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ON THE MASS AND THE MATTER STATE OF THE HDM NEUTRINOS

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Abstract

If there does not exist a low energy phase transition (LEPT) during the evolution process of the universe, the HDM (hot dark matter) neutrinos will be near by a degenerate state, on the contrary, they will be near by a high temperature state. In all of these two states, the neutrino mass is $10^{-1} - 10^{-2}$eV, and the absolute value of the neutrino chemical potential is $10^{-5} - 10^{-4}$eV.

1 Premises

(1) Proceed from the measurement results of COBE

From the measurement results [1]–[3] of COBE satellite about the large angular anisotropy of the cosmic microwave background, it is declared that there exists a hot composition in cosmic dark matter, which portion is about 30% of the recent critical density ρ_c of the universe; but the main candidate of HDM is neutrino ν, so, if the neutrino mass $m_\nu \neq 0$, then $\Omega_\nu = \frac{\rho_\nu}{\rho_c} = 0.3$, where ρ_ν is the recent average density of HDM neutrinos. From now on, all physical quantities will adopt the recent values unless there are any declares.

(2) Non-relativistic approximation

Since there exists the large scale structure in the universe, the recent HDM neutrinos can not be in a relativistic state, or else the large scale structure will be broken by the free stream. So, we arrange that the average thermal velocity of neutrinos is less than 0.1c ($\bar{v} < 0.1c$).

$$\bar{v} = \sqrt{\frac{2kT_\nu}{m_\nu}} \cdot \int_0^\infty \frac{ZdZ}{exp(Z-\gamma)+1} \Big/ \int_0^\infty \frac{\sqrt{Z}dZ}{exp(Z-\gamma)+1}, \tag{1}$$

where $\gamma \equiv \frac{\mu_\nu}{kT_\nu}$, and T_ν is neutrino temperature, and μ_ν is neutrino chemical potential; but the sound velocity v_s in ν medium is

$$v_s = \sqrt{\frac{10}{9}\frac{kT_\nu}{m_\nu} \cdot \int_0^\infty \frac{Z^{\frac{3}{2}}dZ}{exp(Z-\gamma)+1} \Big/ \int_0^\infty \frac{\sqrt{Z}dZ}{exp(Z-\gamma)+1}}. \tag{2}$$

From calculations we know that $\frac{\bar{v}}{v_s} \sim 1$, so $v_s < 0.1c$.

(3) Consider the superstructure of the universe

From the observations [4]–[7] of superstructures, we know the scale of that has being been 1% – 10% of horizon, it will be as well to adopt that $v_s = 0.01c - 0.1c$.

(4) Do not consider the LEFT for the moment

In Ref[8], the mass and the matter state of ν had been preliminarily discussed under a LEFT during evolution; but in this paper, we abandon such restrain for the moment, so T_ν can be estimated as follows

$$T_\nu = \frac{kT_{\nu o}T_{\gamma o}}{m_\nu c^2 + kT_{\gamma o}}, \tag{3}$$

in which $T_{\gamma o}$ is the microwave background temperature, $T_{\gamma o} = 2.7^o K$; $T_{\nu o}$ is the neutrino temperature (as $m_\nu = 0$), $T_{\nu o} = 1.9^o K$. When $m_\nu c^2 \gg kT_{\gamma o}$,

$$T_\nu \doteq \frac{kT_{\nu o} T_{\gamma o}}{m_\nu c^2}. \tag{4}$$

2 Deduction

For the average density of ν, it can be expressed as

$$\rho_\nu = \frac{g m_\nu^{\frac{5}{2}} (kT_\nu)^{\frac{3}{2}}}{2^{\frac{1}{2}} \pi^2 \hbar^3} \cdot \int_0^\infty \frac{\sqrt{Z} dZ}{exp(Z - \gamma) + 1}, \tag{5}$$

where g is the number of neutrino helicity states. If $\rho_c = \frac{3H_{100}^2}{8\pi G}$, $H_{100} = 100 km \cdot sec^{-1} \cdot Mpc^{-1}$, then

$$\rho_\nu = \Omega_\nu \rho_c h^2, \tag{6}$$

where $h = \frac{H_o}{100}$, H_o is the Hubble constant. Make

$$J_a \equiv \int_0^\infty \frac{Z^a dZ}{exp(Z - \gamma) + 1}, \tag{7}$$

from eqs(4),(5),(6), get

$$m_\nu = \frac{\Omega_\nu h^2}{J_{\frac{1}{2}} g} \cdot \frac{2^{\frac{1}{2}} \pi^2 \hbar^3 c^3 \rho_c}{k^3 (T_{\nu o} T_{\gamma o})^{\frac{3}{2}}}; \tag{8}$$

from eqs(2),(4), get

$$m_\nu v_s = \sqrt{\frac{10}{9} \frac{T_{\nu o} T_{\gamma o} J_{\frac{3}{2}}}{J_{\frac{1}{2}}}} \cdot \frac{k}{c}. \tag{9}$$

From eqs(8),(9),(4), we know, after giving the values of $T_{\nu o}$ and γ, the values of m_ν (from Eq(8)), v_s (from Eq(9)), T_ν (from Eq(4)), and μ_ν (from γ) can be deduced. The influence of Ω_ν, h, g on m_ν and etc. can be synthetically considered in a factor w, w$\equiv \frac{g}{\Omega_\nu h^2}$. If g=1, $\Omega_\nu = 1$, h=1, then w=1; if g=6, $\Omega_\nu = 0.3$, h=0.5, then w=80, therefore we shall be discussing in a range of w=1-80.

3 Discussion

(1) The matter state of neutrinos
Taking $T_{\nu o} = 1.9^{o}K$, the calculated results are as follows

w	v_s/c	$m_\nu(eV)$	$T_\nu(^{o}K)$	$\mu_\nu(eV)$	γ
1	0.01	0.17	2.5×10^{-3}	2.6×10^{-5}	120
1	0.1	0.031	1.4×10^{-2}	4.7×10^{-4}	380
40	0.01	0.069	6.4×10^{-3}	1.1×10^{-5}	19
40	0.1	0.012	3.6×10^{-2}	1.9×10^{-4}	60
80	0.01	0.057	7.7×10^{-3}	0.9×10^{-5}	13.5
80	0.1	0.010	4.4×10^{-2}	1.6×10^{-4}	43

From the values of γ, it is indicated that HDM ν are in a degenerate state as the whole, and m_ν is about $10^{-1} - 10^{-2}eV$, and μ_ν is about $10^{-5} - 10^{-4}eV$.

(2) The value of μ_ν during evolution
According to the isoentropic hypothesis in the evolution of the universe, the entropy per neutrino (S/N) is a constant:

$$
\begin{aligned}
S/N \;=\; & \frac{k}{3} \cdot \{ \int_0^\infty Z^{\frac{3}{2}} (Z + \frac{2m_\nu c^2}{kT_\nu})^{\frac{3}{2}} \frac{dZ}{exp(Z-\gamma)+1} \\
& +3 \int_0^\infty (Z + \frac{m_\nu c^2}{kT_\nu}) \sqrt{Z(Z + \frac{2m_\nu c^2}{kT_\nu})} \frac{(Z-\gamma)dZ}{exp(Z-\gamma)+1} \} \div \\
& \int_0^\infty \frac{dZ}{exp(Z-\gamma)+1} \cdot (Z + \frac{m_\nu c^2}{kT_\nu}) \sqrt{Z(Z + \frac{2m_\nu c^2}{kT_\nu})}.
\end{aligned}
$$

Under the relativistic condition, $S/N = \frac{k}{3} \cdot \frac{4J_3 - 3J_2\gamma}{J_2}$; under the non-relativistic condition, $S/N = \frac{k}{3} \cdot \frac{5J_{\frac{3}{2}} - 3J_{\frac{1}{2}}\gamma}{J_{\frac{1}{2}}}$. It is obvious that under the relativistic condition there are two situations for μ_ν: the first is $\gamma \longrightarrow \infty$ (degenerate state), in this time there only exist either neutrinos or antineutrinos $\bar{\nu}$; the second is $\gamma = 0$, then the particle number of ν is equal to that of $\bar{\nu}$. From calculations we know for $\gamma \longrightarrow \infty$ the non-relativistic fermions deduced from relativistic particles will still be in a degenerate state; but for $\gamma = 0$, it will translate to $\gamma = -1.62$ after the deducing. The results for $T_{\nu o} = 1.9^{o}K$ and $\gamma = -1.62$ are as follows

w	v_s/c	$m_\nu(eV)$	$T_\nu(^oK)$	$\mu_\nu(eV)$
1	2.8×10^{-7}	926	4.8×10^{-7}	-6.7×10^{-11}
80	2.2×10^{-5}	11.6	3.8×10^{-5}	-5.3×10^{-9}

Since the values of v_s/c are too small, it suggests that there may exist LEPT, for example the photon cooling process in Ref[9].

(3) LEPT

The LEPT can be described by $T_{\nu o} > 1.9^oK$ and $T_{\gamma o} = (\frac{11}{4})^{\frac{1}{3}} \cdot T_{\nu o}$, for $\gamma = -1.62$ the calculated results are as follows

w	v_s/c	$m_\nu(eV)$	$T_\nu(^oK)$	$\mu_\nu(eV)$	$T_{\nu o}(^oK)$
1	0.01	0.34	0.25	-3.5×10^{-5}	27
1	0.1	0.062	4.3	-6.0×10^{-4}	47
80	0.01	0.12	0.083	-1.2×10^{-5}	9
80	0.1	0.021	1.5	-2.0×10^{-4}	16

In this situation, ν is near by a high temperature state, and m_ν is still about $10^{-1} - 10^{-2}eV$, and the absolute value of μ_ν is still about $10^{-5} - 10^{-4}eV$.

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