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Measurements of α_s in e^+e^- annihilation at TRISTAN

TOPAZ Collaboration

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Abstract

The strong coupling constant α_s was determined from analyses of the thrust, heavy jet mass and, differential 2-jet rate, using e^+e^- hadronic events at $\sqrt{s} = 58$ GeV with the TOPAZ detector at TRISTAN. The NLLjet Monte Carlo simulation(NLLjet) and analytic formulae based on resummation up to the next-to-leading logarithms combined with $O(\alpha_s^2)$ calculations were used to evaluate α_s. The averaged α_s values at $Q^2 = (58\text{ GeV})^2$ from the analyses are $\alpha_s = 0.125 \pm 0.009$ for NLLjet and $\alpha_s = 0.132 \pm 0.008$ for the resummed analytic formulae.

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1 Introduction

The strong interaction has been successfully described by Quantum Chromo Dynamics (QCD). Its coupling constant α_s, however, is much less precisely determined than the couplings in the electroweak sector of the Standard Model.

The strong coupling constant α_s cannot be determined using the parton shower models based on the Leading Logarithmic(LL) approximation, since it cannot specify the renormalization scheme to define the QCD scale parameter Λ. This situation can be overcome by invoking the Next-to-Leading Logarithmic(NLL) approximation. The NLL approximation has been used in two approaches: the NLLjet Monte Carlo program, and resummed analytic QCD calculations.

The NLLjet program(NLLjet)[1], which is a parton shower Monte Carlo program based on the Leading and Next-to-Leading Logarithmic approximation in jet calculus, can be used to derive $\alpha_s(\Lambda_{\overline{MS}})$ by direct comparisons with experimental data[2, 3]. In this generator the modified minimal subtraction scheme($\overline{MS}$) is employed to define the QCD scale parameter Λ.

Analytic QCD calculations up to the second order of α_s and a resummation of the leading and next-to-leading logarithms to all orders can be applied to the distributions of the thrust($d\sigma/dT$)[4], the heavy jet mass($d\sigma/d\rho$, $\rho = M_H^2/s$)[5] with respect to the thrust axis, and the differential 2-jet rate(dR_2/dy_3) with the "k_T " jet clustering algorithm[6] and the E-scheme recombination[7]. Definitions of the variables T, and ρ were described in ref.[8]. The variable y_3[9, 10, 11] is defined as the smallest value of the jet resolution parameter y_{cut} with which the event is recognized as 3-jet event.

The resummation allows us to calculate the effects of soft gluon emissions in the 2-jet region. In addition, it can also predict event shapes in 3- and 4-jet dominated regions when combined with the second order calculations. Recently, measurements of α_s with the resummed analytic formulae have been carried out at the LEP experiments[9, 10, 11]. It is therefore important to measure α_s using the same resummed analytic formulae at different energy points in order to check the running effects of α_s.

We report on measurements of the above three variables(T, ρ, y_3) at $\sqrt{s} =$ 58 GeV using the TOPAZ detector at TRISTAN. The α_s, or $\Lambda_{\overline{MS}}$, was derived by means of the above two approaches regarding the NLL approximation.

We paid particular attention to theoretical uncertainties in those approaches. The NLLjet method has theoretical ambiguities concerning the Q_0 cut-off parameter and δ(the parameter used for distinguishing 2-parton primary vertex from 3-parton primary vertex)[12]. The analytic method, on the other hand, invokes a renormalization scale uncertainty as well as an ambiguity in the combining procedure, though the former is significantly reduced by combining the second order calculations and the resummation. These uncertainties were taken into account in the final results as theoretical errors.

2 Event Selection and Acceptance Correction

The TOPAZ detector is a general purpose 4π detector whose details have been described elsewhere[13]. Since the selection criteria for hadronic events are explained in ref[14], we only describe here additional cuts: (a) the polar angle(θ_T) of the thrust axis of events with respect to the beam axis should satisfy $|\cos\theta_T| < 0.75$; (b) at least three charged tracks exist in each hemisphere divided by the plane which is perpendicular to the thrust axis and includes the interaction point. Cut (a) ensures that the events are well contained in the detector and cut (b) is used to reduce the effect of hard photon radiations from the initial state. These cuts selected 8200 hadronic events at $\sqrt{s} = 58$ GeV, corresponding to an integrated luminosity of 91.7 pb^{-1}.

We used both charged and neutral particles in this analysis. The charged particles were required to satisfy $P_t > 0.15$ GeV/c and $|\cos\theta| \leq 0.83$ with respect to the beam axis, and to come from the interaction region. The neutral particles had to have an energy deposit larger than 100 MeV in the barrel electromagnetic calorimeter. We further demanded that the neutral particles had no charged tracks extrapolated to their vicinity.

In order to make a comparison with theoretical predictions, the measured distributions should be corrected for the acceptance, such as the detector efficiency, resolution, decays, and initial state photon radiation. Here, we use a bin-by-bin correction determined from Monte Carlo studies with the JETSET 7.3 parton shower model[15]:

$$(D^{data}_{hadron})_i = \frac{(D^{RCoff}_{hadron})_i}{(D^{RCon}_{sim.})_i} \cdot (D^{data}_{meas.})_i \qquad i = \text{bin index}, \tag{1}$$

where $D^{RCon}_{sim.}(D^{RCoff}_{hadron})$ is the Monte Carlo prediction of each distribution with(without) the initial state radiation(RC) and the TOPAZ detector simulation, $D^{data}_{meas.}$ and D^{data}_{hadron} are the measured and corrected distributions of the hadrons, respectively. The distributions (T, ρ, y_3) of the normalized differential cross sections after these corrections are summarized in table 1.

3 Determination of α_s from the NLLjet Monte Carlo Simulation

In the NLL order we sum all of the terms in the following form:

$$\sum_{n=0} c_n \alpha_s(Q^2)^n \left(\ln \frac{Q^2}{Q_0^2}\right)^n + \sum_{n=0} d_n \alpha_s(Q^2)^{n+1} \left(\ln \frac{Q^2}{Q_0^2}\right)^n, \tag{2}$$

where Q stands for a momentum scale of the process and Q_0[†] is the cut off parameter to stop parton shower evolution. The first terms are those from the LL approximation and the second terms include any $O(\alpha_s)$ correction to the LL approximation, which is the NLL approximation. These large logarithms originate from a collinear singularity.

NLLjet produces partons by means of the NLL approximation described above. To hadronize partons, the LUND string fragmentation model[15] is applied. Parameters in the LUND fragmentation, a, b, and σ_q, were tuned using several event shape data other than the thrust and heavy jet mass distributions. Since parameter b is strongly correlated to a, we fixed it to be 0.9 GeV^{-2} in the tuning process[16]. The obtained values of the tuned parameters are $a = 0.413$ and $\sigma_q = 0.434$ GeV/c for $Q_0 = 1.0$ GeV. When we set the fragmentation parameters to be these values and varied $\Lambda^{(5)}_{\overline{MS}}$ for 5 flavors from 150 to 300 MeV, χ^2 did not become large and the event shape data were well reproduced. The Q_0(1.0 GeV) was chosen so as to reduce any non-perturbative contributions. In NLLjet there is another parameter δ for separating primary two jets from three jets. We fixed it at 0.5 so that 2- and 3-jet dominated regions in the thrust distribution of partons could be smoothly connected. We found that the dependences of the distributions of T, ρ, and y_3 on Q_0 are small when we set the fit region as $1.2 < -\ln(1-T) < 2.6$, $1.8 < -\ln\ \rho < 2.8$, and $1.6 < -\ln\ y_3 < 4.0$, respectively. In order to determine $\Lambda^{(5)}_{\overline{MS}}$, the distributions obtained from NLLjet for each variable are fitted to data at the hadron level in the above fit region. The fitted distributions of the three observables are shown in Fig.1. The resultant of α_s at $\sqrt{s} = 58$ GeV are listed in table 2. The systematic errors consist of

[†]The definition of Q_0 given in the text is that of LUND. The relation between Q_0 in LUND and Q_0 in NLLjet is $Q_0^2(\text{NLLjet}) = Q_0^2(\text{LUND})/4$.

experimental, hadronization, and theoretical errors. The experimental errors were evaluated as being the differences of the results using both charged and neutral particles and that using charged particles only. The hadronization errors were studied by varying the fragmentation parameters(σ_q and a). To estimate the effect of the Q_0 dependence, $\Lambda^{(5)}_{\overline{MS}}$ was also derived by setting Q_0 to be 2 and 3 GeV with adjusted fragmentation parameters for each Q_0. The variation of the results is considered to be a theoretical error due to the Q_0 dependence. In the ref.[1], it is shown that the physical distribution is independent of δ in the inclusive limit. However, through the detailed study, a slight kink(dip) was found related to δ. In order to reduce this theoretical ambiguity, we have examined the δ dependence over the range of 0.3-0.5. These Q_0 and δ dependences are regarded as being theoretical errors.

4 Determination of α_s from the Resummed Analytic Formula

The fraction $R(y)$ for the event shape variable($y = 1 - T, \rho, y_3$) is defined by

$$R(y) = \frac{1}{\sigma_t}\sigma(y), \tag{3}$$

where σ_t is the total hadronic cross section and $\sigma(y)$ is the partial cross section.

The second order predictions for $R(y)$ have been calculated by Kunszt and Nason[7]. Their results can be cast into the form

$$\ln R(y) = \alpha_s A(y) + \alpha_s^2 B(y), \tag{4}$$

where $A(y)$ and $B(y)$ are the first and the second order calculations. The results heavily depend on the renormalization scale, because higher order terms are not taken into account. This large ambiguity was a serious problem regarding the α_s measurements.

Eq.(3) was recently reexpressed by a power series expansion in $-\ln\ y$, as follows[4, 5, 6, 9]:

$$\ln\ R(y) = L \cdot f_{LL}(\alpha_s L) + f_{NLL}(\alpha_s L) + g(\alpha_s L)/L + \cdots, \tag{5}$$

where $L \equiv -\ln\ y$ and these soft logarithms become large in the 2-jet region. The first and second terms of Eq.(5) represent the LL and NLL terms, respectively, which have been summed up to all orders(resummation techniques). The remaining terms are subleading terms and have not been computed.

When we combine the exact calculation up to $O(\alpha_s^2)$(Eq.(4)) and the resummation formula(Eq.(5)), as shown in table 3, we expect to obtain formulae that are less-dependent on the renormalization scale, and are reliable in the low L region. We call this procedure *matching*. Actually, the following two matching procedures have been proposed[9]. In the first one, one simply adds both calculations for $\ln R$, subtracts the common parts and takes the exponential to obtain R. When we define $\ln \Sigma \equiv L \cdot f_{LL} + f_{NLL}$, this matching would look like Eq.(6), where $\ln \Sigma^{(1)}$ and $\ln \Sigma^{(2)}$ are the $O(\alpha_s)$ and $O(\alpha_s^2)$ parts of $\ln \Sigma$, respectively. This is called $\ln R$-matching. In the second approach, the common parts $\ln \Sigma^{(1)}$ and $\ln \Sigma^{(2)}$ are subtracted from $\ln \Sigma$ in the form of an exponential. After that, the exact formula up to $O(\alpha_s^2)$ is added, as in Eq.(7). This is called R-matching.

$$\ln R\text{-matching} : R(y) = \exp(\ln \Sigma - \ln \Sigma^{(1)} - \ln \Sigma^{(2)} + \alpha_s A(y) + \alpha_s^2 B(y)) \tag{6}$$

$$\begin{aligned} R\text{-matching} : R(y) &= [\exp(\ln \Sigma) - \exp(\ln \Sigma^{(1)}) - \exp(\ln \Sigma^{(2)})] \\ &\quad + \exp(\alpha_s A(y) + \alpha_s^2 B(y)) \\ &= \exp(\ln \Sigma) - \exp(\ln \Sigma^{(1)}) - \exp(\ln \Sigma^{(2)}) \\ &\quad +1 + \alpha_s A(y) + \alpha_s^2 (A(y)^2/2 + B(y)) \end{aligned} \tag{7}$$

The difference between these matching schemes is in the level of $O(\alpha_s^3)$, and should be considered as being a theoretical ambiguity.

The distributions calculated by the resummed analytic formulae at the parton level should be translated to the hadron level as

$$(D^{analytic}_{hadron})_i = \sum_j P_{ij}(D^{analytic}_{parton})_j, \tag{8}$$

where $D^{analytic}_{hadron}$ and $D^{analytic}_{parton}$ are the distributions of hadrons and partons, and P_{ij} is the matrix of the transition probability from partons in the j-th bin to hadrons in the i-th bin, which was obtained using Monte Carlo models. To study the effects of hadronization, we used the JETSET 7.3 parton shower with the string fragmentation model. The LUND parameters were tuned so as to reproduce our several event shape data, as described in section 3. We obtained the following set: $\Lambda_{QCD} = 0.323$ GeV, $a = 0.264$(b was fixed to be 0.9), and $\sigma_q = 0.426$ GeV/c where $Q_0 = 1.0$ GeV.

We used the fit range $1.2 < -\ln(1-T) < 2.6$, $1.8 < -\ln\ \rho < 2.8$ and $1.6 < -\ln\ y_3 < 4.0$, where hadronization corrections were less than typically 10 %, as shown in Fig.1. These fit ranges are the same as the analyses for NLLjet.

In order to derive α_s, we fit the theoretical distributions to the measured distributions at the hadron level for a fixed scale $\mu = \sqrt{s}$. The fit results of α_s are shown in table 4, and three distributions of the thrust, heavy jet mass and differential 2-jet rate are plotted in Fig.1. The value of α_s is taken as the center value of the results using the two matching schemes (ln$R$-matching and R-matching) at the scale $\mu = \sqrt{s}$. We took into account the following effects as systematic errors. The experimental and hadronization errors were considered in a similar way as that described in section 3. The theoretical uncertainties come from the renormalization scale dependence and the matching ambiguity. The theoretical errors were estimated as being the maximum differences between the resultant α_s values, when we switched the matching procedure, changing the renormalization scale factor in the range $-1 < \ln f < +1$ around $\ln f = 0$[9], where $f \equiv \mu^2/s$. The obtained theoretical errors are thus asymmetric.

5 Combined Results

The fitted α_s values can be averaged by taking into account the weights between the three determinations. The averaging procedure[10] is defined as

$$\hat{\alpha_s} = \sum_i w_i \alpha_s^{(i)} / \sum_i w_i \qquad (i = T,\ \rho \text{ and } y_3), \tag{9}$$

where $\alpha_s^{(i)}$ is derived from the i-th observable and w_i is the inverse of the quadratic sums of all the errors given in tables 2 or 4. The correlations in the theoretical errors for the three event shape variables have not been taken

into account.

The results are

$$\left.\begin{aligned} \alpha_s(\text{at 58 GeV}) &= 0.125 \pm 0.009 \\ \Lambda_{\overline{MS}}^{(5)} &= 222^{+118}_{-82} \quad \text{MeV} \end{aligned}\right\} \text{from NLLjet}$$

and

$$\left.\begin{aligned} \alpha_s(\text{at 58 GeV}) &= 0.132 \pm 0.008 \\ \Lambda_{\overline{MS}}^{(5)} &= 315^{+110}_{-98} \quad \text{MeV} \end{aligned}\right\} \text{from the resummed analytic formulae.}$$

We used the solution of the renormalization group equation(B.1) in ref.[17] with the β-function truncated at the second order:

$$\frac{1}{\alpha_s} + b_1 \ln\left(\frac{b_1 \alpha_s}{1 + b_1 \alpha_s}\right) = b_0 \ln \frac{\mu}{\Lambda}, \tag{10}$$

where

$$b_0 = \frac{\beta_0}{2\pi} \quad \text{and} \quad b_1 = \frac{\beta_1}{4\pi\beta_0}. \tag{11}$$

Here, $\beta_0 = 11 - (2/3)n_f$, $\beta_1 = 102 - (38/3)n_f$, and $n_f = 5$. The n_f is the number of quarks with a mass less than the energy scale μ.

6 Conclusions

The thrust, heavy jet mass, and differential 2-jet rate were measured at $\sqrt{s} = 58$ GeV using the TOPAZ detector at TRISTAN. Their acceptance corrected distributions were compared with the distributions obtained from both the NLLjet Monte Carlo simulation and the resummed analytic formulae. The α_s values from the three event shape measurements are summarized in Fig.2 for the two approaches: NLLjet and the resummation with the $O(\alpha_s^2)$ calculations. The results of the α_s values from NLLjet and the resummed analytic formulae are consistent with each other.

We previously studied the asymmetry of the energy-energy correlation at $\sqrt{s} = 59.5$ GeV[18]. The results for α_s are also consistent with this earlier measurement.

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Figure caption

Fig.1 The measured distributions(plot) of thrust(a), heavy jet mass(b), and differential 2-jet rate(c) after acceptance correction. Histograms are distributions from the NLLjet Monte Carlo and solid lines are QCD predictions from the resummed analytic formulae. This is the case of R-matching procedure and the renormalization scale factor $\ln f = 0$ for the resummed analytic formulas. Dashed lines show the fit ranges. Hadron/Parton means the ratio of the number entries in each bin of hadrons to that of partons in the case of the analyses with the resummed analytic formulae, which was obtained from LUND 7.3 PS.

Fig.2 Compilation of final results of α_s at $\sqrt{s} = 58$ GeV. The errors are the square of the quadratic sums of statistical, experimental, hadronization, and theoretical errors.

Table 1: Distributions of the variables of thrust, heavy jet mass, and differential 2-jet rate. The data are corrected for acceptance and resolution of the detector and for initial state photon radiation. The errors are statistical only.

	$(1/\sigma)(d\sigma/dL)$			$(1/\sigma)(d\sigma/dL)$
L	$L = -\ln(1-T)$	$L = -\ln\rho$	L	$L = -\ln y_3$
0.9	0.0009 ± 0.0009		1.4	0.024 ± 0.004
1.1	0.047 ± 0.006	0.0050 ± 0.0015	1.8	0.058 ± 0.005
1.3	0.099 ± 0.008	0.037 ± 0.006	2.2	0.075 ± 0.006
1.5	0.149 ± 0.010	0.092 ± 0.010	2.6	0.100 ± 0.006
1.7	0.188 ± 0.011	0.126 ± 0.011	3.0	0.129 ± 0.006
1.9	0.237 ± 0.012	0.187 ± 0.013	3.4	0.149 ± 0.007
2.1	0.292 ± 0.014	0.226 ± 0.013	3.8	0.159 ± 0.007
2.3	0.347 ± 0.015	0.284 ± 0.015	4.2	0.192 ± 0.008
2.5	0.376 ± 0.015	0.326 ± 0.015	4.6	0.206 ± 0.007
2.7	0.426 ± 0.016	0.382 ± 0.017	5.0	0.251 ± 0.008
2.9	0.473 ± 0.017	0.419 ± 0.017	5.4	0.277 ± 0.009
3.1	0.567 ± 0.019	0.514 ± 0.020	5.8	0.281 ± 0.009
3.3	0.548 ± 0.019	0.596 ± 0.021	6.2	0.246 ± 0.009
3.5	0.464 ± 0.017	0.544 ± 0.019	6.6	0.186 ± 0.008
3.7	0.354 ± 0.015	0.506 ± 0.017	7.0	0.097 ± 0.006
3.9	0.198 ± 0.010	0.334 ± 0.013	7.4	0.041 ± 0.004
4.1	0.134 ± 0.008	0.203 ± 0.008	7.8	0.0101 ± 0.0016
4.3	0.064 ± 0.005	0.108 ± 0.005	8.2	0.0035 ± 0.0009
4.5	0.0223 ± 0.0026	0.0489 ± 0.0028		
4.7	0.0086 ± 0.0015	0.0329 ± 0.0025		
4.9	0.0044 ± 0.0011	0.0047 ± 0.0005		
5.1	0.0018 ± 0.0008	0.0016 ± 0.0003		

Table 2: Fitted α_s values and their errors of α_s at 58 GeV derived from thrust, heavy jet mass, and differential 2-jet rate, using NLLjet Monte Carlo.

variable	α_s at 58 GeV	stat. err.	exp. err.	hadr. err.	theor. err.
thrust	0.1249	±0.0050	±0.0012	±0.0035	±0.0029
jet mass	0.1235	±0.0059	±0.0006	±0.0013	±0.0036
jet rate	0.1309	±0.0050	±0.0024	±0.0026	±0.0103

Table 3: Schematic representation of the expansion in the resummation of the LL, NLL and subleading parts. An expression for Kunszt and Nason calculations(K.N) is also shown.

	Resummed formula			K.N
	$\ln\Sigma$			
	LL	NLL	subleading	
1st	$\alpha_s L^2$	$+\alpha_s L$	$+\alpha_s$	$\alpha_s A(y)$
2nd	$+\alpha_s^2 L^3$	$+\alpha_s^2 L^2$	$+\alpha_s^2 L + \alpha_s^2$	$\alpha_s^2 B(y)$
3rd	$+\alpha_s^3 L^4$	$+\alpha_s^3 L^3$		
:	:	:		

Table 4: Fitted α_s values and their errors at 58 GeV derived from thrust, heavy jet mass, and differential 2-jet rate, using resummed analytic formulae.

variable	α_s at 58 GeV	stat. err.	exp. err.	hadr. err.	theor. err.
thrust	0.1339	±0.0040	±0.0008	±0.0022	$^{+0.0070}_{-0.0059}$
jet mass	0.1287	±0.0041	±0.0005	±0.0020	$^{+0.0057}_{-0.0046}$
jet rate	0.1322	±0.0056	±0.0025	±0.0010	$^{+0.0055}_{-0.0041}$

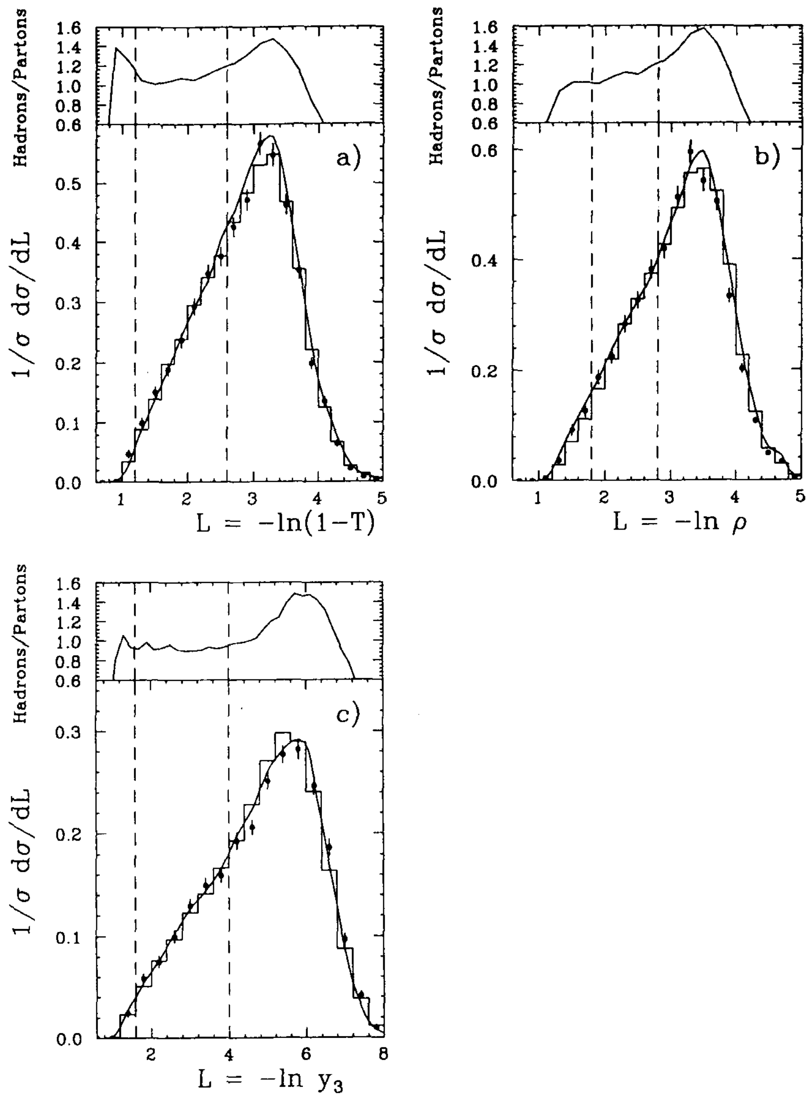

Fig. 1

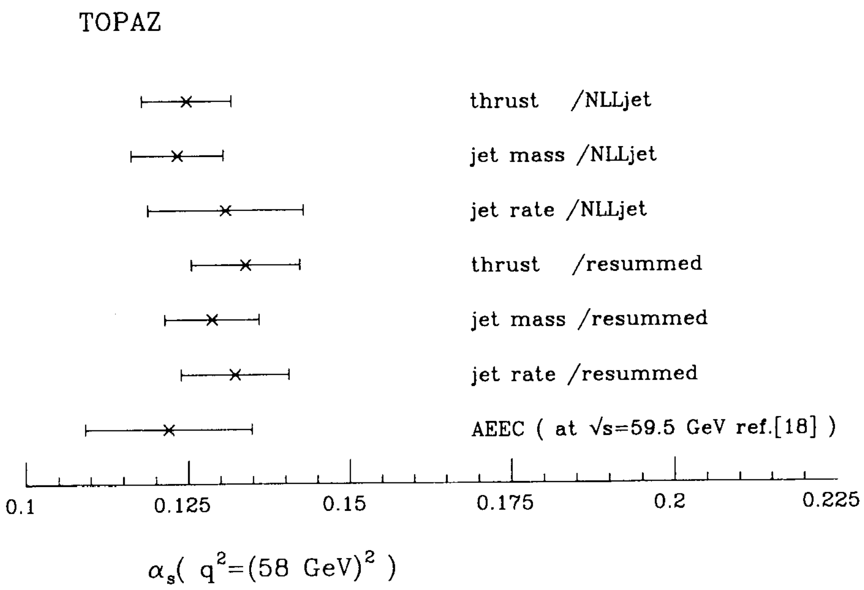
TOPAZ
thrust /NLLjet
jet mass /NLLjet
jet rate /NLLjet
thrust /resummed
jet mass /resummed
jet rate /resummed
AEEC (at √s=59.5 GeV ref.[18])
0.1
0.125
0.15
0.175
0.2
0.225
αs(q²=(58 GeV)²)

Fig. 2