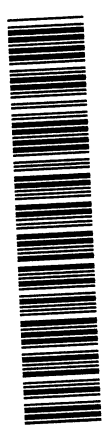


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### Supersymmetric $b \rightarrow s\gamma$ with Large Chargino Contributions

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#### Abstract

Supersymmetric (SUSY) theories are often thought to give large branching ratios for  $b \rightarrow s\gamma$  from charged Higgs loops. We show that in many cases chargino loop contributions can cancel those of the Higgs, and SUSY can give  $B(b \rightarrow s\gamma)$  at or below the Standard Model prediction. We show this occurs because the large stop mass splittings usually found in SUSY break a GIM mechanism suppression. These effects are strongly enhanced by large  $\tan\beta$ , so that  $B(b \rightarrow s\gamma)$  is very sensitive to the value of  $\tan\beta$ , contrary to what has been claimed. We also note that the supergravity relation  $B_0 = A_0 - 1$  is somewhat disfavored over the general case.

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There has been much interest in the decay  $b \rightarrow s\gamma$  because of new results from the CLEO collaboration which bound the inclusive branching ratio,  $B(b \rightarrow s\gamma)$ , below  $5.4 \times 10^{-4}$  at the 95% confidence level, and give a non-zero branching ratio for the exclusive decay  $B \rightarrow K^*\gamma$  of about  $5 \times 10^{-5}$ [1]. One expects this exclusive channel to make up 5% - 40% of the inclusive rate [2], so  $B(b \rightarrow s\gamma)$  must be greater than about  $10^{-4}$ . The Standard Model (SM) contribution depends slowly on the top quark mass and is of order  $4 \times 10^{-4}$  for  $m_t$  of 140 GeV. Given this, some recent works [3, 4] claim that the charged Higgs ( $H^\pm$ ) masses in supersymmetric theories [5] must be very large to avoid exceeding the upper bound on  $B(b \rightarrow s\gamma)$ . We show that this is not always the case—that chargino ( $\chi^\pm$ ) loop contributions can cancel the  $H^\pm$  contributions and give  $B(b \rightarrow s\gamma)$  near or below the SM prediction. In particular, we show that such destructive interference effects are important for large  $\tan\beta$  (which is the ratio of Higgs vacuum expectation values), and when there is a large stop mass splitting. We show that the latter effect is due to the breaking of a GIM cancellation.

The sign of certain mixing angles is also important because one needs the chargino contribution to interfere destructively, rather than constructively. From this one can obtain an approximate condition on the soft SUSY breaking parameters  $A$  and  $B$ , which may have implications for supergravity theories (for a review see [9]).

Calculations for  $B(b \rightarrow s\gamma)$  in SUSY can be found in the literature [6, 7]. Bertolini *et al.* [6] perform a thorough but very constrained analysis which imposes radiative breaking, in the minimal model, with  $B_0 = A_0 - 1$ . They also do not consider large  $\tan\beta$ , where chargino effects can become much more important. Barbieri and Giudice [7] make the important point that  $B(b \rightarrow s\gamma)$  vanishes in the exact supersymmetric limit. However, the scenarios they consider (which are indeed close to the SUSY limit) with gaugino mass ( $m_\lambda$ ) and Higgs mixing mass ( $\mu$ ) set to zero, are not phenomenologically viable because they give chargino and neutralino masses which are too small (one of the higgsinos is even massless in this case). These approaches are understandable since there are many parameters in SUSY theories. Our approach is to concentrate on those parameters which tend to make the chargino contribution large and destructively interfering, so as to make qualitative statements about what areas of parameter space are favored. We show that one cannot neglect the chargino contributions and that there are large areas of parameter space where  $B(b \rightarrow s\gamma)$  in SUSY is at or below the SM prediction. Contrary to what is claimed in [7], we find that  $B(b \rightarrow s\gamma)$  is very sensitive to  $\tan\beta$ , and one can even find regions for large  $\tan\beta$  where the chargino destructive interference is too large [8].

The inclusive decay  $b \rightarrow s\gamma$  comes from the operator  $\bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$ . When one runs the scale from  $M_Z$  to  $m_b$ , this operator mixes with the gluon operator  $\bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a$ , as well as four quark operators. We use the notation of [7] throughout, up to an overall sign in the amplitude. They define the coefficients of the photon (and gluon) operators as  $G_F(\alpha/8\pi^3)^{1/2} \gamma_\mu^* V_{ts}^* V_{tb} A_\gamma$  (and  $A_\gamma \rightarrow A_g$ ). We will concentrate on the photon coefficient  $A_\gamma$ , because the gluon coefficient contribution is relatively suppressed by QCD factors [10], as can be seen from the ratio

of inclusive branching ratios [7]:

$$\frac{B(b \rightarrow s\gamma)}{B(b \rightarrow ce\nu)} = \frac{6\alpha}{\pi} \frac{|\eta^{\frac{15}{8}} A_\gamma + \frac{8}{3}(\eta^{\frac{15}{8}} - \eta^{\frac{15}{8}}) A_\beta + C|^2}{I(m_c/m_b) \left(1 - \frac{2}{3\pi} \alpha_s(m_b) f(m_c/m_b)\right)}, \quad (1)$$

where the inclusive semileptonic branching ratio is  $B(b \rightarrow ce\nu) \simeq 0.107$ ; the QCD factor  $\eta = \alpha_s(M_Z)/\alpha_s(m_b) \simeq 0.546$ ; a QCD correction factor  $f(m_c/m_b) \simeq 2.41$ ; and  $I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$  is a phase space factor. The constant  $C$  comes from mixing of four quark operators as we run down to  $m_b$ , and is about 0.175 [7].

The photon operator coefficient  $A_\gamma$  comes mainly from loops with a  $W^+$  and a top quark, an  $H^+$  and a top quark, and charginos  $\chi_j^\pm$  ( $j=1,2$ ) and up squarks. There are also contributions from flavor changing neutral current (FCNC) vertices due to squark flavor mixing, but these contributions tend to be very small in the minimal model [6]. Squark flavor mixings can be large in certain non-minimal models, but they are constrained to be small by other FCNC observables so that their contribution to  $b \rightarrow s\gamma$  is generally small [11]. Thus we can write

$$A_\gamma \simeq A_\gamma(W^+) + A_\gamma(H^+) + A_\gamma(\chi^+), \quad (2)$$

where  $A_\gamma(W^+)$  and  $A_\gamma(H^+)$  are always greater than zero, while  $A_\gamma(\chi^+)$  can be of either sign. In the limit of degenerate up and charm squark masses, we can write  $A_\gamma(\chi^+) = A_1(\chi^+) + A_2(\chi^+) + A_3(\chi^+) + A_4(\chi^+)$ , with [7]

$$A_1(\chi^+) \simeq + \sum_{j=1}^2 \frac{m_W^2}{\tilde{m}_X^2} |V_{j1}|^2 g^{(1)}(x_{0j}), \quad (3)$$

$$A_2(\chi^+) \simeq - \sum_{j,k=1}^2 \frac{m_W^2}{\tilde{m}_X^2} |V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{v_u}|^2 g^{(1)}(x_{kj}), \quad (4)$$

$$A_3(\chi^+) \simeq - \sum_{j=1}^2 \frac{m_W}{\tilde{m}_X} \frac{U_{j2} V_{j1}}{\sqrt{2} \cos \beta} g^{(3)}(x_{0j}), \quad (5)$$

$$A_4(\chi^+) \simeq + \sum_{j,k=1}^2 \frac{m_W}{\tilde{m}_X} \frac{U_{j2} (V_{j1} T_{k1}^2 - V_{j2} T_{k1} T_{k2} \frac{m_t}{v_u})}{\sqrt{2} \cos \beta} g^{(3)}(x_{kj}), \quad (6)$$

where  $U_{ij}$  and  $V_{ij}$  are the unitary matrices which diagonalize the chargino mass matrix (see [5]),  $T_{kl}$  diagonalizes the stop mass matrix,  $v_u = \sqrt{2} m_W \sin \beta$ , and we define  $x_{0j} \equiv \tilde{m}^2/\tilde{m}_X^2$  and  $x_{kj} \equiv \tilde{m}_t^2/\tilde{m}_X^2$ . We have used

$$g(\tilde{m}_t^2/\tilde{m}_X^2) V_{us}^* V_{ub} + g(\tilde{m}_c^2/\tilde{m}_X^2) V_{cs}^* V_{cb} \simeq -g(\tilde{m}_t^2/\tilde{m}_X^2) V_{ts}^* V_{tb} \quad (7)$$

which follows from the unitarity condition  $V_{cs}^* V_{cb} = 0$  and the condition that the first two generations of up squarks are nearly degenerate. The functions  $g(x)$  are given by [6, 7]:

$$g^{(1)}(x) = \frac{8x^3 - 3x^2 - 12x + 7 + (12x - 18x^2) \ln x}{36(x-1)^4}, \quad (8)$$

$$g^{(3)}(x) = \frac{-7x^2 + 12x - 5 + (6x^2 - 4x) \ln x}{6(x-1)^3}. \quad (9)$$

Both of these are positive, and fall off as  $x$  becomes large. One finds that  $g^{(3)}(x)$  is bigger than  $g^{(1)}(x)$  by at least a factor of 4, for all  $x$ .

We have broken  $A_\gamma(\chi)$  up into four pieces to see when it can significantly reduce  $B(b \rightarrow s\gamma)$ . The sum  $A_1(\chi) + A_2(\chi)$  is almost never large enough to cancel the  $H^+$  contribution. On the other hand,  $A_3(\chi^+)$  and  $A_4(\chi^+)$  can be large because they are enhanced by large  $\tan \beta$ . However, if the stop squarks are degenerate in mass with the other up squarks, these large contributions exactly cancel, due to a GIM cancellation.

To see how the sum  $A_3(\chi) + A_4(\chi)$  depends upon the stop mass splittings, let us define  $f_{0j} \equiv g^{(3)}(x_{0j})$  and  $f_{kj} \equiv g^{(3)}(x_{kj}) \equiv f_{0j} + \Delta f_{kj}$  ( $j, k = 1, 2$ ). Defining  $\sin \theta_i \equiv T_{i2}$ , we can write

$$A_3(\chi^+) + A_4(\chi^+) = - \sum_{j=1}^2 \frac{m_W}{\tilde{m}_X} \frac{1}{\sqrt{2} \cos \beta} \left[ -U_{j2} V_{j1} \left( \cos^2 \theta_i \Delta f_{1j} + \sin^2 \theta_i \Delta f_{2j} \right) + U_{j2} V_{j2} \frac{m_t}{v_u} \sin \theta_i \cos \theta_i \left( \Delta f_{1j} - \Delta f_{2j} \right) \right]. \quad (10)$$

One sees immediately that if all the squark masses are degenerate ( $\tilde{m}_{t_1} = \tilde{m}_{t_2} = \tilde{m}$ ), then  $\Delta f_{1j} = \Delta f_{2j} = 0$  which means that  $A_3(\chi^+) + A_4(\chi^+) = 0$ . From (7) it is clear that this cancellation arises from a GIM mechanism; if  $x_{0j} = x_{kj}$ , the unitarity of the CKM matrix ensures that  $A_3(\chi^+) + A_4(\chi^+) = 0$ .

One can get a sense for the behavior of (10) by considering only the light chargino piece ( $j = 1$ ), which tends to contribute more than the heavier chargino since  $g^{(3)}(x)$  is larger for small  $x$ . A careful analysis of the chargino mass matrix diagonalization reveals that

$$\text{sign} U_{12} V_{11} = -\text{sign} \mu, \quad \text{sign} U_{12} V_{12} = +\text{sign} \mu. \quad (11)$$

The only exception is for  $\mu < -\tilde{m}_{\text{win}} \tan \beta$ , where  $U_{12} V_{12}$  is positive but very small. If we use the large  $\tan \beta$  approximation  $\cos^{-1} \beta \simeq \tan \beta$ , we can estimate that

$$A_3(\chi^+) + A_4(\chi^+) \sim -\frac{1}{\sqrt{2}} \frac{m_W}{\tilde{m}_X} \tan \beta$$

$$\begin{aligned} & \left[ \text{sign}\mu |U_{12}V_{11}| \left( \cos^2\theta_i \Delta f_1 + \sin^2\theta_i \Delta f_2 \right) \right. \\ & \left. + \text{sign}\theta_i \mu |U_{12}V_{12}| \sin\theta_i \cos\theta_i \frac{m_t}{v_u} (\Delta f_1 - \Delta f_2) \right]. \end{aligned} \quad (12)$$

This allows one to understand the gross behavior of the sum. For moderate to large stop splittings,  $|\theta_i| \sim 45^\circ$ ,  $\tilde{m}_t$  will be less than  $\tilde{m}$ , and  $\tilde{m}_t$  will be greater than or of order  $\tilde{m}$  [12]. One sees that the chargino contribution tends to have a large destructive interference with the  $W^+$  and  $H^+$  pieces if  $f_1$  is light (*i.e.* there is a large stop mass splitting),  $\tan\beta$  is large, and if  $\theta_i \mu > 0$ , *i.e.*

$$\mu < 0, \theta_i < 0, \text{ or } \mu > 0, \theta_i > 0. \quad (13)$$

The  $\mu > 0$  case gives a smaller  $B(b \rightarrow s\gamma)$  because both pieces in (12) help to reduce the overall amplitude. One can show that  $\text{sign}\theta_i = -\text{sign}(A m_0 - \mu \cot\beta)$ , so that  $A\mu < 0$  implies that  $\theta_i \mu > 0$  (though the converse is not necessarily true). Finally we note that the sign of  $\mu$  is just the sign of  $B$ —one rotates the Higgs fields so as to make the Higgs potential coefficient  $\mu^2$  positive, and then  $\text{sign}\mu$  equals the sign  $B$  before that rotation [13]. Thus  $AB < 0$  implies  $\theta_i \mu > 0$ , which is the favorable region for destructive interference from the chargino loops. If  $|A m_0| \tan\beta > |\mu|$ , the converse is also true.

In the simplest SUGRA theories, one has the relation at the Planck scale  $B_0 = A_0 - 1$  [9]. One can show using general properties of the renormalization group equations that this relation implies one *cannot* have  $A < 0$  and  $B > 0$  at the weak scale, which is the most favored region for small  $B(b \rightarrow s\gamma)$ . If  $m_{H^+}$  and  $\tan\beta$  were found experimentally to be small, it might be possible to rule out minimal SUSY models which satisfy  $B_0 = A_0 - 1$ .

To illustrate these results, we consider some supersymmetric scenarios. In Figure 1, we consider the heuristic parameters  $\Delta\tilde{m}_t$  and  $\theta_i$ . We see that for the given choice of parameters with  $\theta_i \mu < 0$ ,  $B(b \rightarrow s\gamma)$  is always greater than the CLEO bound (in the region allowed by LEP above the unlabeled curves). The case  $\theta_i \mu > 0$  has lower  $B(b \rightarrow s\gamma)$ , especially for the  $\mu > 0$  case, and there are regions where the CLEO bound is satisfied. Increasing  $\Delta\tilde{m}_t$  lowers  $B(b \rightarrow s\gamma)$  in the  $\theta_i \mu > 0$  regions because  $A_3(\chi^+) + A_4(\chi^+)$  becomes more important. Radiative corrections lower both  $\tilde{m}_t$  and  $\tilde{m}_i$ , relative to  $\tilde{m}$  [12], so we take  $(\tilde{m}_t, -\tilde{m})/\Delta\tilde{m}_t$  to be  $-2/3$  and  $1/3$ , respectively. If one raises (lowers)  $\tilde{m}_t$ , while holding  $\tilde{m}_i$  constant, the difference between the  $\mu > 0$  and  $\mu < 0$  regions tends to become less (more) pronounced.

Figures 2-4 show more realistic scenarios where one inputs  $A$  instead of  $\Delta\tilde{m}_t$ , and  $\text{sign}\theta_i$ . Increasing  $\tan\beta$  will increase  $m_{H^+}^2$ , so that large  $\tan\beta$  gives smaller  $B(b \rightarrow s\gamma)$  just by suppressing the  $H^+$  loop contribution. To examine the different values for  $\tan\beta$  on equal footing, we have taken  $|B| = 1.5/\tan\beta$  so that  $m_{H^+}^2$  is about the same in each graph ( $m_{H^+} \simeq 260$  GeV at  $|\mu| = 400$ ). Even so,  $B(b \rightarrow s\gamma)$  gets much smaller in the  $AB < 0$  (*i.e.*  $A\mu < 0$ ) regions as  $\tan\beta$  increases, again because  $A_3(\chi^+) + A_4(\chi^+)$  becomes more important. Larger  $|A|$  also reduces  $B(b \rightarrow s\gamma)$  in those regions. For  $A < 0$ ,  $B > 0$  (which is not allowed if  $B_0 = A_0 - 1$ ) and  $\tan\beta > 10$ , there are even regions where the chargino contribution flips the sign

of the amplitude—cancelling the  $H^+$ ,  $W^+$  and  $C$  contributions—so that certain regions of parameter space are ruled out because the value of  $-A_1(\chi)$  is too large! Conversely, the regions of  $AB > 0$  tend to give larger  $B(b \rightarrow s\gamma)$  due to constructive interference from  $A_1(\chi)$ .

We have shown what happens when one varies  $\tan\beta$ ,  $A$ ,  $\mu$  and  $m_A$ . We took  $|B|$  such that  $m_{H^+}^2$  was of order the weak scale—if  $|B|$  is larger (smaller) than in Figures 1-4,  $m_{H^+}^2$  will be larger (smaller), and all the values for  $B(b \rightarrow s\gamma)$  will be smaller (larger). This simply demonstrates the point stressed by [3, 4] that  $B(b \rightarrow s\gamma)$  can be suppressed by large  $m_{H^+}$ . If  $m_t$  is heavier (lighter) than 140 GeV, all of the values for  $B(b \rightarrow s\gamma)$  will be shifted up (down) slightly, but for the SUSY result of  $m_t \simeq 134 \pm 25$  GeV [14], there is no qualitative change in our results. Lastly, one can make a different choice for  $m_0$ . Increasing  $m_0$  makes both  $\Delta\tilde{m}_t$  and  $m_{H^+}^2$  larger, so that  $B(b \rightarrow s\gamma)$  generally decreases in the  $AB < 0$  regions. One must be careful for large  $A m_0$  that  $\tilde{m}_t^2$  is greater than zero.

The branching ratio for  $b \rightarrow s\gamma$  in SUSY theories is near or below the SM value if the charged Higgs mass is large, or the chargino contribution destructively interferes with the charged Higgs and  $W$  loops. We found that the latter occurs in regions of parameter space where  $AB < 0$  (or equivalently when  $A\mu < 0$ ), and is accentuated by large  $\tan\beta$  and large  $|A|$ . One can have  $B(b \rightarrow s\gamma)$  at or below the SM prediction in supersymmetric models without requiring a large charged Higgs mass. Finally, we have noted that if  $m_{H^+}$  and  $\tan\beta$  were found to be small, it might be possible to rule out minimal SUSY models which satisfy the SUGRA relation  $B_0 = A_0 - 1$ .

#### Acknowledgements

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## REFERENCES

- [1] E. Thorndike, CLEO Collab., talk given at the 1993 Meeting of the American Physical Society, Washington, D.C., April 1993.
- [2] C. Dominguez, N. Paver, & Riazuddin, Phys. Lett. **B214** (1988) 2583; T. Altomari, Phys. Rev. **D37** (1988) 677; N. Deshpande, P. Lo, J. Trampetic, G. Eilam, & P. Singer, Phys. Rev. Lett. **59** (1987) 183.
- [3] V. Barger, M. Berger, & R. Phillips, Phys. Rev. Lett. **70** (1993) 1368.
- [4] J. Hewett, Phys. Rev. Lett. **70** (1993) 1045.
- [5] For a review of supersymmetry, see for example H. Haber & G. Kane, Phys. Repts. **117** (1985) 75.
- [6] S. Bertolini, F. Borzumati, A. Masiero, & G. Ridolfi, Nucl. Phys. **B353** (1991) 591.
- [7] R. Barbieri and G. Giudice, CERN preprint CERN-TH.6830/93.
- [8] It is possible that Barbieri and Giudice [7] miss the  $\tan\beta$  dependence of the chargino contribution because they vary  $\tan\beta$  only in the case of degenerate squark masses where the large  $\tan\beta$  effects exactly cancel by a GIM mechanism.
- [9] H. P. Nilles, Phys. Repts. **110** (1984) 1.
- [10] Note that it is important to include QCD corrections because they enhance the amplitude significantly; see for example B. Grinstein, R. Springer & M. Wise, Phys. Lett. **B 202** (1988) 138; R. Grigjanis, P. O'Donnell, M. Sutherland & H. Navalet, Phys. Lett. **B 213** (1988) 355.
- [11] J. Hagelin, S. Kelley, & T. Tanaka, Maharishi Intl. University preprint MIU-THP-92/59 (1992).
- [12] M. Drees & M. Nojiri, Nucl. Phys. **B369** (1992) 54.
- [13] R. Garisto, TRIUMF preprint TRI-PP-24 (1993).
- [14] P. Langacker, University of Penn. preprint UPR-0555T (1993).

## Figure Captions

Fig. 1. Contour plots of  $B(b \rightarrow s\gamma)$  (labeled lines) in units of  $10^{-4}$  for  $\tan\beta = 140$ ,  $B = 0.3$ , and  $m_0 = |\mu|$ . The current CLEO bound in these units is 5.4. Graphs (a) and (c) ((b) and (d)) use  $\theta_t < 0$  ( $\theta_t > 0$ ). Graphs (a) and (b) ((c) and (d)) use a fixed stop mass splitting of 100 (200). Unlabeled solid lines are  $m_{X_i^0} = 25$ , and  $m_{X_i^\pm} = 45$ . All masses in GeV.

Fig. 2. Contour plots of  $B(b \rightarrow s\gamma)$  (labeled lines) in units of  $10^{-4}$  for  $\tan\beta = 3$ ,  $m_t = 140$ ,  $B = 0.5$ , and  $m_0 = 100$ . Graphs (a), (b), (c), (d) have  $A = +1, -1, +2, -2$ , respectively. Unlabeled solid lines are  $m_{X_i^0} = 25$ , and  $m_{X_i^\pm} = 45$ . All masses in GeV.

Fig. 3. Same as Figure 2 for  $\tan\beta = 10$ . The contours in the  $\mu > 0$  region of (d) curve back near  $m_\lambda \sim 80$  because  $A_\gamma(\chi)$  becomes negative enough to flip the sign of the amplitude.

Fig. 4. Same as Figure 2 for  $\tan\beta = 30$ . As in Figure 3, the  $\mu > 0$  regions of (b) and (d) have regions which are ruled out because  $-A_\gamma(\chi)$  is too large.

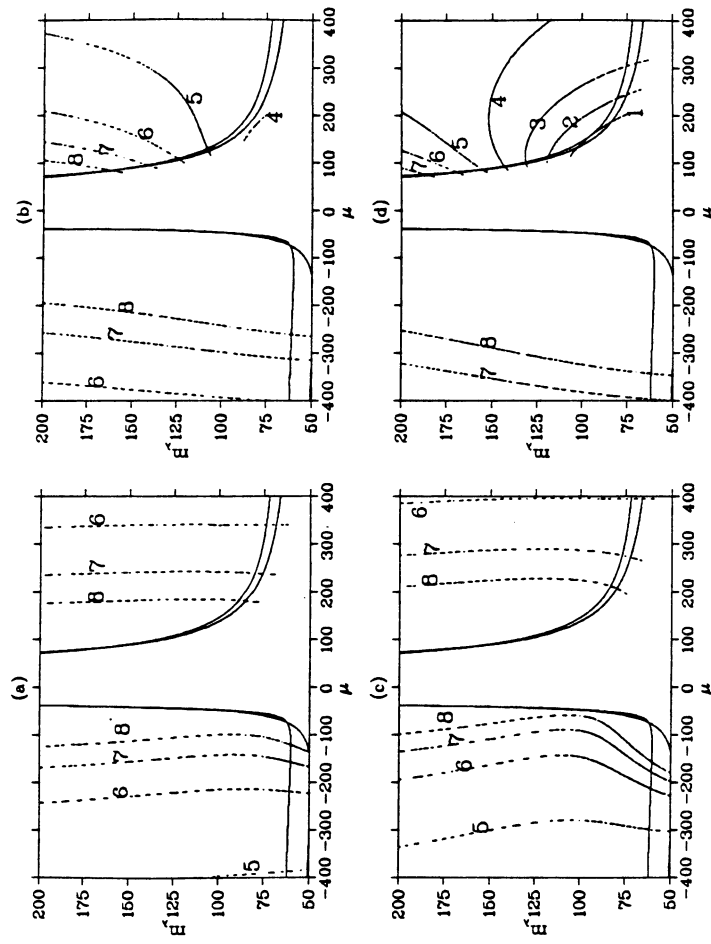


Fig. 1

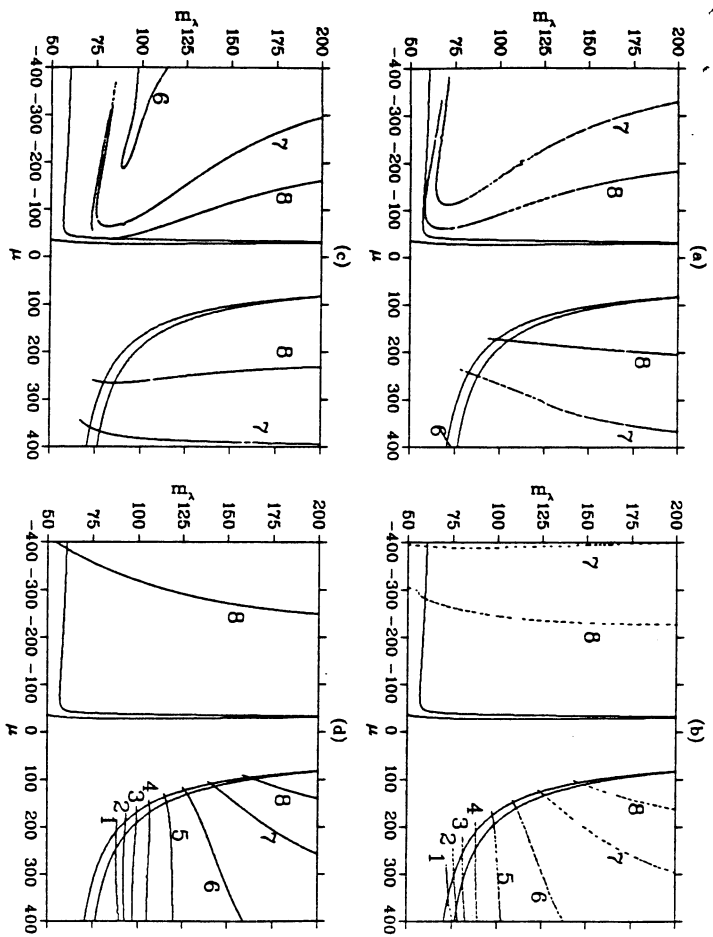


Fig. 2

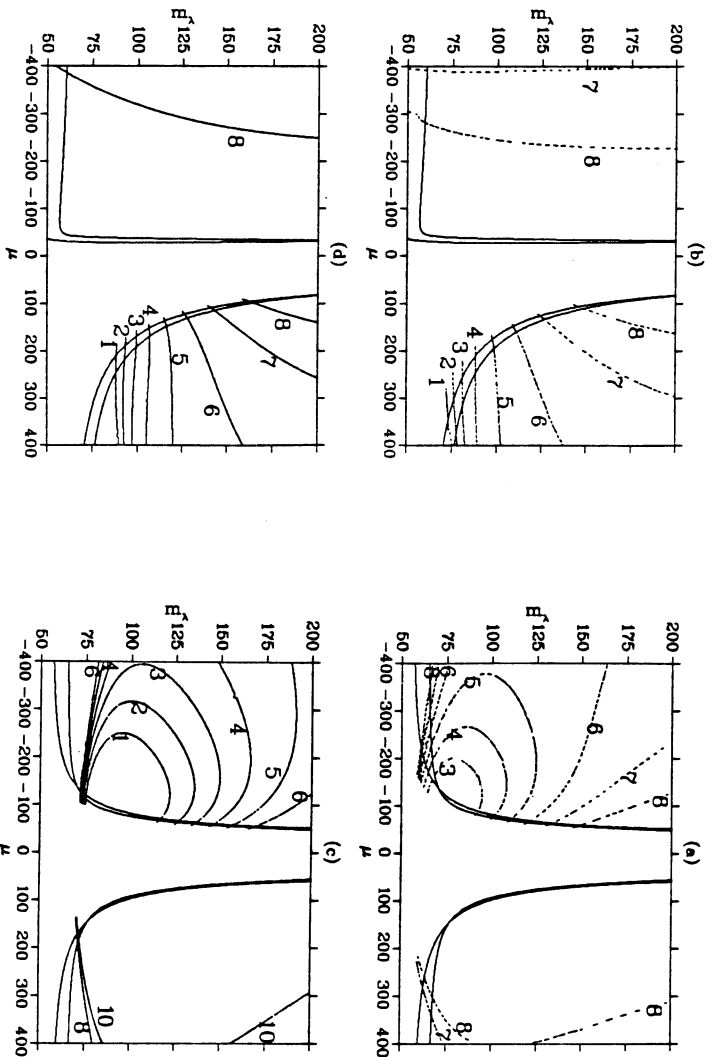


Fig. 3

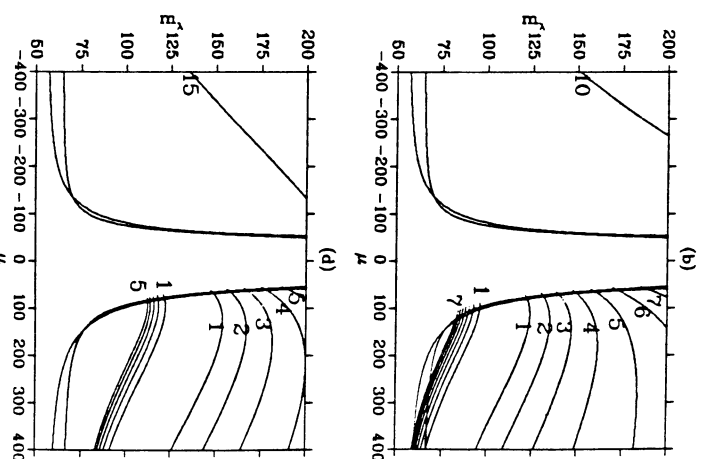


Fig. 4

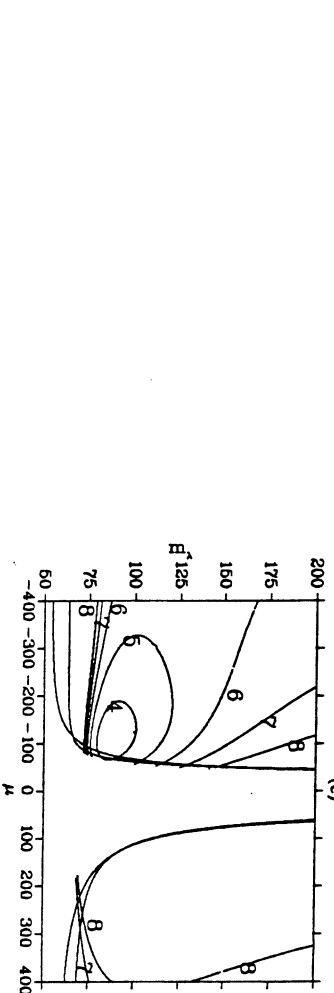


Fig. 2

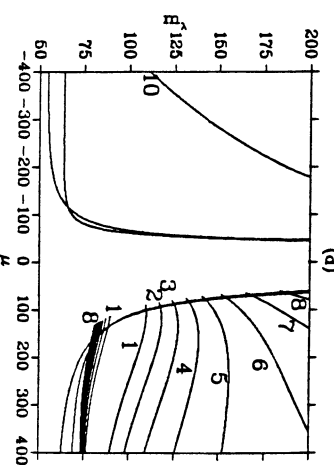


Fig. 3



Fig. 4