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Normal and transverse single tau polarization  
at the  $Z$ -peak

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ABSTRACT

We study normal (to the collision plane) and transverse (within the collision plane) single- $\tau$  polarization in  $\tau$  pairs produced in  $e^+e^-$  unpolarized collisions at the  $Z$ -resonance. The transverse polarization component is sensitive to the anomalous weak-magnetic moment, whereas the normal polarization component is sensitive to a CP-violating weak-electric dipole moment. We show how these components of the single  $\tau$  polarization are accessible from the angular distribution of its decay products. We define a CP violating asymmetry of the  $\tau$  decay products which, with  $10^7 Z$ 's produced, provides a sensitivity of  $2.3 \times 10^{-18} e \cdot \text{cm}$  for the weak-electric dipole moment.

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# 1 Introduction

The spin properties of the  $\tau$ -lepton produced at high-energy colliders allow to obtain important physical magnitudes. Polarization measurements are accessible for the  $\tau$  by means of the energy and angular distributions of its decay products. At the  $Z$ -peak,  $e^+e^-$  collisions produce  $\tau^+\tau^-$  pairs, and each  $\tau$  has a longitudinal polarization [1, 2, 3]. The angular distribution of the  $\tau$ -polarization contains separate information on both the average polarization, measuring parity violation in the  $Z - \tau^+ - \tau^-$  vertex, and the  $Z$ -polarization, measuring parity violation in the  $Z - e^+ - e^-$  vertex. This allows a test of neutral current universality and, within the Standard Model, a precise determination of  $\sin^2\theta_w$ . No other component of the single  $\tau$ -polarization is allowed in the Standard Model in the limit of zero-mass fermions. Spin correlations of the  $\tau$ -pair are, however, allowed in the Standard Model, including a time reversal (T)-odd, parity (P)-odd observable [4, 5, 6] induced by unitarity corrections plus the interference with the  $\tau$ -exchange background.

The third family  $\tau$ -lepton can provide a window to new physics beyond the Standard Model. Up to now, some of the  $\tau$  intrinsic properties are still so loosely restricted that deviations from the Standard Model predictions can not be excluded. In particular, the weak dipole moments are fundamental magnitudes that can be constrained from  $\tau$  physics at the  $Z$ -peak. The P-odd, CP-odd electric and weak-electric dipole moments would be induced in the Standard Model from the Kobayashi–Maskawa mechanism in the hadronic corrections only at very high orders in the loop expansion. So a CP violating signal coming from an appreciable electric or weak-electric dipole moment will unambiguously lead to new physics. In some extended models, dipole moments are enhanced for larger masses, hence the  $\tau$  is a good candidate to look for CP violation in the leptonic sector [7].  $\tau$  pairs produced from  $e^+e^-$  collisions at the  $Z$ -peak allow a detailed scrutiny of their properties, with their decay products in the detectors acting as analyzers. The spin density matrix of the produced  $\tau$  pairs has genuine CP violating terms, both in the single  $\tau$ -polarization and in the spin-spin correlation terms, that translates into the energy and angular distribution of the decay products. A similar study can be performed to isolate a

weak-magnetic dipole moment term, looking for observables sensitive to this property in the spin density matrix of the produced  $\tau$  pairs. In this paper we show that the normal (to the collision plane) and transverse (within the collision plane) components of the single  $\tau$ -polarization at the  $Z$ -peak contain all good requirements in order to search for a weak-electric and a weak-magnetic dipole moment, respectively. Through the angular distribution of the hadronic decay products acting as analyzers [8, 9, 10] of these spin components, we build explicit asymmetries that measure the weak dipole moments at  $q^2 \approx M_Z^2$ .

Genuine CP violating observables arising from correlations in  $\tau^+\tau^-$  decay products have been studied by the Heidelberg group [11, 12, 13]. The proposal to look for a non-vanishing expectation value of a correlated triple product of momenta has been followed by the OPAL and ALEPH Collaborations [14, 15] to obtain an upper limit for the weak-electric dipole moment:

$$|d_x^w(M_Z^2)| \leq 3.7 \times 10^{-17} e \cdot \text{cm} \quad (1)$$

The  $\tau$  weak-magnetic moment was also investigated in [16] by looking to forward and backward transverse asymmetries.

We show here that the normal component of the single  $\tau$ -polarization provides an independent method to have access to the value of  $d_x^w(M_Z^2)$ . The normal polarization is a P-even, T-odd observable which, even if it is not a genuine CP violating quantity, enjoys the following virtues: **i)** it gets a contribution from CP-conserving interactions only through the combined effect of both an helicity-flip transition and the presence of absorptive parts, which are both suppressed in the Standard Model. **ii)** with a CP-violating interaction such as a weak-electric dipole moment, it gets a non-vanishing value without the need of absorptive parts. **iii)** as the observable effect comes from the interference of the CP-violating amplitude with the standard amplitude and the observable is P-even, the sensitivity of the normal polarization to linear terms in  $d_x^w(M_Z^2)$  is enhanced by the leptonic axial neutral current Standard coupling and no need of the suppressed vector coupling of the  $Z$  to taus appears. Although the normal polarization is not a genuine

CP-violating signal these features allow us to conclude that new physics would be needed if a big enough value is measured. If this were the case one has still the possibility to compare the normal polarization for  $\tau^+$  and  $\tau^-$ , thus obtaining a true CP-violating observable but with a half of statistics.

If any of both *taus* decays to leptons no information of the  $\tau$  direction is left in the decay products. In channels where both  $\tau$ 's decay semileptonically, the  $\tau$  direction can only be reconstructed up to a two fold ambiguity [12] if no high precision measurement of both charged hadron tracks is made. It is this ambiguity that destroys the information coming from normal polarization when looking at the decay products, so possible CP violating signals arising from this component would be lost. However, when both hadron momenta are accurately measured, then the  $\tau$  direction can be reconstructed [17]. This opens new possibilities to measure the transverse and normal component of the polarization from the angular distribution of a single  $\tau$  decay product, as shown in this paper. Observing single  $\tau$  decays allows to double statistics and so the bounds will become more stringent.

The rest of the paper is organized as follows. In section 2 we study, from the effective lagrangean approach, the effects of both weak-electric and weak-magnetic dipole moments on the single  $\tau$  polarization at the Z-peak. In section 3 we construct the angular distribution of a single  $\tau$  decay product to identify the terms sensitive to the normal and transverse polarizations. Appropriate asymmetries to disentangle these components are defined. Conclusions are given in section 4.

## 2 Tau polarization at the Z-peak

We consider the following effective interaction lagrangean density:

$$L = -\frac{i}{2} \frac{eF_3^W}{2m_\tau} \psi \sigma^{\mu\nu} \gamma_5 \psi Z_{\mu\nu} + \frac{1}{2} \frac{eF_2^W}{2m_\tau} \bar{\psi} \sigma^{\mu\nu} \psi Z_{\mu\nu}, \quad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \quad (2)$$

where  $e$  is the proton charge,

$$d_\tau^W = \frac{eF_3^W}{2m_\tau}, \quad (3)$$

is the weak-electric dipole moment and  $a_\tau^W \equiv F_2^W$  is the anomalous weak-magnetic dipole moment:

$$\mu_\tau^W = \frac{e}{2m_\tau} 2 \left( \frac{1 - 4s_w^2}{4s_w c_w} + a_\tau^W \right) \quad (4)$$

with  $s_w$ ,  $c_w$  being the weak mixing angle sine and cosine, respectively.  $a_\tau^W$  can have contributions from both new physics or electroweak radiative corrections to the Standard Model.

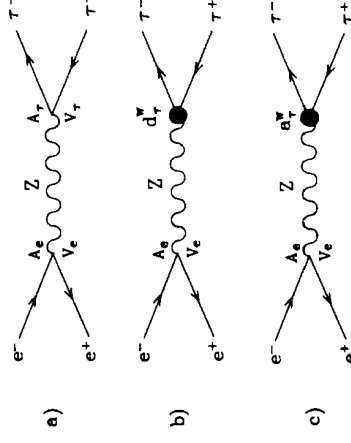


Figure 1: Diagrams considered for the process  $e^+ e^- \longrightarrow \tau^+ \tau^-$

To calculate the  $e^+ e^- \longrightarrow \tau^+ \tau^-$  cross section, the amplitude from this effective lagrangean has to be added to the Standard Model amplitude not included in Eq.(2). At tree level, the relevant diagrams are shown in Fig.1 from the Standard (a) and beyond the Standard (b and c) amplitudes. We retain in the cross section up to linear terms in the weak dipole moments and neglect terms proportional to the electron mass. At the Z-peak the cross section can be written as:

$$\frac{d\sigma}{d\Omega_{\tau^-}} = \frac{d\sigma^0}{d\Omega_{\tau^-}} + \frac{d\sigma^S}{d\Omega_{\tau^-}} + \dots \quad (5)$$

where the first term

$$\frac{d\sigma^0}{d\Omega_{\tau^-}} = \frac{\alpha^2 \beta}{(4s_w c_w)^2 \Gamma_Z^2} \left\{ \beta^2 A^4 (1 + \cos^2 \theta_{\tau^-}) + 2A^2 V^2 (1 + A\beta \cos \theta_{\tau^-} + \beta^2 \cos^2 \theta_{\tau^-}) \right\}$$

$$+ V^4 \left( 1 + \cos^2 \theta_{\tau^-} + \frac{\sin^2 \theta_{\tau^-}}{\gamma^2} \right) \quad (6)$$

collects the leading spin independent terms, whereas the second one takes into account the linear terms in the spin:

$$\frac{d\sigma^S}{d\Omega_{\tau^-}} = \frac{\alpha^2 \beta}{128s_m^3 c_m^2} \frac{1}{F_Z^2} \left[ (s_- + s_+) X_+ + (s_- - s_+) Y_- + (s_- + s_+) Z_+ \right] \quad (7)$$

with

$$X_+ = A \sin \theta_{\tau^-} \left\{ - [2V^2 + (V^2 + A^2)\beta \cos \theta_{\tau^-}] \frac{V}{\gamma s_m c_m} + 2\gamma [2V^2(2 - \beta^2) + (V^2 + A^2)\beta \cos \theta_{\tau^-}] F_3^w \right\} \quad (8)$$

$$Y_- = 2A\gamma\beta \sin \theta_{\tau^-} [2V^2 + (V^2 + A^2)\beta \cos \theta_{\tau^-}] F_3^w \quad (9)$$

$$Z_+ = -\frac{VA}{s_w c_w} \left[ (V^2 + A^2)\beta(1 + \cos^2 \theta_{\tau^-}) + 2(V^2 + \beta^2 A^2) \cos \theta_{\tau^-} \right] + 2A [4V^2 \cos \theta_{\tau^-} + (V^2 + A^2)\beta(1 + \cos^2 \theta_{\tau^-})] F_3^w \quad (10)$$

$\alpha$  is the fine structure constant,  $\Gamma_Z$  is the  $Z$ -width,

$$\gamma = \frac{M_Z}{2m_{\tau^-}}, \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad (11)$$

are the dilation factor and  $\tau$  velocity, respectively, and

$$V \equiv V_e \equiv V_\tau = -\frac{1}{2} + 2s_w^2, \quad A \equiv A_e \equiv A_\tau = -\frac{1}{2}.$$

are the vector and axial vector  $Z - \tau^+ - \tau^-$  couplings. The dots in Eq.(5) symbolize the spin-spin correlations terms which are not considered in this paper. The reference frame is chosen such that the outgoing  $\tau^-$  momenta is along the  $Z$  axis and the incoming  $e^-$  momenta is in the  $X - Z$  plane. The  $s_+$  and  $s_-$  are the  $\tau^\pm$  spin vectors in the  $\tau$  rest system.

The result shown in Eq.(7) is very illuminating: it says that, with the ingredients we have considered, the transverse (within the collision plane) and longitudinal polarizations, proportional to  $X$  and  $Z$  respectively, do not see CP-violating terms, then only  $X_+$  and  $Z_+$  for  $\tau^+$  and  $\tau^-$  are allowed, whereas the normal (to the collision plane) polarization, proportional to  $Y$ , only sees CP-violating terms and then only  $Y_-$  appears.

Terms with  $(s_- - s_+)_{\tau u s}$  factors carry all the information about the CP violating pieces of the Lagrangian. For unpolarized  $e^+ e^- \rightarrow \tau^+ \tau^-$  scattering the normal polarization ( $P_N$ ) of each  $\tau$  (along the  $Y$  axis) is the only component that is a time reversal (T) odd observable. For CP conserving interactions, a non vanishing  $P_N$  would need both helicity-flip and imaginary contributions to the amplitudes –coming from absorptive parts– so their contributions to  $P_N$  are doubly suppressed and one can expect them to be negligible. At the order considered in our calculation, this is illustrated by the fact that  $Y_+ = 0$  in Eq.(7). In the effective Lagrangian (2) the tensorial weak-electric dipole term is chirality flipping and CP-odd, so it does not need any extra absorptive (or helicity flipping) suppressed part to contribute to  $P_N$ . As expected, one sees in Eq.(9) that the anomalous form factor  $F_3^w$  induces a  $(s_- - s_+)_{\tau u}$  dependence in the cross section at the  $Z$ -peak. Looking at observables sensitive to  $P_N$  in the process for unpolarized beams has the advantage that, being  $P_N$  even under parity (P) symmetry, the observable should also be proportional to an odd number of Standard axial couplings in addition to  $F_3^w$ , i.e. either  $A \cdot V^2 \cdot F_3^w$  or  $A^3 \cdot F_3^w$ . In Eq.(9) we have both terms, and the leading one ( $A^3 \cdot F_3^w$ ) is enhanced by a factor  $\frac{1}{\gamma}$  when compared to the leading terms in the spin spin correlation observables [13].

Transverse polarization ( $P_T$ ) of  $\tau$  (along the  $X$  axis) is P-odd and T-even, and can only arise from the interference of both helicity conserving and flipping amplitudes. Then it has to be proportional to the product of an odd number of vector and axial couplings. Furthermore no absorptive terms and effective T-odd couplings like  $F_3^w$  are present here. The first term of  $X_+$  in Eq.(8) comes from helicity flipping suppressed ( $\frac{1}{\gamma} \equiv \frac{2m_{\tau^-}}{M_Z}$ ) amplitudes in the Standard Model and the second one comes from the  $\gamma$ -enhanced chirality flipping weak-magnetic tensorial  $F_3^w$  vertex.

Finally, the longitudinal polarization ( $P_L$ ) of each  $\tau$  (along the  $Z$  axis) is P-odd, so again it carries an odd number of vector and axial couplings both in the Standard terms and in the anomalous, proportional to  $F_2^w$ , ones. As expected, there is no contribution from  $F_3^w$ , which is T-odd, in  $P_L$  and the presence of Standard non suppressed contributions gives a small sensitivity to bound the non dominant weak-magnetic dipole moment

$F_Y^w$  in a measurement of this observable.

In conclusion: **i)** the anomalous weak-magnetic dipole moment  $a_\tau^w$  can be bounded efficiently from measurements of the transverse polarization  $P_T$  of each  $\tau$ ; in this case due to the fact that helicity flipping amplitudes are proportional to fermion masses in the Standard Model, the background is suppressed by  $\frac{1}{\gamma}$ , whereas the  $F_Y^w$  term is proportional to  $\gamma \cdot A^3$ . **ii)** CP invariance can be tested efficiently by selecting observables proportional to the leading CP-odd term, i.e. to the  $\gamma \cdot A^3 \cdot F_Y^w$  term; measurements of the normal polarization  $P_X$  of single  $\tau$  can bound the anomalous weak-electric dipole moment  $d_\tau^w$ . These two effective couplings will be selected by the choice of observables depending on the  $\tau$  decay products that we define in the next section.

If we add the background amplitude due to  $\gamma$ -exchange, then there will also be CP violating vertices contributing to the transverse and longitudinal polarization, arising from the interference of imaginary parts with electric ( $F_3$ ) and weak-electric ( $F_Y^w$ ) dipole moments in the  $\gamma - Z$  interference at the  $Z$ -peak. Considering that  $\tau$  leptons have a non zero anomalous magnetic dipole moment, both  $F_2$  and  $F_2^w$  would contribute to the normal polarization in the  $\gamma - Z$  interference. At the  $Z$ -peak these contributions are suppressed by factors  $\frac{F_2}{M_Z}$  and will not be considered here. Electroweak radiative corrections to the  $Z$ -vertex which generate absorptive parts are of order  $\frac{\alpha}{\pi}$  too.

### 3 Observables

At LEP1  $\tau$  pairs decay before reaching the detectors and the energies and momenta of their hadronic decay products can be measured. In addition vertex detectors allow a high resolution reconstruction of these hadron-tracks. In particular, as it is shown in [17], the  $\tau$  direction can be fully reconstructed if: **i)** both  $\tau$ 's decay into hadrons. **ii)** energies of hadrons are measured and both hadron-tracks are reconstructed. With the  $\tau$  direction identified, there is no more a two fold ambiguity in the azimuthal angle of the hadronic decays (see Fig. 2). This is an essential fact if one is interested in normal and transverse polarization terms. Therefore we will only consider semileptonic decay channels for both

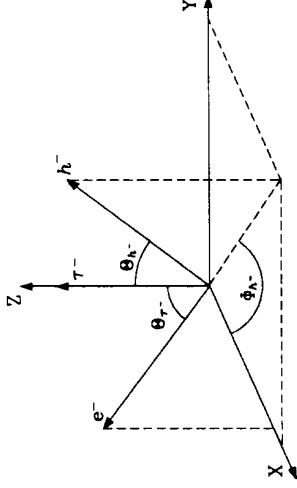


Figure 2: Reference system for the process  $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow h^+ h^-$

*taus*. We define appropriate observables in order to select the most sensitive pieces of the cross section to the anomalous couplings  $F_Y^w$  and  $F_2^w$ .

From Eqs.(5) and (7) and following standard procedures [2, 8], it is straightforward to get the expression for the  $e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow h^+ X h_2^- \nu_\tau$  and  $h^+ \bar{\nu}_\tau h_2^- X$  cross sections, respectively:

$$\frac{d\sigma(e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow h^+ X h_2^- \nu_\tau)}{d(\cos \theta_{\tau^-}) d\phi_{h_2^-}} = B_T(\tau^- \rightarrow h_2^- \nu_\tau) B_T(\tau^+ \rightarrow h^+ X) \times \left[ 2 \frac{d\sigma^0}{d\Omega_{\tau^-}} + \frac{\alpha^2 \beta_\pi}{128 s_w^3 c_w^3 \Gamma_Z^2} \alpha_{h_2^-} (X_+ \cos \phi_{h_2^-} + Y_- \sin \phi_{h_2^-}) \right] \quad (12)$$

$$\frac{d\sigma(e^+ e^- \rightarrow \tau^+ \tau^- \rightarrow h^+ \bar{\nu}_\tau h_2^- X)}{d(\cos \theta_{\tau^-}) d\phi_{h_2^-}} = B_T(\tau^- \rightarrow h_2^- X) B_T(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) \times \left[ 2 \frac{d\sigma^0}{d\Omega_{\tau^-}} + \frac{\alpha^2 \beta_\pi}{128 s_w^3 c_w^3 \Gamma_Z^2} \alpha_{h_2^-} (-X_+ \cos \phi_{h_2^-} + Y_- \sin \phi_{h_2^-}) \right] \quad (13)$$

where the angles  $\theta_\tau, \phi_h$  are defined in Fig. 2, and the polar angle  $\theta_h$  has been integrated out.  $X_+$  and  $Y_-$  are given in Eqs.(8) and (9) respectively, and  $\alpha_{h_i}$  is the analyzer of polarization defined for the  $\tau$  decay angular distribution at rest:

$$\frac{d\Gamma_\tau^\pm}{\Gamma_\tau}(\tau^\pm(s) \rightarrow h^\pm(q) + \nu_\tau) = \frac{1}{4\pi} \left[ 1 + \frac{2m_\tau}{m_\tau^2 - m_{h_i}^2} \alpha_{h_i} \hat{q} \cdot \hat{s} \right] d\Omega_{h_i} \quad (14)$$

In particular, for the  $\pi$  and  $\rho$  channels in *tau*-decay,  $\alpha_h$  takes the following values

$$\alpha_h \equiv \frac{m_\pi^2 - 2m_h^2}{m_\pi^2 + 2m_h^2} \longrightarrow \begin{cases} \alpha_\pi = 0.97 & h = \pi \\ \alpha_\rho = 0.46 & h = \rho \end{cases} \quad (15)$$

The spin correlation terms give no contribution to these angular distributions Eqs.(12) and (13). Looking at the expressions (12) and (13) it is worth to remark that, in addition to the azimuthal angle  $\phi_h$  of the particle that one detects (let us say  $h^-$ ) from the  $\tau^-$  decay products, we always need full reconstruction of the  $\tau$  direction. This opens the hadronic decay of the other  $\tau$  ( $\tau^+$ ) to those channels for which we are able to measure all momenta and one track (i.e. 1-prong, 3-prong,...). For numerical results we will consider  $\tau^+$  decays to  $\pi^+ \nu_\tau$  and  $\rho^+ \nu_\tau$  (i.e.  $h_1, h_2 = \pi, \rho$  in (12) and (13) respectively), while summing up over  $\pi^\mp \nu_\tau, \rho^\mp \nu_\tau$  and  $a_1^\mp \nu_\tau$  hadronic decays for the other  $\tau^\mp$  (this amounts to about 52% of the total decay rate).

The transverse polarization of the  $\tau$  (see Eqs.(7) and (8)) is the most sensitive magnitude to the  $F_2^w$  form factor. The Standard Model background is suppressed by a factor  $\frac{1}{2}$  and  $F_2^w$  is multiplied by  $\gamma$ , with respect to similar terms in the longitudinal polarization. In order to get information about the anomalous weak-magnetic moment, from Eqs.(12) and (13), we define the following asymmetry of the *tau*-decay products:

$$A_{cc}^\mp = \frac{\sigma_{cc}^\mp(+)-\sigma_{cc}^\mp(-)}{\sigma_{cc}^\mp(+)+\sigma_{cc}^\mp(-)} \quad (16)$$

with

$$\sigma_{cc}^\mp(+)=\int_0^1 d(\cos\theta_{\tau^-}) \int_{-\pi/2}^{\pi/2} d\phi_{h^+} + \int_{-1}^0 d(\cos\theta_{\tau^-}) \int_{\pi/2}^{\pi/2} d\phi_{h^+} \left[ \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^+}} \right] \quad (17)$$

and

$$\sigma_{cc}^\mp(-)=\int_0^1 d(\cos\theta_{\tau^-}) \int_{\pi/2}^{\pi/2} d\phi_{h^+} + \int_{-1}^0 d(\cos\theta_{\tau^-}) \int_{-\pi/2}^{\pi/2} d\phi_{h^+} \left[ \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^+}} \right] \quad (18)$$

This asymmetry selects the  $\cos\theta_{\tau^-} \cos\phi_{h^+}$  term of the cross section given in Eqs.(12) and (13), which is the leading one in the anomalous weak-magnetic moment  $F_2^w$  (it comes with the couplings  $A^i$ ). After some algebra one finds

$$A_{cc}^\mp = \mp \alpha_h \frac{s_w c_w}{4\beta} \frac{V^2 + A^2}{A^3} \left[ -\frac{V}{\gamma s_w c_w} + 2\gamma F_2^w \right] \quad (19)$$

with opposite values for  $\tau^-$  and  $\tau^+$ . Selecting the pion channel for the decay of the  $\tau$  (i.e.  $h = \pi$ ) we obtain:

$$A_{cc}^{\pi^-} \equiv -A_{cc}^{\pi^+} = 0.0007 + 10.6 F_2^w \quad (20)$$

while for the  $\rho$  channel (i.e.  $h = \rho$ ) in *tau*-decay one gets:

$$A_{cc}^{\rho^-} \equiv -A_{cc}^{\rho^+} = 0.0003 + 5.0 F_2^w \quad (21)$$

From the measurement of this asymmetry it is then possible to bound the anomalous weak-magnetic moment. Assuming  $10^7 Z$  events, summing up over the  $\pi, \rho$  and  $a_1$  semi-leptonic decay channels of the  $\tau$  for which the angular distribution is not observed, and collecting events from the decay of both *taus*, one gets (within 1s.d.):

$$\begin{aligned} |F_2^w| &\equiv |a_1^w| \leq 5.2 \cdot 10^{-4} && \text{for the } \pi \text{ channel} \\ |F_2^w| &\equiv |a_1^w| \leq 7.3 \cdot 10^{-4} && \text{for the } \rho \text{ channel} \end{aligned} \quad (22)$$

Notice the small value of the background coming from the tree level Standard Model in the asymmetries (20) and (21), which is negligible with the sensitivity given by (22).

The normal polarization of  $\tau$ 's is proportional to the CP violating effective coupling  $F_3^w$  (Eq.(7)). The analysis of the *tau*-decay products allows to select the terms of the cross sections (12) and (13) which carry the relevant information. From there, the leading term with the anomalous weak-electric dipole moment  $F_3^w$  and the  $A^i$  factor is extracted by the asymmetry:

$$A_{cc}^\mp = \frac{\sigma_{cc}^\mp(+)-\sigma_{cc}^\mp(-)}{\sigma_{cc}^\mp(+)+\sigma_{cc}^\mp(-)} \quad (23)$$

where

$$\sigma_{cc}^\mp(+)=\int_0^1 d(\cos\theta_{\tau^-}) \int_0^\pi d\phi_{h^+} + \int_{-1}^0 d(\cos\theta_{\tau^-}) \int_\pi^{2\pi} d\phi_{h^+} \left[ \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^+}} \right] \quad (24)$$

$$\sigma_{cc}^\mp(-)=\int_0^1 d(\cos\theta_{\tau^-}) \int_\pi^{2\pi} d\phi_{h^+} + \int_{-1}^0 d(\cos\theta_{\tau^-}) \int_0^\pi d\phi_{h^+} \left[ \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^+}} \right] \quad (25)$$

From the expressions of the cross sections given in Eqs.(12) and (13) we finally obtain:

$$A_{cc}^\mp = A_{cc}^\pm = \alpha_h \frac{\gamma}{2} s_w c_w \frac{V^2 + A^2}{A^3} F_3^w \quad (26)$$



without any background from the Standard Model within the contributions considered in this paper. A genuine CP violating observable is the asymmetry as compared for the particle and its antiparticle

$$A^{CP} \equiv \frac{1}{2}(A_w^- + A_w^+) \quad (27)$$

What is tested from the  $A^{CP}$ -asymmetry is whether the normal polarizations of both taus are different. Within the contributions considered in this paper, they are opposite. This implies the equality of the decay-product asymmetries (23), so  $A^{CP} = A_w^+ = A_w^-$  and the observable is given only by CP-violating terms. If we take the pion channel (i.e.  $h \equiv \pi$ ) in  $\tau$ -decay, for the asymmetry defined in Eq.(26) we obtain:

$$A_w^+ = A_w^- = -10.6 \cdot F_w^w \quad (28)$$

Then, for  $10^7 Z^0$ 's and collecting events from the decay of both taus, we can get the following bound on the anomalous weak-electric dipole moment from the  $\pi$  channel (within l.s.d.)

$$|d_w^+| < 2.6 \cdot 10^{-16} e \cdot cm \quad (29)$$

Selecting the  $\rho$  channel (i.e.  $\rho \equiv h$ ) in  $\tau$ -decay we get:

$$A_w^+ = A_w^- = -3.8 \cdot F_w^w \quad (30)$$

which gives a bound on  $d_w^+$  of

$$|d_w^+| < 5.4 \cdot 10^{-16} e \cdot cm \quad (31)$$

Notice that possible backgrounds coming from  $\gamma - Z$  interference or electroweak radiative corrections are eliminated in the true CP violating observable (27), as they do change sign when going from the positive charge asymmetry to the negative one.

## 4 Conclusions

To summarize, we have studied the physical content of the normal and transverse  $\tau$  polarization for  $\tau^+ \tau^-$  pairs produced from unpolarized  $e^+ e^-$  collisions at the  $Z$ -peak. The leading (in the dilation factor  $\gamma$ ) transverse polarization term is proportional to the anomalous weak-magnetic dipole moment. We have defined appropriate asymmetries from the  $\tau$  decay products from which one can measure the anomalous weak-magnetic dipole moment. A very small (like  $\frac{1}{2}$ ) tree level Standard Model background, not reachable with the present sensitivity, is expected for these asymmetries. We have also singled out a CP violating asymmetry for the semileptonic decay channels of the single  $\tau$ 's produced with unpolarized  $e^+ e^-$  beams. It is manifested by a non-vanishing value of the normal (to the collision plane) polarization of the outgoing tau-lepton. It is measured from the asymmetry (23) in the azimuthal distribution of the hadron from the  $\tau$ -semileptonic decay. This observable provides a stringent test of CP invariance in the leptonic sector, with a different systematics as that obtained from correlations. It can be used to measure the weak-electric dipole moment of the  $\tau$  lepton in  $e^+ e^-$  experiments at the  $Z$ -peak. We have calculated the values for the weak-electric dipole moment reachable from  $10^7 Z^0$ 's and we have obtained for the resulting sensitivity the value  $2.3 \cdot 10^{-16} e \cdot cm$ .

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