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ZIGZAG BEHAVIOR OF THE YRAST–YRARE INTERACTION STRENGTHS IN ISOTOPE CHAIN $^{166-172}Yb$

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Zigzag behavior of the yrast-yrare interaction strengths in isotope chain $^{166-172}$Yb

Understanding from the Particle-Number-Conserving treatment of the Cranked Shell Model [1]

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I INTRODUCTION

1. In experiment: a drastic oscillation of yrast-yrare interaction strength V exist in the even ytterbium isotope chain.
    - The lighter even ytterbium isotopes ($^{158-166}$Yb) display sharp first backbendings.
    - No backbending has been found in ^{168}Yb up to $I^\pi \sim 38^+$.
    - ^{170}Yb exhibits a sharp backbending at $I^\pi \sim 14^+$.
    - For the heavier istopes (^{172,174,176}Yb) no backbending has been found up to $I^\pi \sim 16^+$.
2. In theory:
    - The cranked shell model (CSM) has proved to be very fruitful for describing the yrast and near-yrast spectra of deformed nuclei.
    - The HFB calculation for a single-j ($j = 13/2$) CSM showed that the yrast-yrare interaction strength V is a periodic function of the degree of shell filling, and thus a sharp backbending may be expected not only at the bottom of the neutron $i13/2$ shell but also may be obtained for appropriate configurations even near the top of the shell. However, the question is that the correctness of the so-called mean field approximation introduced in the HFB formalism is very hard to be justified. In principle, only a smooth result averaged over certain mass region arround the desired mass number can be obtained from the HFB calculation due to the nonconservation of the particle number.

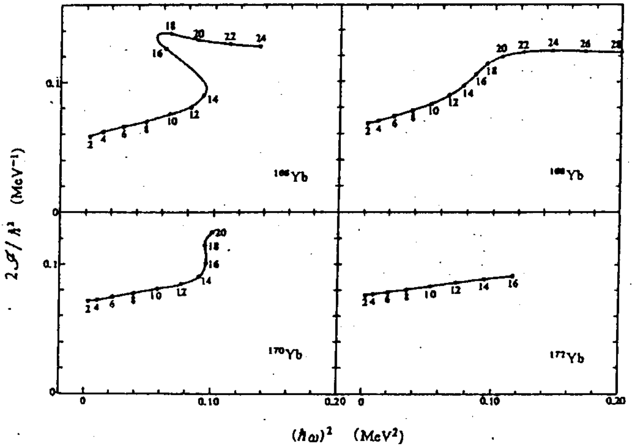

Fig. 1. Observed yrast sequences in ytterbium isotope chain $^{166-172}$Yb in the form of backbending plots.

- The particle-number-conserving (PNC) calculation in a single-j CSM definitely shows that the yarst-yrare interaction is always strong and no periodic oscillation of V with the degree of shell filling is found. Physically, this result is reasonable because all the particles in the high-j shell have strong Coriolis response, and therefore, it is impossible to get an yrast band which has no (or only a very weak) Coriolis response, as the requirement for a quasiparticle vacuum. The single-j model is too simplified to account for the observed sharp backbending.
- Because the observed magnitude of yrast-yrare interaction depends very sensitively on the neutron numbers, any reliable statement should be drawn from the PNC treatment.

II THE PNC FORMALISM

As usual, for an axially symmetric nucleus rotating about an axis (e.g., x-axis) perpendicular to the symmetry axis (z-axis), the CSM Hamiltonian in the rotating frame is expressed as

$$\begin{aligned} H_{\rm CSM} &= H_{\rm intr} + H_C \\ &= H_{\rm sp} + H_{\rm P} + H_C \end{aligned} \qquad (1)$$

where $H_C = -\omega J_x$ is the Coriolis interaction, $H_{\rm sp}$ the single-particle Hamiltonian (Nilsson), and $H_{\rm P}$ the monopole pairing interaction,

$$H_{\rm P} = -G \sum_{\Omega_\mu,\Omega_\nu>0} (-)^{\Omega_\mu-\Omega_\nu} a_\mu^\dagger a_{-\mu}^\dagger a_{-\nu} a_\nu . \qquad (2)$$

$H_{\rm CSM}$ is diagonalized in a sufficiently large many-particle configuration (MPC) space, i.e. all the configurations with energies $(E - E_0) \leq E_C$ are taken into account in the diagonalization of $H_{\rm CSM}$

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to obtain the low-lying eigenstates, where E_0 is the energy of the lowest (ground) configuration and E_C the truncation energy which should be sufficiently large.

For details of the PNC approach, please see:
C. S. Wu and J. Y. Zeng, Phys. Rev. C39, 666 (1989).
C. S. Wu and J. Y. Zeng, Phys. Rev. C40, 998 (1989).
C. S. Wu and J. Y. Zeng, Phys. Rev. C41, 1822 (1990).

III RESULTS AND ANALYSES

A Single particle level scheme and other related parameters

- A common neutron Nilsson level scheme (Lund systematics, $Z = 70, A = 170$ parameters) is adopted in the calculation for all of these nuclei because we are interested in the abrupt change of V with increasing neutron number N rather than their minor details.
- $E_C/\hbar\omega_0 = 0.6$, $G/\hbar\omega_0 = 0.035$ ($\hbar\omega_0 \sim 7.837$MeV). The choice of these values is somewhat arbitrary but seems reasonable because they can reproduce roughly the observed bandhead energies of the low-lying high-K bands.

TABLE I The bandhead energies of some low-lying high-K bands in ytterbium isotopes

Nuclei	I^π	Observed bandhead energies (MeV)	Suggested neutron pair-broken states	Calculated energies (MeV)
^{168}Yb	6^-	1.8422	$7/2^+[633]+5/2^-[523]$	1.80
^{170}Yb	4^-	1.2587	$7/2^+[633]+1/2^-[521]$	1.29
	6^-	1.8514	$7/2^+[633]+5/2^-[523]$	1.64
^{172}Yb	6^-	1.5506	$7/2^+[633]+5/2^-[512]$	1.48
	4^-	1.6406	$7/2^+[633]+1/2^-[521]$	1.86
	4^+	2.5997	$7/2^-[514]+1/2^-[521]$	2.87

B Routhians and aligned angular momenta

TABLE II The calculated bandcrossing frequencies ω_c, yrast-yrare interaction strengths V and spin alignment gains Δi

	$\hbar\omega_c$ (keV)		V (keV)		Δi ($\hbar$)	
Nuclei	Calculated	exptl.	Calculated	exptl.	Calculated	exptl.
^{166}Yb	309	271	52		8.0	$8.1^{8.4}_{7.9}$
^{168}Yb	270	297	450		5.5	$6.4^{7.0}_{5.7}$
^{170}Yb	355	307	109		4.8	$6.1^{7.5}_{5.5}$
^{172}Yb	488		504		~ 1	

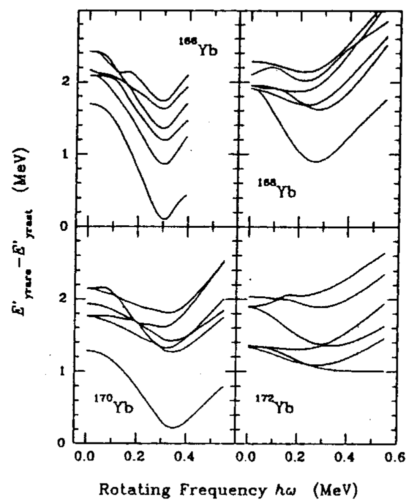

Fig. 2. Calculated $(E'_{\rm yrare} - E'_{\rm yrast})/\hbar\omega_0$ vs ω plots for $^{166-172}$Yb. $G/\hbar\omega_0 = 0.035$.

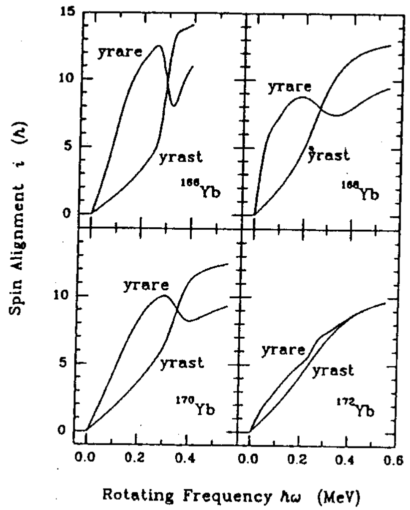

Fig. 3. Spin alignments $i = -dE'/d\omega$ derived from the calculated energy spectra.

C Pair gap

In the HFB (mean field) approximation the pair gap $\Delta = G(P)$ is as used an adjustable input and usually is assumed to be a constant. But in some papers it was also assumed that Δ decreases gradually with increasing ω according to the following formula

$$\Delta(\omega) = \begin{cases} \Delta_0\left[1 - \frac{1}{2}\left(\frac{\omega}{\bar\omega}\right)^2\right], & \omega \le \bar\omega \\ \frac{\Delta_0}{2}\left(\frac{\bar\omega}{\omega}\right)^2, & \omega \ge \bar\omega \end{cases} \tag{3}$$

and $\hbar\bar\omega \approx 0.7$MeV, estimated from the self-consistent calculation with particle number projection before variation, performed for a few nuclei.

In the PNC formalism, the definition of the pair gap $\Delta = G(P)$ should be replaced by

$$\tilde\Delta = G\sqrt{\langle P^\dagger P\rangle}. \tag{4}$$

where the pairing interaction strength G is employed as a direct input instead of the pair gap. Therefore, $\tilde\Delta$ is ω-dependent and different for different bands. The calculated $\tilde\Delta$ of the yrast band decreases with increasing ω and is closer to the behavior described by eq. (3).

D "Quasiparticle" structures

- In fact, the behavior of first backbending is determined mainly by the distribution of the low-lying pair-excitational states ($0_2^+, 0_3^+, \cdots$) and the intruder pair-broken $K^\pi = 1^+$ states.

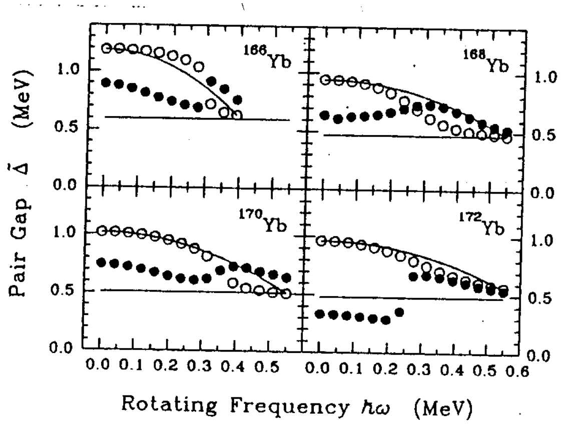

Fig. 4. The calculated pair gap $\tilde{\Delta}(\omega)$. The open and filled circles denote the claculated values for the yrast and yrare bands, respectively, while the curves stand for those estimated by eq. (3) with a proper choice of paramter $\hbar\tilde{\omega}$. $\hbar\tilde{\omega} \sim 0.4$MeV for ^{166}Yb and $\hbar\tilde{\omega} \sim 0.55$MeV for the others.

- For ^{166}Yb and ^{170}Yb, there exist several low-lying pair-excitational states (0_2^+, 0_3^+, $\cdots$) and intruder pair-broken $K^\pi = 1^+$ (and 2^+) states. The strong Coriolis coupling among them results in a rapid drop of the yrare band with increasing ω, hence occur sharp bandcrossings.
- In ^{168}Yb, the pair-excitational states are located rather high because the Fermi surface is just situated in the large gap of the Nilsson level schem ($N = 98$), In ^{172}Yb, though the positions of 0_2^+ and 0_3^+ are not too high, the intruder pair-broken states $K^\pi = 1^+$ are rather high.

E Occupation number distributions for high-j shell

- For ^{170}Yb, exchange of the occupation number distributions for the high-j orbits occurs and results in a sharp backcrossing. A similar situation occurs in ^{166}Yb.
- For ^{168}Yb and ^{172}Yb, the occupation probabilities remain nearly unchanged as ω increases and no significant exchange between the yrast and yrare bands is found.
- The total neutron numbers in the high-j ($i13/2$) shell, $n_{i13/2}$: In ^{168}Yb and ^{172}Yb, the values of $n_{i13/2}$ remain almost unchanged with increasing ω, while in ^{166}Yb and ^{170}Yb abrupt changes of $n_{i13/2}$ appear at $\omega \sim \omega_c$.

F Occupation probabilities of single particle levels

More detailed information can also be obtained from the occupation probabilities of each single particle level.

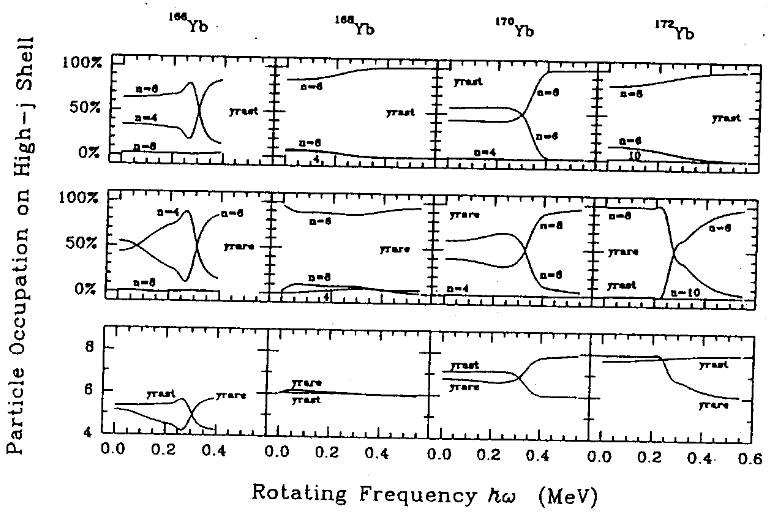

Fig. 5. The occupation number distributions for the high-j ($i13/2$) shell in the yrast and yrare bands. Top: the occupation probabilities for the yrast bands. Medium: those for the yrare bands. Bottom: the total numbers of particles occupying in the $i13/2$ shell.

IV DISCUSSION

The zigzag behavior of yrast-yrare interaction strengths in the ytterbium isotope chain $^{166-172}$Yb was investigated in the framework of the PNC formalism for treating the eigenvalue problem of the CSM Hamiltonian. The PNC calculation shows that the coexistence of high-j intruder orbits and normal orbits is indispensable for emergence of sharp backbendings. The occupation number distribution for the high-j orbits plays a key role for understanding the physics of backbending. A sharp backbending is always accompanied by an abrupt exchange of the occupation number distribution (for the high-j shell) between the yrast and yrare bands. The bandcrossing feature can be qualitatively predicted from the distribution of the low-lying eigenspectrum of $H_{\rm intr} = H_{\rm CSM}(\omega = 0)$ ("quasiparticle spectrum"), particularly, the distribution of the low-lying pair-excitational $v = 0, K^\pi = 0^+$ states and the intruder pair-broken ($v = 2$) $K^\pi = 1^+$ states, which in turn is determined by the single particle level distribution near the Fermi surface. Therefore, systematic investigation of the bandcrossing in the framework of the PNC formalism would provide valuable information on the single particle level distribution.

Open questions: for quantitative reproduction of the experimental data (e.g. bandcrossing frequency), more improvements are needed:

- Deformation changes ?
- Pairing changes ?
- Quadrupole pairing ?
- Spin-dependent interaction ?