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Can μ–e Conversion in Nuclei be a Good Probe for Lepton-Number Violating Higgs Couplings ?

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Motivated by the improving sensitivity, R, of experiments on $\mu\ Ti \to e\ Ti$ and the enhanced Higgs nucleon interaction, we study this lepton number violating process induced by Higgs exchange. Taking the possible sensitivity, $R \simeq 10^{-16}$, we obtain the constraint on the Higgs-muon-electron vertex, $\kappa_{\mu e}$, to be less than 2.4×10^{-7} if the masses of the Higgs scalar and W gauge boson are the same. $\kappa_{\mu e}$ is also calculated for some models.

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In this paper we report on a study of direct muon-electron conversion in nuclei as a probe of new physics represented by an effective μ–e–S vertex where S is a neutral scalar particle. For example, the scalar S can be the Higgs scalars in a one doublet model with an extended fermion sector, or linear combinations of scalar particles in some extended Higgs models such as the supersymmetric standard model. Previous discussion of $\mu - e$ conversion concentrated mostly on the effects of virtual photon and Z-boson exchanges [1]. Effects of an extra Z-boson has also been considered recently [2]. Scalar and pseudoscalar effects were outlined in Ref. [1] where the details of the nuclear effects were emphasized. If the Higgs coupling to nucleon is taken to be proportional to the current masses of the u- and d-quarks, then the effect would be very small. Here, we treat the scalar-nucleon-nucleon via the approach of Shifman, *et al.* [3], which increase the coupling strength to that of $\frac{2}{27}m_N$ where m_N is the mass of the nucleon. This is approximately one order of magnitude enhancement over the use of the current quark masses. A second enhancement of the $S - N - N$ coupling can arise in extended Higgs model where it is multiplied by ratios of vacuum expectation values of scalar fields. In supersymmetry, the ratio $\tan\beta \geq 10$ is certainly acceptable. The third factor comes from the fact that scalar exchange in $\mu - e$ is coherent* over the nuclei [4]. In this respect, it is similar to photon and Z exchange.

On the experimental side, we are encouraged by the on going experiment at PSI [5] of $\mu\ Ti \to e\ Ti$ which will achieve a sensitivity $R(Ti) = \Gamma(\mu Ti \to eTi)/\Gamma(\mu Ti \to \nu_\mu Ti) \simeq 3 \times 10^{-14}$ and prospects of lowering this limit to the level of $10^{-15} - 10^{-16}$ being considered at INS Moscow [6] and TRIUMF [7]. This motivated us to re-examine $\mu - e$ conversion and focus on it as a probe of the non-standard $\mu - e - S$ vertex. Comparison with $\mu \to e\ \gamma$ and/or $\mu \to 3e$ where they apply are also given.

At the quark level, the effective interaction Lagrangian induced by an exchange of a scalar S is given by

$$\mathcal{L}_q(S) = \frac{G_F}{\sqrt{2}} \frac{m_W^2}{m_S^2}\ \bar{e}\ \left[\kappa_{\mu e}\ (1+\gamma_5) + \kappa'_{\mu e}\ (1-\gamma_5)\right]\ \mu \sum_{q=all} \frac{m_q}{m_W}\ \lambda_q\ \bar{q}q\ , \quad (1)$$

where $\kappa_{\mu e}$ and $\kappa'_{\mu e}$ are coefficient of the effective $\mu - e - S$ vertex and G_F is the Fermi coupling constant. m_S and m_W are the masses for the scalar S and the standard W gauge boson whereas m_q's are the current quark masses and the sum is taken over for all quark flavors of a given model. For extended Higgs models, λ_q is not equal to unity as in the standard model. In particular, in the two Higgs doublet extensions of the standard model with natural flavor conservation, we have $\lambda_{up} = \cot\beta$ and $\lambda_{down} = \tan\beta$. These correspond to the linear combination given by $S = -\sin\beta\sqrt{2}Re\phi_1^0 + \cos\beta\sqrt{2}Re\phi_2^0$, where ϕ_1^0 and ϕ_2^0 are the neutral components of the Higgs doublets that provide masses for *down*- and *up*-type quarks separately. In order to see how the discussed factors enter into the study of lepton number

*Such is not the case for pseudoscalar and axial vector exchanges and henceforth we shall ignore them.

violation in general and the $\mu - e - S$ vertex in particular, we first study the cases where the scalar S couples to quark like that of the standard model Higgs boson, H. The effects of Higgs mixing will be illustrated by the minimal supersymmetric model with lepton number violation added in.

To compute the interaction at the nucleon level, we follow the procedure suggested by Shifman *et al.* [3]. Including the effects of the strange and heavy quark contributions [8, 9], we obtain the effective Lagrangian for $\mu - e$ conversion in nuclei as follows,

$$\mathcal{L}_N(S) = \frac{G_F}{\sqrt{2}} \frac{m_W^2}{m_S^2} \bar{e} \left[\kappa_{\mu e} (1+\gamma_5) + \kappa'_{\mu e} (1-\gamma_5) \right] \mu \frac{\tilde{m}_N}{m_W} \bar{\Psi}_N \Psi_N , \qquad (2)$$

and

$$\tilde{m}_N = \frac{2}{27} n_h m_N + \left(1 + \frac{y}{2}\frac{m_s}{\bar{m}}\right)\left(1 - \frac{2}{27} n_h\right) \sigma_{\pi N} , \qquad (3)$$

where Ψ_N is the nucleon wave function. n_h is the number of heavy quarks other than u, d and s. y is the strange content in the nucleon and $\sigma_{\pi N}$ is nucleon matrix element of the σ term in the chiral Lagrangian. m_N is the nucleon mass, $\bar{m} = (m_u + m_d)/2$, and we take $m_s/\bar{m} \simeq 25$. The quantity $\tilde{m}_N$ conveniently expresses the heavy quark effects in the Higgs-nucleon-nucleon coupling. Its value depends on y and $\sigma_{\pi N}$, where $(y, \sigma_{\pi N}) = (0,0)$, $(0.47, 60$ MeV$)$ and $(0.22, 45$ MeV$)$ are used in Refs. [3], [8] and [9] respectively. Particularly, $\tilde{m}_N = 350$ MeV for $(y, \sigma_{\pi N}) = (0.22, 45$ MeV$)$ for $n_h = 3$. Hence, $\tilde{m}_N$ is almost two orders of magnitude bigger than the current quark masses of u and d. This is a larger enhancement factor than originally anticipated as discussed in the introduction.

Using the standard procedure [10], we obtain the conversion rate of $\mu\ N \to e\ N$ as follows,

$$\Gamma(\mu N \to eN) = \frac{G_F^2}{2}\left(\frac{\tilde{m}_N}{m_W}\right)^2 \frac{\alpha^3 m_\mu^5 Z_{eff}^4}{\pi^2 Z} A^2 |F(q^2)|^2 \frac{m_W^4}{m_S^4}$$
$$\int \left[|\kappa_{\mu e}|^2 \frac{(1 - \mathbf{s}_\mu \cdot \hat{\mathbf{p}}_e)}{2} + |\kappa'_{\mu e}|^2 \frac{(1 + \mathbf{s}_\mu \cdot \hat{\mathbf{p}}_e)}{2} \right] d\cos\theta \qquad (4)$$

where A and $Z(Z_{eff})$ are the nucleon and (effective) atomic numbers. $F(q^2)$ is the nucleon form factor. $\mathbf{s}_\mu$ and $\hat{\mathbf{p}}_e$ are the muon spin and the direction of the outgoing electron. Particularly, $Z_{eff} = 17.6$ [11] and $F(q^2 = -m_\mu^2) = 0.54$ [12] for ${}^{22}_{48}Ti$. Using the muon capture rate in Ti, $\Gamma(\mu Ti \to \nu_\mu Ti) = 2.590 \pm 10^6 sec^{-1}$ [13], we obtain

$$\left(|\kappa_{\mu e}|^2 + |\kappa'_{\mu e}|^2\right)^{1/2} \le 2.4 \times 10^{-7} \left(\frac{0.5 \text{ GeV}}{\tilde{m}_N}\right)\left(\frac{R}{10^{-16}}\right)^{1/2} \frac{m_S^2}{m_W^2} . \qquad (5)$$

This is the model independent constraint on $\kappa_{\mu e}$ and $\kappa'_{\mu e}$. In Ref. [1], the author obtained the constraint on $\left(|\kappa_{\mu e}|^2 + |\kappa'_{\mu e}|^2\right)^{1/2} \frac{\tilde{m}_N}{m_W}\frac{m_W^2}{m_S^2} \le 10^{-6}$ for sulphur. If we take the current quark mass approach, namely $\tilde{m}_N = (m_u + m_d)/2 = 5$ MeV, it yields $\left(|\kappa_{\mu e}|^2 + |\kappa'_{\mu e}|^2\right)^{1/2} \le 1.6 \times 10^{-2}$ assuming $m_S = m_W$. Even with the improved sensitivity of two orders of magnitude, the constraint is no better than 10^{-5} if the current quark masses are used. Obviously, the improved calculation of Eq. (3) gives a much better limit as evident from Eq. (5).

Encouraged by the enhancement of the Higgs nucleon interaction, we study three examples to see when this lepton number violating Higgs interaction would be important for the muon–electron conversion in nuclei.

1. Exotic Leptons

In the standard model, the Yukawa interactions of Higgs H and leptons are flavor diagonal, and is given by

$$-\frac{g}{2m_W} H \left[m_e ee + m_\mu \mu\mu + m_\tau \tau\tau\right] . \qquad (6)$$

When we include exotic leptons which mix with the ordinary leptons, there will be lepton flavor changing Higgs interactions. In the lowest order, the coefficient of the $\mu - e - H$ vertex is given by

$$\kappa_{\mu e} \simeq \frac{1}{m_W}\left(m_e U_{e\mu} + m_\mu U_{\mu e} + m_\tau U_{\tau e} U_{\tau\mu}\right) , \qquad (7)$$

where $U_{\alpha\beta}$ is the mixing in the charged lepton sector induced by exotic leptons. Using the constraint in Eq. (5), we obtain $U_{e\mu} \le 0.04$, $U_{\mu e} \le 2 \times 10^{-4}$ and $U_{\tau e} U_{\tau\mu} \le 1 \times 10^{-5}$. From $\mu \to 3e$ and $\tau \to 3\ell$[†] by tree level exchanges of Z gauge boson, the constraints are $U_{\mu e} \le 3 \times 10^{-6}$ and $U_{\tau e} U_{\tau\mu} \le 1.6 \times 10^{-3}$ [14]. Hence, the future $\mu - e$ conversion experiments can improve the constraint on the $\tau - \mu$ and $\tau - e$ mixings by two orders.

In the following two examples, we consider models with lepton flavor conservation at tree level. Hence both the $\mu - e - S$ and $\mu - e - \gamma$ vertices are induced at one-loop level.

2. 4th Generation Standard Model

When the standard model is extended to include the 4th generation, a right-handed neutrino is necessary to provide the mass for the 4th neutrino which must be heavier than 45 GeV from the LEP experiments, leading to three massless and one massive neutrinos. In this model, the scalar S is the standard model Higgs, H. Since the vertex $\mu - e - H$ is induced by the $V - A$ current, hence $\kappa'_{\mu e} = 0$; whereas $\kappa_{\mu e}$ [15] is given by

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[†] Note that when there are tree level lepton flavor changing interactions, the processes $\mu \to e\ \gamma$ induced at one-loop level are less important than $\mu \to 3e$ and $\tau \to 3\ell$.

$$\kappa_{\mu e} = \frac{g^2}{16\pi^2} U_{\mu 4} U_{e4} \frac{m_\mu}{m_W} \left[\frac{3}{4} x + \frac{m_H^2}{m_W^2} \left(\frac{3x - x^2}{8(x-1)^2} + \frac{x^3 - 2x^2}{4(x-1)^3} \ln x \right) \right] , \tag{8}$$

where $x = m_{\nu_4}^2/m_w^2$ and $U_{\alpha\beta}$ is the CKM matrix in the lepton sector of four flavors. For the decay $\mu \to e\ \gamma$, the decay rate is given by

$$\Gamma(\mu \to e\gamma) = \frac{\alpha^3 m_\mu}{64\pi \sin^4\theta_W} \frac{m_\mu^2}{m_W^2} |U_{\mu 4} U_{e4} I(x)|^2 \tag{9}$$

and

$$I(x) = \frac{-x + 5x^2 + 2x^3}{4(1-x)^3} + \frac{3x^3}{2(1-x)^4} \ln x \,. \tag{10}$$

where $U_{\mu 4} U_{e4} \leq 3 \times 10^{-3}$ for $m_{\nu_4} \geq 45$ GeV are obtained [16]. However, $\kappa_{\mu e}$ is suppressed by m_μ/m_W. Unless m_{ν_4} is greater than 2 TeV, the process $\mu \to e\ \gamma$ is more important to probe the lepton number violation mechanism.

3. Minimal Supersymmetric Standard Model

In the minimal supersymmetric extension of the standard model (MSSM), the lepton number processes can be induced through the slepton mixing. In analogy to the Yukawa interactions, there exist soft-breaking terms, $ARe\phi_1^0 \tilde{e}_L^* \tilde{\mu}_R + h.c.$. Therefore the mass matrix in the basis $\{\tilde{e}_L, \tilde{\mu}_R\}$ is given by

$$\begin{pmatrix} \tilde{m}_e^2 & Av_1 \\ Av_1 & \tilde{m}_\mu^2 \end{pmatrix} . \tag{11}$$

yielding the decay rate for $\mu \to e\ \gamma$ to be

$$\Gamma(\mu \to e\gamma) = \frac{\alpha^3 m_\mu}{256\pi^2 \sin^4\theta_W} \sin^2 2\theta |\Gamma_{\mu e\gamma}|^2 , \tag{12}$$

where

$$\Gamma_{\mu e\gamma} = \sum_{i=1}^{4} \tan\theta_W N_{1i} (N_{2i} + N_{1i} \tan\theta_W) \frac{m_\mu}{m_{\chi_i}} [x_i F(x_i) - y_i F(y_i)] , \tag{13}$$

with

$$F(x) = -\frac{1+x}{2(x-1)^2} + \frac{x \ln x}{(x-1)^3} . \tag{14}$$

$\sin\theta$ and N_{ij} are the scalar and neutralino mixing parameters. $x_i = m_{\chi_i}^2/m_1^2$ and $y_i = m_{\chi_i}^2/m_2^2$, where m_{χ_i} and $m_{1,2}$ are the neutralino and slepton masses.

The $\mu - e$ conversion in nuclei is induced [17] by the vertex, $\sqrt{2} Re\phi_1^0 \tilde{e}_L^* \tilde{\mu}_R$, leading to the coefficient of the effective vertex $\mu - e - \sqrt{2} Re\phi_1^0$,

$$\kappa_{\mu e} = \frac{g}{32\pi^2} \sum_{i=1}^{4} \tan\theta_W N_{1i} (N_{2i} + N_{1i} \tan\theta_W) \frac{A}{\sqrt{2} m_{\chi_i}}$$
$$\left\{ \sin^2 2\theta \left[x_i G(x_i) + y_i G(y_i) \right] + 2\cos^2 2\theta \left[x_i H(x_i, m_2^2/m_1^2) \right] \right\} , \tag{15}$$

where

$$G(x) = \frac{1}{1-x} + \frac{x \ln x}{(1-x)^2} , \tag{16}$$

$$H(x,r) = \frac{x \ln x}{(x-1)(x-r)} + \frac{r \ln r}{(r-1)(r-x)} , \tag{17}$$

and

$$\frac{\tilde{m}_N}{m_S^2} = \left[\cos\beta \frac{m_A^2 + m_Z^2 \sin^2 2\beta}{m_A^2 m_Z^2 \cos^2 2\beta} - \sin\beta \frac{\tan 2\beta}{m_A^2} \right] \tilde{m}_{N_1} + \left[\cos\beta \frac{\tan 2\beta}{m_A^2} - \sin\beta \frac{1}{m_A^2} \right] \tilde{m}_{N_2} , \tag{18}$$

where the effective nucleon mass induced by interacting with $\cos\beta\sqrt{2} Re\phi_1^0 + \sin\beta\sqrt{2} Re\phi_2^0$ and $-\sin\beta\sqrt{2} Re\phi_1^0 + \cos\beta\sqrt{2} Re\phi_2^0$ are given by

$$\tilde{m}_{N_1} = \frac{2}{9} m_N + \frac{7}{9} \left(1 + \frac{y}{2} \frac{m_s}{\hat{m}} \right) \sigma_{\pi N} , \tag{19}$$

$$m_{N_2} = -\frac{2}{27} (\tan\beta - 2\cot\beta) m_N - \left(\frac{4}{27} \cot\beta + \frac{25}{27} \tan\beta \right) \left(1 + \frac{y}{2} \frac{m_s}{\hat{m}} \right) \sigma_{\pi N} . \tag{20}$$

The square brackets in Eq. (18) are the effective Higgs propagators.

In table 20, we tabulate the branching ratio for $\mu \to e\ \gamma$ and $\mu\ Ti \to e\ Ti$ in MSSM for different values of A and $\tan\beta$. We take $(y, \sigma_{\pi N}) = (0.22, 45$ GeV). For a large $\tan\beta$, $v_1 = \sqrt{v_1^2 + v_2^2} \cos\beta$ is small. Thus the Higgs interaction would be at least as important as $\mu \to e\ \gamma$. Particularly, for $A = 500$ GeV, $\tan\beta = 50$ and an intermediate mass scalar $m_A = 250$ GeV, the process $\mu\ Ti \to e\ Ti$ is about 4 times below the present experimental limit [19]; whereas the branching ratio for the process $\mu \to e\ \gamma$ is 20 times below the present experimental values [18]. This is especially relevant when the sensitivity of the former is improved by two orders of magnitude; whereas we do not foresee a similar improvement in the $\mu \to e\ \gamma$ measurement.

In conclusion, we have considered the $\mu - e$ conversion in nuclei induced by Higgs exchange for three popular models. This process would be negligible if the Higgs nucleon coupling is taken to be proportional to the current quark masses. Here, we have shown how the Higgs nucleon interaction is enhanced by using the approach first employed by Shifman, *et al.*, and this yields $\kappa_{\mu e} \leq 2.4 \times 10^{-7}$. $\kappa_{\mu e}$ in a model of 4th generation lepton is small because it is suppressed by the muon mass. On the other hand, with the existence of the soft breaking terms in the

MSSM, the Higgs induced $\mu - e$ conversion is at least as important as $\mu \to e\ \gamma$. The process will be more important for a larger $\tan\beta$ as the rate increases as the square of this parameter. Furthermore, we have shown that $\mu - e$ conversion can be a sensitive probe to scalar particles in the mass range of hundreds of GeV even when the lepton-number violation is a one-loop effect. The minimal supersymmetric standard model is used as an illustrative example.

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Table 1. The branching ratio for $\mu \to e\ \gamma$ and $\mu\ Ti \to e\ Ti$ in MSSM. we take $\tan\beta = 10(50)$, $m_A = 250$ GeV and $\tilde{m}_{e,\mu} = 5$ TeV. For the gaugino masses, we take $2M_1 = M_2 = \mu = 250$ GeV.

A (GeV)	$\mu \to e\gamma$ [a]	$R(\mu Ti \to eTi)$ [b]
500	$4.7(0.2) \times 10^{-11}$	$0.05(1.0) \times 10^{-12}$
250	$11(0.5) \times 10^{-12}$	$0.13(2.5) \times 10^{-13}$
50	$46(1.9) \times 10^{-14}$	$0.05(1.0) \times 10^{-14}$

[a]The present experiment limit [18] for the branching ratio is $\leq 4.9 \times 10^{-11}$.
[b]The present experiment limit [19] for the process relative to the muon capture in Ti is $\leq 4.6 \times 10^{-12}$.

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