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Semileptonic Decays of B Meson into Charmed Higher Resonances in the Heavy Quark Effective Theory

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Abstract : Heavy quark effective theory is applied to B mesons semileptonic decays into charmed higher resonances D_X, where subscript X of D_X denotes various spin-parity states of resonances. We calculate the differential decay rates and branching ratios of the decay $B \to D_X \ell \nu_\ell$. We find that the largest fraction of the semileptonic process $B \to D_X \ell \nu_\ell$ other than $D(1869)$ and $D^*(2010)$ is the radially excited state $D^*_{C_2}$ whose value is 0.26%. It is also estimated that among the unidentified modes in the semileptonic decays of B meson other than $D(1896)$ and $D^*(2010)$ the total contribution to the branching ratios from exclusive higher resonance modes is about 0.6%. The results are compared to the recent experimental data and the other theoretical analyses. We discuss also the unidentified modes in the semileptonic decays of B meson which can not be reduced to the exclusive higher resonance modes.

1 Introduction

Recently considerable experimental and theoretical effort has been devoted to understand the dynamics of hadrons containing a heavy quark. In the case that a heavy quark Q has a enough large mass compared to the QCD scale parameter Λ_{QCD} (*i.e.* $m_Q \gg \Lambda_{QCD}$), it is good approximation that a heavy quark has no recoil by QCD interaction in the heavy flavored hadrons. Then the angular momenta of the heavy and light degrees of freedom decouple (due to the above approximation) in the limit of $m_Q \to \infty$ [1]. The heavy quark effective theory(HQET) has simplified the analysis of various phenomena of heavy flavored hadrons such as weak decays, mass spectra and so on [2]. HQET enables us to describe semileptonic decay processes by use of only a single form factor so called as the Isgur-Wise function $\xi(y)$ [1], where y is the invariant product of four-velocities of initial and final heavy flavored mesons. This theory is very useful to determine the Kobayashi-Maskawa matrix element $|V_{cb}|$ with the normalization $\xi(1) = 1$ for $B \to D^* \ell \nu_\ell$ even though the differential decay rate is singular at $y = 1$ [3]. In this paper we study the semileptonic decay processes of B meson with the final state including higher resonance of charmed meson D_X by using HQET, where subscript X of D_X means various spin-parity states of charmed meson resonances.

The present experimental data for inclusive and exclusive branching ratios of B meson semileptonic decays are listed as [4]

$$\mathrm{Br}(B \to X e \nu_e) = 10.7 \pm 0.5\% , \quad \mathrm{Br}(B \to X \mu \nu_\mu) = 10.3 \pm 0.5\% , \tag{1}$$

and

$$\begin{aligned} \mathrm{Br}(B^0 \to D^- \ell^+ \nu_\ell) &= 1.8 \pm 0.5\% , \quad \mathrm{Br}(B^+ \to \bar{D}^0 \ell^+ \nu_\ell) = 1.6 \pm 0.7\% , \\ \mathrm{Br}(B^0 \to D^{*-} \ell^+ \nu_\ell) &= 4.9 \pm 0.8\% , \quad \mathrm{Br}(B^+ \to \bar{D}^{*0} \ell^+ \nu_\ell) = 4.6 \pm 0.7\% , \end{aligned} \tag{2}$$

respectively, where ℓ indicates electron or muon. The unidentified exclusive modes other than the above, whose fraction seems to be about 4%, are possibly reduced to charmed higher resonances D_X, direct production of two or more uncorrelated hadrons and non-charmed hadrons. The direct decay mode into non-charmed meson, however, is expected to be negligibly small due to the small Kobayashi-Maskawa matrix element $|V_{ub}|$ [5]

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.05 - 0.10 .$$

Therefore in order to understand whole of B meson semileptonic decays, from both theoretical and experimental sides it is important to study how the higher resonance modes are participated.

This paper is organized as follows. In section 2 we formulate $B \to D_X \ell \nu_\ell$ processes including kinematics by use of HQET. In section 3 we obtain the Isgur-Wise function on the basis of constituent quark model [6]. Then we calculate the decay rates and the branching ratios of the processes $B \to D_X \ell \nu_\ell$. Finally we give brief discussion and summary in section 4.

2 Semileptonic decay $B \to D_X \ell \nu_\ell$

Heavy flavored mesons are classified by the angular momentum J_ℓ and parity P of the light degrees of freedom denoted as J_ℓ^P. According to heavy quark spin symmetry of HQET, for each J_ℓ^P there are two heavy flavored meson states with $J^P = J_\ell^P \pm \frac{1}{2}$. We denote these two states with spin-parity of the light degrees of freedom $J_\ell^P = \frac{1}{2}^-, \frac{1}{2}^+, \frac{3}{2}^+, \frac{3}{2}^-, \ldots$ by D_X with subscripts $X = C, E, F, G, \ldots$ and the radially excited state of ground state C with $J_\ell^P = \frac{1}{2}^-$ by D_{C_2}, etc. following the notation given by Ali, Ohl and Mannel [7]. In this paper we study the semileptonic decays into the following ten charmed mesons:

$$C : D(1869)(1\,^1S_1),\ D^*(2010)(1\,^3S_1)\,; \qquad E : D_0^*(1\,^3P_0),\ D_{1E}(1\,^{(1,3)}P_1)\,;$$

$$F : D_{1F}(1\,^{(1,3)}P_1),\ D_2^*(2460)(1\,^3P_2)\,; \qquad G : D_1^*(1\,^1D_1),\ D_2(1\,^{(1,3)}D_2)\,;$$

$$C_2 : D_{C_2}(2\,^1S_0),\ D_{C_2}^*(2\,^3S_1)\,.$$

In the parentheses the mesons are also assigned by the notation $n\,^{2S+1}L_J$. We investigate the decays into D, D^* and these eight charmed meson higher resonances. Here we notice that both of heavy flavored meson states D_{1E} and D_{1F} are the mixing states of $1\,^1P_1$ and $1\,^3P_1$ with the mixing angle to be about 35°, that is $|D_{1E}\rangle = \sqrt{1/3}\,|^1P_1\rangle - \sqrt{2/3}\,|^3P_1\rangle$, and $|D_{1F}\rangle = \sqrt{2/3}\,|^1P_1\rangle + \sqrt{1/3}\,|^3P_1\rangle$ [8]. The observed state $D_1(2420)$ should be assigned to either D_{1E} or D_{1F}. Following with Falk [9], we represent these D_X fields by the operator

$$\mathcal{H}_X(v) = \sqrt{m_X}\,\bar{\ell}_X(v)\,H_X(v)\,h(v) \tag{3}$$

which annihilates the heavy flavored meson D_X with the four-velocity v, where $h(v)$ and $\ell_X(v)$ are the fields of the heavy quark and the light degrees of freedom of the heavy flavored meson D_X with the four-velocity v, respectively. The matrices $H_X(v)$ for ten D_X states are given in terms of the γ matrices in Table 1. These representation is very useful to calculate the transition elements with suitable form factors for the processes by using trace formalism as shown later.

Now we concentrate to the semileptonic decays $B \to D_X \ell \nu_\ell$. We denote the momenta of B, ℓ, ν_ℓ and D_X as P, p_1, p_2 and p_3, respectively. In the rest frame of B meson, the decay rate of $B \to D_X \ell \nu_\ell$ is given by

$$d\Gamma(B \to D_X \ell \nu_\ell) = \frac{1}{2m_B}|A(B \to D_X \ell \nu_\ell)|^2 d\pi_3\,, \tag{4}$$

where the phase space

$$d\pi_3 = (2\pi)^4 \delta^{(4)}(P - p_1 - p_2 - p_3) \prod_{i=1}^{3} \frac{d^3p_i}{(2\pi)^3 2E_i} \tag{5}$$

is taken over all final state momenta and spin sum is assumed. The decay amplitude $A(B \to D_X \ell \nu_\ell)$ is given by a product of the leptonic current L^μ and the hadronic current H^μ with the Kobayashi-Maskawa matrix element V_{cb} as

$$A(B \to D_X \ell \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cb} L^\mu H_\mu\,. \tag{6}$$

In the followings we put lepton masses to be zero, because the masses of muon and electron are negligibly small compared to these heavy flavored mesons. The leptonic and hadronic currents are given by

$$L^\mu = \bar{u}_\ell \gamma^\mu (1 - \gamma_5) v_{\nu_\ell}\,, \tag{7}$$
$$H^\mu = \langle D_X(p_3)|J^\mu(0)|B(P)\rangle\,, \tag{8}$$

where

$$J^\mu = V^\mu - A^\mu = \bar{c}\gamma^\mu(1 - \gamma_5)b.$$

The decay rate is Lorentz invariant and we split the phase space into Lorentz invariant pieces so that it takes on a particularly simple form :

$$\begin{aligned} d\pi_3 &= \frac{1}{(4\pi)^5}\frac{K}{m_B} dq^2 d\Omega_\ell d\tilde{\Omega}_{D_X}\,, \\ q^2 &\equiv (P - p_3)^2\,, \\ K^2 &\equiv \left(\frac{m_B^2 + m_{D_X}^2 - q^2}{2m_B}\right)^2 - m_{D_X}^2\,, \end{aligned} \tag{9}$$

where $d\Omega_\ell$ is the solid angle of the charged lepton in the $\ell \nu_\ell$ center of mass frame ($\ell \nu_\ell$ frame), $d\tilde{\Omega}_{D_X}$ is the solid angle of the final meson in the rest frame of B meson and K is the magnitude of the final meson momentum in the rest frame of B meson [10]. After summing over the spins of charged lepton and neutrino the matrix element squared is given by

$$|A(B \to D_X\, \ell\, \nu_\ell)|^2 = \frac{1}{2} G_F^2 |V_{cb}|^2 L^{\mu\nu} H_\mu H_\nu^\dagger \ , \tag{10}$$

where

$$L^{\mu\nu} = 8\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1\cdot p_2 + i\epsilon^{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta\right) .$$

The matrix element of the hadronic current H_μ should be constructed by Lorentz invariant form factors and the four-velocities. Now we set the four-velocity v_μ and v'_μ by the convention of HQET,

$$P_\mu = m_B v_\mu \ ,$$
$$p_{3\mu} = m_{D_X} v'_\mu \ ,$$

and introduce an invariant variable $y \equiv v^\mu \cdot v'_\mu$. The momentum transfer squared quantity is

$$q^2 = (P - p_3)^2 = (p_1 + p_2)^2 = m_B^2 + m_{D_X}^2 - 2 m_B m_{D_X}\, y \,. \tag{11}$$

Then the differential decay rate $d\Gamma/dy$ is given by

$$\frac{d\Gamma}{dy} = \frac{G_F^2 |V_{cb}|^2}{2(4\pi)^5} \frac{m_{D_X}}{m_B} K\, L^{\mu\nu} H_\mu H_\nu^\dagger d\Omega_\ell d\tilde{\Omega}_{D_X} \ . \tag{12}$$

Non-zero matrix elements of the vector and axialvector parts of H_μ in Eq.(8) are expressed in terms of form factors as follows [1],

$$\frac{\langle D(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_D}} = \tilde{f}_+^C(y)(v+v')_\mu + \tilde{f}_-^C(y)(v-v')_\mu \tag{13a}$$

$$\frac{\langle D^*(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = i\tilde{g}^C(y)\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu} v'^\alpha v^\beta \tag{13b}$$

$$\frac{\langle D^*(v')|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = \tilde{f}^C(y)(y+1)\varepsilon_\mu^* - (\varepsilon^*\cdot v)\left\{\tilde{a}_+^C(y)(v+v')_\mu + \tilde{a}_-^C(y)(v-v')_\mu\right\} \tag{13c}$$

$$\frac{\langle D_0^*(v')|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D_0^*}}} = \tilde{f}_+^E(y)(v+v')_\mu + \tilde{f}_-^E(y)(v-v')_\mu \tag{13d}$$

$$\frac{\langle D_{1E,1F}(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_{D_1}}} = \tilde{f}^{1E,1F}(y)(y+1)\varepsilon_\mu^* - (\varepsilon^*\cdot v)\left\{\tilde{a}_+^{1E,1F}(y)(v+v')_\mu + \tilde{a}_-^{1E,1F}(y)(v-v')_\mu\right\} \tag{13e}$$

$$\frac{\langle D_{1E,1F}(v')|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D_1}}} = i\tilde{g}^{1E,1F}(y)\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu} v'^\alpha v^\beta \tag{13f}$$

$$\frac{\langle D_2^*(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_{D_2^*}}} = i\tilde{g}^{2F}(y)\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu\rho} v_\rho v'^\alpha v^\beta \tag{13g}$$

$$\frac{\langle D_2^*(v')|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D_2^*}}} = \tilde{f}^{2F}(y)(y+1)\varepsilon_{\mu\nu}^* v^\nu - (\varepsilon_{\alpha\beta}^* v^\alpha v'^\beta)\left\{\tilde{a}_+^{2F}(y)(v+v')_\mu + \tilde{a}_-^{2F}(y)(v-v')_\mu\right\} \tag{13h}$$

$$\frac{\langle D_1^*(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_{D_1^*}}} = i\tilde{g}^{1G}(y)\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu} v'^\alpha v^\beta \tag{13i}$$

$$\frac{\langle D_1^*(v')|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D_1^*}}} = \tilde{f}^{1G}(y)(y+1)\varepsilon_\mu^* - (\varepsilon^*\cdot v)\left\{\tilde{a}_+^{1G}(y)(v+v')_\mu + \tilde{a}_-^{1G}(y)(v-v')_\mu\right\} \tag{13j}$$

$$\frac{\langle D_2(v')|V_\mu|B(v)\rangle}{\sqrt{m_B m_{D_2}}} = \tilde{f}^{2G}(y)(y+1)\varepsilon_{\mu\nu}^* v^\nu - (\varepsilon_{\alpha\beta}^* v^\alpha v'^\beta)\left\{\tilde{a}_+^{2G}(y)(v+v')_\mu + \tilde{a}_-^{2G}(y)(v-v')_\mu\right\} \tag{13k}$$

$$\frac{\langle D_2(v')|A_\mu|B(v)\rangle}{\sqrt{m_B m_{D_2}}} = i\tilde{g}^{2G}(y)\epsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu\rho} v_\rho v'^\alpha v^\beta , \tag{13l}$$

where ε_μ^* and $\varepsilon_{\mu\nu}^*$ are polarization vector and tensor for final state meson respectively. The matrix elements for radially excited states D_{C_2} and $D_{C_2}^*$ are given by the same expressions with those of the ground states D and D^* by replacing masses and form factors.

The polarization sums for spin-1 and spin-2 mesons are

$$M_{\mu\nu}^{(1)}(v') \equiv \sum_{pol} \varepsilon_\mu^* \varepsilon_\nu^* = -g_{\mu\nu} + v'_\mu v'_\nu \ , \tag{14a}$$

$$M_{\mu\nu,\rho\sigma}^{(2)}(v') \equiv \sum_{pol} \varepsilon_{\mu\nu}^* \varepsilon_{\rho\sigma}^* = \frac{1}{2} M_{\mu\rho}^{(1)}(v') M_{\nu\sigma}^{(1)}(v') + \frac{1}{2} M_{\mu\sigma}^{(1)}(v') M_{\nu\rho}^{(1)}(v') - \frac{1}{3} M_{\mu\nu}^{(1)}(v') M_{\rho\sigma}^{(1)}(v') \ , \tag{14b}$$

respectively. In general the matrix elements of bilinear currents of two heavy quarks are calculated by taking the trace

$$\langle \mathcal{H}'_X(v')|\bar{h}'(v')\,\Gamma\, h(v)|\mathcal{H}(v)\rangle = \mathrm{Tr}\{\mathcal{H}'_X(v')\,\Gamma\,\mathcal{H}(v)\mathcal{M}_X(v,v')\} \ , \tag{15}$$

where $\mathcal{H}(v) = \sqrt{m_B}\,\bar{\ell}_B(v)\,\gamma_5\, h_B(v)$ is the B meson field and $\mathcal{H}'_X(v')$ is D_X fields given by Eq.(3) and Table 1, and Γ is γ matrix. The matrix $\mathcal{M}_X(v,v')$ represents overlapping of the light degrees of freedom. Due to the heavy quark symmetry for each spin symmetry doublet these matrices $\mathcal{M}_X(v,v')$ can be expressed in terms of only one independent form factor called as the Isgur-Wise function $\xi_X(y)$ [1, 9]. For the processes $0^- \to X$ ($X = C$, E and C_2)

$$\mathcal{M}_X(v,v') = \xi_X(y) , \tag{16a}$$

and for $0^- \to X$ ($X = F$ and G)

$$\mathcal{M}_X(v,v') = \xi_X(y)\, v_\mu . \tag{16b}$$

The vector index μ of v_μ in Eq.(16b) is contracted with that of $\mathcal{H}'_X(v')$ of the final excited mesons. $\xi_X(y)$ are different for different X, because the states of the light degrees of freedom are different. Form factors in the matrix elements given by Eqs.(13a)-(13l) with the same J_ℓ^P final state are related to the corresponding Isgur-Wise function $\xi_X(y)$. The relations among form factors are given in Table 2.

Through a straight forward calculation we get the following expression for the decay rate,

$$\frac{d\Gamma_X}{dy} = \frac{G_F^2|V_{cb}|^2}{48\pi^3} m_B^2 \sqrt{y^2-1}\; m_{D_X}^3 W_X(y,r_X)\, \xi_X(y)^2 , \tag{17}$$

where the $W_X(y,r_X)$ is given by

$$\begin{aligned}
W_X(y,r_X) &\\
&= (y-1)(y+1)(1+r_X)^2 \qquad (\text{for } 0^- \to D \text{ and } D_{C_2}) \qquad (18a)\\
&= (y+1)\{(y+1)(1-r_X)^2 + 4y(1-2r_X y + r_X^2)\}\\
&\qquad\qquad (\text{for } 0^- \to D^* \text{ and } D^*_{C_2}) \qquad (18b)\\
&= (y-1)(y+1)(1-r_X)^2 \qquad (\text{for } 0^- \to D_0^*) \qquad (18c)\\
&= (y-1)\{(y-1)(1+r_X)^2 + 4y(1-2r_X y + r_X^2)\}\\
&\qquad\qquad (\text{for } 0^- \to D_{1E}) \qquad (18d)\\
&= \frac{2}{3}(y-1)(y+1)^2\{(y-1)(1+r_X)^2 + y(1-2r_X y + r_X^2)\}\\
&\qquad\qquad (\text{for } 0^- \to D_{1F}) \qquad (18e)\\
&= \frac{2}{3}(y-1)^2(y+1)\{(y+1)(1-r_X)^2 + y(1-2r_X y + r_X^2)\}\\
&\qquad\qquad (\text{for } 0^- \to D_1^*) \qquad (18f)
\end{aligned}$$

respectively and $r_X \equiv m_{D_X}/m_B$.

The exclusive decay rates for the processes $B \to D_X$ can be calculated also in terms of helicity decomposition method developed by Gilman and Singleton [10]. In Appendix

we show the helicity decomposition method for the spin-2 final mesons as well as for spin-1 mesons.

3 Isgur-Wise functions from constituent quark model

In order to estimate the decay rates for $B \to D_X \ell \nu_\ell$, we have to determine these Isgur-Wise functions by use of specific model. Before the model dependent arguments, we point out the following property of the Isgur-Wise function. Form factor $\xi_C(y)$ of the transition between ground states of B meson and D or D^* meson is normalized as $\xi_C(1) = 1$. The other form factors $\xi_X(y)$ for $X = E, F, G, C_2$ are $\xi_X(1) = 0$ in the heavy quark limit due to the orthogonality of the wave functions of each light degrees of freedom. These conditions for $\xi_X(y)$ at $y = 1$ should be satisfied in any models.

In the following in order to obtain the Isgur-Wise functions we employ the method which is systematically analysed by Grinstein, Isgur, Scora and Wise (GISW model) [6] on the basis of the constituent quark model. In the rest frame of the initial meson, the form factors are defined by the overlap integral of wave functions of the initial and final mesons as the bound states of a constituent quark and antiquark as,

$$I(\vec{v'}) = \int d^3\vec{x}\, \Phi_F^*(\vec{x})\Phi_I(\vec{x}) \exp(-i\Lambda \vec{v'}\cdot\vec{x}) , \tag{19}$$

where $\Phi_I(\vec{x})$ and $\Phi_F(\vec{x})$ are the wave functions of the initial and final mesons, respectively, and

$$\Lambda = \frac{m_{D_X} m_d}{m_c + m_d} .$$

We assume that the constituent quarks are bounded by a potential given by

$$V(r) = -\frac{4\alpha}{3r} + c + br$$

where α is the squared coupling constant of QCD and b and c are the parameters of the confinement potential. From the standpoint of HQET, heavy flavored hadrons are classified by spin-parity of the light degrees of freedom J_ℓ^P as shown in section 2. However, since we employ the constituent quark model, the wave functions are chosen to be eigenfunction of the orbital angular momentum L.

The wave functions of initial and final meson states are given by those of the harmonic oscillator potential model as the trial functions;

$$\Phi_I(\vec{x}) = Y_0^0(\frac{\vec{x}}{|\vec{x}|})\, \phi_I(|\vec{x}|) , \tag{20a}$$

$$\Phi_F(\vec{x}) = Y_L^m(\frac{\vec{x}}{|\vec{x}|})\, \phi_F(|\vec{x}|) , \tag{20b}$$

with the normalization

$$1 = \int d^3\vec{x}\, \Phi^*_{I/F}(\vec{x})\Phi_{I/F}(\vec{x}) = \int dr r^2 \phi^*_{I/F}(r)\phi_{I/F}(r) .$$

Inserting the wave functions Eq.(20a) and Eq.(20b) into the overlap integral Eq.(19) and integrating over angular variables, we obtain

$$I(\vec{v'}) = i^L\sqrt{2L+1}\int dr r^2 \phi^*_F(r)\phi_I(r) j_L(\Lambda r|\vec{v'}|) , \tag{21}$$

where j_L is the spherical Bessel function of order L. This expression holds in the rest frame of the initial heavy flavored meson. In a general frame Eq.(21) becomes

$$\xi(y) \equiv I(v\cdot v') = i^L\sqrt{2L+1}\int dr r^2 \phi^*_F(r)\phi_I(r) j_L\left(\Lambda r\sqrt{(v\cdot v')^2-1}\right) . \tag{22}$$

The relevant wave functions $\phi_F(r)$ are

$$\phi_F^{(1S)}(r) = \sqrt{4\pi}\,\frac{\beta_{SD}^{3/2}}{\pi^{3/4}}\exp\left[-\beta_{SD}^2 r^2/2\right] , \tag{23a}$$

$$\phi_F^{(1P)}(r) = \sqrt{\frac{8}{3}}\,\frac{\beta_{PD}^{5/2}}{\pi^{1/4}}\, r\exp\left[-\beta_{PD}^2 r^2/2\right] , \tag{23b}$$

$$\phi_F^{(1D)}(r) = \frac{4}{\sqrt{15}}\,\frac{\beta_{DD}^{7/2}}{\pi^{1/4}}\, r^2\exp\left[-\beta_{DD}^2 r^2/2\right] , \tag{23c}$$

$$\phi_F^{(2S)}(r) = \sqrt{4\pi}\sqrt{\frac{2}{3}}\,\frac{\beta_{2SD}^{7/2}}{\pi^{3/4}}\left(\frac{3}{2}\beta_{2SD}^{-2} - r^2\right)\exp\left[-\beta_{2SD}^2 r^2/2\right] , \tag{23d}$$

where the parameters β_{SD}, β_{PD}, β_{DD} and β_{2SD} are obtained by using variational method [6]. Since the initial state is the pseudoscalar B meson, its wave function $\phi_I(r)$ is

$$\phi_I(r) = \sqrt{4\pi}\,\frac{\beta_{SB}^{3/2}}{\pi^{3/4}}\exp\left[-\beta_{SB}^2 r^2/2\right] , \tag{24}$$

where we take β_{SB} as a variational parameter. In the following we fix the squared strong coupling constant as $\alpha = 0.5$ and the constituent quark masses as $m_u = m_d = 0.33$GeV. To determine the other parameters of the potential and trial wave function and constituent mass of bottom and charm quark, we use the following values as the inputs:

$$m_D = 1.869\,\mathrm{GeV}, \qquad m_{D^*} = 2.010\,\mathrm{GeV}, \qquad m_{D_1} = 2.424\,\mathrm{GeV}$$

$$m_{D_2^*} = 2.459\,\mathrm{GeV}, \qquad m_B = 5.279\,\mathrm{GeV},$$

where we assign $D_1(2420)$ as $D_{1F}(J_\ell^P = \frac{3}{2}^+)$.

The adjusted values of variational parameters which reproduce these input data are;

$$b = 0.158\,\mathrm{GeV}^2, \qquad c = -0.563\,\mathrm{GeV}, \qquad \beta_{SD} = 0.373\,\mathrm{GeV},$$
$$\beta_{PD} = 0.316\,\mathrm{GeV}, \qquad \beta_{DD} = 0.293\,\mathrm{GeV}, \qquad \beta_{2SD} = 0.293\,\mathrm{GeV},$$
$$\beta_{SB} = 0.393\,\mathrm{GeV}, \qquad m_c = 1.63\,\mathrm{GeV}, \qquad m_b = 4.98\,\mathrm{GeV} .$$

Inserting thus obtained wave functions into Eq.(22) we get the Isgur-Wise functions, $\xi_X(y)$,

$$\xi_C(y) = \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2+\beta_{SB}^2}\right)^{\frac{3}{2}}\exp\left[\frac{-\Lambda^2}{2(\beta_{SD}^2+\beta_{SB}^2)}(y^2-1)\right] \tag{25}$$

and so on. The calculated Isgur-Wise function by Eq.(25) does not reproduce the experimental form factor and gives too large branching ratio compared with the experimental values Eq.(2). Then we have to modify the function with compensation factor κ according to GISW [6], so that

$$\xi_C(y) = \left(\frac{2\beta_{SD}\beta_{SB}}{\beta_{SD}^2+\beta_{SB}^2}\right)^{\frac{3}{2}}\exp\left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{SD}^2+\beta_{SB}^2)}(y^2-1)\right] \tag{26a}$$

$$\xi_{E,F}(y) = \frac{1}{\sqrt{2}}\left(\frac{2\beta_{PD}\beta_{SB}}{\beta_{PD}^2+\beta_{SB}^2}\right)^{\frac{5}{2}}\exp\left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{PD}^2+\beta_{SB}^2)}(y^2-1)\right]\frac{\Lambda}{\beta_{SB}}\sqrt{y^2-1} \tag{26b}$$

$$\xi_G(y) = \frac{1}{2\sqrt{3}}\left(\frac{2\beta_{DD}\beta_{SB}}{\beta_{DD}^2+\beta_{SB}^2}\right)^{\frac{7}{2}}\exp\left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{DD}^2+\beta_{SB}^2)}(y^2-1)\right] \times\left(\frac{\Lambda}{\beta_{SB}}\sqrt{y^2-1}\right)^2 \tag{26c}$$

$$\xi_{C_2}(y) = \sqrt{\frac{2}{3}}\left(\frac{2\beta_{2SD}\beta_{SB}}{\beta_{2SD}^2+\beta_{SB}^2}\right)^{\frac{3}{2}}\exp\left[\frac{-\Lambda^2/\kappa^2}{2(\beta_{2SD}^2+\beta_{SB}^2)}(y^2-1)\right] \times\left\{\frac{3}{2}\sqrt{1-\left(\frac{2\beta_{2SD}\beta_{SB}}{\beta_{2SD}^2+\beta_{SB}^2}\right)^2}+\frac{1}{4}\left(\frac{2\beta_{2SD}\beta_{SB}}{\beta_{2SD}^2+\beta_{SB}^2}\right)^2\left(\frac{\Lambda}{\beta_{SB}}\sqrt{y^2-1}\right)^2\right\} \tag{26d}$$

We choose $\kappa = 0.61$ so that the sum of the obtained branching ratios $\mathrm{Br}(B\to D\,\ell\,\nu_\ell)$ and $\mathrm{Br}(B\to D^*\ell\,\nu_\ell)$ are consistent with the experiment, $\mathrm{Br}(B\to D\,\ell\,\nu_\ell)+\mathrm{Br}(B\to D^*\ell\,\nu_\ell) = 6.7\%$.

The calculated form factors $\xi_X(y)$ are shown in Fig. 1. For comparison we also show in Fig. 1 the function $\xi(y) = \exp\left[-\rho^2(y-1)\right]$, with $\rho = 1.08$ [12]. which is given by fitting the experimental data of the decay $B\to D^*\ell\,\nu_\ell$. From these figures it is shown that the Isgur-Wise functions for high angular momentum states are very small due to the little overlap integral and we can expect that contributions from the D_X states higher than D-wave (D_1^* and D_2) are negligibly small. Though the form factor for the first radially excited states $\xi_{C_2}(y)$ is rather large compared with those of the first angular momentum excitation states, the form factors for the second or higher radial excitation states become negligibly small.

The calculated results of decay rates are listed in Table 3. To calculate these decay rates we have use the physical parameters

$$G_F = 1.166 \times 10^{-5}(\mathrm{GeV})^{-2}, \quad |V_{cb}| = 0.042 \pm 0.001\ [5], \quad m_B = 5.279 \pm 0.002(\mathrm{GeV}),$$

and the degenerate masses of D_X ($X = E$, F, G and C_2) calculated from the constituent quark model which are also listed in Table 3. We also summarize the branching ratios of the decay modes $B \to D_X$ in the Table 4, where we take the total decay rate of B meson to be 5.10×10^{-13}GeV from the mean life time $\tau_B = 12.9 \pm 0.5 \times 10^{-13}$sec [4].

As shown in Table 3 and 4 the dominant decay modes of the semileptonic decay $B \to D_X \ell \nu_\ell$ are of course the decays to the ground states D and D^* which contribute about 6% of the total decay of the B meson and account for the major part of the inclusive semileptonic decay $B \to X \ell \nu_\ell$. The next contribution to the semileptonic decay other than D and D^* mesons comes from that of the first radial excitation states D_{C_2} and $D^*_{C_2}$ which are 0.1% and 0,26%, respectively.

Sum of the contributions of the semileptonic decays into D_X of the P- states ($X = E$ and F) and the D-states ($X = G$) are about 0.2% and 0.003%. From these result we may conclude that the contributions from the higher resonance states of D meson with orbital angular momentum $L \geq 3$ will be completely negligible due to the small overlapping integrals and small phase volumes. Contributions from the decays into radially excited states are also rapidly decrease with increase of the principal quantum number. The contribution of the $2S$ states less than one tenth of those of the $1S$ states. Then the total contribution from other than $1S$ states are only one tenth of that of the ground states.

The decay rates of semileptonic processes $B \to D_X \ell \nu_\ell$ are sensitive to the mass of D_X due to the form factor and the phase volume. If the masses of D_X are changed by ± 0.2GeV with the other parameters fixed, then the decay rate into the P-wave resonant states change by $\mp 160\%$ while into radially excited $2S$ states change by $\mp 130\%$. The ratios of longitudinal to transverse, however, are rather insensitive to the mass of D_X. We can get $L/T = 1.4$ for D_{1E} and $L/T = 5.8$ for D_{1F}. The decay rate of multiplet G is negligibly small so that its uncertainty due to the masses of G states could not give sensitive effects to the branching ratios of the inclusive decay $B \to X \ell \nu_\ell$. From Table 4 the total sum of the branching ratio of $B \to D_X \ell \nu_\ell$ other than the ground states D and D^* is about 0.6%.

4 Discussions and Summary

In the preceding sections we evaluated the decay rate and the branching ratio for the exclusive process $B \to D_X \ell \nu_\ell$ on the basis of the HQET. As shown in Table 4, the result for branching ratio of exclusive semileptonic processes to the S-wave ground states are $\mathrm{Br}(B \to D \ell \nu_\ell) = 1.7\%$ and $\mathrm{Br}(B \to D^* \ell \nu_\ell) = 5.0\%$. These calculated branching ratios reproduce the experimental branching ratios. The sum of the branching ratios of the processes into the S-wave ground states is $\mathrm{Br}(B \to S\text{-states}) = \mathrm{Br}(B \to D \ell \nu_\ell) + \mathrm{Br}(B \to D^* \ell \nu_\ell) = 6.7\%$, the one into P-wave states $\mathrm{Br}(B \to P\text{-states}) = 0.25\%$, and the one into D-wave states is estimated about 0.03%, which is negligible. For the radially excited states ($2S$-states) the sum of the branching ratios is $\mathrm{Br}(B \to 2S\text{-states}) = 0.36\%$. From these results, as expected, the exclusive branching raios $\mathrm{Br}(B \to D_X \ell \nu_\ell)$ decrease rapidly as the orbital angular momentum and the radial quantum number of excited D_X states increase. So the contribution of higher resonance to branching ratio may be enough converge with $1S$, $1P$, $1D$ and $2S$ state.

The sum of the calculated branching ratios of $\mathrm{Br}(B \to D_X \ell \nu_\ell)$ other than the ground states D and D^*, $\mathrm{Br}(B \to D_X \ell \nu_\ell;\ X \neq C) \cong 0.6\%$, is only one tenth of the branching ratio into D and D^* mesons, $\mathrm{Br}(B \to S\text{-states})$. This means that $\mathrm{Br}(B \to X_c \ell \nu_\ell) \cong 7.3\%$ and the remains of $\mathrm{Br}(B \to X \ell \nu_\ell) \cong 3.4\%$ may be continuum and non-charmed processes. Our result of the total branching fractions into charmed higher resonances is consistent with those given by GISW [6] and CNP [11]. We obtain the ratio

$$\begin{aligned} R &= \frac{\mathrm{Br}(B \to D_X \ell \nu_\ell;\ X \neq C)}{\mathrm{Br}(B \to X \ell \nu_\ell)} \\ &\simeq 0.06 . \end{aligned} \tag{27}$$

This value should be compared to the recent experimental value $R = 0.21 \pm 0.08$ [5]. From our result in Table 4 we can also evaluate the ratio between observed numbers of $N(D_X \ell^-)$ and $N(D^* \ell^-)$;

$$\begin{aligned} \frac{N(D_X \ell^-)}{N(D^* \ell^-)} &= \sum_{X \neq C} \frac{\mathrm{Br}(\bar{B}^0 \to D_X^+ \ell^- \bar{\nu}_\ell)\,\mathrm{Br}(D_X^+ \to D^*)\,\epsilon_X}{\mathrm{Br}(\bar{B}^0 \to D^{*+} \ell^- \bar{\nu}_\ell)} \\ &\lesssim 0.05 , \end{aligned} \tag{28}$$

where D_X means the higher resonance state other than D and D^* and $\mathrm{Br}(D_X \to D^*)$ are the decay branching ratios of D_X which are listed in Table 4. Though this value 0.05 is given as the efficiency $\epsilon_X = 1$, it is vary small in comparison with ARGUS's data 0.27 [12]. On the other hand, as shown in Table 4, the ones resulted from branching fractions into charmed higher resonances calculated by GISW and CNP are between 0.05 and 0.15, which are comparable with our result. Grinstein pointed out that the processes like $B \to D^{(*)}\pi\ell\nu_\ell$ could be as important correction to the inclusive semileptonic rate and it may be accounted by some of the anomalously large $D_X(X \neq C)$ contribution [13], which is not accounted in this paper. The expected large contribution to the continuum of the inclusive process $B \to X\ell\nu_\ell$ will come from the virtual B^* processes such as $B \to (B^*\pi) \to D^{(*)}\pi\ell\nu_\ell$, which is investigated in Ref.[14].

It is still open problem what causes this difference of the contribution of the excited states D_X between experimental data and theoretical predictions, however, to solve this problem is important for the applicability of HQET. We have to make theoretical investigation as well as to wait the further detailed experimental data.

In this paper we use the constituent quark model and it happens that $\xi_E(y) = \xi_F(y)$. This is caused from that the wave function of each meson is chosen to be eigenfunctions of orbital angular momentum L, not J_ℓ^P. The $1/m_Q$ correction may be important as the energy of light degrees of freedom increases. For example, in Ref.[11] the branching ratios for $1P$ states with $1/m_Q$ correction are $2 \sim 3$ times as ones in the heavy quark limit. And also it is meaningful to clarify the model-dependence and the $1/m_Q$ correction in the Isgur-Wise functions and four-body semileptonic decay from analyses of the experimental data of exclusive decay $B \to D_X\ell\nu_\ell$.

Appendix

In this Appendix we formulate the helicity decomposition method to obtain the decay rate for the D meson resonances with spin-2 as well as spin-0 and -1 which is developed by Gilman and Singleton [10]. The differential decay rate is given in terms of the helicity decomposition as:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{cb}|^2}{96\pi^3}\frac{K}{m_B^2}q^2\left(|\overline{H}_+|^2 + |\overline{H}_-|^2 + |\overline{H}_0|^2\right) , \qquad (29)$$

where subscripts of helicity amplitude denote the helicity of final meson state. In the $\ell\nu_\ell$ frame the helicity amplitudes become

$$\overline{H}_\pm = 0 , \qquad (30a)$$

$$\overline{H}_0 = -2\frac{K}{\sqrt{q^2}}m_B f_+(q^2) , \qquad (30b)$$

for $J^P = 0^-$ final meson, and

$$\overline{H}_\pm = f(q^2) \mp 2m_B K g(q^2) , \qquad (31a)$$

$$\overline{H}_0 = \frac{m_B^2}{2m_{D_X}\sqrt{q^2}}\left\{\left[1 - \frac{m_{D_X}^2}{m_B^2} - \frac{q^2}{m_B^2}\right] f(q^2) + 4K^2 a_+(q^2)\right\} , \qquad (31b)$$

for $J^P = 1^-$ final meson. The form factors $f_+(q^2)$, $g(q^2)$, $f(q^2)$ and $a_+(q^2)$ in above equations are related to those in Eqs.(13a)–(13c) by

$$f_+(q^2) = \frac{1}{2}\sqrt{\frac{m_B}{m_{D_X}}}\left\{\left(1 + \frac{m_B}{m_{D_X}}\right)\tilde{f}_+^C(y) - \left(1 - \frac{m_B}{m_{D_X}}\right)\tilde{f}_-^C(y)\right\} , \qquad (32a)$$

$$f(q^2) = \sqrt{m_B m_{D_X}}(1 + y)\tilde{f}^C(y) , \qquad (32b)$$

$$g(q^2) = \frac{1}{2\sqrt{m_B m_{D_X}}}\tilde{g}^C(y) , \qquad (32c)$$

$$a_+(q^2) = \frac{-1}{2\sqrt{m_B m_{D_X}}}\left\{\left(1 + \frac{m_B}{m_{D_X}}\right)\tilde{a}_+^C(y) - \left(1 - \frac{m_B}{m_{D_X}}\right)\tilde{a}_-^C(y)\right\} . \qquad (32d)$$

Also for $J^P = 0^+$, 1^+ final meson we obtain the decomposition the same as for $J^P = 0^-$, 1^- final meson replacing the Isgur-Wise form factors by corresponding ones. Then using Eqs(29)–(32d) and Table 2 we have the function $W_X(y, r_X)$ in the decay rate Eq.(17):

$$\frac{d\Gamma_X}{dy} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}m_B^2\sqrt{y^2 - 1}\, m_{D_X}^3 W_X(y, r_X)\xi_X(y)^2 , \qquad (33)$$

for each spin and the polarization (longitudinal and transverse), and $W_X(y, r_X)$ are listed in Table 5.

We proceed to the final meson with spin $J^P = 2^\pm$ by extending the method given in Ref.[10]. In the expression of the decay amplitude squared Eq.(10) only the spacial components of $L^{\mu\nu}$ are non zero in the $\ell\nu_\ell$ frame, because we neglect the mass of the leptons. So we only need the spacial components $\vec{H}$ of H_μ in the $\ell\nu_\ell$ frame,

$$\begin{aligned}
\vec{H} &= 2i\sqrt{q^2}m_B g(q^2)(\overrightarrow{\varepsilon^* P}) \times \vec{p}_3 - f(q^2)(\overrightarrow{\varepsilon^* P}) - 2(\varepsilon^* PP)a_+(q^2)\vec{p}_3 , \\
(\overrightarrow{\varepsilon^* P}) &\equiv (\varepsilon_{1\lambda}^* P^\lambda, \varepsilon_{2\lambda}^* P^\lambda, \varepsilon_{3\lambda}^* P^\lambda), \\
(\varepsilon^* PP) &\equiv (\varepsilon_{\nu\lambda}^* P^\nu P^\lambda),
\end{aligned} \qquad (34)$$

where $g(q^2)$, $f(q^2)$ and $a_+(q^2)$ are form factors such as

$$\begin{aligned}
\langle D_2(p_3)|J^\mu(0)|B(P)\rangle = &-ig(q^2)\epsilon_{\mu\nu\rho\sigma}(\varepsilon^{*\nu\lambda}P_\lambda)(P + p_3)^\rho(P - p_3)^\sigma + f(q^2)(\varepsilon_{\mu\lambda}^* P^\lambda) \\
&+ a_+(\varepsilon_{\nu\lambda}^* P^\nu P^\lambda)(P + p_3)_\mu + a_-(\varepsilon_{\nu\lambda}^* P^\nu P^\lambda)(P - p_3)_\mu , \qquad (35)
\end{aligned}$$

so on. To proceed further, we express the polarization tensor $\varepsilon^*_{\nu\lambda}$ in the concrete form. Because the degree of freedom of polarization is five, we put

$$\varepsilon^{*\mu\nu} = \sum_{a=1}^{5} \alpha_a \lambda_a^{\mu\nu}, \qquad \sum_{a=1}^{5} \alpha_a^2 = 5,$$

where α_a is coefficient for each polarization tensor $\lambda_a^{\mu\nu}$ (a =1,2,...,5) which is transverse to p_3^μ, symmetric and traceless tensor and which is normalized as $g_{\mu\rho} g_{\nu\sigma} \lambda_a^{\mu\nu} \lambda_b^{\rho\sigma} = \delta_{ab}$. In the rest frame of final meson, $p_3^\mu = (m_{D_X}, 0, 0, 0)$, λ_a (a =1, 2,...,5) are given by

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&1&0&0 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix}, \quad \sqrt{\frac{2}{3}}\begin{pmatrix} 0&0&0&0\\ 0&-1/2&0&0\\ 0&0&-1/2&0\\ 0&0&0&1 \end{pmatrix},$$
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0&0&0&0\\ 0&0&1&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&0&-1&0\\ 0&0&0&0 \end{pmatrix}. \qquad (36)$$

In the $\ell \nu_\ell$ frame, where we define the positive z axis as the direction of the $-\vec{p}_3$ and let $p \equiv |\vec{p}_3| = MK/\sqrt{q^2}$, the momenta of neutrino and the B meson p_3^μ and P_μ become as

$$\begin{aligned} p_3^\mu &= (E_{D_X}, 0, 0, -p), \\ P^\mu &= (E_B, 0, 0, -p), \end{aligned} \qquad (37)$$

where

$$\begin{aligned} E_{D_X} &= \frac{M^2}{2\sqrt{q^2}}(1 - r_X^2 - \frac{q^2}{M^2}), \\ E_B &= \frac{M^2}{2\sqrt{q^2}}(1 - r_X^2 + \frac{q^2}{M^2}). \end{aligned} \qquad (38)$$

In this frame the matrices λ_a are given by

$$\frac{1}{\sqrt{2}m_{D_X}}\begin{pmatrix} 0&p&0&0\\ p&0&0&-E_{D_X}\\ 0&0&0&0\\ 0&-E_{D_X}&0&0 \end{pmatrix}, \quad \frac{1}{\sqrt{2}m_{D_X}}\begin{pmatrix} 0&0&p&0\\ 0&0&0&0\\ p&0&0&-E_{D_X}\\ 0&0&-E_{D_X}&0 \end{pmatrix},$$

$$\sqrt{\frac{2}{3}}\frac{p}{m_{D_X}^2}\begin{pmatrix} p&0&0&-E_{D_X}\\ 0&-1/2&0&0\\ 0&0&-1/2&0\\ -E_{D_X}&0&0&E_{D_X}^2/p \end{pmatrix},$$
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0&0&0&0\\ 0&0&1&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&0&-1&0\\ 0&0&0&0 \end{pmatrix}. \qquad (39)$$

So in the $\ell \nu_\ell$ frame

$$\begin{aligned} (\overrightarrow{\varepsilon^* P}) &= \frac{m_B K}{m_{D_X}}\left(\frac{\alpha_1}{\sqrt{2}}, \frac{\alpha_2}{\sqrt{2}}, -\alpha_3\sqrt{\frac{2}{3}}\frac{E_{D_X}}{m_{D_X}}\right), \\ (\varepsilon^* PP) &= \alpha_3\sqrt{\frac{2}{3}}\left(\frac{m_B K}{m_{D_X}}\right)^2 \end{aligned} \qquad (40)$$

Then in the same sense of Ref.[10] we have that for spin-2 final meson

$$\overline{H}_\pm = \frac{m_B K}{\sqrt{2}m_{D_X}}\left[f(q^2) \pm 2m_B K\, g(q^2)\right], \qquad (41a)$$
$$\overline{H}_0 = \sqrt{\frac{2}{3}}\frac{m_B^3 K}{2m_{D_X}^2\sqrt{q^2}}\left[\left(1 - r_X^2 - \frac{q^2}{m_B^2}\right) f(q^2) + 4K^2 a_+(q^2)\right]. \qquad (41b)$$

The relation of the form factors $f(q^2)$, $g(q^2)$ and $a_+(q^2)$ to ours are

$$f(q^2) = \sqrt{\frac{m_{D_X}}{m_B}}\,\tilde{f}^X(y), \qquad (42a)$$
$$g(q^2) = \frac{1}{2m_B\sqrt{m_B m_{D_X}}}\,\tilde{g}^X(y), \qquad (42b)$$
$$a_+(q^2) = \frac{-1}{2m_B\sqrt{m_B m_{D_X}}}\left\{(1 + r_X)\tilde{a}_+^X(y) - (1 - r_X)\tilde{a}_-^X(y)\right\}. \qquad (42c)$$

Inserting Eqs.(41a)–(42c) into Eq.(29) we get the function $W_X(y, r_X)$ for the each polarization (longitudinal and transverse) of spin-2 as in Table 5. These result in Table 5 agree with ones in section 2.

We notice here that in the case of longitudinal component with spin-1 and -2 D_X states the factor q^2 in Eq.(29) is cancelled by $1/\sqrt{q^2}$ in $\overline{H}_0$ but in the case of transverse component with D_X the factor q^2 remains so that only transverse components have the factor $(1-2r_X\,y+r^2)$ as seen in Table 5. Factor $1/\sqrt{q^2}$ of $\overline{H}_0$ is due to the Lorentz boost of polarization vector or tensor from the rest frame of final meson to the $\ell\,\nu_\ell$ frame.

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Figure caption

Fig. 1 : Isgur-Wise function $\xi_X(y)$.
ξ_C, $\xi_{E,F}$, ξ_G and ξ_{C_2}, which are calculated with the degenerate masses, are shown as the upper solid line, the upper dashed line, the lower dashed line and the lower solid line, respectively. For comparison, the dotted line is the Isgur-Wise function $\xi_C(y)=\exp\left[-\rho^2(y-1)\right]$ ($\rho=1.08$, $|V_{cb}|=0.045$) given by fitting the experimental data [12].

Table 1: The heavy flavored meson field

	J_ℓ^P	J^P	D_X	$H_X(v)$ $[\mathcal{H}_X(v) = \sqrt{m_X}\, \bar\ell_X(v)\, H_X(v)\, h(v)]$
C	$\frac{1}{2}^-$	0^-	D	γ_5
		1^-	D^*	$\not\epsilon$
E	$\frac{1}{2}^+$	0^+	D_0^*	1
		1^+	D_{1E}	$\gamma_5 \not\epsilon$
F	$\frac{3}{2}^+$	1^+	D_{1F}	$\sqrt{\frac{3}{2}}\, \gamma_5 \left[\epsilon^\mu - \frac{1}{3} \not\epsilon\, (\gamma^\mu - v^\mu)\right]$
		2^+	D_2^*	$\gamma_\nu\, \epsilon^{\mu\nu}$
G	$\frac{3}{2}^-$	1^-	D_1^*	$\sqrt{\frac{3}{2}} \left[\epsilon^\mu - \frac{1}{3} \not\epsilon\, (\gamma^\mu - v^\mu)\right]$
		2^-	D_2	$\gamma_5\, \gamma_\nu\, \epsilon^{\mu\nu}$
C_2	$\frac{1}{2}^-$	0^-	D_{C_2}	γ_5
		1^-	$D_{C_2}^*$	$\not\epsilon$

Table 2: The relation of form factors to $\xi_X(y)$

X	C $(J_\ell^P = \frac{1}{2}^-)$		E $(J_\ell^P = \frac{1}{2}^+)$	
J^P	0^-	1^-	0^+	1^+
		$\tilde g^C(y) = \xi_C(y)$		$\tilde g^E(y) = \xi_E(y)$
		$\tilde f^C(y) = \xi_C(y)$		$\tilde f^E(y) = \frac{y-1}{y+1}\, \xi_E(y)$
	$\tilde f_+^C(y) = \xi_C(y)$	$\tilde a_+^C(y) = \frac{1}{2}\, \xi_C(y)$	$\tilde f_+^E(y) = 0$	$\tilde a_+^E(y) = \frac{1}{2}\, \xi_E(y)$
	$\tilde f_-^C(y) = 0$	$\tilde a_-^C(y) = -\frac{1}{2}\, \xi_C(y)$	$\tilde f_-^E(y) = -\xi_E(y)$	$\tilde a_-^E(y) = \frac{1}{2}\, \xi_E(y)$

X	F $(J_\ell^P = \frac{3}{2}^+)$		G $(J_\ell^P = \frac{3}{2}^-)$	
J^P	1^+	2^+	1^-	2^-
	$\tilde g^F(y) = \frac{y+1}{\sqrt{6}}\, \xi_F(y)$	$\tilde g^F(y) = \xi_F(y)$	$\tilde g^G(y) = \frac{-y+1}{\sqrt{6}}\, \xi_G(y)$	$\tilde g^G(y) = \xi_G(y)$
	$\tilde f^F(y) = \frac{y-1}{\sqrt{6}}\, \xi_F(y)$	$\tilde f^F(y) = \xi_F(y)$	$\tilde f^G(y) = \frac{-y+1}{\sqrt{6}}\, \xi_G(y)$	$\tilde f^G(y) = \frac{y-1}{y+1}\, \xi_G(y)$
	$\tilde a_+^F(y) = \frac{y-5}{2\sqrt{6}}\, \xi_F(y)$	$\tilde a_+^F(y) = \frac{1}{2}\, \xi_F(y)$	$\tilde a_+^G(y) = \frac{-y+1}{2\sqrt{6}}\, \xi_G(y)$	$\tilde a_+^G(y) = \frac{1}{2}\, \xi_G(y)$
	$\tilde a_-^F(y) = \frac{-y-1}{2\sqrt{6}}\, \xi_F(y)$	$\tilde a_-^F(y) = -\frac{1}{2}\, \xi_F(y)$	$\tilde a_-^G(y) = \frac{y+5}{2\sqrt{6}}\, \xi_G(y)$	$\tilde a_-^G(y) = -\frac{1}{2}\, \xi_G(y)$

X	C_2 $(J_\ell^P = \frac{1}{2}^-)$			
J^P	0^-	1^-		
		$\tilde g^C(y) = \xi_C(y)$		
		$\tilde f^C(y) = \xi_C(y)$		
	$\tilde f_+^C(y) = \xi_C(y)$	$\tilde a_+^C(y) = \frac{1}{2}\, \xi_C(y)$		
	$\tilde f_-^C(y) = 0$	$\tilde a_-^C(y) = -\frac{1}{2}\, \xi_C(y)$		

Table 3: $\Gamma(B \to D_X \ell \nu_\ell)$ in unit $|V_{cb}/0.042|^2$ GeV

		J_ℓ^P	J^P	D_X	m_{D_X}(GeV)	$\Gamma(B \to D_X \ell \nu_\ell)$
1S	C	$\frac{1}{2}^-$	0^-	D	1.869	8.60×10^{-15}
			1^-	D^*	2.010	25.54×10^{-15}
1P	E	$\frac{1}{2}^+$	0^+	D_0^*	2.446	0.13×10^{-15}
			1^+	D_{1E}	2.446	0.17×10^{-15}
	F	$\frac{3}{2}^+$	1^+	D_{1F}	2.424	0.40×10^{-15}
			2^+	D_2^*	2.459	0.57×10^{-15}
1D	G	$\frac{3}{2}^-$	1^-	D_1^*	2.800	0.0083×10^{-15}
			2^-	D_2	2.800	0.0085×10^{-15}
2S	C_2	$\frac{1}{2}^-$	0^-	D_{C_2}	2.673	0.52×10^{-15}
			1^-	$D_{C_2}^*$	2.673	1.34×10^{-15}

Table 4: $Br(B \to D_X \ell \nu_\ell)$ in unit $(\tau_B/1.29ps)$%

		J_ℓ^P	J^P	D_X	$Br(B \to D_X \ell \nu_\ell)$	GISW	CNP	$Br(D_X \to D^*)$
1S	C	$\frac{1}{2}^-$	0^-	D	1.69		1.54	-
			1^-	D^*	5.00		4.47	-
1P	E	$\frac{1}{2}^+$	0^+	D_0^*	0.025	0.11	0.06	0
			1^+	D_{1E}	0.033	0.21†	0.08	1
	F	$\frac{3}{2}^+$	1^+	D_{1F}	0.079	0.41‡	0.11	1
			2^+	D_2^*	0.11	0.14	0.22	0.29
1D	G	$\frac{3}{2}^-$	1^-	D_1^*	0.0016			0.15
			2^-	D_2	0.0017			1
2S	C_2	$\frac{1}{2}^-$	0^-	D_{C_2}	0.10	0.07		1
			1^-	$D_{C_2}^*$	0.26	0.06		0.37

The values of GISW are estimated for $Br(B \to D, D^* \ell\nu) = 6.7\%$, and the relative ratio of the each mode are referred from Ref.[12]. The values of CNP are estimated for $|V_{cb}| = 0.042$.

† value for $^3P_1$ state.

‡ value for $^1P_1$ state.

Table 5: Integrand $W_X(y, r_X)$

	J_ℓ^P	$J_{L,T}^P$	$W_X(y, r_X)$
C & C_2	$\frac{1}{2}^-$	0^-	$(y-1)(y+1)(1+r)^2$
		1_L^-	$(y+1)^2(1-r)^2$
		1_T^-	$4y(y+1)(1-2ry+r^2)$
E	$\frac{1}{2}^+$	0^+	$(y-1)(y+1)(1-r)^2$
		1_L^+	$(y-1)^2(1+r)^2$
		1_T^+	$4y(y-1)(1-2ry+r^2)$
F	$\frac{3}{2}^+$	1_L^+	$\frac{2}{3}(y-1)^2(y+1)^2(1+r)^2$
		1_T^+	$\frac{2}{3}y(y-1)(y+1)^2(1-2ry+r^2)$
		2_L^+	$\frac{2}{3}(y-1)(y+1)^3(1-r)^2$
		2_T^+	$2y(y-1)(y+1)^2(1-2ry+r^2)$
G	$\frac{3}{2}^-$	1_L^-	$\frac{2}{3}(y-1)^2(y+1)^2(1-r)^2$
		1_T^-	$\frac{2}{3}y(y-1)^2(y+1)(1-2ry+r^2)$
		2_L^-	$\frac{2}{3}(y-1)^3(y+1)(1+r)^2$
		2_T^-	$2y(y-1)^2(y+1)(1-2ry+r^2)$

The subscripts X of r_X are omitted in the table. L and T mean longitudinal and transverse components, respectively.

Fig.1 Isgur-Wise Function