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Reduction of Vector and Axial–Vector Fields in a Bosonized Nambu–Jona–Lasinio Model

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Abstract

We derive the effective action for pseudoscalar mesons by integrating out vector and axial–vector collective fields in the generating functional of the bosonized NJL–model. The corresponding modifications of the nonlinear effective Lagrangian and the bosonized currents, arising at $O(p^4)$, are discussed.

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I. Introduction

A renewal of interest in chiral Lagrangian theory was excited by recent progress in the construction of realistic effective chiral meson Lagrangians including higher order derivative terms as well as the gauge Wess–Zumino term from low-energy approximations of QCD. The program of bosonization of QCD, which was started about 20 years ago, in the strong sense is of course also beyond our present possibilities. Nevertheless there is some success related to the application of functional methods to approximate forms of QCD (see [1]-[10] and references therein) or to QCD–motivated effective quark models [11]-[18] which are extensions of the well-known Nambu–Jona–Lasinio (NJL) model [19]. These functional methods can be applied also to the bosonization of the effective four–quark nonleptonic weak and electromagnetic–weak interactions with strangeness change $|\Delta S| = 1$ by using the generating functional for Green functions of quark currents introduced in [20], [21].

The NJL model, which we consider in this paper, incorporates not only all relevant symmetries of the quark flavour dynamics of low-energy QCD, but also offers a simple scheme of the spontaneous breakdown of chiral symmetry arising from the explicit symmetry breaking terms due to the quark masses. In this scheme the current quarks transit into constituent ones due to the appearance of a nonvanishing quark condensate, and light composite pseudoscalar Nambu–Goldstone bosons emerge accompanied also by heavier dynamical vector and axial–vector mesons with correct relative weights arising from renormalization. The composite vector and axial–vector resonances are naturally required if one wants to describe the low energy aspects of QCD in a wider energy range up to typical masses of O(1GeV). (For other approaches to the problem of introducing chiral couplings for vector and axial–vector mesons to Goldstone bosons into the effective chiral Lagrangian, see for example refs.[22]-[24] and references therein.)

Independently from the method of including the vector and axial–vector fields in the effective chiral Lagrangian, integrating out the heavy meson resonances essentially modifies the coupling constants of the pseudoscalar low energy interactions. In particular, in refs.[23], [24] it was shown that the structure constants L_i of the Gasser–Leutwyler general expression for the $O(p^4)$ pseudoscalar Lagrangian [25] are largely saturated by the resonance exchange contributions giving a product of terms of $O(p^2)$. But in this case, if the $O(p^4)$ Lagrangian contains meson resonances, their elimination can lead to the double counting mentioned in ref.[23]. The resonance contributions to the purely pseudoscalar chiral weak Lagrangian and the modification of its structure, induced by integrating out the heavy meson exchanges, were discussed recently in refs.[26],[27] in both the frame of the factorization approximation and in the weak deformation approach.

In this paper we consider the effective nonlinear Lagrangian for pseudoscalar mesons which arises after integrating out the explicit vector and axial–vector resonances in the generating functional of the bosonized NJL model. To perform such integration we use a method based on the invariance of the modulus of the quark determinant under a chiral transformations and on the application of the static equations of motion to a special configuration of the chiral rotated fields. The elimination of vector and axial–vector degrees of freedom from the modulus of the quark determinant leads to a modification of the general structure of the effective strong

Lagrangian for the pseudoscalar sector at $O(p^4)$ and to a redefinition of the corresponding Gasser-Leutwyler structure coefficients L_i. This method of reduction of meson resonances can be extended to the procedure [21] of chiral bosonization of weak and electromagnetic–weak currents and can be used for obtaining the corresponding reduced meson currents entering to the bosonized nonleptonic weak Lagrangians.

In such approximation the problem of double counting does not arise. The effect of πA_1–mixing, being most important for the description of radiative weak decays, is taken into account by the corresponding πA_1–diagonalization factor. This factor appears explicitly in the bosonized strong Lagrangians and weak and electromagnetic-weak currents after reduction of the vector and axial-vector fields. With such modification of the strong Lagrangian and currents it is possible to reproduce within the nonlinear parameterization of chiral symmetry most of the results of the linear Lagrangian approach [14] concerning πA_1–mixing effects and meson resonance exchange contributions.

In Section 1 we discuss the basic formalism and display all definitions and constants related to the bosonization of quarks in NJL model. For convenience, all cumbersome expressions resulting from the heat-kernel computation of the quark determinant are given in the Appendix A. The total expressions for the bosonized effective Lagrangians including vector and axial-vector fields are presented in the Appendix B up to $O(p^6)$ terms. In Section 3 we consider the static equations of motion for chiral rotated collective meson fields in unitary gauge. Applying these equations of motion we eliminate the heavy meson resonances from the modulus of the quark determinant and obtain in such a way the effective pseudoscalar strong Lagrangian with reduced vector and axial-vector degrees of freedom. The reduced pseudoscalar $(V-A)$ and $(S-P)$ currents corresponding to the respective quark currents and quark densities are obtained in Section 4. In Section 5 we discuss the results of some numerical estimations and phenomenological analysis of the structure constants for the reduced strong Lagrangian and currents.

II. Bosonization of the NJL model

The starting point of our consideration is the NJL Lagrangian of the effective four–quark interaction which has the form [19]:

$$\mathcal{L}_{NJL} = \bar{q}(i\hat{\partial} - m_0)q + \mathcal{L}_{int} \tag{1}$$

with

$$\mathcal{L}_{int} = 2G_1\left\{\left(\bar{q}\frac{\lambda^a}{2}q\right)^2 + \left(\bar{q}i\gamma^5\frac{\lambda^a}{2}q\right)^2\right\} - 2G_2\left\{\left(\bar{q}\gamma^\mu\frac{\lambda^a}{2}q\right)^2 + \left(\bar{q}\gamma^\mu\gamma^5\frac{\lambda^a}{2}q\right)^2\right\}.$$

Here G_1 and G_2 are some universal coupling constants; $m_0 = diag(m_0^1, m_0^2, ..., m_0^n)$ is the current quark mass matrix (summation over repeated indices is assumed), and λ^a are the generators of the $SU(n)$ flavour group normalized according to $\mathrm{tr}\,\lambda^a\lambda^b = 2\delta_{ab}$. Using a standard quark bosonization approach based on path integral techniques one can derive an effective meson action from the NJL Lagrangian (1). First one has to introduce collective fields for the scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) colorless mesons associated to the following quark bilinears:

$$S^a = -4G_1\bar{q}\frac{\lambda^a}{2}q\,, \quad P^a = -4G_1\bar{q}i\gamma^5\frac{\lambda^a}{2}q\,, \quad V^a_\mu = i4G_2\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\,, \quad A^a_\mu = i4G_2\bar{q}\gamma_\mu\gamma^5\frac{\lambda^a}{2}q\,.$$

After substituting these expressions into $\mathcal{L}_{NJL}$ the interaction part of the Lagrangian is of Yukawa form. The part of $\mathcal{L}_{NJL}$ which is bilinear in the quark fields can be rewritten as

$$\mathcal{L} = \bar{q}i\widehat{\mathbf{D}}q$$

with $\widehat{\mathbf{D}}$ being the Dirac operator:

$$\begin{aligned} i\widehat{\mathbf{D}} &= i(\hat{\partial} + \hat{V} + \hat{A}\gamma^5) - P_R(\Phi + m_0) - P_L(\Phi^+ + m_0) \\ &= [i(\hat{\partial} + \hat{A}_R) - (\Phi + m_0)]P_R + [i(\hat{\partial} + \hat{A}_L) - (\Phi^+ + m_0)]P_L. \end{aligned} \tag{2}$$

Here $\Phi = S + iP$, $\hat{V} = V_\mu\gamma^\mu$, $\hat{A} = A_\mu\gamma^\mu$; $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$ are chiral projectors; $\hat{A}_{L,R} = \hat{V} \pm \hat{A}$ are right and left combinations of fields, and

$$S = S^a\frac{\lambda^a}{2}, \quad P = P^a\frac{\lambda^a}{2}, \quad V_\mu = -iV^a_\mu\frac{\lambda^a}{2}, \quad A_\mu = -iA^a_\mu\frac{\lambda^a}{2}$$

are the matrix-valued collective fields.

After integration over quark fields the generating functional, corresponding to the effective action of the NJL model for collective meson fields, can be presented in the following form:

$$\mathcal{Z} = \int \mathcal{D}\Phi\,\mathcal{D}\Phi^+\,\mathcal{D}V\,\mathcal{D}A\,\exp[i\mathcal{S}(\Phi, \Phi^+, V, A)]\,, \tag{3}$$

where

$$\mathcal{S}(\Phi, \Phi^+, V, A) = \int d^4x\Big[-\frac{1}{4G_1}\mathrm{tr}[(\Phi - m_0)^+(\Phi - m_0)] - \frac{1}{4G_2}\mathrm{tr}(V_\mu^2 + A_\mu^2)\Big] - i\,\mathrm{Tr}'\,[\log(i\widehat{\mathbf{D}})] \tag{4}$$

is the effective action for scalar, pseudoscalar, vector and axial-vector mesons. The first term in (4), quadratic in meson fields, arises from the linearization of the four–quark interaction. The second term is the quark determinant describing the interaction of mesons. The trace Tr' is to be understood as a space-time integration and a "normal" trace over Dirac, colour and flavour indices:

$$\mathrm{Tr}' = \int d^4x\,\mathrm{Tr}\,, \quad \mathrm{Tr} = \mathrm{tr}_\gamma \cdot \mathrm{tr}_c \cdot \mathrm{tr}_f\,.$$

The quark determinant can be evaluated either by expansion in quark loops [11]-[13] or by the heat-kernel technique with proper-time regularization [28]. Then, the real part of $\log(\det i\widehat{\mathbf{D}})$ contributes to the non–anomalous part of the effective Lagrangian while the imaginary part of it gives the anomalous effective Lagrangian of Wess and Zumino which is related to chiral anomalies [29].

The modulus of the quark determinant is presented in the heat-kernel method as the expansion over the so-called Seeley-deWitt coefficients h_k:

$$\log|\det i\widehat{\mathbf{D}}| = -\frac{1}{2}\frac{\mu^4}{(4\pi)^2}\sum_k \frac{\Gamma(k-2, \mu^2/\Lambda^2)}{\mu^{2k}}\,\mathrm{Tr}'\,h_k, \tag{5}$$

where

$$\Gamma(\alpha, x) = \int_x^\infty dt\,e^{-t}t^{\alpha-1}$$

is incomplete gamma function; μ plays the role of some empirical mass scale parameter which will fix the regularization in the region of low momenta, and Λ is the intrinsic regularization cutoff parameter. There are some technical reasons for using the heat kernel method instead of the "straight" method of calculating quark loops. The main advantage of this method is that its recursive algorithms can be adopted on Computer Algebra Systems such as FORM or REDUCE quite effectively. In particular higher order derivative contributions can conveniently be calculated, too. The formulae for the Seeley-deWitt coefficients h_k up to $k = 6$ obtained in [30] and the full expressions for p^4– and p^6–contributions to the bosonized meson Lagrangian are presented in the Appendices A,B.

We will consider a nonlinear parameterization of chiral symmetry corresponding to the following representation of Φ:

$$\Phi = \Omega\,\Sigma\,\Omega,$$

where $\Sigma(x)$ is the matrix of scalar fields belonging to the diagonal flavour group while matrix $\Omega(x)$ represents the pseudoscalar degrees of freedom φ living in the coset space $U(n)_L\times U(n)_R/U_V(n)$, which can be parameterized by the unitary matrix

$$\Omega(x) = \exp\left(\frac{i}{\sqrt{2}F_0}\varphi(x)\right)\,, \quad \varphi(x) = \varphi^a(x)\frac{\lambda^a}{2}\,,$$

with F_0 being the bare π decay constant. Under chiral rotations

$$q \to \tilde{q} = (P_L\xi_L + P_R\xi_R)\,q$$

the fields Φ and $A^\mu_{R/L}$ are transformed as

$$\Phi \to \tilde{\Phi} = \xi_L\Phi\xi_R^+$$

and

$$A^\mu_R \to \tilde{A}^\mu_R = \xi_R(\partial^\mu + V^\mu + A^\mu)\xi_R^+, \quad A^\mu_L \to \tilde{A}^\mu_L = \xi_L(\partial^\mu + V^\mu - A^\mu)\xi_L^+. \qquad (6)$$

For the unitary gauge $\xi_L^+ = \xi_R = \Omega$ the rotated Dirac operator (2) gets the form

$$i\widehat{\mathbf{D}} \to i\widehat{\widetilde{\mathbf{D}}} = (P_L\Omega + P_R\Omega^+)i\widehat{\mathbf{D}}(P_L\Omega + P_R\Omega^+) = i(\hat{\partial} + \hat{\tilde{V}} + \hat{\tilde{A}}\gamma_5) - \Sigma. \qquad (7)$$

It is worth noting that under local $U_L(n)\times U_R(n)$ transformations the modulus of the quark determinant is invariant, while the quadratic terms of V_μ, A_μ and the chiral anomaly do not respect this invariance.

Note that there arises a quark condensate $<\bar{q}q>\neq 0$ owing to the nonvanishing vacuum expectation value of the scalar meson field S_0 realizing spontaneous breakdown of chiral symmetry. In fact, assuming approximate flavor symmetry of the condensate and using the equation of motion $S_0 = -2G_1\sqrt{\frac{2}{n}}\bar{q}q$ one gets

$$<S_0> = -2G_1\sqrt{\frac{2}{n}}<\bar{q}q> \equiv i2G_1\sqrt{\frac{2}{n}}\,\mathrm{Tr}\,\{i\widehat{\mathbf{D}}(\Phi = \mu, V = A = 0)\}^{-1}\,.$$

Thus, our mass scale $\mu = \frac{1}{\sqrt{2n}} <S_0>$ is determined by G_1 and $<\bar{q}q>$ or (using the explicit expression for the condensate by a loop integral) by G_1, the cutoff Λ and the current mass m_0.

Taking into account the equations of motion for nonrotated scalar and pseudoscalar meson fields in nonlinear parameterization one can derive from (4) and eqs.(41,42) of the Appendix B the following general expression of the effective meson Lagrangian including p^2– and p^4–interactions:

$$\begin{aligned}\mathcal{L}^{(nred)}_{eff} = &-\frac{F_0^2}{4}\,\mathrm{tr}\,(L_\mu L^\mu) + \frac{F_0^2}{4}\,\mathrm{tr}\,(MU + U^+M)\\ &+\left(L_1 - \frac{1}{2}L_2\right)(\,\mathrm{tr}\,L_\mu L^\mu)^2 + L_2\,\mathrm{tr}\left(\frac{1}{2}[L_\mu, L_\nu]^2 + 3(L_\mu L^\mu)^2\right) + L_3\,\mathrm{tr}\,((L_\mu L^\mu)^2)\\ &+L_4\,\mathrm{tr}\,(D_\mu U\,\overline{D}^\mu U^+)\,\mathrm{tr}\,M(U + U^+) + L_5\,\mathrm{tr}\,D_\mu U\,\overline{D}^\mu U^+(MU + U^+M)\\ &+L_6\left(\mathrm{tr}\,(MU + U^+M)\right)^2 + L_7\left(\mathrm{tr}\,(MU - U^+M)\right)^2\\ &+L_8\,\mathrm{tr}\,(MUMU + U^+MU^+M)\\ &+L_9\,\mathrm{tr}\left(F^{(+)}_{\mu\nu}D^\mu U\,\overline{D}^\nu U^+ + F^{(-)}_{\mu\nu}\overline{D}^\mu U^+\,D^\nu U\right) - L_{10}\,\mathrm{tr}\left(U^+F^{(+)}_{\mu\nu}UF^{(-)\mu\nu}\right)\\ &+H_1\,\mathrm{tr}\left((F^{(+)}_{\mu\nu})^2 + (F^{(-)}_{\mu\nu})^2\right) + H_2\,\mathrm{tr}\,M^2\,,\end{aligned} \qquad (8)$$

where the dimensionless structure constants $L_i (i = 1, ..., 10)$ and $H_{1,2}$ were introduced by Gasser and Leutwyler in ref. [25]. Here we have introduced the notations

$$U = \Omega^2\;;\quad L_\mu = D_\mu U\,U^+\;;\quad F_0^2 = y\frac{N_c\mu^2}{4\pi^2}\,,$$

with

$y = \Gamma(0, \mu^2/\Lambda^2)\;;\quad M = diag(\chi_u^2, \chi_d^2, ..., \chi_n^2)\,,\quad \chi_i^2 = m_0^i\mu/(G_1F_0^2) = -2m_0^i<\bar{q}q>F_0^{-2}\,;$

$<\bar{q}q>$ is the quark condensate;

$$F^{(\pm)}_{\mu\nu} = \partial_\mu A^{(\pm)}_\nu - \partial_\nu A^{(\pm)}_\mu + [A^{(\pm)}_\mu, A^{(\pm)}_\nu]$$

are field-strength tensors, and

$$D_\mu * = \partial_\mu * + (A^{(-)}_\mu * - * A^{(+)}_\mu)\,,\quad \overline{D}_\mu * = \partial_\mu * + (A^{(+)}_\mu * - * A^{(-)}_\mu) \qquad (9)$$

are the covariant derivatives; $A^{(\pm)}_\mu = V_\mu \pm A_\mu$. Moreover, the coefficients L_i and $H_{1,2}$ are given by $L_1 - \frac{1}{2}L_2 = L_4 = L_6 = 0$ and

$$\begin{aligned}L_2 &= \frac{N_c}{16\pi^2}\frac{1}{12}\,,\quad L_3 = -\frac{N_c}{16\pi^2}\frac{1}{6}\,,\\ L_5 &= \frac{N_c}{16\pi^2}x(y-1)\,,\quad L_7 = -\frac{N_c}{16\pi^2}\frac{1}{6}\left(xy - \frac{1}{12}\right)\,,\\ L_8 &= -\frac{N_c}{16\pi^2}yx^2\,,\quad L_9 = \frac{N_c}{16\pi^2}\frac{1}{3}\,,\quad L_{10} = -\frac{N_c}{16\pi^2}\frac{1}{6}\,,\\ H_1 &= -\frac{N_c}{16\pi^2}\frac{1}{6}\left(y - \frac{1}{2}\right)\,,\quad H_2 = -\frac{N_c}{16\pi^2}2yx^2\,,\end{aligned} \qquad (10)$$

where $x = -\mu F_0^2/(2 <\bar{q}q>)$.

The effective (nonreduced) Lagrangian for the pseudoscalar sector, taking into account also the emission of the "structural" photons $\mathcal{A}_\mu$, can be obtained from (8) when $V_\mu = A_\mu = 0$ and tensor $F^{(\pm)}_{\mu\nu}$ is replaced by $ie(\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu)$. In the following section we will discuss the reduced nonlinear Lagrangian for pseudoscalar fields, which arises from generating functional (3) after integrating out the vector and axial-vector degrees of freedom in the modulus of quark determinant.

III. Strong Lagrangians with reduced vector and axial-vector fields

To perform the integration over vector and axial-vector fields we will use the fact that the modulus of quark determinant is invariant under chiral rotations. Then, the pseudoscalar fields can be eliminated from the modulus of quark determinant in the effective action (4) by using the rotated Dirac operator (7) for unitary gauge. After such transformation the pseudoscalar degrees of freedom still remain in the mass term of eq.(4), quadratic in meson fields, which are not invariant under chiral rotations. Since the masses of the vector and axial-vector mesons are large compared to the pion mass it is possible to integrate out the rotated fields $\tilde{V}_\mu$ and $\tilde{A}_\mu$ (6) in the effective meson action using the equations of motion which arise from the mass terms of the effective action (4) in the static limit [31]. In such an approximation the kinetic terms $(\tilde{F}^{(\pm)}_{\mu\nu})^2$ for the rotated fields $\tilde{V}_\mu$ and $\tilde{A}_\mu$ as well as higher order derivative nonanomalous and Wess-Zumino terms are treated as a perturbation.

In terms of the rotated fields $\tilde{V}_\mu$, $\tilde{A}_\mu$ (6) the quadratic part of the effective action (4) leads to the Lagrangian

$$\mathcal{L}_0 = \frac{F_0^2}{4}\,\mathrm{tr}\,(MU + h.c.) - \left(\frac{m_V^0}{g_V^0}\right)^2 \mathrm{tr}\,[(\tilde{V}_\mu - v_\mu)^2 + (\tilde{A}_\mu - a_\mu)^2]\,, \qquad (11)$$

where $(m_V^0/g_V^0)^2 = 1/(4G_2)$, with m_V^0 and g_V^0 being the bare mass and coupling constant of the vector gauge field, and

$$v_\mu = \frac{1}{2}\left(\Omega\partial_\mu\Omega^+ + \Omega^+\partial_\mu\Omega\right), \quad a_\mu = \frac{1}{2}\left(\Omega\partial_\mu\Omega^+ - \Omega^+\partial_\mu\Omega\right).$$

The modulus of quark determinant contributes to divergent and finite parts of the effective meson Lagrangian. In terms of the rotated fields, taking into account that for unitary gauge $\Phi \to \Sigma$, the divergent part of the quark determinant (eq.(41) from the Appendix B) gives

$$\mathcal{L}_{div} = \frac{F_0^2}{4\mu^2}\,\mathrm{tr}\left\{D_\mu\Sigma\overline{D}^\mu\Sigma^+ + \frac{1}{6}[(\tilde{F}^{(+)}_{\mu\nu})^2 + (\tilde{F}^{(-)}_{\mu\nu})^2]\right\}$$
$$= \frac{F_0^2}{4\mu^2}\,\mathrm{tr}\left\{[\tilde{V}_\mu, m_0]^2 - \{\tilde{A}_\mu, m_0\}^2 - 4\mu(2m_0+\mu)\tilde{A}_\mu^2 + \frac{1}{6}[(\tilde{F}^{(+)}_{\mu\nu})^2 + (\tilde{F}^{(-)}_{\mu\nu})^2]\right\} \qquad (12)$$

where the approximation $\Sigma = \mu + m_0$ was used.

The p^4-terms of the finite part of the effective meson Lagrangians (eq.(42) from the Appendix B) are of the form

$$\mathcal{L}^{(p^4)}_{fin} = \frac{N_c}{32\pi^2\mu^4}\,\mathrm{tr}\left\{\frac{1}{3}\left[\mu^2 D^2\Sigma\overline{D}^2\Sigma^+ - (D^\nu\Sigma\overline{D}_\mu\Sigma^+)^2\right] + \frac{1}{6}(D_\mu\Sigma\overline{D}^\nu\Sigma^+)^2\right.$$
$$-\mu^2\left(\mathcal{M}D_\mu\Sigma\overline{D}^\mu\Sigma^+ + \overline{\mathcal{M}}\,\overline{D}^\mu\Sigma^+D_\mu\Sigma\right)$$
$$+\mu^2\frac{2}{3}\left(D^\mu\Sigma\overline{D}^\nu\Sigma^+\tilde{F}^{(-)}_{\mu\nu} + \overline{D}^\mu\Sigma^+D^\nu\Sigma\tilde{F}^{(+)}_{\mu\nu}\right)$$
$$\left.+\mu^2\frac{1}{3}\tilde{F}^{(+)}_{\mu\nu}\Sigma^+\tilde{F}^{(-)\mu\nu}\Sigma - \frac{1}{6}\mu^4\left[(\tilde{F}^{(-)}_{\mu\nu})^2 + (\tilde{F}^{(+)}_{\mu\nu})^2\right]\right\}$$
$$= \frac{N_c}{32\pi^2}\,\mathrm{tr}\left\{-\frac{4}{3}\left[2\tilde{A}_\mu^2\left(\{\tilde{A}_\nu,\{\tilde{A}^\nu,\overline{m}_0\}\} - [\tilde{V}_\nu,[\tilde{V}^\nu,\overline{m}_0]]\right)\right.\right.$$
$$\left.+[\tilde{V}_\mu,\tilde{A}^\mu]\left(\{\tilde{A}_\nu,[\tilde{V}^\nu,\overline{m}_0]\} + [\tilde{V}_\nu,\{\tilde{A}^\nu,\overline{m}_0\}]\right) + [\tilde{V}_\mu,\tilde{A}^\mu]^2\right]$$
$$+\frac{8}{3}\left((\tilde{A}_\mu\tilde{A}_\nu)^2 + \{\{\tilde{A}_\mu,\overline{m}_0\},\tilde{A}_\nu\tilde{A}^\mu\tilde{A}^\nu\}\right) + 16\mu^2\overline{m}_0\tilde{A}_\mu^2$$
$$-\frac{4}{3}\left[\left(2\tilde{A}^\mu\tilde{A}^\nu + [\tilde{A}^\mu,\{\tilde{A}^\nu,\overline{m}_0\}]\right)(\tilde{F}^{(-)}_{\mu\nu} + \tilde{F}^{(+)}_{\mu\nu}) - \{\tilde{A}^\mu,[\tilde{V}^\nu,\overline{m}_0]\}(\tilde{F}^{(+)}_{\mu\nu} - \tilde{F}^{(-)}_{\mu\nu})\right]$$
$$\left.+\frac{1}{3}\left(\tilde{F}^{(+)}_{\mu\nu}\tilde{F}^{(-)\mu\nu} + \overline{m}_0\{\tilde{F}^{(+)}_{\mu\nu},\tilde{F}^{(-)\mu\nu}\}\right) - \frac{1}{6}\left[(\tilde{F}^{(-)}_{\mu\nu})^2 + (\tilde{F}^{(+)}_{\mu\nu})^2\right]\right\} + O(m_0^2), \qquad (13)$$

where $\overline{m}_0 \equiv m_0/\mu$, and we used the approximation $\mathcal{M} = \overline{\mathcal{M}} = \Sigma^2 - \mu^2 \approx 2\mu^2\overline{m}_0$ for matrices $\mathcal{M}$ and $\overline{\mathcal{M}}$ in unitary gauge.

The kinetic terms $(\tilde{F}^{(V,A)}_{\mu\nu})$, arising from the sum of Lagrangians (12) and (13), lead to the standard form after rescaling the rotated nonphysical vector and axial-vector fields $\tilde{V}_\mu, \tilde{A}_\mu$:

$$\tilde{V}_\mu = \frac{g_V^0}{(1+\tilde{\gamma})^{1/2}}\tilde{V}^{(ph)}_\mu, \quad \tilde{A}_\mu = \frac{g_V^0}{(1-\tilde{\gamma})^{1/2}}\tilde{A}^{(ph)}_\mu. \qquad (14)$$

Here

$$g_V^0 = \left[\frac{N_c}{48\pi^2}\left(\frac{8\pi^2F_0^2}{N_c\mu^2} - 1\right)\right]^{-1/2}, \quad \tilde{\gamma} = \frac{N_c(g_V^0)^2}{48\pi^2}, \qquad (15)$$

and $\tilde{V}^{(ph)}_\mu, \tilde{A}^{(ph)}_\mu$ are the physical fields of vector and axial-vector mesons with masses

$$m_\rho^2 = \frac{(m_V^0)^2}{1+\tilde{\gamma}}, \quad m_{A_1}^2 = \frac{(m_V^0)^2}{1-\tilde{\gamma}}Z_A^{-2}, \qquad (16)$$

where $Z_A^2 = 1 - (F_0 g_V^0/m_V^0)^2$ is the πA_1-mixing factor.

Since in the following we also want to investigate the radiative processes with "structural" photon emission in addition to inner bremsstrahlung ones, it is necessary to include electromagnetic interactions in the bosonization procedure. Obviously, one then simply has to use the replacements $\tilde{V}^{(ph)}_\mu \to \tilde{V}^{(ph)}_\mu + ie^{(ph)}\mathcal{A}^{(ph)}_\mu Q$, or $\tilde{V}_\mu \to \tilde{V}_\mu + ie_0\mathcal{A}_\mu Q$, where Q is the matrix of electric quark charges and

$$\mathcal{A}^{(ph)}_\mu = \frac{g_V^0}{(1+\tilde{\gamma})^{1/2}}\mathcal{A}_\mu, \quad e^{(ph)} = e_0\frac{(1+\tilde{\gamma})^{1/2}}{g_V^0}$$

are the physical electromagnetic field and charge respectively.

The static equations of motion arise from variation the mass terms of eq.(11) in chiral limit over rotated fields $\tilde{V}_\mu$, $\tilde{A}_\mu$ and lead to the relations

$$\tilde{V}_\mu = v_\mu^{(\gamma)}, \quad \tilde{A}_\mu = Z_A^2\, a_\mu^{(\gamma)} \tag{17}$$

and

$$\tilde{F}_{\mu\nu}^{(\pm)} = (Z_A^4 - 1)[a_\mu^{(\gamma)}, a_\nu^{(\gamma)}] + ie_0 Q\mathcal{F}_{\mu\nu}^{(\gamma)} + ie_0(\mathcal{A}_\mu[Q, v_\nu^{(\gamma)}] - \mathcal{A}_\nu[Q, v_\mu^{(\gamma)}])$$
$$\pm ie_0 Z_A^2(\mathcal{A}_\mu[Q, a_\nu^{(\gamma)}] - \mathcal{A}_\nu[Q, a_\mu^{(\gamma)}]). \tag{18}$$

Here

$$v_\mu^{(\gamma)} = \frac{1}{2}\Big(\Omega\partial_\mu^{(\gamma)}\Omega^+ + \Omega^+\partial_\mu^{(\gamma)}\Omega\Big), \quad a_\mu^{(\gamma)} = \frac{1}{2}\Big(\Omega\partial_\mu^{(\gamma)}\Omega^+ - \Omega^+\partial_\mu^{(\gamma)}\Omega\Big) = -\frac{1}{2}\xi_R^+ L_\mu^{(\gamma)}\xi_R\,;$$

$\partial_\mu^{(\gamma)} * = \partial_\mu * + ie_0\mathcal{A}_\mu[Q,*] = \partial_\mu * + ie^{(ph)}\mathcal{A}_\mu^{(ph)}[Q,*]$ is the prolonged derivative describing the emission of the inner bremsstrahlung photon while the electromagnetic field strength tensor $\mathcal{F}_{\mu\nu}^{(\gamma)} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$ corresponds to the structural photon ($e_0\mathcal{F}_{\mu\nu}^{(\gamma)} = e^{(ph)}\mathcal{F}_{\mu\nu}^{(\gamma,ph)}$); and $L_\mu^{(\gamma)} = (\partial_\mu^{(\gamma)}U)U^+$. Further, we will omit for simplicity the upper indices (γ) corresponding to the inner bremsstrahlung photon and only tensors $\mathcal{F}_{\mu\nu}^{(\gamma)}$ will be kept explicitly. We will also omit everywhere the upper indices (ph) assuming that all photons and electromagnetic charges in further formulae are physical.

Applying the equations of motion (17) to the terms of the effective actions (11,12), quadratic in vector and axial-vector fields, one reproduces the standard kinetic term for the pseudoscalar sector:

$$L_{kin} = -\frac{F_0^2}{4}\,\mathrm{tr}\,(L_\mu L^\mu). \tag{19}$$

In the same way the p^4-terms of the actions (12,13) lead to the reduced Lagrangians for pseudoscalar mesons of the types

$$\mathcal{L}^{(p^4,red)} = \frac{1}{2}\tilde{L}_2\,\mathrm{tr}\,\Big([L_\mu, L_\nu]^2\Big) + (3\tilde{L}_2 + \tilde{L}_3)\,\mathrm{tr}\,\Big((L_\mu L^\mu)^2\Big)$$
$$-2\tilde{L}_5\,\mathrm{tr}\,\Big(L_\mu L^\mu\xi_R M\xi_R^+\Big) - 2ie\mathcal{F}_{\mu\nu}^{(\gamma)}\tilde{L}_9\,\mathrm{tr}\,\Big(Q\xi_R^+ L^\mu L^\nu\xi_R\Big)$$
$$-2(ie)^2\tilde{L}_{10}\,\mathrm{tr}\,\Big[\mathcal{A}_\mu^2\Big(Q\xi_R^+ L_\nu\xi_R\, Q\xi_R^+ L^\nu\xi_R - Q^2\xi_R^+ L_\nu^2\xi_R\Big)$$
$$-\mathcal{A}_\mu\mathcal{A}_\nu\Big(Q\xi_R^+ L^\mu\xi_R\, Q\xi_R^+ L^\nu\xi_R - Q^2\xi_R^+ L^\mu L^\nu\xi_R\Big)\Big], \tag{20}$$

$$\mathcal{L}_{m_0}^{(p^4,red)} = \frac{N_c}{32\pi^2}\frac{4}{3}\,\mathrm{tr}\Big\{-4Z_A^4\Big[(L_\mu L^\mu)^2 - 2(L_\mu L_\nu)^2 + L_\nu L_\mu L^\mu L^\nu\Big](\xi_R\overline{m}_0\xi_R^+)$$
$$+(Z_A^4 - 1)^2\Big[(L_\mu L_\nu)^2 - L_\nu L_\mu L^\mu L^\nu\Big](\xi_R\overline{m}_0\xi_R^+)\Big\}$$
$$-ie\mathcal{F}_{\mu\nu}^{(\gamma)}\frac{N_c}{32\pi^2}\frac{1}{3}\,\mathrm{tr}\,\Big\{Q\Big[(1 + Z_A^4)\{\xi_R^+ L^\mu L^\nu\xi_R, \overline{m}_0\} + 4Z_A^4(\xi_R^+ L^\mu\xi_R\overline{m}_0\xi_R^+ L^\nu\xi_R)\Big]\Big\}$$
$$-(ie)^2\frac{N_c}{32\pi^2}\frac{1}{3}Z_A^4\,\mathrm{tr}\,\Big\{\overline{m}_0\Big[\mathcal{A}_\mu^2\Big(2(Q\xi_R^+ L_\nu\xi_R)^2 - \xi_R^+ L_\nu\xi_R\{\xi_R^+ L_\nu\xi_R, Q^2\}\Big)$$
$$-\mathcal{A}_\mu\mathcal{A}_\nu\Big(2Q\xi_R^+ L^\mu\xi_R\, Q\xi_R^+ L^\nu\xi_R - \xi_R^+ L^\mu\xi_R\{\xi_R^+ L^\nu\xi_R, Q^2\}\Big)\Big]\Big\} + O(\overline{m}_0^2), \tag{21}$$

Here $\mathcal{L}^{(p^4,red)}$ represents the part, corresponding to the effective p^4-Lagrangian in the Gasser–Leutwyler representation with the structure coefficients $\tilde{L}_i$ defined by the relations,

$$\tilde{L}_2 = \frac{N_c}{16\pi^2}\Big[\frac{1}{12}Z_A^8 + \frac{1}{6}(Z_A^4 - 1)\Big((Z_A^4 - 1)\frac{6\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2} - Z_A^4\Big)\Big],$$
$$\tilde{L}_3 = -\frac{N_c}{16\pi^2}\Big[\frac{1}{6}Z_A^8 + \frac{1}{2}(Z_A^4 - 1)\Big((Z_A^4 - 1)\frac{6\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2} - Z_A^4\Big)\Big],$$
$$\tilde{L}_5 = \frac{N_c}{16\pi^2}Z_A^4 x(4y - 1),$$
$$\tilde{L}_9 = \frac{N_c}{16\pi^2}\Big[\frac{1}{3}Z_A^4 - (Z_A^4 - 1)\frac{4\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2}\Big],$$
$$\tilde{L}_{10} = -Z_A^4\frac{1}{4}\frac{1+\tilde{\gamma}}{(g_V^0)^2}, \tag{22}$$

where

$$\frac{1+\tilde{\gamma}}{(g_V^0)^2} = \frac{F_0^2}{6\mu^2}.$$

The Lagrangian $\mathcal{L}_{m_0}^{(p^4,red)}$ describes the additional corrections arising from the expansion over the quark mass m_0.

To take into account also the p^6-terms of the finite part of the effective meson Lagrangian (eq.(43) from Appendix B) we will restrict ourselves only by consideration of the terms which additionally contribute to m_0-corrections of type of eq.(21):

$$\mathcal{L}_{fin}^{(p^6)} = \frac{N_c}{32\pi^2\mu^4}\,\mathrm{tr}\,\Big\{\frac{1}{6}\Big[\mathcal{M}\Big(D_\mu D_\nu\Sigma\,\overline{D}^\mu\overline{D}^\nu\Sigma^+ + D_\mu\Sigma\,\overline{D}^2\overline{D}^\mu\Sigma^+ + D^2 D_\mu\Sigma\,\overline{D}^\mu\Sigma^+\Big)$$
$$+\overline{\mathcal{M}}\Big(\overline{D}^\mu\overline{D}^\nu\Sigma^+ D_\mu D_\nu\Sigma + \overline{D}_\mu\Sigma^+ D^2 D^\mu\Sigma + \overline{D}^2\overline{D}^\mu\Sigma^+ D_\mu\Sigma\Big)\Big]$$
$$-\frac{1}{12\mu^2}\Big[\mathcal{M}\Big((D_\mu\Sigma\,\overline{D}^\mu\Sigma^+)^2 - (D_\mu\Sigma\,\overline{D}_\nu\Sigma^+)^2 + D_\mu\Sigma\,\overline{D}_\nu\Sigma^+ D^\nu\Sigma\,\overline{D}^\mu\Sigma^+\Big)$$
$$+\overline{\mathcal{M}}\Big((\overline{D}_\mu\Sigma^+ D^\mu\Sigma)^2 - (\overline{D}_\mu\Sigma^+ D_\nu\Sigma)^2 + \overline{D}_\mu\Sigma^+ D_\nu\Sigma\,\overline{D}^\nu\Sigma^+ D^\mu\Sigma\Big)\Big]$$
$$-\frac{1}{3}\Big[\tilde{F}_{\mu\nu}^{(-)}\Big(D^\mu\Sigma\,\overline{D}^\nu\Sigma^+\mathcal{M} + \mathcal{M}\,D^\mu\Sigma\,\overline{D}^\nu\Sigma^+ + D^\mu\Sigma\,\overline{\mathcal{M}}\,\overline{D}^\nu\Sigma^+\Big)$$
$$+\tilde{F}_{\mu\nu}^{(+)}\Big(\overline{D}^\mu\Sigma^+ D^\nu\Sigma\,\overline{\mathcal{M}} + \overline{\mathcal{M}}\,\overline{D}^\mu\Sigma^+ D^\nu\Sigma + \overline{D}^\mu\Sigma^+\mathcal{M}\,D^\nu\Sigma\Big)\Big]$$
$$-\frac{5}{6}\mu^2\Big[\mathcal{M}(\tilde{F}_{\mu\nu}^{(-)})^2 + \overline{\mathcal{M}}(\tilde{F}_{\mu\nu}^{(+)})^2\Big] + (\text{other terms}). \tag{23}$$

All other omitted terms after reducing the vector and axial-vector fields in eq.(43) will contribute only to the interaction of six or more pseudoscalar mesons.

The final expression for the reduced Lagrangian $\mathcal{L}_{m_0}^{(p^4,red)}$ can be presented then in the general form

$$\mathcal{L}_{m_0}^{(p^4,red)} = \mathrm{tr}\,\Big[\Big(\tilde{Q}_1(L_\mu L^\mu)^2 + \tilde{Q}_2(L_\mu L_\nu)^2 + \tilde{Q}_3 L_\nu L_\mu L^\mu L^\nu\Big)(\xi_R\overline{m}_0\xi_R^+)\Big]$$
$$-ie\mathcal{F}_{\mu\nu}^{(\gamma)}\,\mathrm{tr}\,\Big[Q\Big(\tilde{Q}_4\{\xi_R^+ L^\mu L^\nu\xi_R, \overline{m}_0\} + \tilde{Q}_5(\xi_R^+ L^\mu\xi_R\overline{m}_0\xi_R^+ L^\nu\xi_R)\Big)\Big]$$

$$
\begin{aligned}
&-(ie)^2\tilde{Q}_6\,\mathrm{tr}\Big\{\overline{m}_0\Big[\mathcal{A}_\mu^2\Big(2(Q\xi_R^\dagger L_\nu\xi_R)^2-\xi_R^\dagger L_\nu\xi_R\{\xi_R^\dagger L_\nu\xi_R,Q^2\}\Big)\\
&-\mathcal{A}_\mu\mathcal{A}_\nu\Big(2Q\xi_R^\dagger L^\mu\xi_R Q\xi_R^\dagger L^\nu\xi_R-\xi_R^\dagger L^\mu\xi_R\{\xi_R^\dagger L^\nu\xi_R,Q^2\}\Big)\Big]\Big\}
\end{aligned}
\tag{24}
$$

with $\tilde{Q}_i$ being the structure coefficients:

$$
\begin{aligned}
\tilde{Q}_1 &= \frac{N_c}{16\pi^2}\frac{1}{6}Z_A^4(5Z_A^4-17),\\
\tilde{Q}_2 &= \frac{N_c}{16\pi^2}\frac{1}{3}\Big[Z_A^8+16Z_A^4+\frac{3}{16}(Z_A^4-1)(17Z_A^4-9)\Big],\\
\tilde{Q}_3 &= -\frac{N_c}{16\pi^2}\frac{1}{6}\Big[3Z_A^8+16Z_A^4+\frac{3}{8}(Z_A^4-1)(7Z_A^4-9)\Big],\\
\tilde{Q}_4 &= \frac{N_c}{16\pi^2}\frac{1}{12}(5Z_A^4+7),\quad \tilde{Q}_5=0,\quad \tilde{Q}_6=\frac{N_c}{16\pi^2}\frac{3}{8}Z_A^4.
\end{aligned}
\tag{25}
$$

IV. Reduced currents

The path-integral bosonization method can be applied to the weak and electromagnetic-weak currents by using a generating functional for Green functions of quark currents introduced in [20] and [21]. After transition to collective fields in such a generating functional the latter is determined by the analog of formula (5) where now $i\widehat{\mathbf{D}}$ is replaced by

$$
\begin{aligned}
i\widehat{\mathbf{D}}(\eta) = &\,[i(\hat\partial+\hat{A}_R-i\hat\eta_R)-(\Phi+m_0-\eta_R)]P_R\\
&+[i(\hat\partial+\hat{A}_L-i\hat\eta_L)-(\Phi^+ +m_0-\eta_L)]P_L\,.
\end{aligned}
\tag{26}
$$

Here $\hat\eta_{L,R}=\eta^a_{L,R\mu}\gamma^\mu\frac{\lambda^a}{2}$ and $\eta_{L,R}=\eta^a_{L,R}\frac{\lambda^a}{2}$ are the external sources coupling to the quark currents $\overline{q}P_{L,R}\gamma^\mu\frac{\lambda^a}{2}q$ and quark densities $\overline{q}P_{L,R}\frac{\lambda^a}{2}q$ respectively. The quark densities define the contributions of the penguin-type four-quark operators of the effective nonleptonic weak Lagrangian [32] to the matrix elements of relevant kaon decays. The bosonized $(V\mp A)$ and $(S\mp P)$ meson currents, corresponding to the quark currents $\overline{q}P_{L,R}\gamma_\mu\frac{\lambda^a}{2}q$ and quark densities $\overline{q}P_{L,R}\frac{\lambda^a}{2}q$, can be obtained by variating the quark determinant with redefined Dirac operator (26) over the external sources coupling with these quark bilinears [21].

For further discussions it is convenient to present the bosonized weak and electromagnetic-weak $(V-A)$ current for pseudoscalar sector, generated by the nonreduced Lagrangian (8) and including the electromagnetic weak structural photon emission, in the form:

$$
\begin{aligned}
J^{(nred)a}_{L\mu} = &\, i\frac{F_0^2}{4}\,\mathrm{tr}\Big(\lambda^a L_\mu\Big)\\
&-i\,\mathrm{tr}\Big\{\lambda^a\Big[\frac{1}{2}R_1U^+\{(MU+U^+M),L_\mu\}U+R_2L_\nu L_\mu L^\nu\\
&+R_3\{L_\mu,L_\nu L^\nu\}+R_4\partial_\nu([L_\mu,L^\nu])\Big]\Big\}\\
&+e\mathcal{F}^{(\gamma)}_{\mu\nu}R_5\,\mathrm{tr}\Big(\lambda^a[UQU^+,L^\nu]\Big)\,.
\end{aligned}
\tag{27}
$$

Here, the first term is the kinetic current and all other terms originate from the p^4-part of Lagrangian (8); R_i are the structure coefficients, associated with the corresponding parameters $\tilde{R}_i$ of the representation (30) for the reduced $(V-A)$-currents:

$$
R_1=-L_5\,,\quad R_2=2L_2\,,\quad R_3=2L_2+L_3\,,\quad R_4=-\frac{1}{2}L_9\,,\quad R_5=L_9+L_{10}\,. \tag{28}
$$

The bosonized $(S-P)$ current for pseudoscalar sector, generated by the Lagrangian (8) and including the structural photons, has the form:

$$
\begin{aligned}
J_L^{(nred)a} = &\,\frac{F_0^2}{8\mu}\,\mathrm{tr}\,(\lambda^a\partial^2U)+\frac{F_0^2}{4}\mu R\,\mathrm{tr}\,(\lambda^aU)\\
&-\frac{1}{\mu}\,\mathrm{tr}\,\Big\{\lambda^a\Big[L_2\partial_\mu(L_\nu L^\mu L^\nu)+(2L_2+L_3)\partial_\mu(L_\nu L^\nu L^\mu)\\
&-\frac{1}{2}L_5\Big(\partial_\mu((MU+U^+M)L^\mu U)+2\mu^2RL_\mu L^\mu\Big)\Big]\Big\}\\
&-\frac{ie}{2\mu}L_9\,\mathrm{tr}\,\Big\{\lambda^a\partial^\mu(\mathcal{F}^{(\gamma)}_{\mu\nu}[Q,L^\nu U])\Big\}-\frac{1}{2\mu}L_{10}(ie\mathcal{F}^{(\gamma)}_{\mu\nu})^2\,\mathrm{tr}\,(\lambda^aQUQ)\,,
\end{aligned}
\tag{29}
$$

where $R=\langle\overline{q}q\rangle/(\mu F_0^2)$. Here, the first and second terms are generated at p^2-level by the kinetic and mass terms of Lagrangian (8), respectively, while all other terms originate from its p^4-part.

Combining the method of the chiral bosonization of quark currents with the static equations of motions it is possible to obtain the bosonized meson currents for pseudoscalar sector with the reduced vector and axial vector degrees of freedom. In this way one can reproduce the standard kinetic $(V-A)$ current for pseudoscalar mesons

$$
J^{(kin)a}_{L\mu}=i\frac{F_0^2}{4}\,\mathrm{tr}\,(\lambda^aL_\mu)\,,
$$

which arises from the terms of effective actions (11,12), quadratic in vector and axial-vector rotated fields, after redefinition of the rotated fields

$$
\tilde{V}_\mu\to\tilde{V}_\mu-i(\xi_L\eta_{L\mu}\xi_L^\dagger+\xi_R\eta_{R\mu}\xi_R^\dagger)\,,\quad \tilde{A}_\mu\to\tilde{A}_\mu+i(\xi_L\eta_{L\mu}\xi_L^\dagger-\xi_R\eta_{R\mu}\xi_R^\dagger)\,,
$$

and variation over $\eta_{L\mu}$ with applying the static equations of motion.

Applying the same procedure to the p^4- and p^6 terms (13,23) of the effective action we also obtain the bosonized weak and electromagnetic weak $(V-A)$ currents for pseudoscalar sector with the reduced vector and axial-vector degrees of freedom. It is convenient to present these reduced currents in the form:

$$
\begin{aligned}
J^{(p^4,red)a}_{L\mu} = &-i\,\tilde{R}_1\,\mathrm{tr}\,\Big(\lambda^a\{\xi_RM\xi_R^\dagger,L_\mu\}\Big)\\
&-i\,\mathrm{tr}\,\Big\{\lambda^a\Big[\tilde{R}_2L_\nu L_\mu L^\nu+\tilde{R}_3\{L_\mu,L_\nu L^\nu\}+\tilde{R}_4\xi_R\partial_\nu(\xi_R^\dagger[L_\mu,L^\nu]\xi_R)\xi_R^\dagger\Big]\Big\}\\
&+e\mathcal{F}^{(\gamma)}_{\mu\nu}\tilde{R}_5\,\mathrm{tr}\,\Big(\lambda^a[\xi_RQ\xi_R^\dagger,L^\nu]\Big)\,,
\end{aligned}
\tag{30}
$$

$$J^{(p^4,red)a}_{m_0,L\mu} = -i\,\mathrm{tr}\left\{\lambda^a\left[\tilde{R}_6\{\xi_R\overline{m}_0\xi_R^\dagger, L_\nu L_\mu L_\nu\} + \tilde{R}_7\left(\xi_R\overline{m}_0\xi_R^\dagger L_\nu^2 L_\mu + L_\mu L_\nu^2\xi_R\overline{m}_0\xi_R^\dagger\right)\right.\right.$$
$$+\tilde{R}_8\left(\xi_R\overline{m}_0\xi_R^\dagger L_\mu L_\nu^2 + L_\nu^2 L_\mu\xi_R\overline{m}_0\xi_R^\dagger + \{L_\mu, L_\nu\xi_R\overline{m}_0\xi_R^\dagger L^\nu\}\right)$$
$$+\tilde{R}_9\left(L_\mu\xi_R\overline{m}_0\xi_R^\dagger L_\nu^2 + L_\nu^2\xi_R\overline{m}_0\xi_R^\dagger L_\mu\right)$$
$$+\tilde{R}_{10}\left(L_\nu L_\mu\xi_R\overline{m}_0\xi_R^\dagger L^\nu + L^\nu\xi_R\overline{m}_0\xi_R^\dagger L_\mu L_\nu\right)$$
$$+\tilde{R}_{11}\xi_R\partial_\nu\left(\xi_R^\dagger(L^\nu\xi_R\overline{m}_0\xi_R^\dagger L_\mu - L_\mu\xi_R\overline{m}_0\xi_R^\dagger L^\nu)\xi_R\right)\xi_R^\dagger$$
$$\left.+\tilde{R}_{12}\xi_R\{\overline{m}_0, \partial_\nu(\xi_R^\dagger[L_\mu, L^\nu]\xi_R)\}\xi_R^\dagger\right]\Big\}$$
$$+e\mathcal{F}^{(\gamma)}_{\mu\nu}\,\mathrm{tr}\left\{\lambda^a\left[\tilde{R}_{13}(\xi_R Q\xi_R^\dagger L^\nu\xi_R\overline{m}_0\xi_R^\dagger - \xi_R\overline{m}_0\xi_R^\dagger L^\nu\xi_R Q\xi_R^\dagger)\right.\right.$$
$$+\tilde{R}_{14}(\xi_R\overline{m}_0 Q\xi_R^\dagger L^\nu - L^\nu\xi_R Q\overline{m}_0\xi_R^\dagger)$$
$$\left.\left.+\tilde{R}_{15}(\xi_R Q\overline{m}_0\xi_R^\dagger L^\nu - L^\nu\xi_R\overline{m}_0 Q\xi_R^\dagger)\right]\right\} \tag{31}$$

with $\tilde{R}_i$ being the structure coefficients:

$$\tilde{R}_1 = -\frac{N_c}{16\pi^2}\frac{1}{2}Z_A^2 x(y-1),$$
$$\tilde{R}_2 = \frac{N_c}{16\pi^2}\frac{1}{12}Z_A^2\left(Z_A^4 + 1 - (Z_A^4 - 1)\frac{12\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2}\right),$$
$$\tilde{R}_3 = \frac{1}{2}\tilde{R}_4 = -\frac{N_c}{16\pi^2}\frac{1}{24}Z_A^2\left(1 - (Z_A^4 - 1)\frac{12\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2}\right),$$
$$\tilde{R}_5 = -\frac{N_c}{16\pi^2}\frac{1}{6}Z_A^2\left(1 - \frac{12\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2}\right),$$
$$\tilde{R}_6 = \frac{N_c}{16\pi^2}\frac{1}{96}Z_A^2(19Z_A^4 - 1), \quad \tilde{R}_7 = \frac{5}{6}\tilde{R}_9 = -\frac{N_c}{16\pi^2}\frac{5}{24}Z_A^6,$$
$$\tilde{R}_8 = -\frac{N_c}{16\pi^2}\frac{1}{96}Z_A^2(19Z_A^4 - 5), \quad \tilde{R}_{10} = \frac{N_c}{16\pi^2}\frac{1}{96}Z_A^2(23Z_A^4 - 1),$$
$$\tilde{R}_{11} = \frac{N_c}{16\pi^2}\frac{1}{12}Z_A^4(Z_A^2 - 2),$$
$$\tilde{R}_{12} = \frac{N_c}{16\pi^2}\frac{1}{24}Z_A^2\left(Z_A^4 + 4Z_A^2 - \frac{9}{2}Z_A^{-2}(Z_A^4 - 1)\right),$$
$$\tilde{R}_{13} = \frac{3}{5}\tilde{R}_{14} = \frac{1}{2}\tilde{R}_{15} = -\frac{N_c}{16\pi^2}\frac{1}{8}Z_A^2. \tag{32}$$

Thus, the reduction of the vector and axial vector fields does not change the kinetic term of the bosonized $(V-A)$ current while the structure of the p^4 part of $(V-A)$ current is strongly modified (compare (27) and (30)).

Using the bosonization procedure of ref.[21] and the equations of motion (17) we obtain also the reduced $(S-P)$ meson currents. After redefinition of scalar fields

$$\Sigma \to \Sigma - 2\xi_L\eta_R\xi_R^\dagger, \qquad \Sigma^+ \to \Sigma^+ - 2\xi_R\eta_L\xi_L^\dagger \tag{33}$$

and variation over η_L with applying the static equations of motion the divergent part of the effective action (12) leads to the scalar current

$$J_L^{(div,red)a} = \frac{F_0^2}{8\mu}Z_A^2\,\mathrm{tr}\left\{\lambda^a\left[L_\mu^2 U + 4\mu^2\xi_R\overline{m}_0\xi_R\right.\right.$$

$$+\frac{1}{4}\left(2L_\mu\xi_R\overline{m}_0\xi_R^\dagger L^\mu U + \xi_R\{\overline{m}_0, \xi_R^\dagger L_\mu^2\xi_R\}\xi_R\right)\Big]\Big\}$$
$$+\frac{F_0^2}{4}\mu R Z_A^{-2}\,\mathrm{tr}\left(\lambda^a(U + \xi_R\overline{m}_0\xi_R)\right). \tag{34}$$

The p^4– and p^6–terms of the finite part of the effective action (13,23) generate the scalar meson current which can be present in the form

$$J_L^{(p^4,red)a} = -\mu\tilde{G}_1\,\mathrm{tr}\,(\lambda^a L_\mu^2 U)$$
$$-\frac{1}{\mu}\mathrm{tr}\left\{\lambda^a\left[\tilde{G}_2\xi_R\partial^2(\xi_R^\dagger L_\mu^2\xi_R)\xi_R\right.\right.$$
$$+\tilde{G}_3\left(\xi_R(\partial_\mu(\xi_R^\dagger L_\nu\xi_R))^2\xi_R + \xi_R\{\xi_R^\dagger L_\mu\xi_R, \partial^2(\xi_R^\dagger L^\mu\xi_R)\}\xi_R\right)$$
$$+\tilde{G}_4\left(\xi_R\partial_\mu(\xi_R^\dagger L_\nu\xi_R)\,\xi_R^\dagger L^\mu L^\nu U + L^\nu L^\mu\xi_R\,\partial_\mu(\xi_R^\dagger L_\nu\xi_R)\xi_R\right)$$
$$+\tilde{G}_5\left(L^\mu\xi_R\,\partial_\mu(\xi_R^\dagger L_\nu\xi_R)\,\xi_R^\dagger L^\nu U - L^\nu\xi_R\,\partial_\mu(\xi_R^\dagger L_\nu\xi_R)\,\xi_R^\dagger L^\mu U\right.$$
$$\left.+L^\mu L^\nu\xi_R\,\partial_\mu(\xi_R^\dagger L_\nu\xi_R)\xi_R - L^\nu L^\mu\xi_R\,\partial_\mu(\xi_R^\dagger L_\nu\xi_R)\xi_R\right)$$
$$+\tilde{G}_6\xi_R\partial_\mu(\xi_R^\dagger[L_\nu, [L^\nu, L^\mu]]\xi_R)\xi_R + \tilde{G}_7 L_\nu L_\mu^2 L^\nu U$$
$$\left.+\tilde{G}_8 L_\mu^2 L_\nu^2 U + \tilde{G}_9(L_\mu L_\nu)^2 U\right]\Big\}$$
$$-ie\frac{1}{\mu}\,\mathrm{tr}\left\{\lambda^a\left[\tilde{G}_{10}\xi_R\partial^\mu(\mathcal{F}^{(\gamma)}_{\mu\nu}[Q, \xi_R^\dagger L^\nu\xi_R])\xi_R\right.\right.$$
$$\left.+\mathcal{F}^{(\gamma)}_{\mu\nu}\left(\tilde{G}_{11}L^\mu\xi_R Q\xi_R^\dagger L^\nu U + \tilde{G}_{12}\xi_R\{\xi_R^\dagger L^\mu L^\nu\xi_R, Q\}\xi_R\right)\right]\Big\}$$
$$+(ie\mathcal{F}^{(\gamma)}_{\mu\nu})^2\frac{1}{\mu}\tilde{G}_{13}\,\mathrm{tr}\,(\lambda^a\xi_R Q^2\xi_R) + O(\overline{m}_0) \tag{35}$$

where $\tilde{G}_i$ are the structure coefficients:

$$\tilde{G}_1 = 6\tilde{G}_2 = 6\tilde{G}_3 = \frac{3}{2}\tilde{G}_{11} = \frac{N_c}{128\pi^2}Z_A^4, \quad \tilde{G}_4 = \tilde{G}_5 = \frac{N_c}{256\pi^2}\frac{1}{3}Z_A^6,$$
$$\tilde{G}_6 = \frac{N_c}{256\pi^2}\frac{1}{6}Z_A^2(Z_A^4 - 1),$$
$$\tilde{G}_7 = \frac{N_c}{256\pi^2}\frac{1}{6}\left(2Z_A^8 + (Z_A^4 - 1)(2Z_A^4 + 1)\right),$$
$$\tilde{G}_8 = \frac{N_c}{256\pi^2}\frac{1}{6}\left(2Z_A^8 + (Z_A^4 - 1)(Z_A^4 + 1)\right),$$
$$\tilde{G}_9 = \frac{N_c}{256\pi^2}\frac{1}{6}\left(5Z_A^8 - (Z_A^4 - 1)(3Z_A^4 + 2)\right),$$
$$\tilde{G}_{10} = \frac{N_c}{256\pi^2}\frac{2}{3}Z_A^2, \quad \tilde{G}_{12} = -\frac{N_c}{256\pi^2}\frac{1}{3}(5Z_A^4 - 2), \quad \tilde{G}_{12} = \frac{N_c}{256\pi^2}\frac{4}{3}. \tag{36}$$

It is important to mention that the p^6 terms (23) of the finite part of the effective action contributes only to the m_0 corrections (31) for $(V-A)$ current while in the case of $(S-P)$ currents due to the fact that redefinition (33) leads to the replacement

$$\mathcal{M} \to \mathcal{M} - 2(\xi_L\eta_R\xi_R^\dagger\Sigma^+ + \Sigma\xi_R\eta_L\xi_L^\dagger), \qquad \overline{\mathcal{M}} \to \overline{\mathcal{M}} - 2(\Sigma^+\xi_L\eta_R\xi_R^\dagger + \xi_R\eta_L\xi_L^\dagger\Sigma),$$

the same p^6-terms give contributions also to the part of current (35) do not containing quark mass m_0.

It can be easily shown that at the p^2 level the reduction of the vector and axial-vector fields does not change the physical results for matrix elements of the bosonized gluonic penguin operator. In fact, both for the reduced current (35) and for the first two terms of the nonreduced current (29) the corresponding p^2-contributions to the penguin operator matrix element, dominating by the interference of the kinetic and mass terms of the scalar current, can be presented effectively in the same form:

$$< T^{(peng)} > \propto -\frac{F_0^4}{32} R < (\partial_\mu U \, \partial^\mu U^+)_{23} > .$$

On the other hand the structure of the p^4-part of the pseudoscalar meson $(S-P)$ current proves to be strongly modified by the reduction of the vector and axial-vector fields (compare the expressions (29) and (35)).

V. Numerical estimations

To discuss some physical consequences for pseudoscalar nonet of mesons we have to fix initially the numerical values of the various parameters entering in the reduced Lagrangian and currents. The parameters χ_i^2 can be fixed by the spectrum of pseudoscalar mesons. Here we use the values $\chi_u^2 = 0.0114\mathrm{GeV}^2$, $\chi_d^2 = 0.025\mathrm{GeV}^2$, and $\chi_s^2 = 0.47\mathrm{GeV}^2$. To fix other empirical constants of our model we will use the experimental parameters, listed in Table 1: the masses of ρ- and A_1 mesons, the coupling constants of $\rho \to \pi\pi$ and $\pi, K \to \mu\nu$ decays, the $\pi\pi$ scattering lengths a_l^I, the pion electromagnetic squared radii $< r_{em}^2 >_{\pi^+}$ and pion polarizability $\alpha_{\pi^\pm}$. We also include in our analysis the data on the $\gamma\gamma \to \pi^+\pi^-$ cross section near to the threshold (see Fig.1). We will use the relations (15), (16), $g_V = g_V^0(1+\tilde{\gamma})^{-1/2}$ and

$$g_{\rho\pi\pi} = g_V\Big[1 + \frac{m_\rho^2}{2F_0^2}\Big(\frac{N_c}{48\pi^2}Z_A^4 - \frac{F_0^2}{24\mu^2}Z_A^{-4}(1-Z_A^2)^2\Big)\Big].$$

Then, using the connection $m_s^0 = \widehat{m}_0 \chi_s^2/m_\pi^2$, where $\widehat{m}_0 \equiv (m_u^0 + m_d^0)/2$, the splitting of the constants F_{π,K^+} can be presented in the form:

$$F_{\pi,K^+} = Z_{\pi,K^+} F_0\Big[1 + (\overline{m}_u^0 + \overline{m}_{d,s}^0)\frac{1}{2}Z_A^2\Big(1 - \frac{N_c\mu^2}{4\pi^2 F_0^2}\Big)\Big]$$
$$\approx F_0\Big[1 + (\overline{m}_u^0 + \overline{m}_{d,s}^0)\frac{1}{2}Z_A^2(1+4Z_A^2)\Big(1 - \frac{N_c\mu^2}{4\pi^2 F_0^2}\frac{1+Z_A^2}{1+5Z_A^2}\Big)\Big] + O(\overline{m}_0^2).$$

Here

$$Z_{\pi,K^+} = \Big[1 + (\overline{m}_u^0 + \overline{m}_{d,s}^0)4Z_A^4\Big(1 - \frac{N_c\mu^2}{16\pi^2 F_0^2}\Big)\Big]^{1/2}$$

is the factor arising from the renormalization of the pseudoscalar meson fields

$$\varphi \to \varphi + \Big\{4Z_A^4\Big(1 - \frac{N_c\mu^2}{16\pi^2 F_0^2}\Big)\overline{m}_0, \varphi\Big\}$$

which leads the bilinear kinetic part, including terms up to order m_q^0, to the standard form of the kinetic Lagrangian:

$$\mathcal{L}_{kin}^{bil} = \frac{1}{2}\,\mathrm{tr}\Big\{\Big[1 - 8Z_A^4\Big(1 - \frac{N_c\mu^2}{16\pi^2F_0^2}\Big)\overline{m}_0\Big]\partial_\mu\varphi\partial^\mu\varphi\Big\} \to \frac{1}{2}\,\mathrm{tr}\,(\partial_\mu\varphi\partial^\mu\varphi).$$

The $\pi\pi$-scattering lengths are defined by the structure coefficients $\tilde{L}_2$ and $\tilde{L}_3$. For $\pi\pi$-scattering lengths a_l^I (indices I and l refer here to the isotopic spin and orbital momentum, respectively) in one-loop approximation we obtained [33]

$$a_0^0 = \frac{\pi}{2}\alpha_0(9-5\delta) + \frac{\pi}{2}\alpha_0^2\big[5A + 3B + 2D + 3C - 6(\delta^2 + 4b + 3)\big],$$
$$a_0^2 = -\frac{\pi}{2}\alpha_0 2\delta + \frac{\pi}{2}\alpha_0^2 2\big[A + D - 3(\delta^2 + b + 3)\big],$$
$$a_1^1 = \frac{\pi}{2}\alpha_0 + \frac{\pi}{2}\alpha_0^2\frac{1}{3}\Big[B + 6\delta + a - 3 + \frac{1}{3}(\delta^2 - b - 3)\Big],$$
$$a_2^0 = \frac{\pi}{2}\alpha_0^2\Big[\frac{1}{15}(C + 4D) - \frac{2}{5}\Big(5 + \frac{3\delta - 2a + 6}{9} - \frac{\delta^2 + 4b + 3}{15}\Big)\Big],$$
$$a_2^2 = \frac{\pi}{2}\alpha_0^2\Big[\frac{1}{15}(C + D) - \frac{1}{5}\Big(4 + \frac{6\delta - a + 3}{9} - \frac{2}{45}(\delta^2 + b + 3)\Big)\Big].$$

Here $\alpha_0 = \frac{1}{3}(m_\pi/(2\pi F_0))^2$; $\delta = \frac{3}{2}(1-\beta)$, with β being the parameter of chiral symmetry breaking which takes here the value $\beta = 1/2$; $a = 21(1-\delta)$ and $b = (11\delta^2 - 15\delta + 3)$. The parameters

$$A = A^B + A^{loop}, \quad B = B^B + B^{loop}, \quad C = C^B + C^{loop}, \quad D = D^B + D^{loop}$$

include in themselves the Born contributions

$$A^B = -144\pi^2(\tilde{L}_2 - \tilde{L}_3), \quad B^B = -576\pi^2\tilde{L}_3, \quad C^B = 576\pi^2(\tilde{L}_2 + \tilde{L}_3), \quad D^B = 576\pi^2\tilde{L}_2$$

and the pion-loop contributions calculated, using the superpropagator method [34], in ref.[35]:

$$A^{loop} = -1.5, \quad B^{loop} = 3, \quad C^{loop} = 5.5, \quad D^{loop} = 11.$$

The pion electromagnetic squared radius is defined as the coefficient of the q^2-expansion of the electromagnetic form factor $f_\pi^{em}(q^2)$:

$$< \pi(p_2)|V_\mu^{em}|\pi(p_1) > = f_\pi^{em}(q^2)(p_1 - p_2)_\mu,$$
$$f_\pi^{em}(q^2) = 1 + \frac{1}{6} < r_{em}^2 >_\pi q^2 + \ldots$$

Being restricted only by pion loops, one gets in the SP regularization the corresponding contribution to the electromagnetic squared radius [40,41]:

$$< r_{em}^2 >_{\pi^+}^{(loop)} = -\frac{1}{(4\pi F_0)^2}\Big[3C + \ln\Big(\frac{m_\pi}{2\pi F_0}\Big)^2 - 1\Big] = 0.062 fm^2,$$

where $C = 0.577$ is the Euler constant. Because the main contribution to this value arises from the logarithmic term, the kaon loop contribution, containing the small logarithm $\ln\left(m_K/(2\pi F_0)\right)^2$, can be neglected. At the Born level, the contribution to the pion electromagnetic squared radius originates from the $\tilde{L}_9$-term of the reduced Lagrangian (20):

$$< r^2_{em} >^{(Born)}_{\pi^+} = \frac{12}{F_0^2}\tilde{L}_9$$

The pion polarizability can be determined through the Compton-scattering amplitude:

$$< \pi_1(p_1)\pi_2(p_2)|S|\gamma_{\lambda_1}(q_1)\gamma_{\lambda_2}(q_2) >= T_1(p_1p_2 \mid q_1q_2) + T_2(p_1p_2|q_1q_2)\,,$$

$$T_1^{(\pm)} = 2e^2\varepsilon^\mu_{\lambda_1}\varepsilon^\nu_{\lambda_2}\left(g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1q_1} - \frac{p_1^\nu p_2^\mu}{p_2q_1}\right), \quad T_1^{(0)} = 0;$$

$$T_2 = \varepsilon^\mu_{\lambda_1}\varepsilon^\nu_{\lambda_2}((q_1q_2)g_{\mu\nu} - q_{1\nu}q_{2\mu})\beta(q_1q_2),$$

where $\beta(q_1q_2)$ is the so-called dynamical polarizability function. Defining the polarizability of a meson as the coefficient of the effective interaction with the external electromagnetic field

$$V_{int} = -\alpha_\pi(E^2 - H^2)/2$$

one obtains

$$\alpha_\pi = \left.\frac{\beta_\pi(q_1q_2)}{8\pi m_\pi}\right|_{(q_1q_2)=0}.$$

The pion-loops give the finite contributions without UV-divergences [6,7,11]:

$$\beta^{(loop)}_{\pi\pm} = \frac{e^2}{8\pi^2F_0^2}\left(1 - \frac{4\delta-3}{3s_\pi}\right)f(s_\pi)\,, \qquad \beta^{(loop)}_{\pi^0} = \frac{e^2}{4\pi^2F_0^2}\left(1 - \frac{\delta}{3s_\pi}\right)f(s_\pi)\,,$$

where $s_\pi = (q_1q_2)/(2m_\pi^2)$, $f(\zeta) = \zeta^{-1}J^2(\zeta) - 1$, and

$$J(\zeta) = \begin{cases} \arctan(\zeta^{-1}-1)^{-1/2}, & 0 < \zeta < 1; \\ \frac{1}{2}\left(\ln\frac{1+\sqrt{1-\zeta^{-1}}}{1-\sqrt{1+\zeta^{-1}}} - i\pi\right), & \zeta > 1; \\ \frac{1}{2}\ln\frac{1+\sqrt{1-\zeta^{-1}}}{-1+\sqrt{1-\zeta^{-1}}}, & \zeta < 0. \end{cases}$$

The meson-loop contributions to the pion polarizabilities are

$$\alpha^{(loop)}_{\pi\pm} = 0, \qquad \alpha^{(loop)}_{\pi^0} = -\frac{e^2}{384\pi^3F_0^2m_\pi^2} = -5.43\cdot 10^{-5}fm^3\,.$$

At the Born level, the $\tilde{L}_9$ and $\tilde{L}_{10}$ terms of the reduced Lagrangian (20) give:

$$\beta^{(Born)}_{\pi\pm} = \frac{8e^2}{F_0^2}(\tilde{L}_9 + \tilde{L}_{10})\,, \qquad \beta^{(Born)}_{\pi^0} = 0\,.$$

In our analysis the constants F_0, $\widehat{m}_0$, μ and m^0_V are treated as the independent empirical parameters and their values are fixed as

$$F_0 = 91.7\text{MeV}\,, \quad \widehat{m}_0 = 2.2\text{MeV}\,, \quad \mu = 186\text{MeV}\,, \quad m^0_V = 840\text{MeV}\,. \qquad (37)$$

The corresponding calculated values of the input parameters are also presented in Table 1. The results for the $\gamma\gamma \to \pi^+\pi^-$ cross sections are shown in Fig.1. All other constants can be calculated using the values (37):

$$g^0_V = 5.4\,, \quad \tilde{\gamma} = 0.185\,, \quad Z_A^2 = 0.653\,, \quad <\bar{q}q> = -(330\text{MeV})^3\,, \quad m^0_s = 53\text{MeV}\,.$$

The values for the current quark masses seem to be by a factor of $2 \div 3$ too small and the quark condensate, respectively, by the same factor too large as compared with the corresponding value from the usual phenomenological analysis, based on nonreduced Lagrangian and currents. A similar shift of the current masses and the condensate was already observed, for example, in ref.[42] after taking into account the vector–scalar and axial-vector–pseudoscalar mixing in the analysis of the collective mesons mass spectrum within the extended NJL model.

Using the values of the parameters $Z_A^2, \tilde{\gamma}$ and $(g^0_V)^2$ which were fixed above, one can compare numerically the structural parameters $\tilde{L}_i$ (22) of the reduced effective Lagrangian (20) with the corresponding parameters L_i of the nonreduced Lagrangian (8):

$$\tilde{L}_2 = 1.20L_2 = 1.90\cdot 10^{-3}\,, \quad \tilde{L}_3 = 1.71L_3 = -5.41\cdot 10^{-3}\,, \quad \tilde{L}_5 = 1.99\cdot 10^{-3}\,,$$
$$\tilde{L}_9 = 1.35L_9 = 8.53\cdot 10^{-3}\,, \quad \tilde{L}_{10} = 1.36L_{10} = -4.33\cdot 10^{-3}\,, \qquad (38)$$

After substituting the values of $Z_A^2, \tilde{\gamma}$ and $(g^0_V)^2$ into eqs.(32) one can also compare numerically the structure parameters $\tilde{R}_i$ and R_i:

$$\tilde{R}_1 = -0.285\cdot 10^{-3}\,, \quad \tilde{R}_2 = 0.76R_2 = 2.42\cdot 10^{-3}\,, \quad \tilde{R}_3 = -0.992\cdot 10^{-3} \quad (R_3 = 0)\,,$$
$$\tilde{R}_4 = 2\tilde{R}_3 = 0.62R_4 = -1.98\cdot 10^{-3}\,, \quad \tilde{R}_5 = 0.39R_5 = 1.23\cdot 10^{-3}\,. \qquad (39)$$

The electromagnetic–weak part of the non reduced current (27) corresponding to the structural constant R_5 (respectively, the $\tilde{R}_5$ term of the reduced current (30)) describes the axial-vector form factor F_A of the radiative decay $\pi \to l\nu\gamma$. The form factors of this decay are defined by the parameterization of the amplitude

$$T_\mu(K, \pi \to l\nu\gamma) = \sqrt{2}e\left[F_V\varepsilon_{\mu\nu\alpha\beta}k^\nu q^\alpha\varepsilon^\beta + iF_A\left(\varepsilon_\mu(kq) - q_\mu(k\varepsilon)\right)\right],$$

where k is the 4-momentum of the decaying meson, q and ε are the 4-momentum and polarization 4-vector of the photon, and the vector form factor F_V is determined by the anomalous Wess–Zumino electromagnetic–weak current, originating from the anomalous part of the effective meson action, which is related to the phase of the quark determinant. The ratio of the axial-vector and vector form factors is determined by the relation

$$\frac{F_A}{F_V} = 32\pi^2R_5\,.$$

The theoretical value of the ratio $F_A/F_V = 32\pi^2(L_9 + L_{10}) = 1$ arising from non reduced current (27) with structure constants $L_{9,10}$ (10) is in disagreement with the experimental results on this ratio:

$$\left(\frac{F_A}{F_V}\right)^{(exp)} = \begin{cases} 0.25 \pm 0.12 & [43], \\ 0.41 \pm 0.23 & [44]. \end{cases}$$

At the same time the $\tilde{R}_5$ gives the value

$$\frac{F_A}{F_V} = -Z_A^2\Big(1 - \frac{12\pi^2}{N_c}\frac{1+\tilde{\gamma}}{(g_V^0)^2}\Big) = 0.39$$

in agreement with the experimental data.

This result reproduces in nonlinear realization of chiral symmetry the well known result of ref.[14], where the role of πA_1 mixing in $\pi \to l\nu\gamma$ decay was considered in linear parameterization. Thus, after reducing the vector and axial-vector degrees of freedom it proves to be possible to remove the inselfconsistency in the description of the ratio F_A/F_V and pion polarizability which arises seemingly in the pseudoscalar sector of the non reduced effective Lagrangian (8) with the $L_{9,10}$ terms * (see the detailed discussion of this inselfconsistency, for example, in ref. [46-48]). The same problem was also considered in ref.[49], where the values of the structure constants combination $(L_9 + L_{10})$ and pion polarizability $\alpha_{\pi\pm}$ determined from the fit of $\gamma\gamma \to \pi^+\pi^-$ cross section data were discussed. Fig.1 shows that within the experimental errors the MARK-II data [50] are consistent with the experimental result for pion polarizability obtained from radiative π scattering in nuclear Coulomb fields [39]. We have taken into account one loop corrections, while this was not done in ref.[49]. The description of the $\gamma\gamma \to \pi^+\pi^-$ cross section data above $m_{\pi\pi} = 500$MeV can be improved if one takes into account the unitary corrections in a more complete way [51].

VI. Conclusion

In this paper we considered the modification of the bosonized Lagrangian and of the currents for the pseudoscalar sector obtained after integrating out the vector and axial-vector collective fields in the generating functional of the NJL model. It has been shown, that the reduction of the meson resonances does not affect the kinetic terms of the strong Lagrangian and the bosonized $(V-A)$ current as well as the $(S-P)$ current, generated by the divergent part of quark determinant. On the other hand, the reduction of the vector and axial-vector fields leads to an essential modification of those part of the pseudoscalar strong Lagrangian and of the currents, which originate from $O(p^4)$ terms of the quark determinant. The reduced Lagrangians and currents allow us to take into account in a simple way all effects arising from resonance exchange contributions and πA_1 mixing when calculating the amplitudes of various processes with pseudoscalar mesons in the initial and final states.

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*This point was already noted shortly in our resent paper [45].

Appendix

Appendix A. Heat-kernel computation of quark determinant

The logarithm of the modulus of the quark determinant is defined in "proper-time" regularization as

$$\log|\det i\widehat{\mathbf{D}}| = -\frac{1}{2}\mathrm{Tr}'\log(\widehat{\mathbf{D}}^+\widehat{\mathbf{D}}) = -\frac{1}{2}\int_{1/\Lambda^2}^{\infty} d\tau\,\frac{1}{\tau}\,\mathrm{Tr}'\exp(-\widehat{\mathbf{D}}^+\widehat{\mathbf{D}}\tau) \qquad (40)$$

with Λ being the intrinsic regularization parameter. The main idea of the heat-kernel method is to evaluate

$$< x \mid \exp(-\widehat{\mathbf{D}}^+\widehat{\mathbf{D}}\tau) \mid y >$$

around its nonperturbated part

$$< x \mid \exp(-(\Box + \mu^2)\tau) \mid y >= \frac{1}{(4\pi\tau)^2}e^{-\mu^2\tau + (x-y)^2/(4\tau)}$$

in powers of proper-time τ with the so-called Seeley-deWitt coefficients $h_k(x,y)$

$$< x \mid \exp(-\widehat{\mathbf{D}}^+\widehat{\mathbf{D}}\tau) \mid y >= \frac{1}{(4\pi\tau)^2}e^{-\mu^2\tau + (x-y)^2/(4\tau)}\sum_k h_k(x,y)\cdot\tau^k.$$

After integration over τ in (40) one gets the expression (5) for $\log\mid\det i\widehat{\mathbf{D}}\mid$. Using the definition of the gamma function $\Gamma(\alpha,x)$, one can separate the divergent and finite parts of the quark determinant

$$\frac{1}{2}\log(\det\widehat{\mathbf{D}}^+\widehat{\mathbf{D}}) = B_{\mathrm{pol}} + B_{\mathrm{log}} + B_{\mathrm{fin}},$$

where

$$B_{\mathrm{pol}} = \frac{1}{2}\frac{e^{-x}}{(4\pi)^2}\Big[-\frac{\mu^4}{2x^2}\mathrm{Tr}'h_0 + \frac{1}{x}(\frac{\mu^4}{2}\mathrm{Tr}'h_0 - \mu^2\mathrm{Tr}'h_1)\Big]$$

has a pole at $x = \mu^2/\Lambda^2 = 0$,

$$B_{\mathrm{log}} = -\frac{1}{2}\frac{1}{(4\pi)^2}\Gamma(0,x)\Big[\frac{1}{2}\mu^4\mathrm{Tr}'h_0 - \mu^2\mathrm{Tr}'h_1 + \mathrm{Tr}'h_2\Big]$$

is logarithmic divergent, and the finite part has the form

$$B_{\mathrm{fin}} = -\frac{1}{2}\frac{1}{(4\pi)^2}\sum_{k=3}^{\infty}\mu^{4-2k}\Gamma(k-2,x)\,\mathrm{Tr}'h_k.$$

The very lengthy calculations of the Seeley-deWitt coefficients h_k can be performed only by computer support. The calculation of the heat-coefficients is a recursive process which can be done by Computer Algebra Systems such as FORM and REDUCE very conveniently. In ref.[30] we have calculated the full coefficients up to the order

$k = 6$. After voluminous computations one gets the complex expressions for heat-coefficients $h_1, \dots h_4$:

$$h_0(x) = 1,$$
$$h_1(x) = -a,$$
$$\mathrm{Tr}\,[h_2(x)] = \mathrm{Tr}\left\{\frac{1}{12}(\Gamma_{\mu\nu})^2 + \frac{1}{2}a^2\right\},$$
$$\mathrm{Tr}\,[h_3(x)] = -\frac{1}{12}\mathrm{Tr}\left\{2a^3 - S_\mu S^\mu + a(\Gamma_{\mu\nu})^2 - \frac{2}{45}(K_{\alpha\beta\gamma})^2 - \frac{1}{9}(K^\alpha{}_{\alpha\beta})^2\right\},$$
$$\begin{aligned}\mathrm{Tr}\,[h_4(x)] = \mathrm{Tr}\Big\{ & \frac{1}{24}a^4 + \frac{1}{12}a^2 S^\mu{}_\mu + aS_\mu S^\mu + \frac{1}{720}7(S^\mu{}_\mu)^2 - (S_{\mu\nu})^2 \\ & + \frac{1}{30}a^2(\Gamma_{\mu\nu})^2 + \frac{1}{120}(a\Gamma_{\mu\nu})^2 + \frac{1}{180}a(K^\alpha{}_{\alpha\mu})^2 + \frac{1}{75}a\Gamma_{\mu\nu}K_\beta{}^{\beta\mu\nu} + \frac{7}{900}\Gamma_{\mu\nu}S^\mu K_\alpha{}^{\alpha\nu} \\ & + \frac{1}{50}aK_\beta{}^{\beta\mu\nu}\Gamma_{\mu\nu} - \frac{1}{300}\Gamma_{\mu\nu}K_\alpha{}^{\alpha\mu}S^\nu + \frac{1}{3600}K^\alpha{}_{\alpha\mu}\left(S_\beta{}^{\beta\mu} + S_\beta{}^{\mu\beta}\right) + \frac{1}{72}S_\mu{}^\mu(\Gamma_{\alpha\beta})^2 \\ & + \frac{1}{180}S^{\mu\nu}\{\Gamma_{\mu\alpha}, \Gamma_\nu{}^\alpha\} + \frac{1}{40}a\left(\Gamma_{\mu\nu}S^{\mu\nu} + \frac{11}{9}S_{\mu\nu}\Gamma^{\mu\nu}\right) + \frac{1}{144}a\,[K^\mu{}_{\mu\nu}, S^\nu] \\ & + \left(\frac{2}{135}aK_{\beta\mu\nu} + \frac{11}{900}\Gamma_{\mu\nu}S_\beta + \frac{1}{100}S_\beta\Gamma_{\mu\nu} + \frac{1}{4725}\left[\Gamma_{\mu\nu}, K^\alpha_{\alpha\beta}\right]\right)(K^{\beta\mu\nu} - K^{\mu\nu\beta}) \\ & + \frac{1}{1260}\Gamma_{\mu\nu}K_\alpha{}^{\alpha\mu}K_\beta{}^{\beta\nu} - \frac{1}{12600}\left(29\Gamma^\beta{}_\alpha\Gamma^{\mu\alpha} + 27\Gamma^{\mu\alpha}\Gamma^\beta{}_\alpha\right)\left(K^\nu_{\mu\beta\nu} + K^\nu_{\mu\nu\beta}\right) \\ & + \Gamma_{\alpha\beta}\Gamma_{\mu\nu}\left(\frac{83}{25200}K^{\mu\nu\alpha\beta} + \frac{4}{1575}K^{\alpha\beta\mu\nu} - \frac{127}{5040}K^{\alpha\mu\nu\beta} - \frac{1}{600}K^{\mu\alpha\beta\nu}\right) \\ & + \frac{13}{12600}\Gamma_{\mu\beta}\Gamma^\beta{}_\nu\Gamma^\nu{}_\alpha\Gamma^{\alpha\mu} + \frac{47}{16800}(\Gamma_{\mu\nu})^2(\Gamma_{\alpha\beta})^2 + \frac{17}{25200}(\Gamma_{\mu\nu}\Gamma_{\alpha\beta})^2 \\ & + \frac{4}{1575}(\Gamma_{\mu\alpha}\Gamma^\alpha{}_\nu)^2 + \frac{19}{25200}K^\alpha{}_{\alpha\mu\nu}K^\mu{}_\beta{}^{\beta\nu} - \frac{1}{12600}(K^\alpha{}_{\mu\nu\alpha})^2 + \frac{1}{1575}(K_\mu{}^\alpha{}_{\alpha\nu})^2 \\ & + \frac{1}{6300}K_\mu{}^\alpha{}_{\alpha\nu}K^{\beta\mu\nu}{}_\beta + \frac{1}{5600}(K^\alpha{}_{\alpha\mu\nu})^2 - \frac{1}{5040}K^\alpha{}_{\alpha\mu\nu}K_\beta{}^{\mu\nu\beta} + K_{\mu\nu\alpha\beta}K^{\alpha\mu\nu\beta} \\ & - \frac{1}{1800}K_{\mu\alpha}{}^\alpha{}_\nu K^{\mu\beta}{}_\beta{}^\nu - \frac{1}{25200}K_{\mu\nu\alpha\beta}\left[3\left(K^{\mu\nu\alpha\beta} + K^{\nu\alpha\beta\mu}\right)\right. \\ & \left. + 2\left(K^{\mu\alpha\beta\nu} + K^{\alpha\nu\mu\beta}\right)\right]\Big\}\end{aligned}$$

Here,

$$K_{\mu\nu} = [d_\mu, d_\nu] = \Gamma_{\mu\nu}, \quad K_{\lambda\mu\nu} = [d_\lambda, K_{\mu\nu}], \quad K_{\kappa\lambda\mu\nu} = [d_\kappa, K_{\lambda\mu\nu}] \quad \text{, etc.}$$
$$S_\mu = [d_\mu, a], \quad S_{\mu\nu} = [d_\mu, S_\nu], \quad S_{\lambda\mu\nu} = [d_\lambda, S_{\mu\nu}] \quad \text{, etc.}$$

are commutators of the operators d_μ and a which are defined by the relations

$$d_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = V_\mu + A_\mu\gamma^5, \quad a(x) = i\widehat{\nabla}H + H^+H + \frac{1}{4}[\gamma^\mu, \gamma^\nu]\Gamma_{\mu\nu} - \mu^2.$$

We used the following notations:

$$H = P_R\Phi + P_L\Phi^+ = S + i\gamma_5 P, \quad \Gamma_{\mu\nu} = [d_\mu, d_\nu] = \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu + [\Gamma_\mu, \Gamma_\nu] = F^V_{\mu\nu} + \gamma^5 F^A_{\mu\nu}$$

with $F^{V,A}_{\mu\nu}$ as field strength tensors,

$$F^V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] + [A_\mu, A_\nu],$$
$$F^A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [V_\mu, A_\nu] + [A_\mu, V_\nu],$$

and

$$\nabla_\mu H = \partial_\mu H + [V_\mu, H] - \gamma^5\{A_\mu, H\}$$

as the covariant derivative.

In the same way the next orders of heat expansion coefficients h_i can be obtained using the developed computational technique based on the usage of computer algebra [30]. For simplicity we present below expressions only for minimal parts of heat-coefficients, i.e., only for the parts which do not vanish in the pseudoscalar region of the theory when $V_\mu = A_\mu = 0$:

$$\begin{aligned}\mathrm{Tr}\,[h_5(x)^{min}] = -\mathrm{Tr}\Big\{ & \frac{1}{120}a^2(a^3 + S_\mu S^\mu) + \frac{1}{180}a^3 S_\mu{}^\mu + 2(aS_\mu)^2 \\ & + \frac{1}{6300}\left[10aS_\mu(S^{\mu\nu}{}_\nu + S_\nu{}^{\nu\mu}) - 2a(S_{\mu\nu})^2 + 17a(S^\mu_\mu)^2 + S^{\mu\nu}{}_\nu S_\mu + 3aS_\mu{}^{\mu\nu}S_\nu\right] \\ & + \frac{11}{1008}S_\mu S^\mu S_\nu{}^\nu + \frac{19}{2800}S_\mu S_\nu S^{\mu\nu} + \frac{2}{225}S_\mu S_\nu S^{\nu\mu} \\ & + \frac{1}{25200}\left[3(S^\mu{}_{\mu\nu})^2 - 2(S_{\mu\nu\alpha})^2 - 23(S^{\mu\nu}{}_\nu)^2 + 7S^\mu{}_{\mu\nu}S^{\nu\alpha}{}_\alpha\right]\Big\}\end{aligned}$$

$$\begin{aligned}\mathrm{Tr}\,[h_6(x)^{min}] = \mathrm{Tr}\Big\{ & \frac{1}{720}a^2(a^4 + 4S_\mu aS^\mu) + \frac{1}{420}a^3 S_\mu S^\mu \\ & + \frac{1}{20160}a^2\left[20a^2 S_\mu{}^\mu + 5S_\mu(S^{\mu\nu}{}_\nu + S_\nu{}^{\nu\mu}) + S^\mu{}_{\mu\nu}S^\nu - (S_{\mu\nu})^2 + 11(S_\mu{}^\mu)^2 + 9S^{\mu\nu}{}_\nu S_\mu\right] \\ & + \frac{1}{25200}a\left[S_\mu a(73S^{\mu\nu}{}_\nu + 37S_\nu{}^{\nu\mu}) + 5S^{\mu\nu}(S_\mu S_\nu + 4S_\nu S_\mu)\right] + \frac{1}{2016}aS^\mu{}_\mu S^\nu S_\nu \\ & + \frac{1}{9450}aS_\mu(37S^{\mu\nu} + 23S^{\nu\mu})S_\nu + \frac{1}{9072}aS_\mu(S^{\mu\nu}{}_\nu{}^\alpha{}_\alpha + S_\nu{}^{\nu\mu\alpha}{}_\alpha + S_\nu{}^\nu{}_\alpha{}^{\alpha\mu}) \\ & + aS^\mu\left(\frac{23}{4800}S_\mu S_\nu{}^\nu + \frac{937}{302400}S^\nu S_{\mu\nu} + \frac{23}{10800}S^\nu S_{\nu\mu}\right) + \frac{1}{252}aS_\mu S^\nu{}_\nu S^\mu \\ & + \frac{1}{352800}aS^\mu{}_{\mu\nu}(52S_\alpha{}^{\alpha\nu} + 53S^{\nu\alpha}{}_\alpha) + aS^{\mu\nu}{}_\nu\left(\frac{1}{3360}S^\alpha{}_{\alpha\mu} - \frac{1}{11340}S_{\mu\alpha}{}^\alpha\right) \\ & + \frac{17}{226800}a(S_{\mu\nu\alpha})^2 + \frac{1}{317520}a(23S^\mu{}_\mu{}^{\nu\alpha}{}_\alpha + 5S^\mu{}_{\mu\alpha}{}^{\alpha\nu} + 77S^{\nu\mu}{}_\mu{}^\alpha{}_\alpha)S_\nu \\ & - \frac{1}{30240}\left[53(S_\mu S^\mu)^2 + (S_\mu S_\nu)^2\right] + \frac{1}{352800}S_\mu S_\nu(157S^\mu{}_\alpha{}^{\alpha\nu} + 298S^{\mu\nu\alpha}{}_\alpha) \\ & + \frac{1}{70560}S_\mu S_\nu(31S_\alpha{}^{\alpha\nu\mu} + 58S^{\nu\mu\alpha}{}_\alpha + 47S^{\nu\alpha}{}_\alpha{}^\mu) + \frac{1}{720}S^\mu S_\mu S^\nu{}_\nu{}^\alpha{}_\alpha \\ & + \frac{5}{14112}S_\mu S_\nu S_\alpha{}^{\alpha\mu\nu} + \frac{1}{105840}S_\mu[S^{\mu\nu}(37S^\alpha{}_{\alpha\nu} + 70S_{\nu\alpha}{}^\alpha) + 35S^{\nu\mu}S^\alpha{}_{\nu\alpha}] \\ & + \frac{1}{21168}S_\mu[S^{\nu\mu}S^\alpha{}_{\alpha\nu} + S_{\nu\alpha}(5S^{\nu\mu\alpha} + 2S^{\nu\alpha\mu})] + \frac{1}{2880}S_\mu(S^{\mu\nu}{}_\nu + S_\nu{}^{\nu\mu})S_\alpha{}^\alpha\end{aligned}$$

$$+S_\mu S^\nu{}_\nu\left(\frac{83}{141120}S^{\mu\alpha}{}_\alpha+\frac{1}{9408}S_\alpha{}^{\alpha\mu}\right)+\frac{1}{30240}S_\mu\left(17S_{\nu\alpha}{}^\alpha+13S^\alpha{}_{\alpha\nu}\right)S^{\nu\mu}$$
$$+\frac{1}{7560}S_\mu\left[2\left(S^{\mu\nu\alpha}+S^{\nu\mu\alpha}+S^{\nu\alpha\mu}\right)S_{\nu\alpha}+\left(S_{\nu\alpha}{}^\alpha+2S^\alpha{}_{\alpha\nu}\right)S^{\mu\nu}\right]$$
$$+\frac{1}{2160}S_\mu S_{\nu\alpha}S^{\mu\nu\alpha}$$
$$-\frac{1}{635040}\left(701(S_\mu{}^\mu)^3+583S^{\mu\nu}S_{\mu\alpha}S^\alpha_\nu\right)-\frac{689}{316386}S^\alpha{}_\alpha(S^{\mu\nu})^2$$
$$-\frac{2}{2835}S^\mu{}_\mu S^{\nu\alpha}S_{\alpha\nu}-\frac{1}{952560}S^{\mu\nu}\left(619S^\alpha{}_\nu S_{\mu\alpha}+190S_{\mu\alpha}S^\alpha{}_\nu\right)$$
$$+\frac{1}{151200}\left[11(S^\mu{}_\mu{}^\nu{}_\nu)^2-2(S_{\mu\nu}{}^\alpha{}_\alpha)^2\right]+\frac{1}{176400}\left((S^\mu{}_{\mu\nu\alpha})^2+S^\mu{}_{\mu\nu\alpha}S^{\nu\beta}{}_\beta{}^\alpha\right)$$
$$-\frac{1}{226800}(S_{\mu\nu\alpha\beta})^2-\frac{103}{12700800}(S_\mu{}^\alpha{}_{\alpha\nu})^2+\frac{1}{66150}S^\mu{}_{\mu\nu\alpha}S^{\nu\alpha\beta}{}_\beta$$
$$-\frac{1}{52920}S^\mu{}_\mu{}^\nu{}_\nu S^{\alpha\beta}{}_{\beta\alpha}-\frac{13}{604800}S^{\mu\nu\alpha}{}_\alpha S_{\mu\beta}{}^\beta{}_\nu\Big\}.$$

To obtain these expressions for the heat coefficients, we have made extensive use of the cyclic properties of the trace and of the Jacobi identity.

Appendix B. Bosonized effective Lagrangians

The effective meson Lagrangians in terms of collective fields can be obtained from the quark determinant after calculating in $\mathrm{tr}'\, h_i(x)$ the trace over Dirac indices. The "divergent" part of the effective meson Lagrangian is defined by the coefficients h_0, h_1 and h_2 of the expansion (5):

$$\mathcal{L}_{div}=\frac{N_c}{32\pi^2}\,\mathrm{tr}\left\{\Gamma\left(0,\frac{\mu^2}{\Lambda^2}\right)\left[D^\mu\Phi\,\overline{D}_\mu\Phi^+-\mathcal{M}^2+\frac{1}{6}\left((F^{(-)}_{\mu\nu})^2+(F^{(+)}_{\mu\nu})^2\right)\right]\right.$$
$$\left.+2\left[\Lambda^2e^{-\mu^2/\Lambda^2}-\mu^2\Gamma\left(0,\frac{\mu^2}{\Lambda^2}\right)\right]\mathcal{M}\right\},\qquad(41)$$

where D^μ and $\overline{D}_\mu$ are covariant derivatives defined by eq.(9), and $\mathcal{M}=\Phi\Phi^+-\mu^2$.

Applying the properties of the covariant derivatives

$$D_\mu(O_1O_2)=(D_\mu O_1)O_2+O_1(\overline{D}'_\mu O_2)=(D'_\mu O_1)O_2+O_1(D_\mu O_2),$$
$$\overline{D}_\mu(O_1O_2)=(\overline{D}_\mu O_1)O_2+O_1(D'_\mu O_2)=(\overline{D}'_\mu O_1)O_2+O_1(\overline{D}_\mu O_2);$$

$$[D_\mu,D_\nu]O=F^{(-)}_{\mu\nu}O-OF^{(+)}_{\mu\nu},\qquad[\overline{D}_\mu,\overline{D}_\nu]O=F^{(+)}_{\mu\nu}O-OF^{(-)}_{\mu\nu}$$

with

$$D'_\mu*=\partial_\mu*+[A^{(-)}_\mu,*],\qquad\overline{D}'_\mu*=\partial_\mu*+[A^{(+)}_\mu,*],$$

and assuming the approximation $\Gamma(k,\mu^2/\Lambda^2)\approx\Gamma(k)$ (valid for $k\geq1$, and $\mu^2/\Lambda^2\ll1$) one can present the p^4-terms of the finite part of the effective meson Lagrangian in

the form

$$\mathcal{L}^{(p^4)}_{fin}=\frac{N_c}{32\pi^2\mu^4}\,\mathrm{tr}\left\{\frac{1}{3}\left[\mu^2D^2\Phi\,\overline{D}^2\Phi^+-(D^\mu\Phi\,\overline{D}_\mu\Phi^+)^2\right]+\frac{1}{6}(D_\mu\Phi\,\overline{D}_\nu\Phi^+)^2\right.$$
$$-\mu^2(\mathcal{M}D_\mu\Phi\,\overline{D}^\mu\Phi^++\overline{\mathcal{M}}\,\overline{D}_\mu\Phi^+D_\mu\Phi)$$
$$+\frac{2}{3}\mu^2\left(D^\mu\Phi\,\overline{D}^\nu\Phi^+F^{(-)}_{\mu\nu}+\overline{D}^\mu\Phi^+D^\nu\Phi\,F^{(+)}_{\mu\nu}\right)+\frac{1}{3}\mu^2F^{(+)}_{\mu\nu}\Phi^+F^{(-)\mu\nu}\Phi$$
$$\left.-\frac{1}{6}\mu^4\left[(F^{(-)}_{\mu\nu})^2+(F^{(+)}_{\mu\nu})^2\right]\right\},\qquad(42)$$

where $\overline{\mathcal{M}}=\Phi^+\Phi-\mu^2$.

In an analogous way, the p^6-terms of the finite part of the effective meson Lagrangian can be presented as

$$\mathcal{L}^{(p^6)}_{fin}=\frac{N_c}{32\pi^2\mu^6}\,\mathrm{tr}\left\{\frac{1}{30}\mu^2D^2D_\alpha\Phi\,\overline{D}^2\overline{D}^\alpha\Phi^+\right.$$
$$+\frac{1}{6}\mu^2\left[\mathcal{M}\left(D_\mu D_\nu\Phi\,\overline{D}^\mu\overline{D}^\nu\Phi^++D^\mu\Phi\,\overline{D}^2\overline{D}_\mu\Phi^++D^2D_\mu\Phi\,\overline{D}^\mu\Phi^+\right)\right.$$
$$\left.+\overline{\mathcal{M}}\left(\overline{D}^\mu\overline{D}^\nu\Phi^+D_\mu D_\nu\Phi+\overline{D}^\mu\Phi^+D^2D_\mu\Phi+\overline{D}^2\overline{D}_\mu\Phi^+D^\mu\Phi\right)\right]$$
$$+\frac{1}{3}\mu^2\left(\overline{D}^\mu\Phi^+\overline{\mathcal{M}}D_\mu\Phi\,\mathcal{M}+\mathcal{M}^2D_\mu\Phi\,\overline{D}^\mu\Phi^++\overline{\mathcal{M}}^2\overline{D}_\mu\Phi^+D^\mu\Phi\right)$$
$$-\frac{2}{45}\left[(D_\mu\Phi\,\overline{D}_\nu\overline{D}^\mu\Phi^+)^2+D_\mu\Phi\,\overline{D}_\nu\overline{D}_\alpha\Phi^+(D^\alpha\Phi\,\overline{D}^\nu\overline{D}^\mu\Phi^+-D^\mu\Phi\,\overline{D}^\nu\overline{D}^\alpha\Phi^+)\right.$$
$$\left.+(\overline{D}_\mu\Phi^+D_\nu D^\mu\Phi)^2+D_\mu\Phi^+D_\nu D_\alpha\Phi(\overline{D}^\alpha\Phi^+D^\nu D^\mu\Phi-\overline{D}^\mu\Phi^+D^\nu D^\alpha\Phi)\right]$$
$$-\frac{1}{30}\left[D_\mu\Phi\,\overline{D}^\mu\Phi^+D_\nu D_\alpha\Phi\,\overline{D}^\nu\overline{D}^\alpha\Phi^++D_\mu\Phi\,\overline{D}_\nu\Phi^+(D_\alpha D^\nu\Phi\,\overline{D}^\alpha\overline{D}^\mu\Phi^+-D_\alpha D^\mu\Phi\,\overline{D}^\alpha D^\nu\Phi^+)\right.$$
$$\left.+\overline{D}_\mu\Phi^+D^\mu\Phi\,\overline{D}_\nu\overline{D}_\alpha\Phi^+D^\nu D^\alpha\Phi+\overline{D}_\mu\Phi^+D_\nu\Phi(\overline{D}_\alpha\overline{D}^\nu\Phi^+D^\alpha D^\mu\Phi-\overline{D}_\alpha\overline{D}^\mu\Phi^+D^\alpha D^\nu\Phi)\right]$$
$$-\frac{1}{45}\left[D_\mu\Phi\,D^\mu\Phi^+D_\nu\Phi\,\overline{D}^2\overline{D}^\nu\Phi^++D_\mu\Phi\,\overline{D}_\nu\Phi^+(D^\nu\Phi\,\overline{D}^2\overline{D}^\mu\Phi^+-D^\mu\Phi\,D^2D^\nu\Phi^+)\right.$$
$$\left.+D_\mu\Phi^+D^\mu\Phi\,\overline{D}_\nu\Phi^+D^2D^\nu\Phi+\overline{D}_\mu\Phi^+D_\nu\Phi(\overline{D}^\nu\Phi^+D^2D^\mu\Phi-\overline{D}^\mu\Phi^+D^2D^\nu\Phi)\right]$$
$$-\frac{1}{6}\left[\mathcal{M}\left((D_\mu\Phi\,\overline{D}^\mu\Phi^+)^2-(D_\mu\Phi\,\overline{D}_\nu\Phi^+)^2+D_\mu\Phi\,\overline{D}_\nu\Phi^+D^\nu\Phi\,\overline{D}^\mu\Phi^+\right)\right.$$
$$\left.+\overline{\mathcal{M}}\left((\overline{D}_\mu\Phi^+D^\mu\Phi)^2-(\overline{D}_\mu\Phi^+D_\nu\Phi)^2+\overline{D}_\mu\Phi^+D_\nu\Phi\,\overline{D}^\nu\Phi^+D^\mu\Phi\right)\right]$$
$$+\frac{1}{30\mu^2}\left[\overline{D}_\mu\Phi^+D_\nu\Phi\,\overline{D}_\alpha\Phi^+D^\mu\Phi\,\overline{D}^\nu\Phi^+D^\alpha\Phi-2(\overline{D}_\mu\Phi^+D^\mu\Phi)^3+6\overline{D}_\mu\Phi^+D^\mu\Phi(\overline{D}_\nu\Phi^+D_\alpha\Phi)^2\right.$$
$$\left.-3(\overline{D}_\mu\Phi^+D^\mu\Phi\,D_\nu\Phi^+D_\alpha\Phi\,\overline{D}^\alpha\Phi^+D^\nu\Phi+\overline{D}_\mu\Phi^+D_\nu\Phi\,\overline{D}_\alpha\Phi^+D^\mu\Phi\,\overline{D}^\alpha\Phi^+D^\nu\Phi)\right]$$
$$-\frac{1}{6}\mu^6\left[(D'_\mu\mathcal{M})^2+(\overline{D}'_\mu\overline{\mathcal{M}})^2\right]$$
$$+\frac{1}{6}\mu^2\left[F^{(-)}_{\mu\nu}\left(\frac{1}{5}D^\mu D^\alpha\Phi\,\overline{D}^\nu\overline{D}_\alpha\Phi^+-D^2D^\mu\Phi\,\overline{D}^\nu\Phi^+-D^\mu\Phi\,\overline{D}^2\overline{D}^\nu\Phi^+-D^\alpha D^\mu\Phi\,\overline{D}_\alpha\overline{D}^\nu\Phi^+\right.\right.$$
$$\left.-\frac{13}{60}(D_\alpha\Phi\,\overline{D}^\mu\overline{D}^\nu\overline{D}^\alpha\Phi^+-D^\mu D^\nu D^\alpha\Phi\,\overline{D}_\alpha\Phi^+)\right)$$
$$+F^{(+)}_{\mu\nu}\left(\frac{1}{5}\overline{D}^\mu\overline{D}^\alpha\Phi^+D^\nu D_\alpha\Phi-\overline{D}^2\overline{D}^\mu\Phi^+D^\nu\Phi-\overline{D}^\mu\Phi^+D^2D^\nu\Phi-\overline{D}^\alpha\overline{D}^\mu\Phi^+D_\alpha D^\nu\Phi\right.$$

$$
\begin{aligned}
&-\frac{13}{60}(\overline{D}_\alpha\Phi^+ D^\mu D^\nu D^\alpha\Phi - \overline{D}^\mu\overline{D}^\nu\overline{D}^\alpha\Phi^+ D_\alpha\Phi))\Big]\\
&+\frac{11}{180}\mu^2 F^{(+)}_{\mu\nu}\overline{D}_\alpha\Phi^+ F^{(-)\mu\nu}D^\alpha\Phi + \frac{19}{360}\mu^2\Big[(F^{(-)}_{\mu\nu})^2 D_\alpha\Phi\,\overline{D}^\alpha\Phi^+\\
&+(F^{(+)}_{\mu\nu})^2\overline{D}_\alpha\Phi^+ D^\alpha\Phi\Big]\\
&-\frac{1}{3}\mu^2\Big[F^{(-)}_{\mu\nu}(D^\mu\Phi\,\overline{D}^\nu\Phi^+\mathcal{M} + \mathcal{M}D^\mu\Phi\,\overline{D}^\nu\Phi^+ + D^\mu\Phi\,\overline{\mathcal{M}}\,\overline{D}^\nu\Phi^+)\\
&+F^{(+)}_{\mu\nu}(\overline{D}^\mu\Phi^+ D^\nu\Phi\,\overline{\mathcal{M}} + \overline{\mathcal{M}}\,\overline{D}^\mu\Phi^+ D^\nu\Phi + \overline{D}^\mu\Phi^+\mathcal{M}D^\nu\Phi)\Big]\\
&+\frac{1}{3}\mu^2\Big[F^{(-)}_{\mu\alpha}F^{(-)\alpha}_{\nu}(D^\mu\Phi\,\overline{D}^\nu\Phi^+ - D^\nu\Phi\,\overline{D}^\mu\Phi^+)\\
&+F^{(+)}_{\mu\alpha}F^{(+)\alpha}_{\nu}(\overline{D}^\mu\Phi^+ D^\nu\Phi - \overline{D}^\nu\Phi^+ D^\mu\Phi)\\
&-F^{(-)}_{\mu\nu}D_\alpha\Phi\,F^{(+)\nu\alpha}\overline{D}^\mu\Phi^+ - F^{(-)}_{\nu\alpha}D^\alpha\Phi\,F^{(+)\mu\nu}\overline{D}_\mu\Phi^+\Big]\\
&+\frac{1}{6}\Big[F^{(-)}_{\mu\nu}\Big(\{D_\alpha\Phi\,\overline{D}^\alpha\Phi^+, D^\mu\Phi\,\overline{D}^\nu\Phi^+\} + D_\alpha\Phi\,\overline{D}^\mu\Phi^+(D^\nu\Phi\,\overline{D}^\alpha\Phi^+ - D^\alpha\Phi\,\overline{D}^\nu\Phi^+)\\
&+D^\mu\Phi\,\overline{D}_\alpha\Phi^+(D^\alpha\Phi\,\overline{D}^\nu\Phi^+ - D^\nu\Phi\,\overline{D}^\alpha\Phi^+)\Big)\\
&+F^{(+)}_{\mu\nu}\Big(\{\overline{D}_\alpha\Phi^+ D^\alpha\Phi, \overline{D}^\mu\Phi^+ D^\nu\Phi\} + \overline{D}_\alpha\Phi^+ D^\mu\Phi(\overline{D}^\nu\Phi^+ D^\alpha\Phi - \overline{D}^\alpha\Phi^+ D^\nu\Phi)\\
&+\overline{D}^\mu\Phi^+ D_\alpha\Phi(\overline{D}^\alpha\Phi^+ D^\nu\Phi - \overline{D}_\nu\Phi^+ D^\alpha\Phi)\Big)\Big]\\
&-\frac{5}{6}\mu^4\Big[\mathcal{M}(F^{(-)}_{\mu\nu})^2 + \overline{\mathcal{M}}(F^{(+)}_{\mu\nu})^2\Big] + \frac{1}{20}\mu^4\Big[F^{(-)}_{\mu\nu}\{\mathcal{M}, D'^\mu D'^\nu\mathcal{M}\}\\
&+F^{(+)}_{\mu\nu}\{\overline{\mathcal{M}}, \overline{D}'^\mu\overline{D}'^\nu\overline{\mathcal{M}}\}\Big]\\
&-\frac{1}{3}\mu^2\Big(F^{(-)}_{\mu\nu}F^{(-)\mu\alpha}F^{(-)\nu}{}_\alpha + F^{(+)}_{\mu\nu}F^{(+)\mu\alpha}F^{(+)\nu}{}_\alpha\Big)\\
&+\frac{1}{540}\mu^4\Big[41\Big((D^{(-)}_{\mu\nu\alpha})^2 + (D^{(+)}_{\mu\nu\alpha})^2\Big) - 10\Big((D^{(-)\mu}{}_{\mu\alpha})^2 + (D^{(+)\mu}{}_{\mu\alpha})^2\Big)\Big]\Big\}. \qquad (43)
\end{aligned}
$$

VII. REFERENCES

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Table 1. Physical input parameters used for the fixin: of the empirical constants of the model.

Input parameters	Experiment	Theory
m_ρ	770 MeV	772 MeV
m_{A_1}	1260 MeV	1160 MeV
$g_{\rho\pi\pi}$	6.3	6.8
F_π	93 MeV	93.9 MeV
F_K	119 MeV	118.5 MeV
$a_0^0 \cdot m_\pi$	0.23 ± 0.05 [36]	0.20
$a_0^2 \cdot m_\pi$	-0.05 ± 0.03 [36]	-0.04
$a_1^1 \cdot m_\pi^3$	0.036 ± 0.010 [36]	0.038
$a_2^0 \cdot m_\pi^5$	$(17 \pm 3) \cdot 10^{-4}$ [37]	$17 \cdot 10^{-4}$
$a_2^2 \cdot m_\pi^5$	$(1.3 \pm 3) \cdot 10^{-4}$ [37]	$2 \cdot 10^{-4}$
$< r_{em}^2 >_{\pi^+}$	$(0.439 \pm 0.030) fm^2$ [38]	$0.53 fm^2$
$\alpha_{\pi^\pm}$	$(6.8 \pm 1.4) \cdot 10^{-4} fm^3$ [39]	$8.0 \cdot 10^{-4} fm^3$

Figure Caption

1. MARK–II [50] cross section data for $\gamma\gamma \to \pi^+\pi^-$ for CMS production angles $|\cos\theta| \leq 0.6$. The experimental points in the region $m_{\pi\pi}$ <0.5 GeV were only included in the analysis. The dotted line shows the QED Born contribution; the dashed and dash-dotted lines show the results of the successive inclusion of p^4–contributions and one–loop corrections. Both lines are calculated with $(\tilde{L}_9 + \tilde{L}_{10}) = 4.2 \cdot 10^{-3}$, corresponding to the fit of the total cross section data together with the parameters of Table 1. The solid line corresponds to the direct fit of the experimental points for $m_{\pi\pi}$ <0.5 GeV without including the experimental parameters of Table 1.

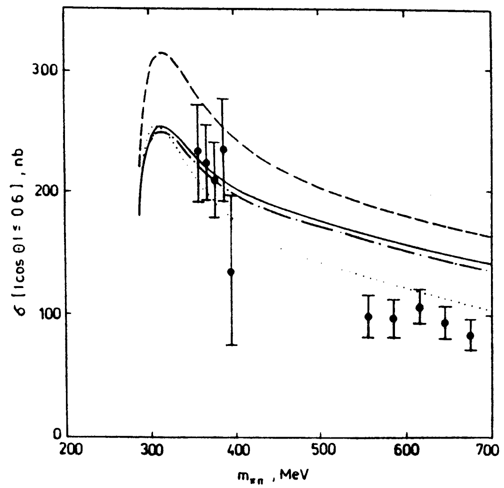

Fig. 1