Induced θ-Term in Nambu-Jona-Lasinio Model at Finite Temperatures

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Abstract

We have studied the induced topological θ-term in the Nambu-Jona-Lasinio model at finite temperatures. It is found that it vanishes at the same critical temperature at which chiral symmetry is restored. This shows that hidden local symmetry works even at finite temperatures

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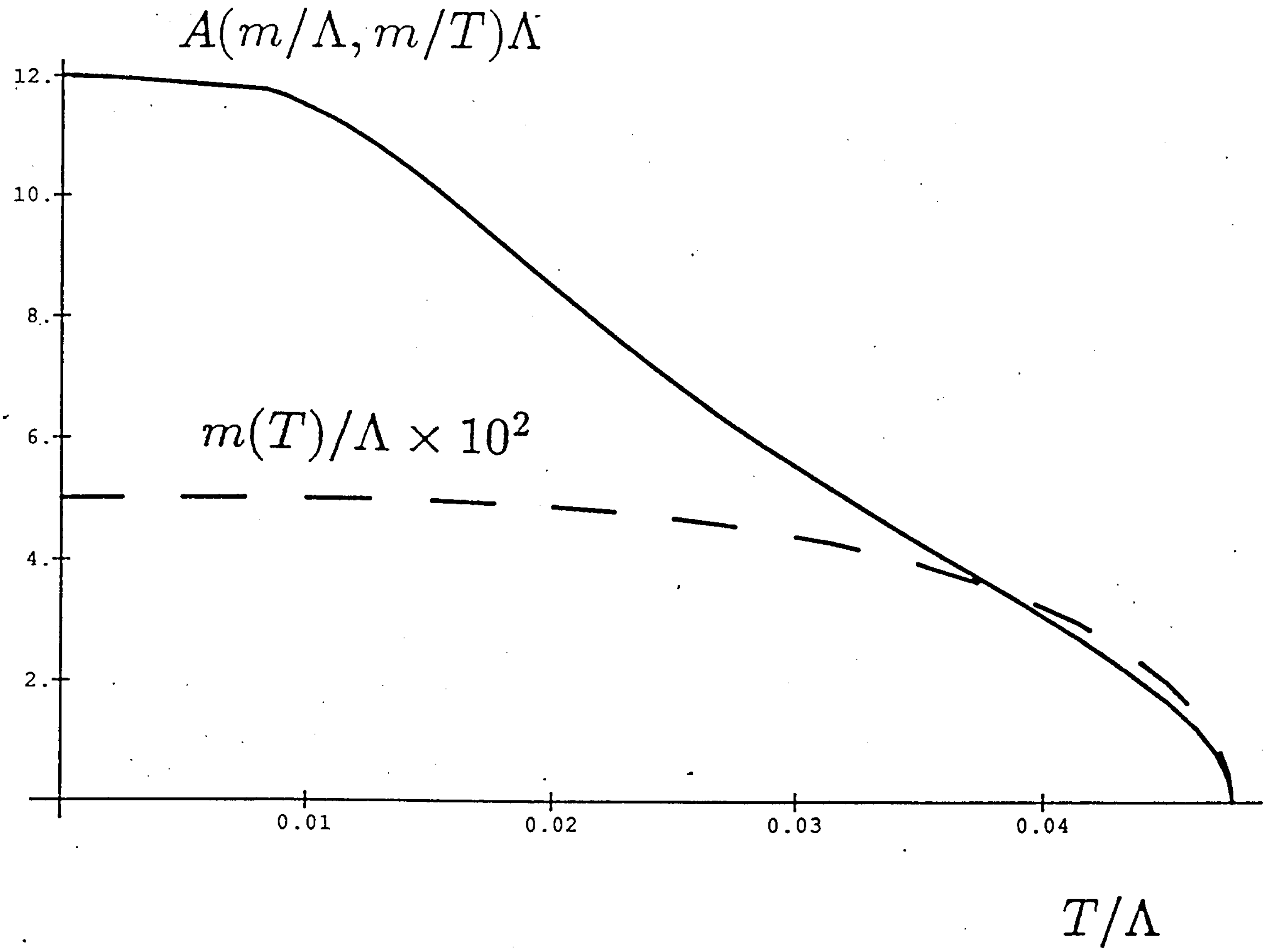
$A(m/\Lambda, m/T)\Lambda$
$m(T)/\Lambda \times 10^2$
12.
10.
8.
6.
4.
2.
0.01
0.02
0.03
0.04
T/Λ

Fig. 1

The apparently simple Nambu-Jona-Lasinio (NJL) model [1] bears tremendous dynamical information and has been intensively discussed since its discovery in the early sixties. Although not renormalizable, as an effective theory it has not only stimulated the idea of spontaneous chiral symmetry breaking of strongly interacting hadronic phenomena, but also provided an important laboratory for generic discussions of non-perturbative dynamical symmetry breaking. In the strongly interacting regime, composite collective modes emerge as the result of spontaneous chiral symmetry breaking. This has a natural connection with the hidden local symmetry (HLS) hypothesis [2]. The HLS hypothesis states that any non-linear sigma model based on the manifold G/H is gauge-equivalent to a linear model with $G_{\mathrm{global}} \otimes H_{\mathrm{local}}$ symmetry. Here H_{local} is the hidden local symmetry which preserves composite gauge bosons as its corresponding gauge fields. When combined with the HLS hypothesis, the NJL model provides a very powerful approximate scheme in the discussion of the spectra of low-lying scalar and vector mesons[2].

It has been argued by us previously [3] that if HLS works, then it should generate correctly all the operators in the theory allowed by the symmetries. There we have demonstrated that this is indeed the case by showing that topological θ-term could be correctly generated in NJL models in $(1+3)$ dimensions. In this paper, we would like to further check the HLS hypothesis by studying these induced topological θ-term at finite temperatures. It is generally believed that broken symmetry at zero temperature will be restored at high temperatures. The chiral symmetry restoration in NJL models had already been demonstrated by Konoue[4]. The purpose of this paper is to check if this induced topological term would also vanish as the broken chiral symmetry is restored. This will provide a serious consistency check on the HLS hypothesis.

We consider a $(1+3)$-dimensional NJL model described by the Lagrangian

$$\mathcal{L} = \bar{\psi} i \not\partial \psi + g\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5\psi)^2\right] - g'\left[(\bar{\psi}\gamma_\mu\lambda_i\psi)^2 + (\bar{\psi} i\gamma_5\gamma_\mu\lambda_i\psi)^2\right] \qquad (1)$$
$$i = 1,\ldots,N^2-1,$$

where λ^i are $SU(N)$ matrices with normalisation Tr $\lambda^i\lambda^j = \frac{1}{2}\delta^{ij}$. This model has a global $SU(N)\times SU(N)\times U(1)\times U(1)$ symmetry. We introduce the auxiliary fields

$$\begin{aligned}
\sigma &= -2g\bar{\psi}\psi - m \ , \\
\theta &= -2gi\bar{\psi}\gamma_5\psi \ , \\
A_\mu &= 2g'\lambda_i\bar{\psi}\gamma_\mu\lambda_i\psi \ , \\
A_{5\mu} &= 2g' i\lambda_i\bar{\psi}\gamma_5\gamma_\mu\lambda_i\psi \ ,
\end{aligned} \qquad (2)$$

To leading order in $1/N$ expansion, the Lagrangian comprising only the σ, θ and A_μ fields relevant for our present purpose is as follows:

$$\begin{aligned}
\mathcal{L}' = \frac{c_1}{2}\Bigg[&\frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}\left(\partial_\mu\theta\right)^2 - 2m^2\sigma^2 - 2m\sigma^3 - \frac{1}{2}\sigma^4 \\
&- \sigma(\sigma+m)\theta^2 - \frac{1}{2}\theta^4\Bigg] \\
+ \frac{c_1}{3}\Bigg[&-\frac{1}{2}\mathrm{Tr}\ F_{\mu\nu}F^{\mu\nu} + \frac{3}{4c_1}\left(\frac{2}{g'} - c_0 - m^2c_1\right)\ \mathrm{Tr}\ A_\mu A^\mu \\
&+ \frac{3}{c_1}mc_2\ \theta\ \mathrm{Tr}\ \widetilde{F}F\Bigg],
\end{aligned} \qquad (3a)$$

where

$$\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \ , \\
c_0 &= 4i\int^\Lambda \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2-m^2} = \frac{1}{4\pi^2}\left[\Lambda^2 - m^2\ln\left(1+\frac{\Lambda^2}{m^2}\right)\right] \ , \\
c_1 &= -4i\int^\Lambda \frac{d^4p}{(2\pi)^4}\,\frac{1}{(p^2-m^2)^2} = \frac{1}{4\pi^2}\left[\ln\left(1+\frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{\Lambda^2+m^2}\right] \ , \\
c_2 &= 2i\int^\Lambda \frac{d^4p}{(2\pi)^4}\,\frac{1}{(p^2-m^2)^3} = \frac{1}{16\pi^2m^2} \ .
\end{aligned} \qquad (3b)$$

In deriving $\mathcal{L}'$, we have imposed the mass-gap condition

$$2gc_0 = 1 \tag{4}$$

to eliminate the linear term in σ so that σ has a vanishing vacuum expectation value. The dynamical mass m in (3) is to be determined self-consistently by (4). A non-trivial solution of (4), *i.e.* $m \neq 0$, which indicates a breakdown of chiral symmetry, is possible if the coupling constant g satisfies $0 < \frac{2\pi^2}{g\Lambda^2} < 1$. To ensure gauge invariance in (4), one imposes the masslessness condition

$$M_2^2 = \frac{3}{4c_1}\left(\frac{2}{g'} - c_0 - m^2 c_1\right) = 0\ . \tag{5}$$

It is interesting to note that a topological θ-term has been induced by quantum fluctuations in this case.

We would like to see how temperature affects the induced θ-term. To this end, we shall evaluate the temperature dependence of c_i's in (3). The standard recipe for evaluating the relevant integrals at finite temperature is to apply the following rules (in Euclidean time) [5]:

$$\begin{aligned} p_0 &\longrightarrow i\omega_n \equiv i\frac{\pi}{\beta}(2n+1), \qquad \beta \equiv \frac{1}{T}, \\ \int \frac{d^d p}{(2\pi)^d} &\longrightarrow \frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^{d-1}p}{(2\pi)^{d-1}}. \end{aligned} \tag{6}$$

For instance, the temperature-dependent integral of c_1 is

$$\begin{aligned} c_1(T) &= -4i\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^3p}{(2\pi)^3}\frac{1}{(\omega_n^2+\omega^2)^2}, \\ \omega^2 &\equiv k^2 + m^2. \end{aligned} \tag{7}$$

There are various ways of evaluating the frequency sum in (7). Here we shall adopt the formula [5]

$$\frac{1}{\beta}\sum_n f(p_0=i\omega_n) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dp_0 f(p_0) - \frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon} dp_0 f(p_0)\frac{1}{e^{\beta p_0}+1} - \frac{1}{2\pi i}\int_{-i\infty-\epsilon}^{i\infty-\epsilon} dp_0 f(p_0)\frac{1}{e^{-\beta p_0}+1}. \tag{8}$$

Applying rule (6) and the frequency sum formula (8) to the integrals c_0, c_1 and c_2 in (3b), we get the results as follows:

$$\begin{aligned}
c_i(T) &= c_i(0)+\bar{c}_i(T)\ , \qquad\qquad i=0,1,2\\
\bar{c}_0(T) &= -4\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega}\frac{1}{e^{\beta\omega}+1}\\
&= -\frac{1}{4\pi^2}\frac{8m^2}{x^2}\int_x^\infty dy\frac{\sqrt{y^2-x^2}}{e^y+1}\ ,\\
\bar{c}_1(T) &= -2\int\frac{d^3p}{(2\pi)^3}\left[\frac{1}{\omega^3}\frac{1}{e^{\beta\omega}+1}+\frac{\beta}{\omega^2}\frac{e^{\beta\omega}}{(e^{\beta\omega}+1)^2}\right]\\
&= -\frac{1}{4\pi^2}4\int_x^\infty dy\frac{\sqrt{y^2-x^2}}{e^y+1}\left[\frac{1}{y^2}+\frac{1}{y}\frac{e^y}{e^y+1}\right]\ ,\\
\bar{c}_2(T) &= -2\int\frac{d^3p}{(2\pi)^3}\left[\frac{3}{8\omega^5}\frac{1}{e^{\beta\omega}+1}+\frac{3\beta}{8\omega^4}\frac{e^{\beta\omega}}{(e^{\beta\omega}+1)^2}\right.\\
&\qquad\left.+\frac{\beta^2}{8\omega^3}\frac{e^{\beta\omega}(e^{\beta\omega}-1)}{(e^{\beta\omega}+1)^3}\right]\\
&= -\frac{1}{4\pi^2}\frac{x^2}{2m^2}\int_x^\infty dy\frac{\sqrt{y^2-x^2}}{e^y+1}\left[3\left(\frac{1}{y^4}+\frac{1}{y^3}\frac{e^y}{e^y+1}\right)+\frac{1}{y^2}\frac{e^y(e^y-1)}{(e^y+1)^2}\right]\ ,\\
x &\equiv \beta m\ , \qquad y\equiv\beta\omega\ ,
\end{aligned} \tag{9}$$

where $c_i(0)$ were denoted by c_i in (3b).

Recall that the dynamical mass m is always a solution to the mass-gap equation (4). It therefore depends on the temperature implicitly through the gap equation. In terms of the parameters $x\equiv\frac{m}{T}=\beta m$ and $z\equiv\frac{m}{\Lambda}$, the gap equation reads

$$1-z^2\ln\left(1+\frac{1}{z^2}\right)-8\left(\frac{z}{x}\right)^2\int_x^\infty dy\frac{\sqrt{y^2-x^2}}{e^y+1}=\frac{2\pi^2}{g\Lambda^2}\ . \tag{10}$$

At zero temperature, the integral in (10) vanishes. Let m_0 be the solution of (9) at $T = 0$, *i.e.*

$$1 - z_0^2 \ln\left(1 + \frac{1}{z_0^2}\right) = \frac{2\pi^2}{g\Lambda^2} , \qquad z_0 = \frac{m_0}{\Lambda} . \tag{11}$$

We shall assume that $z_0 << 1$ (*i.e.* $m_0 << \Lambda$) throughtout this paper. It follows from (11) that in order for a non-trivial solution $m_0 \neq 0$ to exist, we must have $0 < \frac{2\pi^2}{g\Lambda^2} < 1$ as mentioned before. To see how m varies as a function of T, we plot $z = \frac{m}{\Lambda}$ versus $t \equiv \frac{z}{x} = \frac{T}{\Lambda}$ for $\frac{2\pi^2}{g\Lambda^2} = 0.985, z_0 = 0.0500$ in Fig.1. Here z and x satisfy (10) and are solved numerically. The graph shows that there exists a critical temperature T_c beyond which $m = 0$, which corresponds to a restoration of chiral symmetry. An estimate of T_c can be obtained from (10) by setting $x \to 0, z \to 0$. We then have

$$\frac{T_c}{\Lambda} \approx \sqrt{\frac{3}{2\pi^2}\left(1 - \frac{2\pi^2}{g\Lambda^2}\right)} . \tag{12}$$

Near T_c, $m(T)$ scales as $\left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}$ [4].

From the solutions (z, x) to (10), we can study the temperature dependence of the induced θ-term. We shall only be interested in the effective theory in terms of the renormalized fields: $\sqrt{\frac{c_1}{2}}\sigma \to \sigma, \sqrt{\frac{c_1}{2}}\theta \to \theta$ and $\sqrt{\frac{c_1}{3}}A_\mu \to A_\mu$. The coefficient of the induced term becomes

$$\begin{aligned} A(z, x) &= \frac{3\sqrt{2}m(T)}{c_1(T)^{\frac{3}{2}}} c_2(T) \\ &= \frac{3\pi}{\sqrt{2}} \frac{B(x)/z}{D(z, x)^{\frac{3}{2}}} \frac{1}{\Lambda} , \end{aligned} \tag{13}$$

where

$$B(x) \equiv 1 + 16\pi^2 m^2 \bar{c}_2(T) ,$$
$$D(z, x) \equiv \ln\left(1 + \frac{1}{z^2}\right) - \frac{1}{1 + z^2} + 4\pi^2 \bar{c}_1(T) ,$$

and the mass-squared term (5) is

$$\left(\frac{M_2}{\Lambda}\right)^2 = \frac{3}{4}\left[\frac{8\pi^2}{g'\Lambda^2} - \frac{1}{1+z^2} + 4z^2 G(x)\right] \Big/ D(z,x) \tag{14}$$

where

$$G(x) \equiv -\pi^2 \left[\frac{1}{m^2}\bar{c}_0(T) + \bar{c}_1(T)\right].$$

Here we assume g' satisfies

$$\frac{8\pi^2}{g'\Lambda^2} = \frac{1}{1+z_0^2} \tag{15}$$

so that $M_2^2 = 0$ at $T = 0$. In Fig. 1 we also show the graph of $A(z,x)\Lambda$ versus $t = \frac{z}{x} = \frac{T}{\Lambda}$ (solid line) for the same values of $\frac{2\pi^2}{g\Lambda}$ and z_0 as those used in the graph of m/Λ versus t. It shows rather clearly that as the temperature T increases, the topological term becomes smaller and vanishes at exactly the same critical temperature T_c at which chiral symmetry is restored. This is the most important result of this paper. We have demonstrated explicitly that, though the gap equation (10) and the function A in (13) differ greatly, they bear the same critical temperature T_c. This is a remarkable demonstration of the underlying HLS at work even at finite temperatures.

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FIGURE CAPTIONS

1) (a) $z = m/\Lambda(\times 10^2)$ as a function of T/Λ (dashed line); (b) $A(m/\Lambda, m/T)\Lambda$ as a function of T/Λ (solid line). Both plots assume $z_0 \equiv m_0/\Lambda = 0.0500$.

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