SNUTP 93-89

The Self-Energy Corrections to the Light-Cone Two-Body Equation in ϕ^3-Theories

CC

Chueng-Ryong Ji
Department of Physics
North Carolina State University
Raleigh, North Carolina 27695-8202

Abstract

The light-cone ladder approximation in the Wick-Cutkosky model is extended to the lowest order light-cone Tamm-Dancoff approximation which includes the self-energy corrections and counter-terms. The light-cone two-body equation is modified by the term corresponding to the self-energy corrections and counter-terms. The analytic relation between the coupling constant, α, and the binding energy, β, which was previously derived for all nl states with $l = n - 1$ under the light-cone ladder approximation is also modified by these corrections and compared with the numerical results obtained by a variational principle. The numerical estimate of this modification shows that self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects and makes β become frozen as α increases. The probability of finding the three-body component inside the bound-state is also compared with that of finding the two-body component and the ratio of two probabilities is calculated for several different α values investigating the self-energy effects to this ratio.

A relativistic treatment is necessary for the bound-state problems which are characterized by strong coupling constant and small constituent masses. An important physical example is the description of hadrons composed of the light constituents such as up and down quarks and antiquarks as well as gluons in the quantum chromodynamics (QCD). For an accurate calculation of the spectra and wavefunctions of hadrons as well as the form factors and the structure functions in exclusive and inclusive hadronic processes, it is important to include the fundamental relativistic effects such as the correct relativistic energy-momentum relation, retardation effects, particle and antiparticle pair production.

A conventional tool for dealing with the relativistic two-body problem is the Bethe-Salpeter formalism[1] utilizing the Green functions of covariant perturbation theory. Since the Bethe-Salpeter formalism was introduced, much effort has been concentrated on solving the Bethe-Salpeter equation analytically or numerically in the lowest order approximation (ladder approximation). However, this approach is not well suited to the fundamental problem of including higher order irreducible kernels (cross diagrams, vacuum fluctuations, etc.)[2]. The relative time dependence of the covariant formalism adds uncertainties and complexities.

An alternative approach which can remove these difficulties and restore a systematic calculation for obtaining higher accuracy is the reformulation of the covariant Bethe-Salpeter equation at the equal light-cone time, $\tau = t + z/c$ [3]. This is equivalent to expressing the Bethe-Salpeter equation in the infinite momentum frame[4]. The light-cone quantization method[5] provides a hamiltonian formalism and a Fock-state representation at equal τ, which retains all of the simplicity and utility of the Schrödinger nonrelativistic many-body theory. This method not only suppresses the vacuum fluctuations but also systematically includes cross diagrams when higher-state contributions are taken into account.

It may be worth mentioning at this point that the boost problem[6] of the equal t quantization is changed to the rotation problem[7] in the light-cone equal τ quantization. The rotational invariance is violated in the light-cone quantization method, when the Fock-space is truncated for practical calculations. The restoration of the rotational symmetry in the light-cone quantization has been discussed[8,9,10], and the advantage due to the compactness of the rotation group was pointed out[9].

For simplicity, we consider a scalar field model which describes the interaction between two scalar particles $\phi, \bar{\phi}$ with equal mass m exchanging a scalar particle χ with mass λ. This model with $\lambda = 0$ is known as the Wick- Cutkosky model[11]. The interaction lagrangian is given by

$$L = g\phi^2\chi \ , \tag{1}$$

where g is the coupling constant. The first step in solving the full set of coupled Fock-state equations on the light-cone is to find a simple, analytically tractable equation for the lowest-particle-number sector, and to develop a systematic expansion for obtaining higher particle number states and greater accuracy. However, this model does not have a stable vacuum due to the presence of the cubic coupling[12]. The instability of the vacuum would be indicated by the appearance of imaginary masses[13]. Due to the vacuum instability of the cubic theory, the calculation in this model cannot be pressed beyond a certain level of resolution and particle number. Fortunately, a recent numerical calculation[14] based on the descretized light-cone quantization (DLCQ) method[15] indicated that the instability does not appear in this model if the Fock-space is truncated up to the three-body even though the increment of the number of particles more than three may not lead to meaningful results. Thus far, in this model, the light-cone two-body equation was investigated based on the light-cone ladder approximation which corresponds to the truncation of the Fock-space up to the three-body but neglecting the self-energy contribution. The analytic expressions for the eigenfunctions of this equation were obtained by Karmanov[16] and the analytic relation between the coupling constant and the binding energy was derived for all nl states with $l = n - 1$[17] and compared with the numerical results obtained by a variational principle[18]. Recently, the light-cone scattering formalism was also developed[19] and the phase-shifts of the two-body scattering was analyzed in this model[7].

In this short note, the light-cone ladder approximation is extended to the lowest

order light-cone Tamm-Dancoff approximation which includes the self-energy contribution under the truncation of the Fock-space up to the three-body. To go beyond the lowest order approximation and include higher-orders in the Tamm-Dancoff expansion, we would need to change the present model to other model that does not suffer from the vacuum instability. Here, however, we do not change the model but limit our scope of the calculations up to the three-body to remove the vacuum instability. The theory is then sufficient at least for our purposes of investigating the self-energy effects in the lowest order light-cone Tamm-Dancoff approximation of the Wick-Cutkosky model. We apply the renormalization procedure[6] proposed recently to this model and find the modification in the light-cone two-body equation by the term corresponding to the self-energy corrections and the counter-terms. Consequently, we modify the analytic relation between the coupling constant and the binding energy which was previously derived[17] for all nl states with $l = n - 1$ under the light-cone ladder approximation. The numerical estimate of this modification is discussed. However, we note that the derivation of the analytic relation uses an approximation beyond that of the Tamm-Dancoff truncation. In order to provide an evidence of accuracy, we compare the analytic results with the numerical results obtained by a variational principle. We also compute the probability of finding the three-body component inside the bound-state and compare with that of finding the two-body component for several different values of the coupling constant to further investigate the self-energy effects.

The light-cone quantization method provides a Fock-state representation at equal τ for a bound state $|B>$

$$|B> = <\phi\bar{\phi}|B> |\phi\bar{\phi}> + <\phi\bar{\phi}\chi|B> |\phi\bar{\phi}\chi> + \cdots \ . \tag{2}$$

The bound state $|B>$ with mass M and four-momentum p is an eigenstate of the light-cone hamiltonian $H_{LC} = P^- = P^0 - P^3$,

$$(M^2 - H_{LC})|B> = 0 \ , \tag{3}$$

where the light-cone frame $p^+ = p^0 + p^3 = 1$ and $\vec{p}_\perp = 0$ is chosen. Projecting Eq.

(3) on the Fock-space composed of $<\phi\bar{\phi}|, <\phi\bar{\phi}\chi|, \cdots$ results in an infinite number of coupled integral equations,

$$(M^2-\sum_i k_i^-)\begin{bmatrix} <\phi\bar{\phi}|B> \\ <\phi\bar{\phi}\chi|B> \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 & <\phi\bar{\phi}|V|\phi\bar{\phi}\chi> & \cdots \\ <\phi\bar{\phi}\chi|V|\phi\bar{\phi}> & 0 & \cdots \\ \vdots & \vdots & \vdots \end{bmatrix}\begin{bmatrix} <\phi\bar{\phi}|B> \\ <\phi\bar{\phi}\chi|B> \\ \vdots \end{bmatrix}, \tag{4}$$

where k_i^- is the light-cone energy of the i'th constituent and V is the interaction part of H_{LC}. The light-cone wavefunctions $<\phi\bar{\phi}|B>, <\phi\bar{\phi}\chi|B>, \cdots$ provide a physically clear description of a bound state, since the vacuum fluctuations are suppressed in the light-cone quantization and all constituents are on the mass shell where

$$k_i^- = (\vec{k}_{\perp_i}^2 + m_i^2)/x_i , \tag{5}$$

with $x_i = k_i^+/p^+$. The cross diagrams can be included systematically, in principle, when the higher Fock-state contributions such as $|\phi\bar{\phi}\chi\chi>$ are taken into account. However, because of the vacuum instability in the cubic theory, we limit our scope of the calculations up to the three-body state $|\phi\bar{\phi}\chi>$ and remove the vacuum instability as discussed earlier.

The lowest-order light-cone Tamm-Dancoff approximation corresponds to take into account only two-and three-body sectors in Eq. (4). We follow the renormalization procedure proposed recently[15] and eliminate the three-body amplitude $<\phi\bar{\phi}\chi|B>$ to obtain the effective equation for the two-body wavefunction in this approximation ($x_1 = x, x_2 = 1-x, \vec{k}_{\perp_1} = -\vec{k}_{\perp_2} = \vec{k}_\perp$);

$$\begin{aligned} M^2\Psi(x,\vec{k}_\perp) &= \left(\frac{\vec{k}_\perp^2+m^2}{x(1-x)} + \text{C.T.}\right)\Psi(x,\vec{k}_\perp) \\ &+ g^2\int\frac{dy}{y(1-y)}\frac{d^2\vec{l}_\perp}{16\pi^3}\left[\frac{\theta(x-y)}{x-y}\frac{1}{M^2-\frac{\vec{l}_\perp^2+m^2}{y}-\frac{(\vec{k}_\perp-\vec{l}_\perp)^2+\lambda^2}{x-y}-\frac{\vec{k}_\perp^2+m^2}{1-x}} \right.\\ &\left. + \begin{pmatrix} x\leftrightarrow y \\ \vec{k}_\perp\leftrightarrow\vec{l}_\perp \end{pmatrix}\right]\Psi(y,\vec{l}_\perp) \\ &+ g^2\Psi(x,\vec{k}_\perp)\int\frac{d^2\vec{l}_\perp}{16\pi^3}\left[\frac{1}{x}\int_0^x\frac{dy}{y(x-y)}\frac{1}{M^2-\frac{\vec{l}_\perp^2+m^2}{y}-\frac{(\vec{k}_\perp-\vec{l}_\perp)^2+\lambda^2}{x-y}-\frac{\vec{k}_\perp^2+m^2}{1-x}}\right. \\ &\left.+\frac{1}{(1-x)}\int_x^1\frac{dy}{(1-y)(y-x)}\frac{1}{M^2-\frac{\vec{k}_\perp^2+m^2}{x}-\frac{(\vec{k}_\perp-\vec{l}_\perp)^2+\lambda^2}{y-x}-\frac{\vec{l}_\perp^2+m^2}{1-y}}\right], \end{aligned} \tag{6}$$

where the counter terms are obtained by the renormalization condition

$$M^2 = \frac{\vec{k}_\perp^2+m^2}{x(1-x)}, \tag{7}$$

above the scattering threshold and given by

$$\begin{aligned} \text{C.T.} &= -g^2\int\frac{d^2\vec{l}_\perp}{16\pi^3}\left[\frac{1}{x}\int_0^x\frac{dy}{y(x-y)}\frac{1}{\frac{\vec{k}_\perp^2+m^2}{x}-\frac{(\vec{k}_\perp-\vec{l}_\perp)^2+\lambda^2}{x-y}-\frac{\vec{l}_\perp^2+m^2}{y}}\right. \\ &\left.+\frac{1}{(1-x)}\int_x^1\frac{dy}{(1-y)(y-x)}\frac{1}{\frac{\vec{k}_\perp^2+m^2}{1-x}-\frac{(\vec{k}_\perp-\vec{l}_\perp)^2+\lambda^2}{y-x}-\frac{\vec{l}_\perp^2+m^2}{1-y}}\right]. \end{aligned} \tag{8}$$

Here, the last term proportional to $\Psi(x,\vec{k}_\perp)$ in the right-hand-side of Eq. (6) corresponds to the self-energy corrections. The integrations in the self-energy corrections and the counter terms can be explicitly performed by using the variable change $y = xz$ and the following two-body equation is obtained;

$$\left\{M^2-\frac{\vec{k}_\perp^2+m^2}{x(1-x)}-\frac{g^2}{16\pi^2}f(x,\vec{k}_\perp)\right\}\Psi(x,\vec{k}_\perp) = g^2\int\frac{dy}{y(1-y)}\frac{d^2\vec{l}_\perp}{16\pi^3}V(x,\vec{k}_\perp;y,\vec{l}_\perp)\Psi(y,\vec{l}_\perp), \tag{9}$$

where the self-energy corrections and counter terms are summarized by

$$f(x,\vec{k}_\perp) = \frac{1}{x}\int_0^1 dz\log\left[1+\frac{x\left\{\frac{\vec{k}_\perp^2+m^2}{x(1-x)}-M^2\right\}z(1-z)}{\lambda^2 z+m^2(1-z)^2}\right] + (x\leftrightarrow(1-x)). \tag{10}$$

and the two-body kernel is given by

$$V(x,\vec{k}_\perp;y,\vec{l}_\perp) = \frac{\theta(x-y)}{x-y}\frac{1}{M^2-\frac{\vec{l}_\perp^2+m^2}{y}-\frac{(\vec{k}_\perp-\vec{l}_\perp)^2+\lambda^2}{x-y}-\frac{\vec{k}_\perp^2+m^2}{1-x}} + \begin{pmatrix} x\leftrightarrow y \\ \vec{k}_\perp\leftrightarrow\vec{l}_\perp \end{pmatrix}. \tag{11}$$

For $\lambda = 0$, the z-integration in Eq. (10) can be analytically performed and $f(x, \vec{k}_\perp)$ is given by

$$f(x,\vec{k}_\perp) = \frac{\left\{M^2 - \frac{\vec{k}_\perp^2+m^2}{x(1-x)}\right\}}{m^2}\left[\frac{\log\left[\frac{x\left\{\frac{\vec{k}_\perp^2+m^2}{x(1-x)}-M^2\right\}}{m^2}\right]}{1-\frac{x\left\{\frac{\vec{k}_\perp^2+m^2}{x(1-x)}-M^2\right\}}{m^2}} + (x \leftrightarrow (1-x))\right] . \quad (12)$$

In the light-cone ladder approximation considered in the previous analyses[17,18], the self-energy term $f(x, \vec{k}_\perp)$ has been neglected and the calculation of this term is a new contribution obtained in this work.

If we represent the light-cone variables $x, \vec{k}_\perp, y, \vec{l}_\perp$ in terms of the center of momentum variables of the two-body system; *i.e.* the initial and final momenta of the first (second) particle are given by $\vec{k}(-\vec{k})$ and $\vec{l}(-\vec{l})$ respectively, then, in this frame, the light-cone variables are given by

$$\begin{aligned} x &= \frac{1}{2}\left(1+\frac{\hat{n}\cdot\vec{k}}{\epsilon(\vec{k})}\right), \vec{k}_\perp = \vec{k} - (\hat{n}\cdot\vec{k})\hat{n} , \\ y &= \frac{1}{2}\left(1+\frac{\hat{n}\cdot\vec{l}}{\epsilon(\vec{l})}\right), \vec{l}_\perp = \vec{l} - (\hat{n}\cdot\vec{l})\hat{n} , \end{aligned} \quad (13)$$

where $\epsilon(\vec{k}) = \sqrt{m^2+\vec{k}^2}$ and $\hat{n}$ is the direction of the spatial part chosen in the definition of the light-cone time $\tau = t + \hat{n}\cdot\vec{r}/c$; i.e. if $\hat{n} = \hat{z}$, then $\tau = t + z/c$. Using the c.m. variables, Eqs. (9)-(11) are given by

$$\left\{4(\vec{k}^2+\beta^2) + \frac{\alpha m^2}{\pi} f(\vec{k}^2, \hat{n}\cdot\vec{k})\right\}\Psi(\vec{k},\hat{n}) = \frac{2\alpha m^2}{\pi^2}\int\frac{d^3\vec{l}}{\epsilon(\vec{l})}V(\vec{k},\vec{l},\hat{n})\Psi(\vec{l},\hat{n}) , \quad (14)$$

where

$$f(\vec{k}^2,\hat{n}\cdot\vec{k}) = \frac{2}{\left(1+\frac{\hat{n}\cdot\vec{k}}{\epsilon(\vec{k})}\right)}\int_0^1 dz \log\left\{1+\frac{2(\vec{k}^2+\beta^2)z(1-z)}{\lambda^2 z + m^2(1-z)^2}\left(1+\frac{\hat{n}\cdot\vec{k}}{\epsilon(\vec{k})}\right)\right\} + (\hat{n}\leftrightarrow -\hat{n}) , \quad (15)$$

and

$$\begin{aligned} V^{-1}(\vec{k},\vec{l},\hat{n}) = |\vec{k}-\vec{l}|^2 &+ \lambda^2 - (\hat{n}\cdot\vec{k})(\hat{n}\cdot\vec{l})\frac{\left\{\epsilon(\vec{k})-\epsilon(\vec{l})\right\}^2}{\epsilon(\vec{k})\epsilon(\vec{l})} \\ &+ \left\{\epsilon^2(\vec{k})+\epsilon^2(\vec{l}) - \frac{M^2}{2}\right\}\left|\frac{\hat{n}\cdot\vec{k}}{\epsilon(\vec{k})} - \frac{\hat{n}\cdot\vec{l}}{\epsilon(\vec{l})}\right| . \end{aligned} \quad (16)$$

Here, the dimensionless coupling constant α is given by

$$\alpha = \frac{g^2}{16\pi m^2} \quad (17)$$

and the quantity β^2 related with the binding energy is given by

$$\beta^2 = m^2 - \frac{M^2}{4} . \quad (18)$$

In the non-relativistic limit, Eqs. (14)-(16) become the Schrödinger equation;

$$\left(\frac{\vec{k}^2}{2m_r} - \varepsilon\right)\Psi(\vec{k}) = 4\pi\alpha\int\frac{d^3\vec{l}}{(2\pi)^3}\frac{1}{(\vec{k}-\vec{l})^2+\lambda^2}\Psi(\vec{l}) , \quad (19)$$

where $m_r = m/2$ and $\varepsilon = M - 2m$.

In the light-cone ladder approximation (i.e., $f = 0$ in Eq. (14)) with $\lambda = 0$, we have obtained a simple analytic relation between the coupling constant and the binding energy (i.e., α and β, respectively) for all nl states with $l = n-1$[17]. Here, the quantum numbers n and l are the same with those defined in the Wick-Cutkosky model[11]. This analytic relation is consistent with the numerical results obtained by a variational principle for the entire range of the binding energy $0 \le \beta \le m$ [18]. In order to discuss the effect of the self-energy correction numerically, we now include the self-energy correction to the previously derived analytic relation. We follow the same procedures taken in the previous work[17]. First, we take the wavefunction valid both in the small and large asymptotic momentum region;

$$\Psi_{n,l=n-1}(\vec{k},\hat{n}) = N_{nn-1}\left\{\frac{|\vec{k}|^{n-1}}{(\vec{k}^2+\beta^2)^{n+1}\left(1+\frac{|\hat{n}\cdot\vec{k}|}{\epsilon(\vec{k})}\right)^n}\right\}(\sin\theta)^{n-1}\exp\{i(n-1)\phi\} , \quad (20)$$

where N_{nn-1} is the normalization constant and (θ, ϕ) determines the direction of $\vec{k}$. For $n = 1$ and 2, these wavefunctions correspond to the Karmanov solutions for $1s$ and $2p$ states respectively. Next, we substitute $\Psi(\vec{k}, \hat{n})$ given by Eq. (20) into Eq. (14) and perform the $\vec{l}$-integration for some convenient $\vec{k}$ value to find an analytic relation between the coupling constant α and the binding energy.

If we take $\vec{k} = |\vec{k}|\hat{x}$ (or $\sin\theta = 1$) and the limit $|\vec{k}| \to 0$, then Eq. (14) becomes

$$\frac{\pi}{\alpha} = 2\mu\left[\int_0^1 dx(1-x^2)^{n-1}\int_0^\infty \frac{d\mu}{\sqrt{\mu+y^2}}\frac{1}{(y^2+1)^{n+1}}\frac{1}{\left(1+\frac{yx}{\sqrt{\mu+y^2}}\right)^n}\frac{1}{\left(1+\frac{(y^2+2)x}{y\sqrt{\mu+y^2}}\right)} + \frac{1}{\mu-2}\log\frac{2}{\mu}\right], \tag{21}$$

where $\mu = m^2/\beta^2, x = \cos\theta'$ and $y = |\vec{l}|/\beta$. The second term in the right-hand-side of Eq. (21) corresponds to the self-energy correction and does not have an explicit dependence on the quantum number n. In the nonrelativistic limit (i.e. $\mu \to \infty$), Eq. (21) becomes

$$\begin{aligned}\frac{\pi}{\alpha} &= 2\sqrt{\mu}\left[\int_0^1 dx(1-x^2)^{n-1}\int_0^\infty dy\frac{1}{(1+y^2)^{n+1}} + \lim_{\mu\to\infty}\frac{\sqrt{\mu}}{(\mu-2)}\log\frac{2}{\mu}\right] \\ &= 2\sqrt{\mu}\left[\frac{\pi}{4n} + \lim_{\mu\to\infty}\frac{1}{\sqrt{\mu}}\log\frac{2}{\mu}\right] \\ &= \frac{\pi}{2n}\sqrt{\mu},\end{aligned} \tag{22}$$

and we recover the Balmer formula

$$2m - M = \frac{m\alpha^2}{4n^2}. \tag{23}$$

However, we make clear that Eq. (20) is not an exact solution of Eq.(14) and thus our derivation uses an approximation beyond that of the Tamm-Dancoff truncation. In order to check the accuracy of Eq.(21), we solve Eq.(14) numerically by considering the variational principle for the coupling constant g. For fixed binding energy (or mass M of the bound state), the Minimum of the following expression determines the relation between the binding energy and the coupling constant:

$$g^2 = \frac{N}{D_1 + D_2}, \tag{24}$$

where

$$\begin{aligned} N &= \int\frac{dx}{x(1-x)}\left(\frac{d^2\vec{k}_\perp}{16\pi^3}\right)\Psi^\dagger(x,\vec{k}_\perp)\left\{M^2 - \frac{\vec{k}_\perp^2+m^2}{x(1-x)}\right\}\Psi(x,\vec{k}_\perp), \\ D_1 &= \int\frac{dx}{x(1-x)}\left(\frac{d^2\vec{k}_\perp}{16\pi^3}\right)\int\frac{dy}{y(1-y)}\left(\frac{d^2\vec{l}_\perp}{16\pi^3}\right)\Psi^\dagger(x,\vec{k}_\perp)V(x,\vec{k}_\perp;y,\vec{l}_\perp)\Psi(y,\vec{l}_\perp), \\ D_2 &= \int\frac{dx}{x(1-x)}\left(\frac{d^2\vec{k}_\perp}{16\pi^3}\right)\Psi^\dagger(x,\vec{k}_\perp)\frac{f(x,\vec{k}_\perp)}{16\pi^2}\Psi(x,\vec{k}_\perp).\end{aligned}$$

The minimum is found by varying parameters in the variational wavefunction $\Psi(x, \vec{k}_\perp)$. For $1s$ state, we consider the trial function parameterized by

$$\Psi_{1s}(x,\vec{k}_\perp) = \frac{N_{1s}}{\left(C^2 - \frac{\vec{k}_\perp^2+m^2}{x(1-x)}\right)^2(1+|2x-1|)}, \tag{25}$$

where the normalization constant N_{1s} cancels in the ratio given by Eq.(24) and C is the variational parameter. This trial function agrees with the wave function given by Eq. (20) with $n = 1$, when $C = M$.

The numerical estimate for the self-energy correction is shown in Fig. 1. The agreement between the analytic result and the numerical result is reasonable even if the self-energy correction is included. As shown in Fig. 1, the self-energy effects are as repulsive as relativistic kinematic corrections and retardation effects and make β become frozen as α increases. The similar results were obtained in the generalized theory of the Wick-Cutkosky model using DLCQ[14] and in the Yukawa model[20].

Here, we also compare the probability of finding the three-body component inside the bound-state $|B>$ (see Eq. (2)), $P_3 \equiv |<\phi\bar{\phi}\chi|B>|^2$, with that of finding the two-body component, $P_2 \equiv |<\phi\bar{\phi}|B>|^2$. These probabilities can be obtained by the following integrations;

$$P_2 = \int\frac{dx}{x(1-x)}\frac{d^2\vec{k}_\perp}{16\pi^3}|\Psi(x,\vec{k}_\perp)|^2 \tag{26}$$

and

$$P_3 = g^2 \int \frac{dx}{x(1-x)} \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{dy}{y(1-y)} \frac{d^2\vec{l}_\perp}{16\pi^3} \Psi(x,\vec{k}_\perp) \left(\frac{\partial V_{eff}}{\partial M^2}\right) \Psi(y,\vec{l}_\perp) \ , \qquad (27)$$

where $\Psi(x,\vec{k}_\perp)$ is the two-body wavefunction and the effective kernel V_{eff} including the self-energy corrections is given by

$$V_{eff} = \frac{V(x,\vec{k}_\perp;y,\vec{l}_\perp)}{1 - \frac{\alpha m^2 f(x,\vec{k}_\perp)}{\pi\left\{M^2 - \frac{\vec{k}_\perp^2+m^2}{x(1-x)}\right\}}} \ . \qquad (28)$$

If one does not include the self-energy corrections, V_{eff} is same with $V(x,\vec{k}_\perp;y,\vec{l}_\perp)$ given by Eq. (11). For simplicity, we used the wavefunction given by Eq. (25) when $C = M$. The numerical results of the calculation P_3/P_2 are summarized in Table 1. As we can see from Table 1, this ratio with or without self-energy corrections becomes larger as α increases indicating that the three-body sector gets more important as the coupling constant increases. However, it is interesting to note that this ratio is substantially reduced by the self-energy corrections and becomes stabilized as the coupling constant increases. We believe that this result is due to the significant repulsive effects from the self-energy corrections as shown in Fig. 1.

In conclusion, we extended the light-cone ladder approximation in the Wick-Cutkosky model to the first order light-cone Tamm-Dancoff approximation and obtained the light-cone two-body equation including the self-energy corrections. Both the numerical results and the approximate analytic relation between the coupling constant and the binding energy indicate that the self-energy effects are as repulsive as the relativistic effects such as the correct relativistic energy-momentum relation and the retardation effects.Thus it makes the binding energy become frozen as the coupling constant increases. The same repulsive effect in the self-energy corrections also makes the ratio P_3/P_2 be substantially reduced and stabilized as the coupling constant increases.

I acknowledge the useful discussions with A. Harindranath and J.R.Hiller and the help on the numerical computation provided by G.-H. Kim at Seoul National University.

References

[1] E. E Salpeter and H.A. Bethe, Phys. Rev. 84, (1951) 1232; For a review up to 1969 see N. Nakanishi, Prog. Theor. Phys. Suppl. 43 (1969) 1.

[2] An approximate summation of the crossed-ladder diagrams has been considered in E. Brezin, C. Itzykson and J.Zinn-Justin, Phys.Rev. D1 (1970) 2349.

[3] G.P.Lepage, S.J.Brodsky, T. Huang and P.M. Mackenzie, in: Proc. Banff Summer Institute on Particle Physics (Banff, Alberta, Canada, August 1981); S.J.Brodsky, Quarks and Nuclear Forces, Springer Tracts in Modern Physics, Vol. 100, eds. D. Fries and B. Zeitnitz (Springer, Berlin, 1982) p.81; S.J. Brodsky and H.C. Pauli , in Recent Aspects of Quantum Fields, edited by H.Mitter and H. Gausterer, Lecture Notes in Physics Vol. 396 (Springer- Verlag, Berlin, 1991).

[4] P.A.M. Dirac, Rev. Mod. Phys., 21 (1949) 392;S. Weinberg, Rhys. Rev. 150 (1966) 1313; L. Susskind and G. Frye, Phys. Rev. 165 (1968) 1535; J.B.Kogut and D.E. Soper, Phys. Rev. D1 (1970) 2901; J.D.Bjorken, J.B.Kogut and D.E.Soper, Phys. Rev. D3 (1971) 1382; S.J.Brodsky, R.Roskies and R.Suaya, Phys. Rev. D8 (1973)4574.

[5] S.J.Brodsky, C.-R. Ji and M. Sawicki, Phys. Rev. D32 (1985) 1530.

[6] R.J.Perry, A.Harindranath, and K.G.Wilson, Phys. Rev. Lett. 65 (1990) 2959; R.J. Perry and A. Harindranath, Phys. Rev. D43 (1991)4051.

[7] C.-R.Ji and Y.Surya, Phys. Rev. D46 (1992) 3565.

[8] S.Glazek, A. Harindranath, S.Pinsky, J.Shigemitsu and K. Wilson, Phys. Rev. D47 (1993) 1599.

[9] C.-R.Ji, in: DIRKFEST'92, eds. W.Buck, K.Maung and B.Serot (World Scientific Publishing, Singapore, 1992) pp. 147-163.

[10] A Prescription to restore the rotational invariance in the perturbative one loop and two loop calculation using Yukawa model based on the light-cone quantization was discussed by M. Burkardt and A.Langnau, Phys.Rev. D44 (1991) 3857. However, this prescription does not apply to the present work beyond the perturbative calculation. Furthermore, the Wick-Cutkosky model does not involve the two different masses in the numerator and the denominator of the scattering amplitudes which are necessary in their prescription.

[11] G.C.Wick, Phys. Rev. 96 (1954) 1124; R.E. Cutkosky, Phys. Rev. 96 (1954) 1135.

[12] G.Baym, Phys. Rev. 117, (1960) 886.

[13] J.R.Hiller, Phys. Rev. D44, (1991) 2504.

[14] J.J.Wivoda and J.R.Hiller, Phys.Rev. D47 (1993) 4647; J.B. Swenson and J.R. Hiller, Submitted to Phys. Rev. D; a private communication with J.R.Hiller.

[15] H.C.Pauli and S.J.Brodsky, Phys. Rev. D32 (1985) 1993;(1985)2001; T.Eller, H.C.Pauli and S.J.Brodsky, Phys.Rev.D35 (1987) 1493;J. R. Hiller, Phys. Rev. D43 (1991) 2418; K.J.Hornbostel, S.J.Brodsky and H.C. Pauli, Phys. Rev. D41(1990) 3814; A.C.Tang, S.J.Brodsky and H.C.Pauli, Phys.Rev. D44(1991)1842.

[16] V.A.Karmanov, Nucl.Phys. B166 (1980) 378.

[17] C.-R.Ji, Phys. Lett. 167B (1986) 16.

[18] C.-R.Ji and R.J.Furnstahl, Phys.Lett.167B (1986) 11.

[19] M.G.Fuda, Phys.Rev. D44 (1991) 1880; Ann.Phys. (N.Y.) 197 (1990) 2.

[20] A.Harindranath, R.J.Perry, and J. Shigemitsu, Phys. Rev. D46 (1992) 4580.

Figure Caption

Figure 1: Curves of β versus α for $1s$ state in the Wick-Cutkosky model in units $m = 1$. The numerical results with and without the self-energy corrections obtained by the variational principle (solid curves) are compared with the corresponding approximate analytic results from Eq.(21) (dash-dot curves) and the non-relativistic result from Eq.(22) (dashed curve).

Table 1. The numerical results of the ratio P_3/P_2 with or without self-energy corrections for several different values of the coupling constant α.

α	0.1	0.2	0.4	0.6	0.8	1.1
P_3/P_2(with)	0.24	0.33	0.42	0.48	0.51	0.55
P_3/P_2(without)	0.40	0.67	0.90	1.04	1.16	1.27