The top-quark mass in SU(5)xU(1) supergravity

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ABSTRACT

We show that the currently experimentally preferred values of the top-quark mass (*i.e.*, $130 \lesssim m_t \lesssim 180 \, \mathrm{GeV}$) are naturally understood in the context of string models, where the top-quark Yukawa coupling at the string scale is generically given by $\lambda_t = \mathcal{O}(g)$, with g the unified gauge coupling. A detailed study of the Yukawa sector of $SU(5) \times U(1)$ supergravity shows that the ratio of the bottom-quark to tau-lepton Yukawa couplings at the string scale is required to be in the range $0.7 \lesssim \lambda_b/\lambda_\tau \lesssim 1$, depending on the values of m_t and m_b. This result is consistent with $SU(5) \times U(1)$ symmetry, which does *not* require the equality of these Yukawa couplings in the unbroken symmetry phase of the theory. As a means of possibly predicting the value of m_t, we propose a procedure whereby the size of the allowed parameter space is determined as a function of m_t, since all sparticle and Higgs-boson masses and couplings depend non-trivially on m_t. At present, no significant preference for particular values of m_t in $SU(5) \times U(1)$ supergravity is observed, except that high-precision LEP data requires $m_t \lesssim 180 \, \mathrm{GeV}$.

1   Introduction

The origin of elementary particle masses is one of the most profound questions in physics. Modern field theories try to answer this question, in the context of spontaneously broken gauge symmetries, through vacuum expectation values (vevs) of elementary or composite scalar Higgs fields. In general the masses of all particles (scalars, fermions, gauge bosons) are proportional to this (or these) vev(s). The proportionality coefficients are: the quartic couplings (λ) for the scalars, the Yukawa couplings (y) for the fermions, and the gauge couplings (g) for the gauge bosons. Thus, schematically we have:

$$m_s = \lambda^{1/2} \langle \text{vev} \rangle, \tag{1}$$

$$m_f = y \langle \text{vev} \rangle, \tag{2}$$

$$m_g = g \langle \text{vev} \rangle. \tag{3}$$

This general picture looks convincingly simple, but its implementation in realistic models is not. At present, there are several reasons that prevent us from a complete and satisfactory solution of the mass problem. The quark and lepton mass spectrum (neglecting neutrinos) spans a range of at least five orders of magnitude, *i.e.*, from $m_e = 0.5\,\text{MeV}$ to $m_t \gtrsim 130\,\text{GeV}$. If we take as "normal" the electroweak gauge boson masses, $\mathcal{O}(80 - 90)\,\text{GeV}$, then a seemingly "heavy" top quark $\mathcal{O}(150\,\text{GeV})$ looks perfectly reasonable, while all other quark and lepton masses look peculiarly small. Clearly, a natural theory cannot support fundamental Yukawa couplings extending over five orders of magnitude. The hope has always been [1] that several of these Yukawa couplings are *naturally zero* at the classical level, and that quantum corrections generate Yukawa couplings that reproduce reality. A modern version of this program has arisen in string theory, as we discuss shortly. We should point out that in a softly broken supersymmetric theory, several mass parameters arise beyond those in Eqs. (1)–(3). However, these lead to "normal" sparticle masses, and thus do not relate to the light fermion mass puzzle.

Despite the pessimism expressed above, certain features of the fermion mass spectrum have been already explored, most notably in unified theories, where the difference between quark and lepton masses is attributed to the strong interactions that make the quarks much heavier than the leptons (of the same generation). In this context, the successful prediction for the m_b/m_τ ratio [2] led to the highly correlated prediction of $N_f = 3$, which was spectacularly confirmed at LEP: $N_f = 2.980 \pm 0.027$ [3]. An important feature of supergravity unified models is their ability to trigger radiative spontaneous breaking of the electroweak symmetry [4, 5], thus explaining naturally why $m_W/m_{Pl} \approx 10^{-16}$. However, this mechanism only works when the theory contains a Yukawa coupling of the order of "g", *i.e.*, $y = \mathcal{O}(g)$, which is naturally identified with the top-quark Yukawa coupling. In other words, in supergravity models, a "heavy" top quark is not only natural, but it is also needed if we want to have a dynamical understanding of electroweak symmetry breaking. Finally, string theory – more precisely its infrared limit, which naturally encompasses supergravity – is characterized by two features of relevance to us here (see *e.g.*, Refs. [6, 7, 8, 9]):

1. Most of the Yukawa couplings are naturally zero at the lowest order, and acquire non-vanishing values progressively at higher orders (through effective "non-renormalizable" terms), consistent with the spectrum of fermion masses observed in Nature.

2. Non-zero Yukawa couplings, at lowest order, are automatically of $\mathcal{O}(g)$.

Once more, in string theory a "heavy" top quark is a natural possibility and, for the first time, we may even have a dynamical explanation for the origin of its large Yukawa coupling, *i.e.*, $\mathcal{O}(g)$. We should remark that large values of the top-quark Yukawa coupling at very high energies have long been advocated as the explanation for a "heavy" top quark in connection with the infrared quasi fixed point of the corresponding renormalization group equation [10, 11, 12, 13]. However, the origin of such large values has been usually regarded as a remnant of new non-perturbative physics at very high energies [10], or simply left unspecified. In this note we emphasize that string theory provides a natural underlying structure where the experimentally favored values of the top-quark mass can be understood.

2   SU(5)xU(1) Supergravity: bottom-up view

Here we briefly describe the most salient features of string-inspired $SU(5) \times U(1)$ supergravity [14], which constitutes our bottom-up approach to the prediction for m_t. The $SU(5) \times U(1)$ gauge group (also known as "flipped $SU(5)$") can be argued to be the simplest unified gauge extension of the Standard Model. It is unified because the two non-abelian gauge couplings of the Standard Model (α_2 and α_3) are unified into the $SU(5)$ gauge coupling. It is the simplest extension because this is the smallest unified group which provides neutrino masses. In this interpretation, minimal $SU(5)$ would appear as a subgroup of $SO(10)$, if it is to allow for neutrino masses. Moreover, the $SU(5) \times U(1)$ matter representations entail several simplifications, such as the breaking of the gauge group via vacuum expectation values of $\mathbf{10},\overline{\mathbf{10}}$ Higgs fields, the natural splitting of the doublet and triplet components of the Higgs pentaplets and therefore the natural avoidance of dangerous dimension-five proton decay operators, and the natural appearance of a see-saw mechanism for neutrino masses.

We supplement the $SU(5) \times U(1)$ gauge group choice with the minimal matter content which allows it to unify at the string scale $M_U \sim 10^{18}$ GeV, as expected to occur in the string-derived versions of the model [15]. This entails a set of intermediate-scale mass particles: a vector-like quark doublet with mass $m_Q \sim 10^{12}$ GeV and a vector-like charge $-1/3$ quark singlet with mass $m_D \sim 10^6$ GeV [16]. The model is also implicitly constrained by the requirement of suitable supersymmetry breaking. We choose two string-inspired scenarios which have the virtue of yielding universal soft-supersymmetry-breaking parameters, in contrast with non-universal soft-supersymmetry-breaking scenarios which occur quite commonly in string constructions [17, 18] and may be phenomenologically troublesome [19]. These scenarios are examples of the no-scale supergravity framework [20, 5] in which the dimensional

2

parameters of the theory are undetermined at the classical level, but are fixed by radiative corrections, thus including the whole theory in the determination of the low-energy parameters. In the *moduli* scenario, supersymmetry breaking is driven by the vev of the moduli fields (T), and gives $m_0 = A = 0$, whereas in the *dilaton* scenario [18] supersymmetry breaking is driven by the vev of the dilaton field (S) and entails $m_0 = \frac{1}{\sqrt{3}} m_{1/2}$, $A = -m_{1/2}$. Thus, the supersymmetry breaking sector depends on only one parameter (*i.e.*, $m_{1/2}$).

The procedure to extract the low-energy predictions of the models outlined above is rather standard (see *e.g.*, Ref. [21]): (a) the bottom-quark and tau-lepton masses, together with the input values of m_t and $\tan\beta$ are used to determine the respective Yukawa couplings at the electroweak scale; (b) the gauge and Yukawa couplings are then run up to the unification scale $M_U = 10^{18}\,\mathrm{GeV}$ taking into account the extra vector-like quark doublet ($\sim 10^{12}\,\mathrm{GeV}$) and singlet ($\sim 10^6\,\mathrm{GeV}$) introduced above [22, 16]; (c) at the unification scale the soft-supersymmetry breaking parameters are introduced (*i.e.*, moduli and dilaton scenarios) and the scalar masses are then run down to the electroweak scale; (d) radiative electroweak symmetry breaking is enforced by minimizing the one-loop effective potential which depends on the whole mass spectrum, and the values of the Higgs mixing term $|\mu|$ and the bilinear soft-supersymmetry breaking parameter B are determined from the minimization conditions; (e) all known phenomenological constraints on the sparticle and Higgs masses are applied (most importantly the LEP lower bounds on the chargino and Higgs-boson masses), including the cosmological requirement of a not-too-young Universe.

The three-dimensional parameter space of this model (*i.e.*, $m_{1/2}$, $\tan\beta$ and the top-quark mass) has been explored in detail in Refs. [16] and [23] for the moduli and dilaton scenarios respectively. More recently, we have investigated further constraints on the parameter space, including: (i) the CLEO limits on the $b \to s\gamma$ rate [24, 25], (ii) the long-standing limit on the anomalous magnetic moment of the muon [26], (iii) the electroweak LEP high-precision measurements in the form of the ϵ_1, ϵ_b parameters [27, 25], (iv) the non-observation of anomalous muon fluxes in underground detectors ("neutrino telescopes") [28], and (v) the possible constraints from trilepton searches at the Tevatron [29].

3   SU(5)xU(1) Supergravity: top-down view

In the context of string model-building, the $SU(5) \times U(1)$ structure becomes even more important, since the traditional grand unified gauge groups ($SU(5), SO(10), E_6$) cannot be broken down to the Standard Model gauge group in the simplest (and to date almost unique) string constructions, because of the absence of adjoint Higgs representations [30]. This reasoning is not applicable to the $SU(5) \times U(1)$ gauge group, since the required $\mathbf{10}, \overline{\mathbf{10}}$ representations are very common in string model building [6, 8]. As a "descendant" of string theory, $SU(5) \times U(1)$ supergravity is characterized by two basic features: (a) a large top-quark Yukawa coupling: $\mathcal{O}(g)$, and

(b) the no-scale structure. Notice that (b) in conjunction with (a), not only triggers radiative electroweak breaking but, in principle, may also determine *dynamically* the magnitude of the supersymmetry breaking scale [20, 5]. As mentioned above, string unification occurs at the scale $M_U \sim 10^{18}$ GeV [15], and this has been seen to occur in explicit $SU(5) \times U(1)$ string models [8].

Of more relevance to the present discussion is the composition of the Yukawa sector in $SU(5) \times U(1)$ string models. The usual situation [6, 7, 9] is that at the cubic level of superpotential interactions, string symmetries allow only few couplings among the matter fields containing the quarks, leptons, and Higgs bosons of the low-energy theory. A particularly simple solution to the question of how to assign low-energy fields to the string representations consists of having only the top-quark, bottom-quark, and tau-lepton Yukawa couplings be non-vanishing. Further assumptions lead to a scenario with $\lambda_t = \lambda_b = \lambda_\tau = \sqrt{2}g$ at the string scale, where g is the unified gauge coupling determined by the vacuum expectation value of the dilaton field in the top-down approach, or by the unification condition in the bottom-up approach. This is however not a robust prediction since various unknown mixing angles could possibly destroy this relation. Moreover, it is possible that the bottom-quark and tau-lepton Yukawa couplings could be suppressed relative to the top-quark Yukawa coupling [9]. What is a robust prediction is the magnitude of the top-quark Yukawa coupling

$$\lambda_t(M_U) = \sqrt{2}g \cos\theta_t \, , \tag{4}$$

where $\cos\theta_t$ is a possible mixing angle factor. The bottom-quark and tau-lepton Yukawa couplings are not necessarily equal at the string scale, since no obvious symmetry principle is at play in $SU(5) \times U(1)$ (as opposed to the case of $SU(5)$). Nonetheless, equality of these Yukawa couplings does occur in many explicit $SU(5) \times U(1)$ string models [6, 7, 8]. The Yukawa couplings for the first- and second-generation quarks and leptons appear at the quartic or higher non-renormalizable order [31, 7] and are naturally suppressed relative to the cubic level Yukawa couplings, in agreement with the observed hierarchical mass spectrum.

4 The Yukawa sector and the value of m_t

From the bottom-up approach we are able to compute the value of the third-generation Yukawa couplings at the string scale in terms of m_t and $\tan\beta$. (These string-scale Yukawa couplings also depend on m_b and $\alpha_3(M_Z)$.) In Fig. 1 we show the top-quark Yukawa coupling at the string scale versus the top-quark mass for various values of $\tan\beta$. As expected, a Landau pole is encountered in the running of the Yukawa coupling if the top-quark mass exceeds a maximum value at low energies. For example, $m_t \lesssim 170$ GeV is required for $\tan\beta = 2$. Values of $\tan\beta$ larger than those shown are indistinguishable from the $\tan\beta = 10$ curve. The dependence on $\alpha_3(M_Z)$ and m_b is rather small in this case, *i.e.*, comparable to the thickness of the lines for $\alpha_3(M_Z) = 0.118 \pm 0.007$ and $m_b = 4.25 - 4.9$ GeV.

The above are the results of the bottom-up approach. On the other hand, from the top-down approach we expect values of λ_t as given in Eq. (4), which are shown as dashed lines on Fig. 1 for two typical cases. Here $g \approx 0.84$ is obtained from the running of the gauge couplings up to the string scale. These values of λ_t do not exceed the unitarity requirement of Ref. [12] ($\lambda_t < 4.8$) or the perturbative criterion of Ref. [13] ($\lambda_t < 3.3$). Thus, the experimentally preferred top-quark masses (direct Tevatron limits $m_t > 131\,\mathrm{GeV}$ [32] and indirect fits to the electroweak data $m_t \approx 140 \pm 20\,\mathrm{GeV}$ [33]) can be naturally understood in string models, and do not require the existence of new non-perturbative interactions at the unification scale.

From the bottom-up approach we also obtain the values for the bottom-quark and tau-lepton Yukawa couplings at the string scale, as shown in Figs. 2 and 3 (for $\alpha_3(M_Z) = 0.118$). In Fig. 2, the bottom-quark Yukawa coupling is plotted against the top-quark Yukawa coupling for various values of $\tan\beta$ (2,6,10,20). Along the (solid) lines the top-quark mass varies as shown. The two sets of curves for each value of $\tan\beta$ correspond to the representative choices of $m_b = 4.25$ and $4.9\,\mathrm{GeV}$. The dashed lines for $\tan\beta = 10$ show the decrease in $\lambda_b(M_U)$ due to a shift in $\alpha_3(M_Z)$ from 0.118 to 0.125. The corresponding shifts for larger (smaller) values of $\tan\beta$ are proportionally larger (smaller). Values of $\tan\beta$ larger than the ones shown, when allowed by the theoretical constraints on the model, simply yield proportionally larger values of $\lambda_b(M_U)$.

In Fig. 3 the bottom-quark Yukawa coupling is plotted against the tau-lepton Yukawa coupling for various values of $\tan\beta$ (2,6,10,20). Two representative values of m_b have been chosen (4.25 and 4.9 GeV) which are only visibly distinguished for $\tan\beta = 20$, as indicated. Also, $\alpha_3(M_Z) = 0.118$ has been chosen. Along the vertical lines the top-quark mass increases from bottom to top. The effect of shifts in $\alpha_3(M_Z)$ is to extend the vertical lines slightly. It is interesting to note that the traditional $\lambda_b = \lambda_\tau$ relation (as would be required in an $SU(5)$ model) can be obtained for the largest values of m_t and for the larger values of m_b. However, the range

$$0.7 \lesssim \lambda_b/\lambda_\tau \lesssim 1 \tag{5}$$

is a more realistic estimate of what would be required from a string model in the top-down approach. Such deviations from the $\lambda_b = \lambda_\tau$ relation have been explored in the literature [13] and have been shown to weaken significantly the tight constraint on the $(m_t, \tan\beta)$ plane which otherwise results from imposing the $\lambda_b = \lambda_\tau$ relation.

As discussed above, the allowable free parameters are reduced to a minimal number in $SU(5) \times U(1)$ supergravity, allowing severe experimental scrutiny. An interesting exercise along these lines consists of determining the size of the allowed parameter space (in the $(m_{1/2}, \tan\beta)$ plane) as a function of m_t, hoping that the correlations among the model variables and their intricate dependence on m_t may show a preference for particular values of the top-quark mass. The results of this exercise, when only the basic theoretical and experimental LEP constraints are imposed, are shown in Fig. 4 ("theory+LEP" curves). The drop in the curves near $m_t = 190\,\mathrm{GeV}$ has been studied in detail (for $m_t = 180, 185, 187, 188, 189\,\mathrm{GeV}$) and corresponds to

encountering a Landau pole in the top-quark Yukawa coupling below the string scale [12, 21]. Imposing in addition all of the direct and indirect experimental constraints mentioned above (*i.e.*, $b \to s\gamma$, $(g-2)_\mu$, neutrino telescopes, and $\epsilon_{1,b}$) we obtain the curves labelled "ALL" in Fig. 4 [34]. These curves still do not show any obvious preference for particular values of m_t. However, $m_t \lesssim 180\,\mathrm{GeV}$ is now required, basically to fit the precise LEP electroweak data [34]. This exercise is rather interesting and should be repeated as present experimental constraints are tightened or new constraints arise.

5 Conclusions

We have shown that the currently experimentally preferred values of the top-quark mass are naturally understood in a top-down approach in the context of string models. We have studied this point explicitly in the context of $SU(5) \times U(1)$ supergravity, which is to be viewed as the bottom-up approach to physics at the string scale. Using the bottom-up approach we have also found that the ratio of the bottom-quark to tau-lepton Yukawa couplings at the string scale is required to be in the range $0.7 \lesssim \lambda_b/\lambda_\tau \lesssim 1$, depending on the values of m_t and m_b. This result is consistent with $SU(5) \times U(1)$ symmetry, which does *not* require the equality of these Yukawa couplings in the unbroken symmetry phase of the theory. Finally, as a means of possibly predicting the value of m_t, we have proposed a procedure whereby the size of the allowed parameter space is determined as a function of m_t. Since all sparticle and Higgs-boson masses and couplings, and therefore all observables calculated from them, depend non-trivially on m_t (mostly through the radiative breaking mechanism), such procedure could show a preference towards particular values of m_t. At present no such preference is clearly observed, except for the high-precision LEP data requirement of $m_t \lesssim 180\,\mathrm{GeV}$. Nonetheless, future more sensitive experimental constraints may produce more clear effects. This present relative insensitivity to the value of m_t should not obscure the fact that all experimentally preferred values of m_t are allowed in $SU(5) \times U(1)$ supergravity, even after the many theoretical and experimental constraints have been applied to the model. We should remark that this procedure could also be applied to more general classes of supergravity models, which have been recently studied in the literature [35], as a means of gauging the experimental viability of these models.

Acknowledgements

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Figure Captions

1. The top-quark Yukawa coupling at the string scale in $SU(5) \times U(1)$ supergravity versus the top-quark mass for fixed values of $\tan\beta$ (larger values of $\tan\beta$ overlap with the $\tan\beta = 10$ curve). The dashed lines indicate typical string-like predictions for the Yukawa coupling.

2. The bottom-quark Yukawa coupling versus the top-quark Yukawa coupling at the string scale in $SU(5) \times U(1)$ supergravity for various values of $\tan\beta$ (2,6,10,20), two values of m_b (4.25 and 4.9 GeV), and $\alpha_3(M_Z) = 0.118$. The top-quark mass varies along the curves as indicated. The dashed lines for $\tan\beta = 10$ show the effect of varying $\alpha_3(M_Z)$ from 0.118 to 0.125. The magnitude of this effect scales with $\tan\beta$.

3. The bottom-quark Yukawa coupling versus the tau-lepton Yukawa coupling at the string scale in $SU(5) \times U(1)$ supergravity for various values of $\tan\beta$ (2,6,10,20), two values of m_b (4.25 and 4.9 GeV), and $\alpha_3(M_Z) = 0.118$. The value of m_t increases from bottom to top along the vertical lines. Note that $0.7 \lesssim \lambda_b/\lambda_\tau \lesssim 1$ is obtained.

4. The number of allowed points in parameter space of no-scale $SU(5) \times U(1)$ supergravity in the moduli and dilaton scenarios, as a function of m_t when the basic theoretical and experimental LEP constraints have been imposed ("theory+LEP"), and when all known direct and indirect experimental constraints have been additionally imposed ("ALL"). Note that $m_t \lesssim 180$ GeV is required.

SU(5)×U(1) supergravity

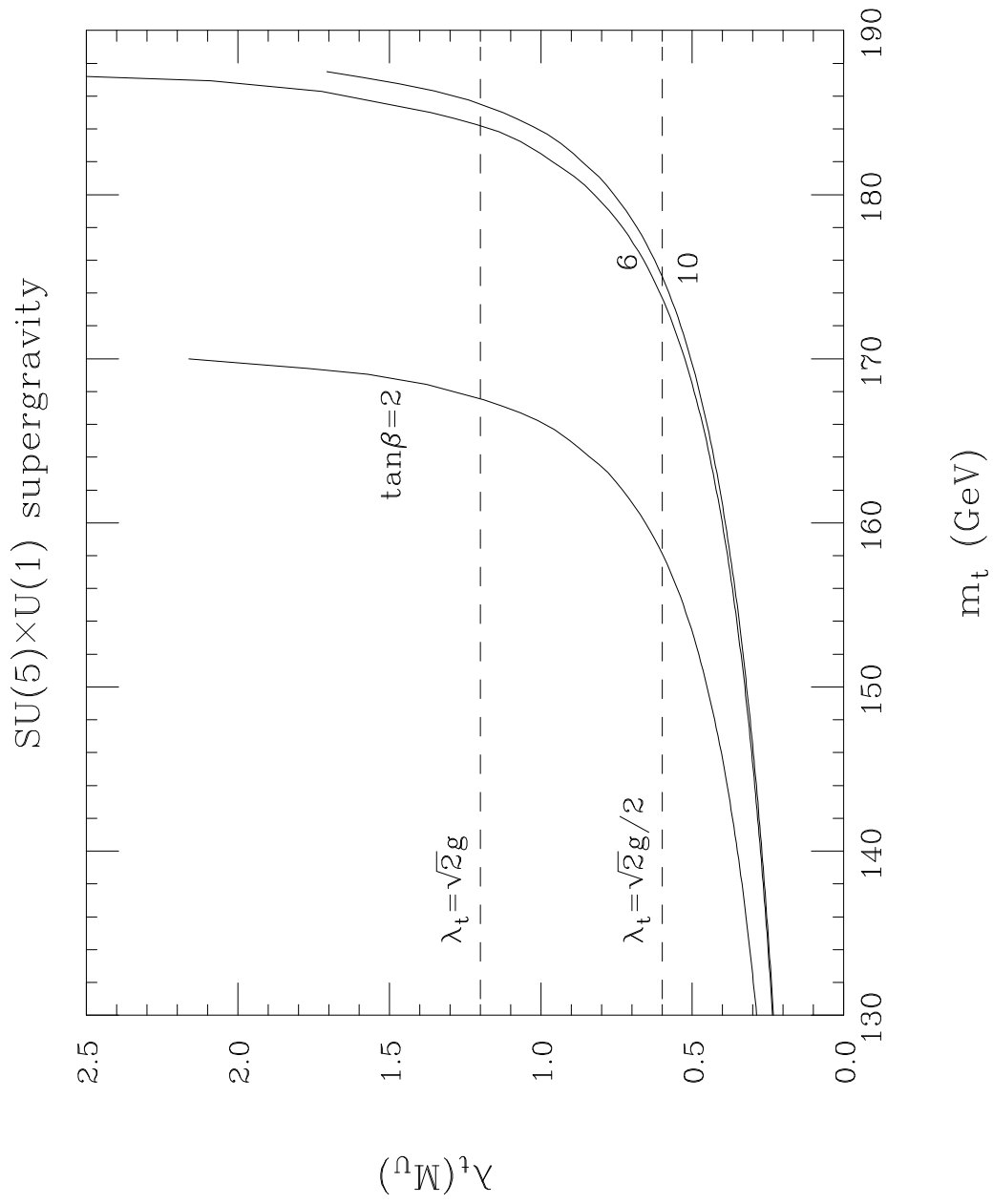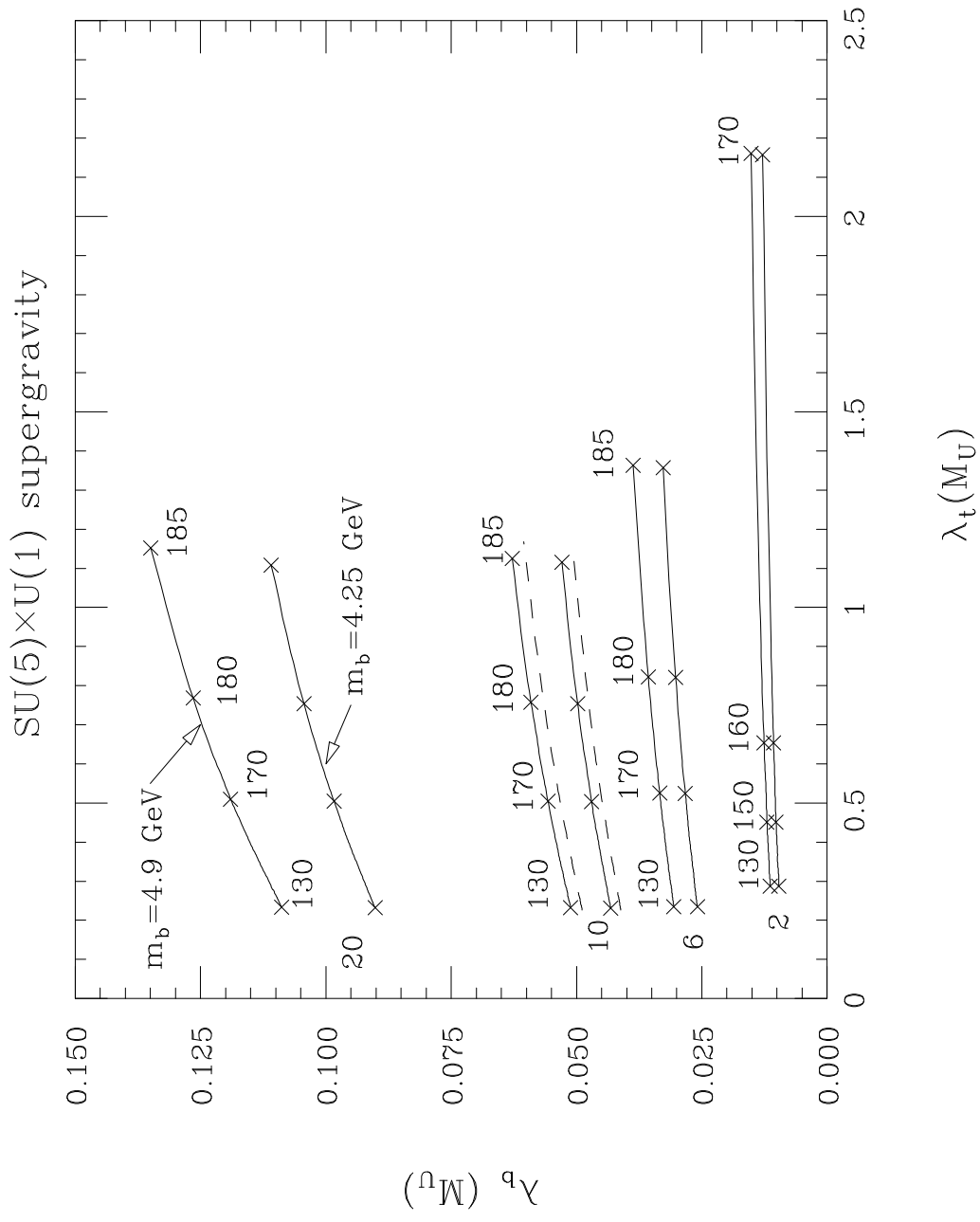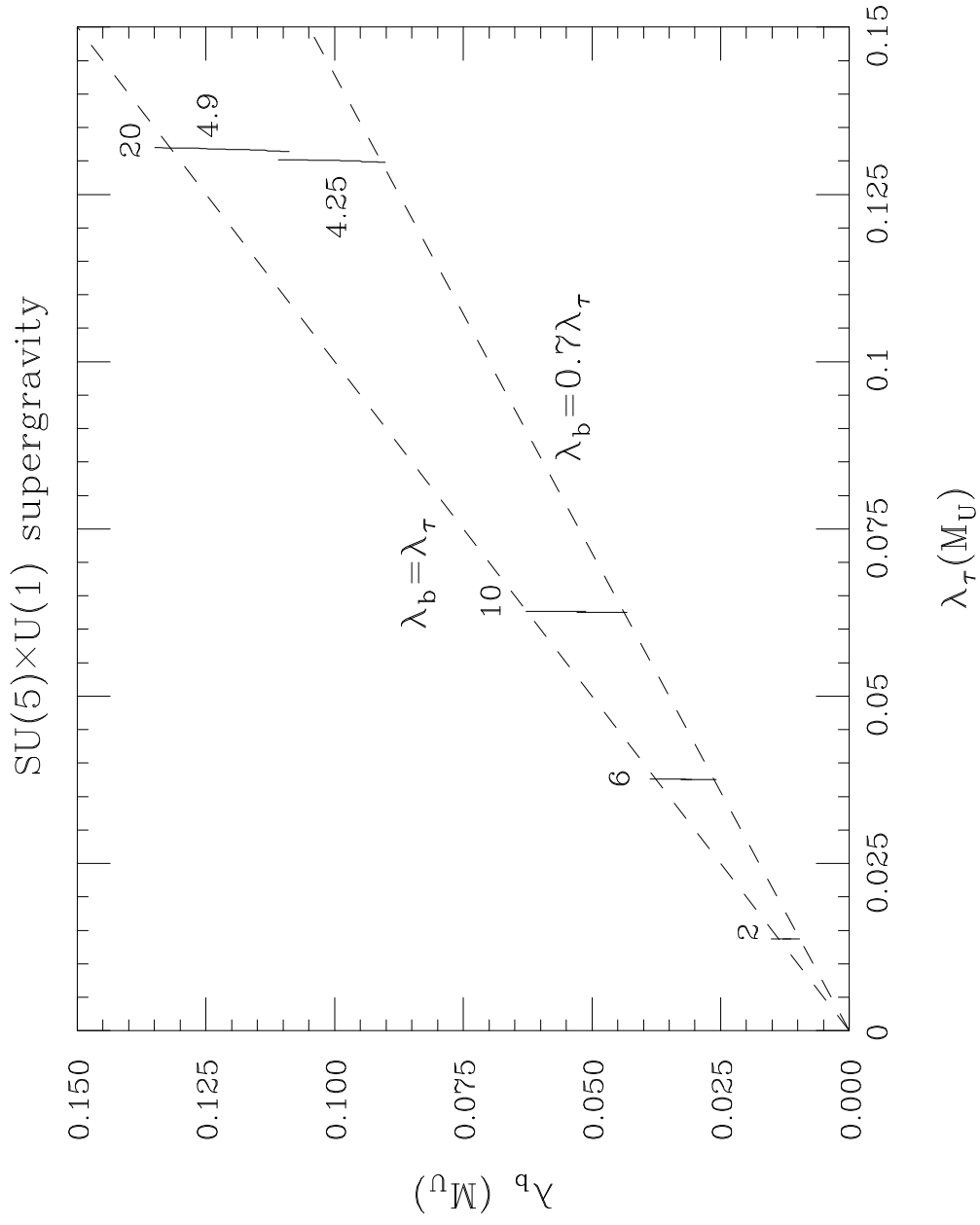$\lambda_t = \sqrt{2}g$

$\lambda_t = \sqrt{2}g/2$

$\tan\beta = 2$

6

10

$\lambda_t(M_U)$

m_t (GeV)

Figure 1

Figure 2

Figure 3

Figure 4