sum rule [11] (which were confirmed in [12]) and the corresponding HT corrections [13] estimated by the QCD sum rules method in Refs. [14, 10]. However, it should be stressed that the explicit Q_0^2 -dependence of the CCFR data for the GLS sum rule remained non-investigated.

In this work this important problem is studied with the help of the method of the SF reconstruction over their Mellin moments, which is based on the expansion of the SF over

the Jacobi polynomials [15]. This method was developed [16, 17] (see also [18]), discussed [19] and previously used by the BCDMS collaboration in concrete physical applications [20] (see also [21]).

2. The Method of the QCD Analysis of SF

We first recall the basic steps of the method used in this work to reconstruct of SF in the x -representation over their Mellin moments.

Let us define the Mellin moments for the non-singlet SF $xF_3(x, Q^2)$:

$$M_n^{NS}(Q^2) = \int_0^1 x^{n-1} F_3(x, Q^2) dx, \quad (4)$$

where $n = 2, 3, 4, \dots$.⁴ The Q^2 -evolution of the moments is given by the solution of the corresponding renormalization-group equation. In the NLO approximation of perturbative QCD it can be presented in the following form [22]:

$$M_n^{NS}(Q^2) = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{d_n} H_n(Q_0^2, Q^2) M_n^{NS}(Q_0^2), \quad (5)$$

where $d_n = -\gamma_{NS}^0/2\beta_0$ and

$$H_n(Q_0^2, Q^2) = \frac{1 + C_{NS}^1(n) \frac{\alpha_s(Q^2)}{4\pi}}{1 + C_{NS}^1(n) \frac{\alpha_s(Q_0^2)}{4\pi}} \left[\frac{1 + \beta_1 \frac{\alpha_s(Q^2)}{4\pi\beta_0}}{1 + \beta_1 \frac{\alpha_s(Q_0^2)}{4\pi\beta_0}} \right]^{p(n)} \quad (6)$$

$$p(n) = \frac{1}{2} \left[\frac{\gamma_{NS}^{(1)}}{\beta_1} - \frac{\gamma_{NS}^{(0)}}{\beta_0} \right]. \quad (7)$$

The NLO approximation of the QCD coupling constant $\alpha_s(Q^2)$ can be expressed through the scale parameter $\Lambda_{\overline{MS}}$ as

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln(Q^2/\Lambda_{\overline{MS}}^2)} - \frac{\beta_1 \ln \ln(Q^2/\Lambda_{\overline{MS}}^2)}{\beta_0^3 \ln^2(Q^2/\Lambda_{\overline{MS}}^2)} \quad (8)$$

where β_0 and β_1 , namely

$$\beta_0 = 11 - \frac{2}{3}f \quad \text{and} \quad \beta_1 = 102 - \frac{38}{3}f, \quad (9)$$

are the leading-order (LO) and NLO coefficients of the QCD β -function, which was originally calculated at the next-next-to-leading order (NNLO) level [23] and confirmed in Ref. [24].

The analytic expressions for the LO coefficient $\gamma_{NS}^{(0)}$ of the anomalous dimension function of the non-singlet operator and the corresponding expression for the NLO coefficient $\gamma_{NS}^{(1)}$ can be found, e.g. in the textbook of Ref. [22]. For the neutrino–nucleon deep-inelastic

⁴Note that the first moment $M_1^{NS}(Q^2)$ is nothing more than the GLS sum rule defined in Eq. (2).

scattering, the NLO coefficient $C_{NS}^1(n)$ of the coefficient function is known from the results of Ref. [25].

Following the methods of [15]-[18], one can expand the SF in the set over Jacobi polynomials $\Theta_n^{\alpha,\beta}(x)$:

$$xF_3^{N_{max}}(x, Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{max}} a_n(Q^2) \Theta_n^{\alpha,\beta}(x), \quad (10)$$

where N_{max} is the number of polynomials and $a_n(Q^2)$ are the coefficients of the corresponding expansion.

The Jacobi polynomials $\Theta_n^{\alpha,\beta}(x)$ obey the orthogonality relation

$$\int_0^1 dx x^\alpha(1-x)^\beta \Theta_k^{\alpha,\beta}(x) \Theta_l^{\alpha,\beta}(x) = \delta_{k,l}, \quad (11)$$

and can be expressed as the series in powers of x :

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) x^j, \quad (12)$$

where $c_j^{(n)}(\alpha, \beta)$ are the coefficients that expressed through Γ -functions.

Using now Eqs. (11), (12) and (4), one can relate the SF with its Mellin moments

$$xF_3^{N_{max}}(x, Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha, (x) \sum_{j=0}^n c_j^{(n)}(\alpha,\beta) M_n^{NS}(Q^2), n=2,3,\dots} \quad (13)$$

The relations of Eqs. (5), (6) and (13) form the basis of the computer program created by the authors of Ref. [17]. It was previously tested and used by the members of the BCDMS collaboration in the course of detailed QCD analysis of the experimental data for the SF of the deep-inelastic muon–nucleon scattering [20, 19].

3. The Procedure of the QCD Fit of xF_3 Data

In accordance with the original non-singlet fit of the CCFR collaboration [3, 5] in the proces of the studies of their experimental data, we choose the parametrization of the parton distributions at fixed momentum transfer Q_0^2 in the simplest form :

$$xF_3(x, Q_0^2) = Ax^b(1-x)^c, \quad (14)$$

which was originally used by the CCFR collaboration to get the result of Eq. (3) for the GLS sum rule.

The constants A , b , c in Eq. (14) and the QCD scale parameter Λ are considered as free parameters, which should be determined for concrete values of Q_0^2 . The values of the parameters A , b and c depend on the value of Q_0^2 . In order to avoid the influence of the HT effects and the TM corrections, we use the experimental points of the concrete CCFR data [5] in the plane (x, Q^2) with $0.015 < x < 0.65$ and $10 \text{ GeV}^2 < |Q^2| < 501 \text{ GeV}^2$.

It should be stressed that for deep-inelastic processes with charged currents, which were dealt with in the CCFR experiment, the former region of momentum transfer implies that there are four active flavours and that we have to use $f = 4$ in formulae (5),(6) and (8). In view of this fact we will not take threshold effects into account in the process of the present analysis.

We are now ready to discuss the main steps of our analysis :

- The parametrization of Eq. (14) allows us to calculate the concrete expression for the Mellin moments through Γ -functions that depend on the parameters A , b and c , namely the expression $M_n^{NS}(Q_0^2, A, b, c)$. As was estimated in [17], in order to get the accuracy better than 10^{-3} in the procedure of the SF reconstructions, it is sufficient to use in Eq. (10) $N_{max} = 10$. However to make the analysis even more reliable we will use $N_{max} = 12$.
- The next step is to use the QCD theoretical evolution of Eqs. (5) and (6) for the calculation of each Mellin moment at Q^2 values that corresponds to concrete experimental points Q_{exp}^2 for the SF $xF_3(x, Q^2)$. At this stage the essential dependence of the Mellin moments from the QCD scale parameter Λ appears :

$$M_n^{NS}(Q_0^2, A, b, c) \xrightarrow{QCD} M_n^{NS}(Q_{exp}^2, A, b, c, \Lambda), \quad (15)$$

- Using now Eq. (13) we can reconstruct the theoretical expression for the SF $xF_3(x, Q^2)$, namely $xF_3^{(N_{max}=12)}(x, Q^2, A, b, c, \Lambda)$ for all experimental points (x_{exp}, Q_{exp}^2) .
- The numerical values of the parameters α and β which define the corresponding Jacobi polynomials $\Theta_n^{\alpha, \beta}(x)$, can be chosen such as to achieve the fastest convergence of the series in the r.h.s. of Eq. (10). This procedure was discussed in Ref. [17]. In accordance with the results of Ref. [17] we use $\alpha = 0.12$ and $\beta = 2.0$.
- The determination of the free parameters of the fit (namely A , b , c and Λ) from the CCFR experimental data for $xF_3(x_{exp}, Q_{exp}^2)$ is made by minimization of χ^2 by the MINUIT program, which automatically calculates the statistical errors of the parameters also.
- The obtained values of the parameters A , b and c depend on the reference scale Q_0^2 , which enters into the expression for $M_n^{NS}(Q_{exp}^2, A, b, c, \Lambda)$ (and thus for $xF_3^{(N_{max}=12)}(x, Q^2, A, b, c, \Lambda)$) through Eqs. (5) and (6)).
- In order to evaluate the numerical value of the GLS sum rule at the reference scale Q_0^2 , it is necessary now to substitute $A(Q_0^2)$, $b(Q_0^2)$ and $c(Q_0^2)$ for their concrete values in Eq. (14) and to calculate the integral of Eq. (2).

- Repeating the above procedure for the different values of Q_0^2 , we determine the experimental dependence of the GLS sum rule from the momentum transfer.
- The described fit will be made both in the LO and NLO of perturbative QCD. In the process of the LO fit we will use the LO approximations of the anomalous dimension function, coefficient function and the QCD coupling constant α_s defined through the corresponding scale parameter Λ_{LO} as $\alpha_s(Q^2) = 4\pi/\beta_0 \ln(Q^2/\Lambda_{LO}^2)$.

4. The Results of the QCD Fit of the CCFR xF_3 Data

The fitting procedure discussed in Section 3 was applied by us to the analysis of the CCFR data for the non-singlet SF measured in the neutrino deep-inelastic scattering [5]. The results of the fit at different values of Q_0^2 are presented in Table 1.

Several comments are in order:

- The stable value of Λ for fits with different Q_0^2 both for LO and NLO indicates the stability and the self-consistence of the method used.
- It is easy to see from Table 1, that within the statistical errors, the results of our NLO fit of the parameter $\Lambda_{\overline{MS}}^{(4)}$ in the wide region of Q_0^2 are in agreement with the result (1), obtained by the CCFR group with a little bit more complicated parametrization of the SF $x F_3(x, Q_0^2) = Ax^b(1-x)^c + Dx^e$, in the same kinematic region (see ref.[5]). The estimation of the systematic error in Eq. (1) remains true for our results.
- Our results for $\Lambda_{\overline{MS}}^{(4)}$ are in exact agreement with the outcome of the combined non-singlet fit of the CCFR data for the $x F_3(x, Q^2)$ and $F_2(x, Q^2)$ SFs [4, 5], namely $\Lambda_{\overline{MS}}^{(4)} = 210 \pm 28(stat) \pm 41(syst) MeV$.
- The inequality $\chi_{d.f.}^{2(LO)} > \chi_{d.f.}^{2(NLO)}$ indicates that the NLO is preferable, for the description of the experimental data. Notice, that even if HT-effects and the TM corrections have been neglected, $\chi_{d.f.}^2$ is rather good in the wide region of Q_0^2 , which includes even low momentum transfer.
- The results of our fit for the parameters of quark distributions can be compared with the results obtained by the CCFR group with the help of another program [6] at the reference scale $|Q_0^2| = 3 GeV^2$. This comparison is presented in Table 2. One can see that the agreement between the NLO results is satisfactory.

5. The Q^2 -Dependence of the GLS Sum Rule VS Experiment

We consider the results of Table 1 for the GLS sum rule as the experimental points in the wide region of Q_0^2 . The corresponding statistical errors can be estimated using the statistical errors of the parameters A , b and c of quark distributions as presented in the second column

of Table 2. Using the concrete expression for the first Mellin moment through the quark distributions of Eq. (14), we find that the statistical errors for the GLS sum rule are within 4%–5%. The systematical uncertainty was determined by the CCFR group itself [3, 5] (see Eq. (3)).

Taking into account these estimates of the statistical and experimental uncertainties of the experimental outcomes of our NLO fit, we get the following value for the GLS sum rule at the scale $|Q_0^2| = 3 \text{ GeV}^2$:

$$GLS(|Q_0^2| = 3 \text{ GeV}^2) = 2.446 \pm 0.100(stat) \pm 0.078(syst) \quad (16)$$

which is in agreement with the result (3) obtained by the CCFR group. The smaller statistical error of the CCFR result of Eq. (3) comes from their more refined analysis of this type of experimental uncertainties.

Let us now compare the experimental behaviour of the GLS sum rule with the corresponding perturbative QCD predictions for the first Mellin moment, which determines the theoretical expression for the GLS sum rule. The result we are interested in has the form

$$GLS_{QCD}(Q^2) = 3 \left[1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2(Q^2)) + O\left(\frac{1}{Q^2}\right) \right]. \quad (17)$$

The estimates of the HT corrections are presented in Refs. [14, 10] and the NLO corrections of order $O(\alpha_s^2(Q^2))$ and the NNLO corrections of order $O(\alpha_s^3(Q^2))$ were analytically calculated in [11, 12] and [26], respectively.

It is worth emphasizing that putting $n = 1$ in the LO QCD expression for the moment $M_n^{NS}(Q^2)$ we obtain the quark–parton prediction for the GLS sum rule. In order to obtain LO and NLO expressions for the GLS sum rule one should consider the NLO and NNLO approximations of the moments $M_n^{NS}(Q^2)$ correspondingly. Therefore, in order to make a self-consistent study of the results of Table 1 for the GLS sum rule within the framework of perturbative QCD, it is necessary to compare the results of the LO fit with the quark–parton expression of the GLS sum rule and the results of the NLO fit with the LO expression of the GLS sum rule, but with the coupling constant α_s defined through Eq. (8), with $\Lambda_{\overline{MS}}^{(4)}$ taken from the results of the NLO fit.

Figures 1 demonstrate the experimental results for the GLS sum rule for both LO and NLO fits (see Table 1) with the statistical experimental errors discussed above. On the same Figures we present the quark–parton and LO theoretical expressions for the GLS sum rule (17). In this work we are neglecting the contributions of the HT corrections (which are known to be quite important for the analysis of the GLS sum rules results in the low-energy region [14, 9, 10]) since in the process of both our fit and of the one made by the CCFR group itself these corrections were not taken into account in the expression for the SF $xF_3(x, Q^2)$.

The values of the parameter $\Lambda_{\overline{MS}}^{(4)}$ in the LO perturbative QCD predictions depicted in Fig. 1b, namely

$$\Lambda_{\overline{MS}}^{(4)} = 213 \pm 31(stat) \pm 54(syst) \text{ MeV} \quad (18)$$

are taken in accordance with the results of our analysis of the CCFR data for the SF xF_3 at the reference point $|Q_0^2| = 3 \text{ GeV}^2$ (see Table 1 and discussions beyond it). The statistical errors in Eq. (18) determine the corresponding errors bars of the theoretical GLS sum rule predictions (see Figs. 1).

It should be stressed that for the NLO fit the experimental values of the GLS sum rule tend slowly to 3 from below, in qualitative agreement with the theoretical expectations (see Fig. 1b). Moreover, within the systematical experimental uncertainties our results are consistent with the analysed perturbative QCD predictions.

However, at the quantitative level the tendency is toward the manifestation of a certain disagreement between the perturbative QCD predictions and the experimental results of Table 1 obtained by us:

1. The results of Fig. 1a demonstrate the slight Q^2 -dependence of the experimental data for the GLS sum rule obtained in the process of the LO fit. The obtained results lie below the quark-parton prediction $GLS = 3$.
2. The NLO fit minimizes the disagreement presented at Fig. 1a. However, even in this case the discrepancy between the results of the NLO fit and the LO QCD prediction for the GLS is surviving (see Fig. 1b)⁵. The most surprising fact is that the minor discrepancy takes place in the perturbative QCD region $|Q_0^2| > 10 \text{ GeV}^2$

where we can safely follow our approximation of neglecting the effects of the HT contributions and TM corrections.

6. Discussion

It seems to us that at the level of the statistical error bars the results depicted in Fig. 1b reveal certain problems of the explanation of the experimental data for the GLS sum rule within the framework of the analysed QCD predictions. Indeed, the LO theoretical QCD expression of Eq. (17) is approaching the asymptotic value $GLS_{As} = 3$ (which corresponds to the number of valence quarks inside nucleon) somehow faster than the results of our NLO fit of the the CCFR experimental data. In view of this conclusion it is necessary to make several comments:

1. It seems problematic to describe the deviation that we observed between theoretical and experimental results by taking into account threshold effects, e.g. following the

⁵We have checked that this disagreement does not disappear even after the brute-force inclusion of the NLO corrections into the theoretical expression for the GLS sum rule.

lines of the results of recent studies [27]. In the case of charged currents, radiated by neutrinos, the thresholds of production of new flavours manifest themselves in generations. Indeed, the production of the s -quark is mainly accompanied by the production of the c -quark in the whole region of momentum transfer. Therefore we are taking $f = 4$ in this region. The mixing with the quarks from the third generation are damped by the small values of the Kobayashi–Maskawa matrix elements V_{sb} , V_{cb} and V_{ct} .

2. In accordance with the discussions presented above, b - and t -quarks appear simultaneously in the processes with charged currents. In the neutrino–nucleon deep-inelastic scattering, their contributions should be studied in the region of very high Q^2 .
3. The deviation of the experimental result (3), the corresponding statistical uncertainties taken into account, from the pure perturbative QCD predictions for the GLS sum rule, with $\Lambda_{\overline{MS}}^{(4)}$ defined by Eq. (1), was previously noticed even at the scale $|Q_0^2| = 3 \text{ GeV}^2$ in the process of phenomenological [3, 5] and theoretical [9] studies. We confirm this observation and stress that a similar inconsistency takes place in a wide region of momentum transfer (see Figs. 1).
4. We can try to avoid this discrepancy by choosing $\Lambda_{\overline{MS}}^{(4)}$ as the free parameter and making a fit of the experimental data presented in Fig. 1b on the GLS sum rule for $|Q_0^2| < 10 \text{ GeV}^2$. In this case we can describe the Q^2 -behaviour of the obtained experimental data satisfactorily using $\Lambda_{\overline{MS}}^{(4)} = 724 \pm 153 \text{ MeV}$, which is too large to support this procedure.
5. Notice once more that taking the HT contributions into account [14, 10] in the analysis can improve the agreement with the theoretical predictions for the GLS sum rule at low Q_0^2 [9, 10]. However, at $|Q_0^2| > 10 \text{ GeV}^2$ these corrections cannot remove the observed deviation between the experimental and theoretical results for the GLS sum rule.
6. It should be noted that in the region of small values of x ($x < 0.015$) not considered in the CCFR experiment, the more-complicated parametrization of the SF can be used. The extrapolation of our simplest parametrization (14) to this region can be a source of errors on the experimental values of the GLS sum rule as calculated by us.
7. It is worth mentioning that the results of the NLO fits of the CCFR data for $\Lambda_{\overline{MS}}^{(4)}$, made with the help of the Altarelli–Parisi equation (see Eq. (1)) and using Mellin moments (see Eq. (18)), are in exact agreement with the central values from the results of analogous fits of the $xF_3(x, Q^2)$ less precise data obtained at Protvino [28]: $\Lambda_{\overline{MS}}^{(4)} = 170 \pm 60(\text{stat}) \pm 120(\text{syst}) \text{ MeV}$ (AP) and $\Lambda_{\overline{MS}}^{(4)} = 230 \pm 40(\text{stat}) \pm 100(\text{syst}) \text{ MeV}$ (Moments).

7. Conclusions

In conclusion we would like to stress several points.

1. Our NLO result (18) for $\Lambda_{\overline{MS}}^{(4)}$ is in agreement with the world average value of this parameter extracted from the deep-inelastic scattering data [1].
2. The obtained experimental Q^2 -behaviour of the GLS sum rule is consistent with the analysed perturbative QCD predictions, within systematical experimental uncertainties. However at the level of the statistical experimental uncertainties there is a certain discrepancy between theory and experiment.
3. We do not exclude the possibility that taking into account, in our procedure, of the effects of the different QCD corrections, namely of the α_s^2 corrections to the coefficient function of $xF_3(x, Q^2)$ [12], the NNLO coefficients of the even anomalous dimensions [29], still unknown NNLO coefficients of odd anomalous dimensions and of odd and even moments of xF_3 , might remove the discrepancy we found between the experimental data for the GLS sum rule and the corresponding theoretical prediction in the region of high-momentum transfer. We hope that our work will push ahead the necessary calculations and studies.
4. The third important problem is related to the necessity of experimental studies of the behaviour of the $xF_3(x, Q^2)$ SF in the region of small x : $x < 0.015$. Hopefully this problem can be studied in the future at HERA, where it is planned to reach the region $x \approx 10^{-4}$.

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NLO				LO		
$ Q_0^2 $ [GeV ²]	$\Lambda_{\overline{MS}}^{(4)}$ [MeV]	$\chi_{d.f.}^2$	GLS sum rule	$\Lambda_{LO}^{(4)}$ [MeV]	$\chi_{d.f.}^2$	GLS sum rule
2	209 ±32	71.5/62	2.401	154 ±16	87.6/62	2.515
3	213 ±31	71.5/62	2.446	154 ±29	87.7/62	2.525
5	215 ±32	71.8/62	2.496	154 ±28	88.0/62	2.537
7	215 ±34	72.2/62	2.525	155 ±27	88.3/62	2.549
10	215 ±35	72.6/62	2.553	154 ±29	88.5/62	2.558
15	215 ±34	73.2/62	2.583	155 ±28	88.8/62	2.569
25	214 ±31	74.1/62	2.618	155 ±17	89.2/62	2.583
50	213 ±33	75.4/62	2.661	155 ±27	90.2/62	2.603
70	212 ±34	76.1/62	2.680	155 ±26	90.3/62	2.614
100	211 ±33	76.8/62	2.699	154 ±29	90.7/62	2.623
150	210 ±34	77.6/62	2.720	154 ±29	91.2/62	2.635
200	209 ±33	78.2/62	2.735	154 ±29	91.5/62	2.643
300	209 ±33	79.0/62	2.755	153 ±29	92.0/62	2.655
500	207 ±35	80.1/62	2.779	153 ±29	92.7/62	2.664

Table 1. The results of the LO and NLO QCD fit of the CCFR xF_3 SF data for $f = 4$, $|Q^2| > 10 \text{ GeV}^2$, $N_{max} = 12$ with the corresponding statistical errors. $\chi_{d.f.}^2$ is the χ^2 parameter normalized to the degree of freedom $d.f.$

	Our analysis		CCFR [3, 5]
	LO	NLO	NLO
A	6.86 ± 0.09	6.423 ± 0.088	5.976 ± 0.148
b	0.795 ± 0.008	0.794 ± 0.012	0.766 ± 0.010
c	3.38 ± 0.03	3.218 ± 0.035	3.101 ± 0.036

Table 2. The parameters of quark distributions $xF_3(x, Q_0^2) = Ax^b(1-x)^c$ at $|Q_0^2| = 3 \text{ GeV}^2$.

Figure captions

Fig. 1a: The comparison of the results of the LO fit of the Q^2 evaluation of the GLS sum rule with the statistical error bars taken into account with the quark–parton prediction.

Fig. 1b: The comparison of the result of the NLO fit of the Q^2 evaluation of the GLS sum rule with the statistical error bars taken into account with the LO perturbative QCD prediction.