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The Influence of Bose-Einstein Correlations on Intermittency in $p\bar{p}$ Collisions at $\sqrt{s} = 630$ GeV

UA1-MINIMUM BIAS-Collaboration

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Abstract

The influence of Bose-Einstein correlations on the rise of factorial moments is small in the 1-dimensional phase space given by the pseudorapidity η , where the 2-body correlation function is dominated by unlike sign particle correlations. Contrarily, the influence is dominant in the higher dimensional phase space. This is shown by using correlation integrals. They exhibit clear power law dependences on the four-momentum transfer Q^2 for all orders investigated ($i = 2-5$). When searching for the origin of this behaviour, we found that the Bose-Einstein ratio itself shows a steep rise for $Q^2 \rightarrow 0$, compatible with a power law.

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1 Introduction

Recent searches for intermittency in the two- and three dimensional phase space show a strong rise of the factorial moments (FM) with decreasing phase space volumes v [1]. This rise is not only seen in hard scattering processes, but also in hadron-hadron reactions [2, 3] and in nuclear collisions [4, 5]. It indicates, that a singularity might occur for $v \rightarrow 0$ in the two and multiparticle correlation function. In hadron-hadron and nuclear collisions, the question is still open, if this behavior can be explained by known effects. One candidate is the Bose-Einstein (BE) effect [6, 7, 8]. All investigations of the influence of this effect are rather indirect up to now¹: Usually the slopes of the FM measured for all particles (φ_i^{all}) are compared to those where only like-sign particles contribute (φ_i^{ls}). However, experimental investigations in the 1-dimensional rapidity space show either no differences between φ_i^{all} and φ_i^{ls} , indicating that the Bose-Einstein effect has no major influence [9] or are not conclusive (see e.g. refs. [1, 10]). Studies of intermittency and multifractality with Monte Carlo models including the BE effect [11] could not explain the experimental results [9]. Perhaps, the implementation has been too crude and so the question is still open.

The aim of this paper is to study the influence of the Bose-Einstein correlations with some new experimental methods. We cannot distinguish between the quantum statistical symmetrization effect and other short range correlations of like-sign particles (e.g. from the decay of higher resonances). Therefore, more precisely, we want to study the contribution of the very short range correlations observed in the correlation function of like-sign particles (when analysed e.g. in Q^2) to the rise of the FM. We will call these correlations B.E. effect throughout this paper².

The paper is organized as follows: after giving the definitions and the specification of the data sample in sections 2 and 3, we present the analysis in section 4. We used three methods: a) comparison of φ_i^{ls} with φ_i^{all} , b) the method of “pair subtraction” and c) a detailed study of the two particle correlation function. For our analysis we used two different variables: the pseudorapidity η and the four-momentum difference Q^2 between pairs. The conclusions of our results are given in section 5.

2 Definitions

We use the usual “vertical” FM of order i :

$$\langle F_i \rangle = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m-1)\cdots(n_m-i+1) \rangle}{\langle n_m \rangle^i} = \frac{1}{M} \sum_{m=1}^M \frac{\int_{\Omega_m} \prod_k d\eta_k \rho_i(\eta_1, \dots, \eta_i)}{\int_{\Omega_m} \prod_k d\eta_k \rho_1(\eta_1) \cdots \rho_1(\eta_i)} \quad (1)$$

¹We will refer to new investigations which occurred during the preparation of this manuscript in our conclusions

²The question about the relative contributions of different effects is still discussed, however, recent analysis [12] of higher order like sign correlations [13] in the framework of quantum statistics has led to a most consistent description of all orders $i = 2 - 5$.

where ρ_i is the inclusive i -particle density function. For the computation of the integrals a binning of the original region $\Delta\eta$ into M subintervals of the size $\delta\eta$ is introduced. The number of particles in the m -th bin n_m is counted. The integration domain $\Omega_B = \sum_{m=1}^M \Omega_m$ thus consists of M i -dimensional boxes Ω_m of edge length $\delta\eta$. The brackets $\langle \ \rangle$ denote the averages over the event sample.

Selfsimilar density fluctuations at all scales $\delta\eta$ would lead to a power law dependence of $\langle F_i \rangle$ on $\delta\eta$:

$$\langle F_i \rangle \propto \left(\frac{1}{\delta\eta} \right)^{\varphi_i} \quad (2)$$

$$\log \langle F_i \rangle = a_i + \varphi_i \cdot \log \left(\frac{1}{\delta\eta} \right)$$

This behavior is called intermittency [14] and the parameters φ_i (slopes of the $\langle F_i \rangle$ in a log-log scale) are called intermittency exponents.

Recently a considerable improvement of the factorial moment method to study correlations has been proposed in [15] with the measurement of the correlation integrals $\langle C_i \rangle$. These quantities are closely related to the $\langle F_i \rangle$. The main difference is that the integration domain $\Omega_B = \sum_{m=1}^M \Omega_m$ is extended to a strip domain Ω_S which depends only on $\delta\eta$:

$$\langle C_i(\delta\eta) \rangle = \frac{\int_{\Omega_S} \prod_k d\eta_k \rho_i(\eta_1, \dots, \eta_i)}{\int_{\Omega_S} \prod_k d\eta_k \rho_1(\eta_1) \dots \rho_1(\eta_i)} \quad (3)$$

The counting procedure for the correlation integral requires, that all i -tuples in $[0, \Delta\eta]$ which are separated by a distance less than $\delta\eta$ are counted. In [15] a detailed discussion of the implementation of the $\langle C_i \rangle$ has been given. The method of counting i -tuples which is used in this paper is given by the ‘‘GHP’’ integral [16]:

$$\langle C_i(\delta\eta) \rangle = \frac{1}{Norm} \left\langle i! \sum_{j_1 < \dots < j_i} \prod_{k_1, k_2} \Theta(\delta\eta - |\eta_{j_{k_1}} - \eta_{j_{k_2}}|) \right\rangle \quad (4)$$

where Θ is the usual Heaviside step function and $Norm$ is obtained by ‘‘event mixing’’ [15].

We have verified, that the values of $\langle F_i \rangle$ and $\langle C_i \rangle$ are almost identical in the case of the analysis in $\delta\eta$ [17, 18] (differences are of the order of the statistical errors in our data sample of 160.000 events, see fig. 1a). One advantage of $\langle C_i \rangle$ is the better statistical accuracy. We use here another advantage: since (4) depends only on differences of phase space variables, we can replace $|\eta_{j_{k_1}} - \eta_{j_{k_2}}|$ by $-(p_{j_{k_1}} - p_{j_{k_2}})^2$, and $\delta\eta$ by Q^2 , where p is the four-momentum of a particle. Thus we are able to measure the $\langle C_i \rangle$ as a function of Q^2 , which is the theoretically preferred variable in jet evolution. In (4) the product extends over all possible pairs of an i -tuple. It contributes to $\langle C_i \rangle$ only if all pairs satisfy the condition $|\eta_{j_{k_1}} - \eta_{j_{k_2}}| < \delta\eta$. In the case of the Q^2 -analysis we have modified this condition: only i -tuples with $q_{12}^2 + q_{13}^2 + \dots + q_{(i-1)i}^2 < Q^2$, where $q_{ab}^2 = -(p_{j_{k_a}} - p_{j_{k_b}})^2$, contribute to $\langle C_i \rangle$ and we obtain:

$$\langle C_i(Q^2) \rangle = \frac{1}{Norm} \left\langle i! \sum_{j_1 < \dots < j_i} \Theta(Q^2 - \sum_{k_1, k_2} q_{j_{k_1}, j_{k_2}}^2) \right\rangle \quad (5)$$

In analogy to the usual analysis with FM, we will search for a power law of the $\langle C_i \rangle$ as a function of Q^2 :

$$\langle C_i \rangle \propto \left(\frac{1}{Q^2} \right)^{\varphi_i} \quad (6)$$

Eqn. (5) is conceptually different from eqn. (4) for $i \geq 3$. However, the search for a power law is motivated by the desire to search for selfsimilar dynamics in the production of particles, not knowing a priori in which variable it might show up. The variable Q^2 defined above has been proposed in [19] and used in the analysis of higher order Bose-Einstein correlations [13]. In choosing this variables, we are able to demonstrate the close connection between intermittency analysis and the analysis of Bose-Einstein correlations. Moreover, we want to remind the reader that there exists the simple relation between Q^2 and the invariant mass M_i of the i -tuple: $Q^2 = M_i^2 - (im_\pi)^2$ in the case all particles are pions. The Bose-Einstein correlations are given in the differential form. Let's denote $N^{(i)}(Q^2) dQ^2$ as the number of i -tuples found in $[Q^2, Q^2 + dQ^2]$ where $N^{(i)}(Q^2)$ is the i -body density function $\rho_i(k_1, k_2, \dots, k_i)$, integrated over all phase space variables k_i except Q^2 , and let's define $N_{mix}^{(i)}(Q^2) dQ^2$ as the expected number of i -tuples in the same interval in absence of correlations. $N_{mix}^{(i)}(Q^2)$ is the product of single particle densities $\rho_1(k_1) \dots \rho_1(k_i)$ integrated in the same manner as $\rho_i(k_1, k_2, \dots, k_i)$. It can be obtained by Monte Carlo integration, or simply by event mixing³. The proper normalization of the event mixing term is achieved by demanding the total number of mixed i -tuples in the overall phase space region (in our case: $|\eta| \leq 3, \phi \leq 2\pi, p_T > 0.15 GeV$) to be $N_{evt} \cdot \langle n \rangle^i$ where $\langle n \rangle$ is the measured mean number of particles per event in this overall region, and N_{evt} the total number of events. This can be obtained e.g. by generating a Poissonic multiplicity distribution of the mixed events with the mean value $\langle n \rangle$. The Bose Einstein correlations are usually presented in the form $f_{BE}^{(i)} = N^{(i)} / (const \cdot N_{mix}^{(i)})$, where *const* is chosen such that this ratio is equal to 1 in a suitably chosen Q^2 region (usually $Q^2 > 1$). The connection with $\langle C_i \rangle$ is given by eqn. (7):

$$\langle C_i(Q^2) \rangle = \frac{\int_0^{Q^2} N^{(i)}(Q_1^2) dQ_1^2}{\int_0^{Q^2} N_{mix}^{(i)}(Q_1^2) dQ_1^2} \quad (7)$$

We will measure in the following also the properly normalized differential 2-body density correlation function

$$f(l) = \frac{N^{(2)}(l)}{N_{mix}^{(2)}(l)}, \quad l = Q^2 \quad or \quad \delta\eta \quad (8)$$

³The event mixing technique has been recently discussed and justified in refs. [15, 20].

3 Data Sample

The data sample consists of approximately 160.000 non-single-diffractive events at $\sqrt{s}= 630$ GeV. All data were taken using a “minimum bias” trigger [21], requiring at least one charged particle in the pseudorapidity range of $1.5 < |\eta| < 5.6$ in each of the downstream arms of the detector. All information used for this analysis was obtained from reconstructed trajectories measured by the UA1 central detector [22]. Only vertex associated charged tracks with transverse momentum $p_T \geq 0.15$ GeV/c, $|\eta| \leq 3$, good measurement quality and fitted length ≥ 30 cm have been used. To calculate Q^2 , we assumed that all charged particles are pions.

Acceptance loss: All tracks recorded in the central detector (CD) are reconstructed first from the drift time measurements in the xy -plane which includes the beam axis (x) and is normal to the direction of the magnetic field (z). The dip angle γ with respect to this plane is determined from the charge division on the wires which are spanned parallel to the magnetic field of 0.7 Tesla [22]. There is a multiplicity dependent loss of tracks with large angle γ . Using the independence of particle production from the azimuthal angle around the beam axis, we estimated this loss and corrected the data.

Double counting of tracks: A visual examination of a sample of nearby like sign track pairs has been done on the graphic device MEGATEK to investigate possible double counting of tracks. About 1% of all tracks are misidentified as two tracks by the pattern recognition program. A special algorithm has been applied which searches for like sign pairs which are close together in phase space. If the two tracks belong to only one splitted track, then this algorithm removes the one that has more distance to the vertex. In addition we require for each pair with $\Delta\gamma < 2.5^\circ$ a separation $d \geq 15$ mm in the xy plane along a track length of ≥ 30 cm. Finally, we require $Q^2 \geq 0.0001$ (GeV/c)² for all accepted like sign pairs. The accompanying visual control asserts, that the remaining fake pairs occur at a rate of 0.03% (5% in the region $0.001 \leq Q^2 \leq 0.003$) and are compensated by approximately the same rate of visual identifiable real pairs (two separated chains of hits in the CD) which are cut away by the software cuts.

Loss of nearby track pairs: Due to the limited signal resolution in the CD and since the tracks are reconstructed only in the xy plane, most of the pairs with $d < 15$ mm are not recorded as two distinct tracks, irrespectively of $\Delta\gamma$. This has been confirmed by the visual examination: the number of detected real pairs with $d < 15$ mm is only 1/3 of those found in the neighbouring region with $15 \leq d \leq 30$ mm, independent from $\Delta\gamma$. The correction for this loss is dependent on the (unknown) physics at small separation of pairs in phase space. We therefore will present the data without a correction for this loss (unless it is stated explicitly). Since the $\langle F_i \rangle$ and $\langle C_i \rangle$ are integrated quantities, their sensitivity on track-pair losses grows rapidly with decreasing phase space bins. We therefore restricted our investigations to $\delta\eta > 0.05$ and $Q^2 > 0.05$ (GeV/c)² and

verified by cutting away successively also pairs in the region $15 \leq d \leq 30$ cm, that our data are not yet sensitive to these cuts. In the case of the differential two particle density correlation functions $f(Q^2)$ the data turned out to be much less sensitive. We therefore show this function also in the low Q^2 region ($Q^2 > 0.001$) and discuss the influence of the pair loss separately in section 4.3.

γ conversions: The influence of Dalitz pairs and γ -conversions in the beam pipe on the $\langle F_i \rangle$'s has been estimated previously [9] to be less than 10% in the case of the $\delta\eta$ analysis. We have verified that it is negligible for $C_i(Q^2 > 0.05)$. In the case of the differential two particle correlation function of unlike sign pairs, our Monte Carlo generations show, that they mainly contribute at $Q^2 < 0.001$, a region which is not investigated here (see also ref. [23]).

Resolution: The error of the pseudorapidity measurement varies between $\bar{\sigma}_\eta = 0.007$ ($|\eta| < 1.5$) and $\bar{\sigma}_\eta = 0.034$ ($1.5 \leq |\eta| \leq 3.5$). The error of Q^2 has been estimated from the errors of track-fits and has been also determined directly at $Q^2 = 0.17$ from the width of K_s^0 -decays. It is given by $\Delta Q^2 = 2Q \cdot \Delta Q + (\Delta Q)^2$ with $\Delta Q \simeq 8$ MeV, where ΔQ is approximately constant over the whole region of investigation.

4 Analysis

4.1 Charge dependence of slope parameters

Fig. 1 shows the rise of $\langle F_i \rangle$ or $\langle C_i \rangle$ for two different data samples with decreasing bin size⁴ in $\delta\eta$ (fig. 1a) and Q^2 (fig. 1b) in a log-log plot. The first data sample contains only like-sign particles and the second one all charged particles. The comparison in fig. 1 shows, that $\varphi_i^{all} = \frac{1}{2}\varphi_i^{ls}$ is fulfilled approximately in the Q^2 representation (table 1) whereas at the same time only small differences are visible in the $\delta\eta$ analysis⁵. This demonstrates that the influence of the BE correlations is strongly dependent on the variable used and turns out to be more important in the higher dimensional phase space. It has been conjectured [24, 25, 26] that intermittency occurs in the higher dimensional phase space and the bending of the $\langle F_i \rangle$ or $\langle C_i \rangle$ in fig. 1a is due to the projection to the 1-dimensional pseudorapidity space. However, figs. 1a,b demonstrate, that with projections we may also select different mechanisms: in fig. 1a the like sign particle correlations are significantly smaller than the correlations of all charged particles, but they dominate (for small Q^2) in fig. 1b. It should be stressed that in fig. 1b a good linearity shows up in agreement with eqn. (6) and the conjecture of

⁴In all figures of this paper bin sizes increase from left to right. This differs from the usual convention of drawing $\langle F_i \rangle$ or $\langle C_i \rangle$ values where the bin sizes increase from right to left. However, our present notation most naturally merges with the usual way to draw the correlation functions and in particular the B.E. correlations and thus provides us with a consistent presentation of its influence.

⁵This is in agreement with an earlier UA1 analysis [9], where in a very central ($|\eta| < 1.5$) region even no difference between φ_i^{all} and φ_i^{ls} has been found.

intermittency. Slight deviations from this law (a bending upwards of $\langle C_2 \rangle$ and $\langle C_3 \rangle$ for the like-sign sample) vanish, if all charged particles are considered (open circles). This indicates, that the linearity is due to an interplay of all correlations, irrespectively of the distinct dynamical origin.

In the following we want to investigate in a more sensitive way the contribution of Bose-Einstein correlations to the rise in $\delta\eta$, therefore we applied another method [17] in the next section.

4.2 The method of “pair subtraction”

With this method we attempt to measure the rise of the $\langle C_i \rangle$ in absence of the Bose-Einstein effect. To this end, like-sign pairs with small Q^2 were cut away until a data sample is (artificially) achieved which exhibits no Bose-Einstein effect as shown in fig. 2.

It should be mentioned, however, that studying the sample with subtracted pairs provides us only with a lower limit of what the influence of the BE correlations might be. If there exist also genuine higher order correlations they will contribute additionally. Recently higher order BE density correlations (i -tuple counts in bins $[Q^2, Q^2 + dQ^2]$) have been measured up to the 5th order for the first time [13]. However, to get rid of the contributions from lower orders, we need to measure the cumulant correlation functions [27].

We restrict in the following our investigation to the influence of 2-particle BE correlations. The behavior of the $\langle C_i \rangle$ before (open circles) and after (full circles) the subtraction of the BE pairs is shown in figs. 3a-d. Let's concentrate first on $\langle C_2 \rangle$ (figs. 3a,b). In the case of the analysis in Q^2 (fig. 3b) and the sample of like-sign pairs no residual rise is left after the subtraction as expected, since $\langle C_2 \rangle$ is (apart from a normalization constant) the integral over the BE ratio shown in fig. 2 (see eqn. (7)). There is some residual rise in the sample of all particles in fig. 3b but it vanishes for $Q^2 \leq 0.2$ which indicates that in the region of small Q^2 only the BE correlations contribute to the overall 2-body correlation function.

The situation is different in the case of the analysis in $\delta\eta$ (fig. 3a): there is after the subtraction of BE pairs still some rise also in the case of the like-sign particles for $\delta\eta \gtrsim 1$ indicating the presence of some correlations which do not originate from the very short range BE correlations in Q^2 . Only a small rise is left for $\delta\eta \lesssim 1$. We conclude therefore, that the rise in that region in the like-sign sample is mainly due to BE correlations.

In the case of all particles (fig. 3a) the subtraction of BE pairs has only a small influence and we conclude that the rise is mainly due to strong correlations of unlike-sign pairs.

The region in $\delta\eta$ which can be populated by BE pairs is given by the following relation [17]:

$$Q^2 = - \left(m_{T_1}^2 + m_{T_2}^2 - 2m_{T_1}m_{T_2} \cosh(\delta y) \right) + \left(p_{T_1}^2 + p_{T_2}^2 - 2p_{T_1}p_{T_2} \cos(\delta\phi) \right) \quad (9)$$

with $\delta\eta \approx \delta y$ and $m_T^2 = m^2 + p_T^2$.

Eqn. (9) shows that pairs with the same Q^2 can contribute at different $\delta\eta$ values, dependent on the transverse momenta p_T and the difference of azimuthal angles $\delta\phi$.

We turn now to the influence on higher order $\langle C_i \rangle$ in the case of all particles in figs. 3c,d. (The like-sign sample (not shown here) has very similar behaviour.) After the subtraction of the BE pairs they all show still a significant rise (see table 2), indicating the presence of other correlations too. A difference before and after the subtraction of BE pairs is seen in both the Q^2 and $\delta\eta$ analysis, but it is weaker in the case of $\delta\eta$.

4.3 A detailed study of the two-particle correlation functions

If one assumes that intermittency is indeed present and - as fig. 1b suggests - is dominated (in Q^2) by the B.E. correlations, one would expect that the shape of the BE ratio itself should be represented by a power law rather than by an exponential (or Gaussian). Therefore we present a measurement of differential 2-body density correlation functions $f(l)$, ($l = \delta\eta$ or Q^2) for like-sign pairs and unlike-sign pairs separately and search for a singularity⁶ for $l \rightarrow 0$, as an indication for intermittency. Fig. 4 shows the ratio $f_{BE}^{(2)} = (1/const) \cdot f(Q^2)$. This is the usual form in which BE correlations are presented. Fig. 4 shows a comparison of the samples of like-sign pairs with unlike sign pairs and all charged particles. Each sample is normalized to 1 separately for $Q^2 > 1$ by choosing "const" [3, 28]. One observes a strong dominance of unlike-sign pair correlations for $0.03 \leq Q^2 \leq 1$ which is at least partly due to resonance and particle decays (e.g. there is a broad peak at $Q^2 \approx 0.5 \text{ GeV}/c^2$ which is due to ρ decays ($m_\rho = \sqrt{Q^2 + 4m_\pi^2}$) and a peak at $Q^2 \approx 0.17 \text{ GeV}/c^2$ which is due to remaining K_S^0 decays, where the decay point could not be resolved from the vertex). However, at very small $Q^2 (\leq 0.03 \text{ GeV}/c^2)$ this function is nearly constant. Contrarily, the like-sign particle correlation function rises above one only for small $Q^2 (\lesssim 0.24 \text{ GeV}/c^2)$. For very small $Q^2 (\lesssim 0.03 \text{ GeV}/c^2)$ there is a cross-over and the function is rising very rapidly for $Q^2 \rightarrow 0$. Fig. 4 suggests that a possible singularity in the correlation function would be due to the like-sign particle contributions only. To resolve the small Q^2 region we choose again the log-log scale in fig. 5. Fig. 5b shows $f(Q^2)$, the same functions as fig. 4, the only difference - besides the different binning - is the proper normalization to the uncorrelated sample as described in section 2. Fig. 5b confirms the observations in fig. 4. The unlike sign correlation function stays approximately constant for $Q^2 \lesssim 0.17$. The rise near $Q^2 = 0.001$ can be attributed partly to the onset of γ -conversions (which contribute mainly to the region $Q^2 < 0.001$) but may be also at least partly due to the Coulomb attraction of the unlike sign pairs (+ 10% increase at $Q^2 = 0.001$ expected). The like sign correlation function continues to rise at least until $Q^2 = 0.001 \text{ GeV}^2$. Once more we show the same analysis in $\delta\eta$ (fig. 5a). A comparison between figs. 5a and 5b confirms the results of the previous

⁶not in the mathematical sense: either due to the limited detector resolution or because of physical reasons there will be a cut-off at finite l .

investigations: the 2-body correlation function of all charged particles is dominated by unlike-sign particle correlations when analysed in $\delta\eta$ but dominated by the like-sign correlation function when analysed at small Q^2 .

The good resolution of the functions presented in fig. 5b permits to study the functional form of $f(Q^2)$ of the like-sign sample, and especially to search for a power law dependence. In fig. 6 we show a comparison with the following functions⁷:

$$f(Q^2) = a + b \cdot (Q^2)^{-\varphi} \quad (10)$$

$$f(Q^2) = a' + b' \exp(-rQ) \quad (11)$$

$$f(Q^2) = a''(1 + 2\lambda(1 - \lambda) \exp(-rQ) + \lambda^2 \exp(-2rQ)) \quad (12)$$

Each of them has 3 free parameters: a , b , φ (eqn. (10)), a' , b' , r (eqn. (11)) and a'' , λ , r (eqn. (12)). The best agreement with the data (at small Q^2) is obtained by the power law of eqn. (10). Subsamples, with positive pairs or negative pairs only, agree within their statistical errors, each showing the excess of pairs at small Q^2 over an exponential ansatz separately. We have also studied the systematic uncertainties which arise from the inclusion of residual fake pairs on one side, and from the loss of real pairs on the other side (see section 3) by varying the selection criteria for accepted pairs. The result of this study gives a systematic uncertainty of +9.0%, +3.2% and -8.0%, -2.2% at $Q^2 = 0.001, 0.005$ (GeV/c)², respectively. It should be stressed that with each selection and in particular with a sample (called sample 2) where all fake pairs have been removed by rigorous cuts, and which has been corrected for the severe loss of real pairs by Monte Carlo afterwards, we come to the same conclusion as above: the best agreement is obtained by a power law.

Table 3 contains the fit parameters of eqns. (10), (11) and (12) for the data as defined in section 3 and shown in fig. 6, and for sample 2 (in brackets).

In conclusion, the data of fig. 6 indicate, that one scale might be not enough to describe them satisfactorily, but they are in agreement with the conjecture of scale invariance.

Three remarks should be added: The first concerns the question how the present data compare with previously published ones [13] and with those of other experiments [30] which also show a steep rise for $Q^2 \rightarrow 0$ incompatible with a Gaußian function but still in agreement with an exponential ansatz. The answer is, that with a larger binning ($\Delta Q^2 = 0.05$ (GeV/c)² and $Q^2 > 0.05$ in the case of [13], and $\Delta Q = 25$ MeV in the case of [30]) the data became smeared out and show a less steep rise for $Q^2 \rightarrow 0$. We have compared in detail with the statistically excellent data of the AFS collaboration [30] from pp reactions at $\sqrt{s} = 63$ GeV. It turned out, that our data are (after the correction for pair loss) in quantitative agreement with the AFS data in our lowest Q^2 -region, if we adopt the same (larger) binning (AFS: $f_{BE} = 1.79 \pm 0.1$, UA1: $f_{BE} = 1.77 \pm 0.035$, $25 \text{ MeV} < Q < 50 \text{ MeV}$). To obtain the Bose-Einstein

⁷The functional form of (10) has been proposed in [29] for a 3-dimensional analysis, a possible contribution from long range correlations can be absorbed in the parameter a .

ratio, we have put $const = 1.36$, the value of the correlation function at $Q^2 = 1$. Our data show (again with the same binning) a comparable steeper fall off towards higher Q^2 -values than the AFS data. It is not clear, if this can be attributed to the much higher CMS energy of our experiment, since the data of ref. [31] at $\sqrt{s} = 26$ GeV also show a steeper fall off than the AFS data, as discussed in ref. [32]. We want furthermore to refer to an early experiment [33] where 2-body correlations have been successfully compared to power laws. Secondly, we want to point out, that we did not correct our data for Coulomb repulsion, because the validity of the Gamov correction factor [34] (which would amount +10% at $Q^2 = 0.001$ (GeV/c)²) has been questioned recently [35]. The third remark concerns the fact that the Bose-Einstein ratio of our experiment, but also that of refs. [30, 31] reach or nearly reach the value $f_{BE} = 2$ in the lowest bins, leaving not much room for “coherence”. We question therefore the procedure to deduce coherence from extrapolations to $Q^2 = 0$ of exponential or (even worse) Gaußian fits, obtained in a region of much larger Q^2 .

5 Summary and Conclusions

- We studied the contribution of the (very short range) like sign particle correlations which we call the BE effect to the rise of factorial moments (or correlation integrals) with decreasing phase space bins.
- We used two variables for this study:
 - (i) the 1-dimensional variable $\delta\eta$,
 - (ii) the squared 4-momentum difference Q^2 between two particles.

A study with a similar formalism in both variables was possible with the help of the correlation integrals [15], quantities which are closely related to the factorial moments, but which depend only on differences of phase space variables.

- Three methods have been applied:
 1. a comparison of slope parameters φ_i^{all} and φ_i^{ls} of the rise of the $\langle C_i \rangle$,
 2. the method of “pair subtraction”,
 3. a search for a singularity in the two-particle density correlation function for like-sign pairs and unlike-sign pairs separately.

Our conclusions:

- The influence of the BE effect depends on the variable used. Whereas it is weak in the case of $\delta\eta$, it is the dominant contribution to the rise of $\langle C_i \rangle$ with decreasing Q^2 ($Q^2 \rightarrow 0$). This is shown most clearly by method 1: In the $\delta\eta$ -analysis we found $\varphi_i^{all} \approx \varphi_i^{ls}$, but in the Q^2 -analysis the condition $\varphi_i^{all} \approx \frac{1}{2}\varphi_i^{ls}$ is fulfilled for all orders i .

- Method 2 shows, that there is also a weak, but non-negligible contribution of the BE effect in the case of $\delta\eta$ -analysis.
- Different dynamical mechanisms are dominant in the $\delta\eta$ and Q^2 analysis, this is confirmed directly by method 3: when analysed in $\delta\eta$, the two particle correlation function is dominated by the contribution of unlike-sign pairs whereas when analysed in Q^2 the dominance of the like-sign pairs shows up very clearly for small Q^2 ($< 0.03 \text{ GeV}/c^2$).
- The correlation integrals in fig. 1b show an almost perfect power law dependence of $\langle C_i \rangle$ on Q^2 over the whole region of analysis⁸ It is due to the interplay of different mechanisms, however, for small Q^2 the $\langle C_i \rangle$ s are dominated by the like sign particle correlations⁹ for all orders $i = 2 - 5$. Therefore the question, if intermittency (in Q^2) can be explained by known effects, will only be answered with "yes", if we can explain the correlations of like sign particles for $Q^2 \rightarrow 0$. Restricting to the two-particle density correlation function, we observe a steep rise compatible with a power law down to $Q^2 = 0.001 \text{ (GeV}/c)^2$. This deviates from the usual Gaußian parametrisations and it even exceeds (only at very small Q^2) the exponential ansatz, but it is in agreement with an early measurement [33] where power laws as a function of the invariant mass have been successfully compared to the data in the framework of the Müller-Regge approach. But it has also been pointed out in ref. [33] that the predictive power of this approach is limited to large masses only. More recently another attempt has been made to understand intermittency [38]. Let's assume that all like sign particle correlations at small Q^2 are due to the quantum statistical symmetrisation effect. Then arguments have been given in [38], that a power law in the correlation function would imply strong fluctuations of the size of the interaction region. One possibility could be, that the interaction region is itself a fractal [38, 39, 40], extending over a large volume. According to our smallest $Q^2 = 0.001 \text{ (GeV}/c)^2$ this extension would be as large as 6 fm. It would be interesting to check, if the contribution to the B.E. effect from known resonance decays [41, 42] can explain this behaviour.

⁸This region is not very large yet because of the reasons discussed in section 3.

⁹Similar results have been found recently with hh reactions at lower energy [36] and with a 3-dimensional intermittency analysis of μp reactions [37].

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Tables

Table 1: The results of fitting the $\langle C_i \rangle$ ($i = 2, \dots, 5$) as a function of Q^2 to a power law (6). In fig. 1b the fitted functions superimposed to the data are shown. The fitted slope parameters are given for two different data samples. The errors indicated are only statistical.

slope parameters	φ_2	φ_3	φ_4	φ_5
all charged particles	0.0348 ± 0.0006	0.078 ± 0.001	0.213 ± 0.004	0.338 ± 0.019
like-sign particles	0.0522 ± 0.0009	0.147 ± 0.001	0.443 ± 0.010	0.855 ± 0.051

Table 2: A comparison of the slope parameters before and after the BE-cut, obtained by fitting the $\langle C_i \rangle$ of all charged particles as a function of Q^2 to a power law (6). In fig. 3d the fitted functions superimposed to the data are shown. The errors are only statistical.

slope parameters	φ_2	φ_3	φ_4	φ_5
before BE-cut	0.0348 ± 0.0006	0.078 ± 0.001	0.213 ± 0.004	0.338 ± 0.019
after BE-cut	0.0108 ± 0.0005	0.046 ± 0.001	0.172 ± 0.004	0.305 ± 0.018

Table 3: Parameters of the fits, shown in fig. 6. The errors include statistical and systematic uncertainties. The values in the brackets are obtained with sample 2, see text. Q^2 is in units $[(\text{GeV}/c)^2]$, r in [fm]. The data are not corrected for Coulomb repulsion.

fit to Eq.10	fit to Eq.11	fit to Eq.12
$a = 1.25 \pm 0.02$ (1.27)	$a' = 1.357 \pm 0.003$ (1.359)	$a'' = 1.355 \pm 0.003$ (1.357)
$b = 0.08 \pm 0.02$ (0.07)	$b' = 0.84 \pm 0.10$ (0.96)	$\lambda = 0.43 \pm 0.13$ (0.56)
$\varphi = 0.39 \pm 0.06$ (0.43)	$r = 1.39 \pm 0.11$ (1.50)	$r = 1.26 \pm 0.06$ (1.31)
$\chi^2/NF = 2.14$ (2.05)	$\chi^2/NF = 2.61$ (3.65)	$\chi^2/NF = 2.23$ (2.90)

Figure Captions

Fig. 1: The rise of the factorial moments and correlation integrals a) with decreasing $\delta\eta$, b) with decreasing Q^2 . The indicated errors are statistical only. Additional

systematic errors arise from uncertainties of acceptance corrections. Their magnitudes are: $\pm 1.5\%$ ($i = 2$), $\pm 3\%$ ($i = 3$), $\pm 7\%$ ($i = 4$), $\pm 14\%$ ($i = 5$). Since these numbers are independent from $\delta\eta$ or Q^2 , they concern only the absolute values of the FM or C_i but not the slopes.

Fig. 2: The Bose-Einstein ratio before and after the subtraction of pairs with small Q^2 .

Fig. 3: The effect of the pair-subtraction (BE-cut)

a) b) on the second order correlation integrals depending on $\delta\eta$ or Q^2 respectively,

c) d) on the higher order correlation integrals depending on $\delta\eta$ or Q^2 respectively.

Fig. 4: Bose-Einstein ratios f_{BE} for different samples, as indicated.

Fig. 5: The normalized two-body density correlation function f , eqn. (8)

a) as a function of $\delta\eta$,

b) as a function of Q^2 .

Fig. 6: The two-body density correlation function for like-sign particles fitted to (10), (11), (12), see text. The indicated errors are statistical only.

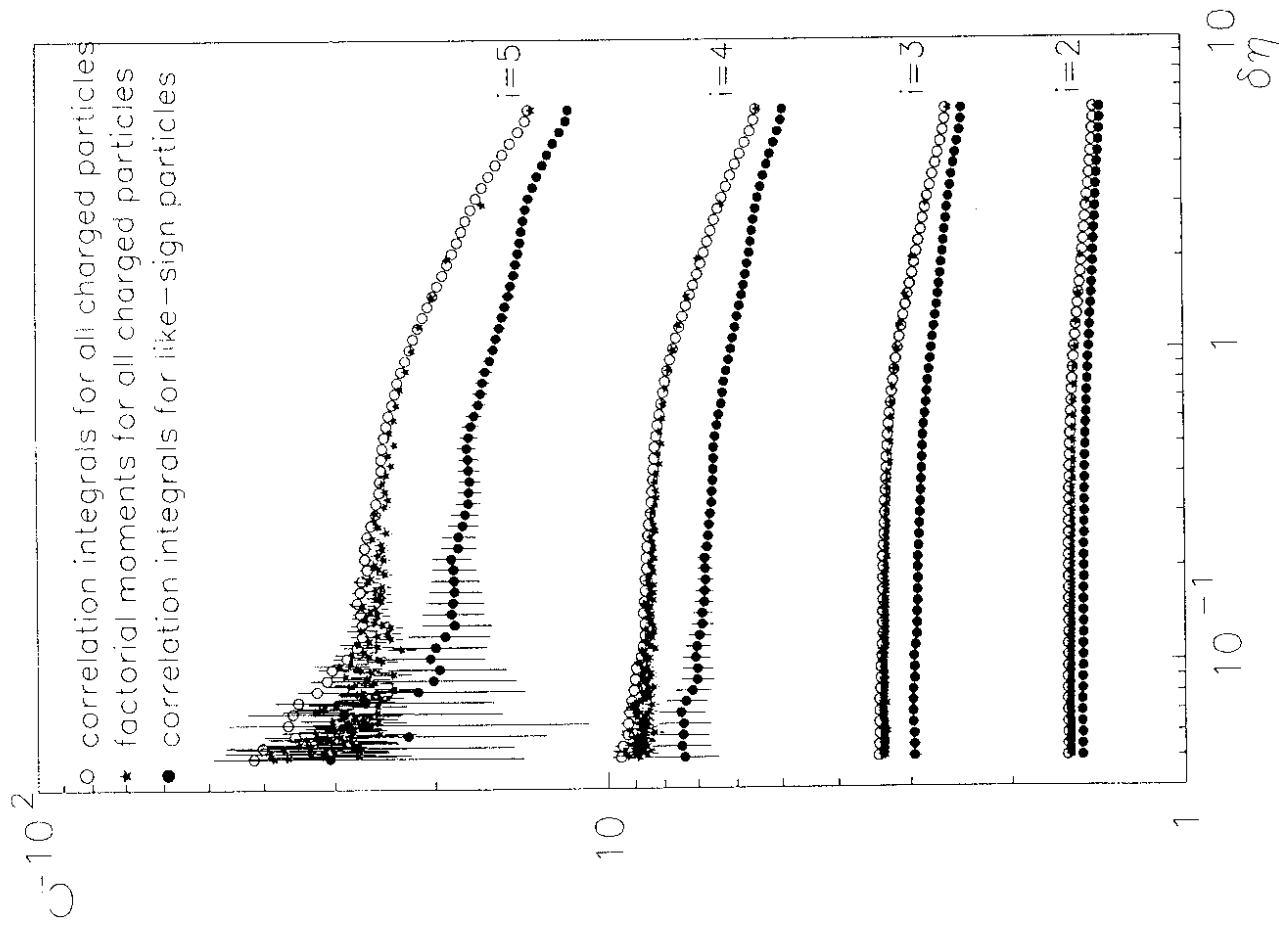


Fig. 1a

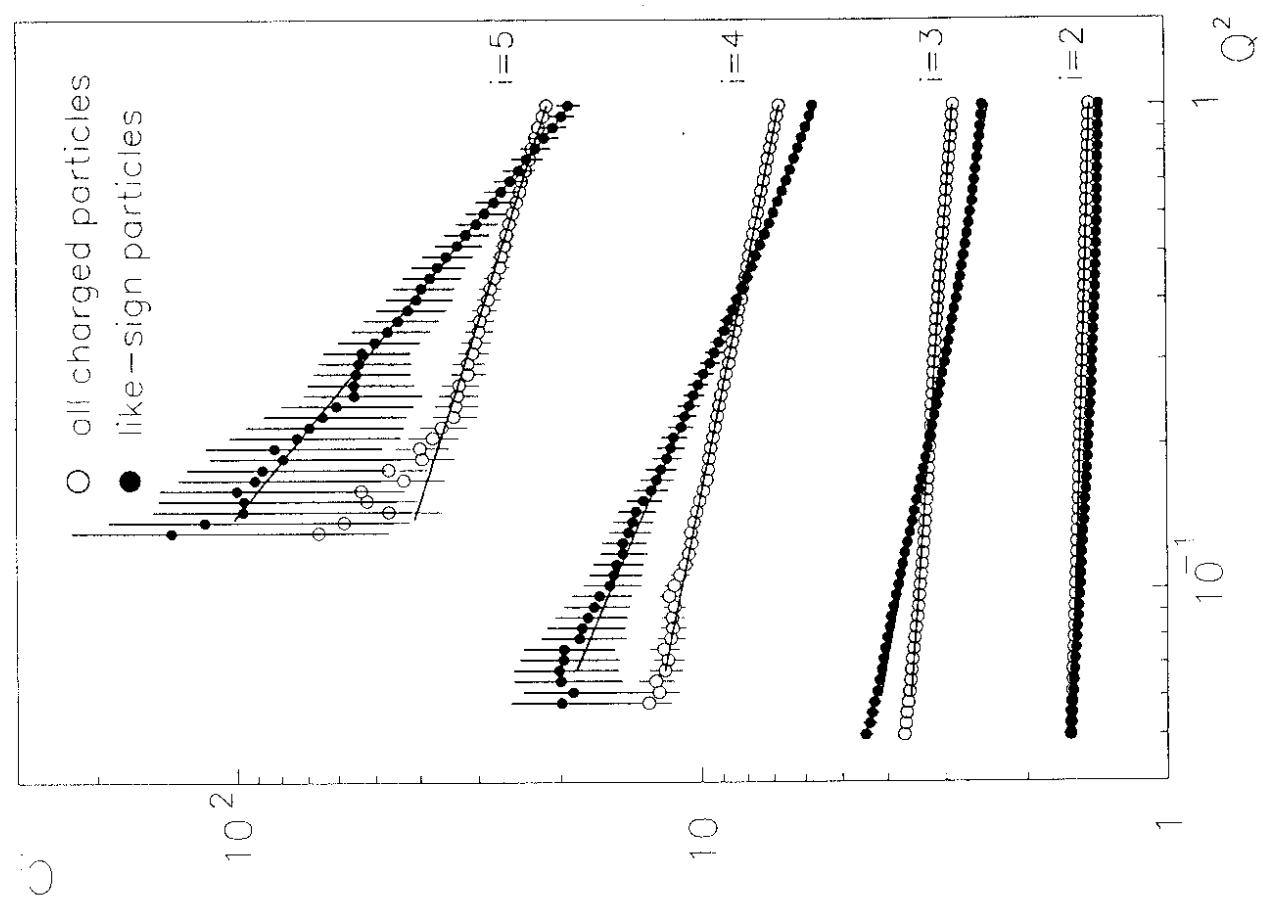


Fig. 1b

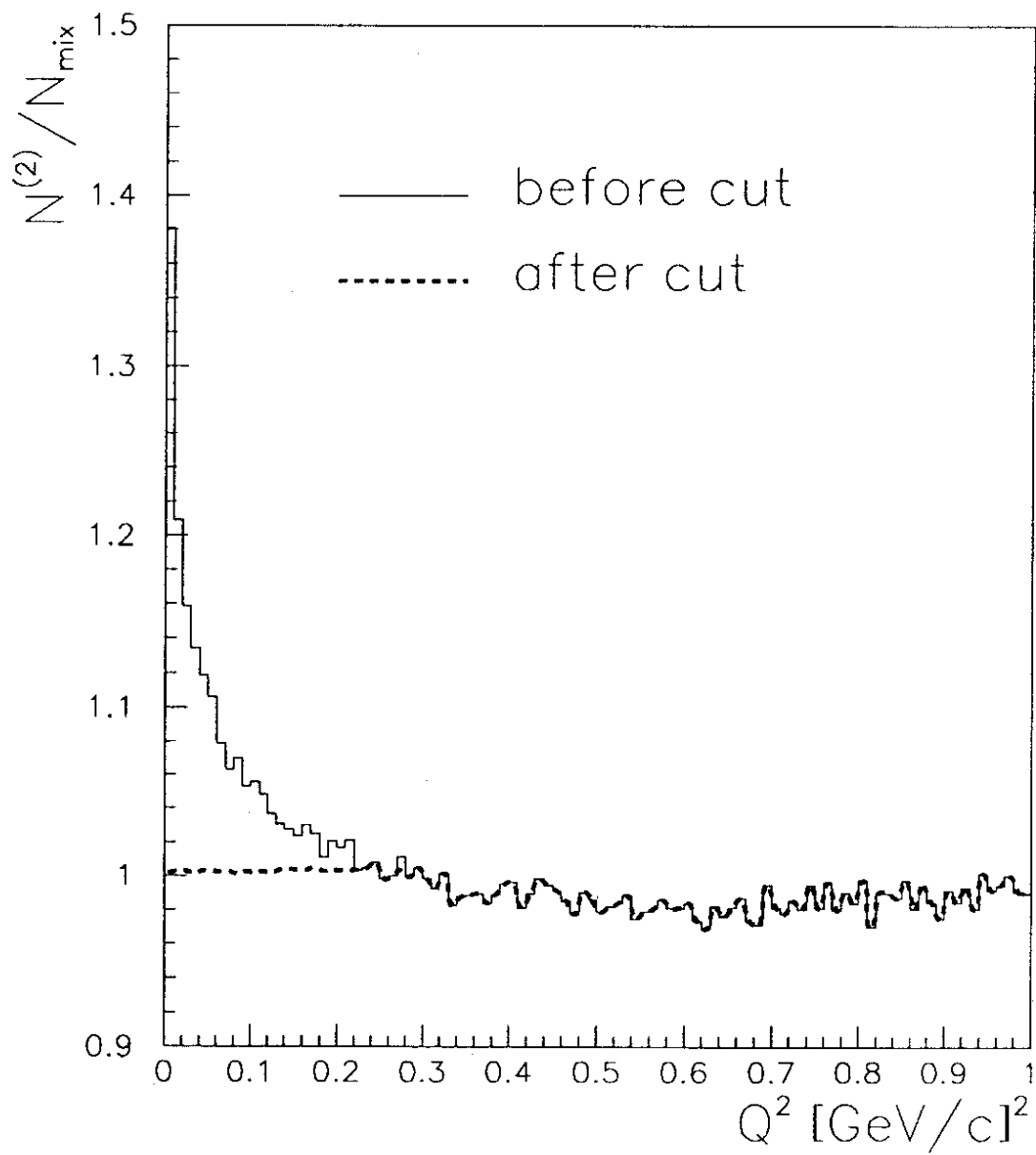


Fig. 2

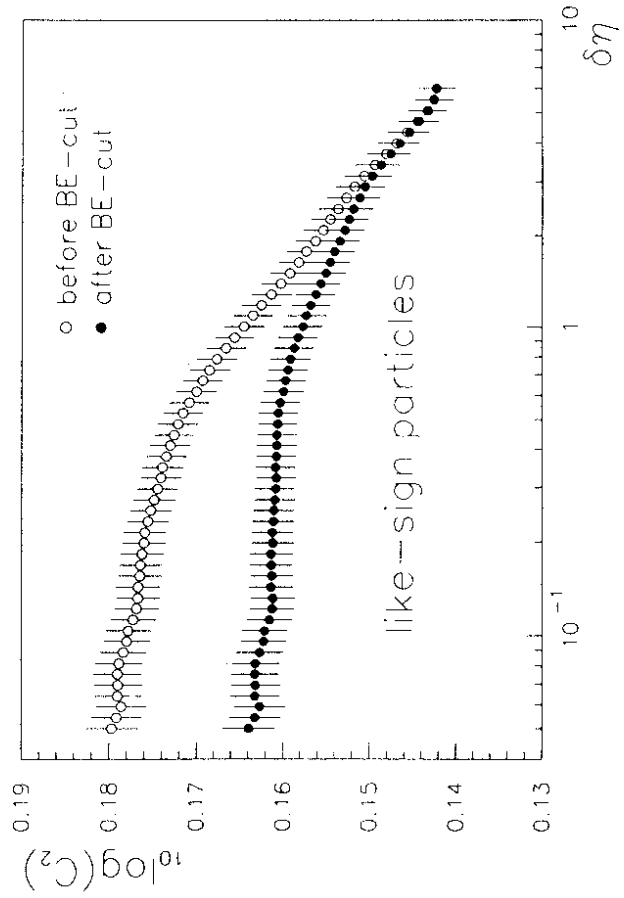
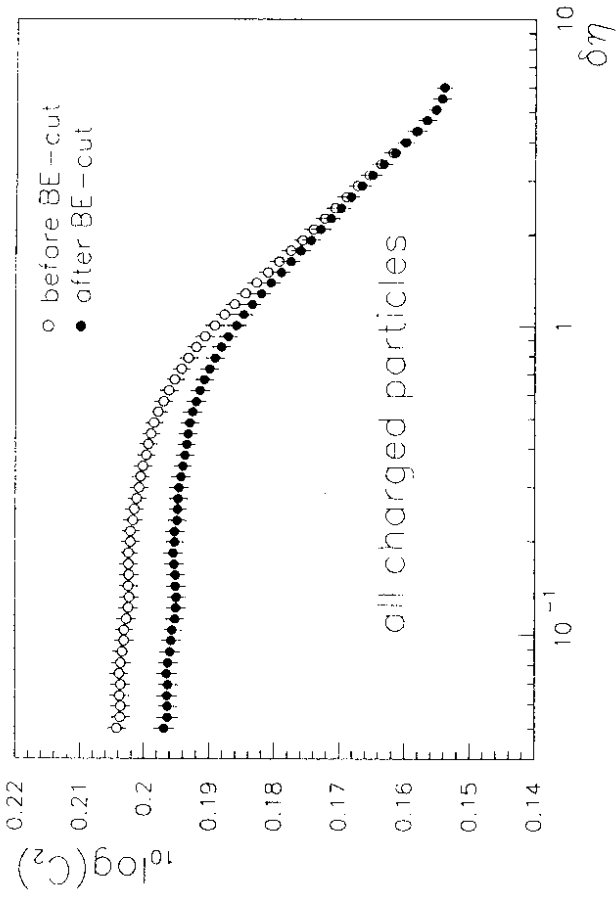


Fig. 3a

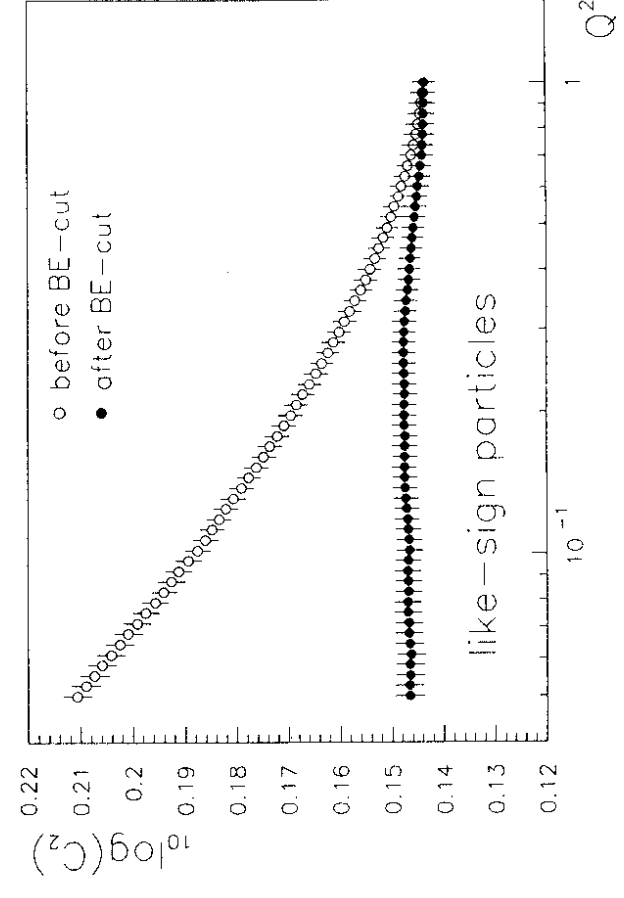
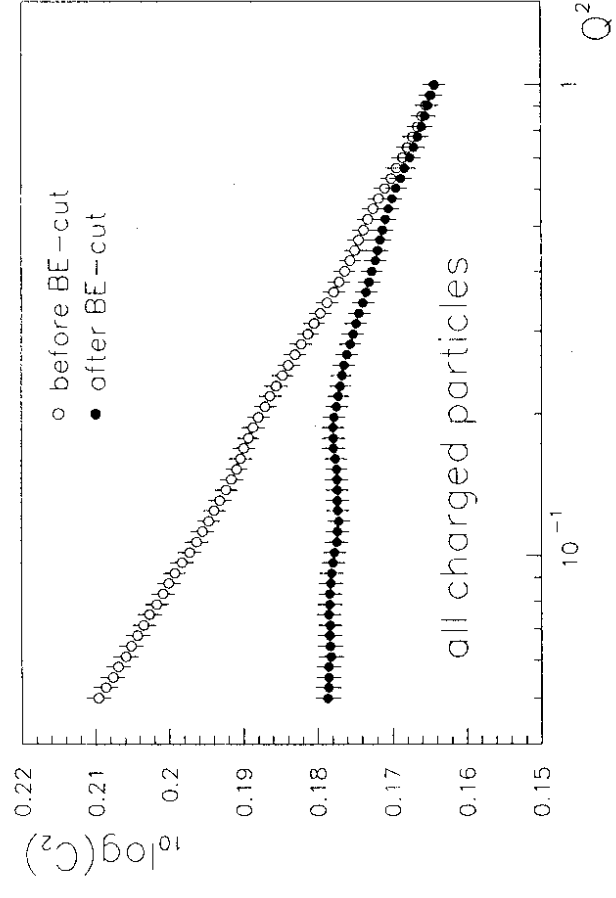


Fig. 3b

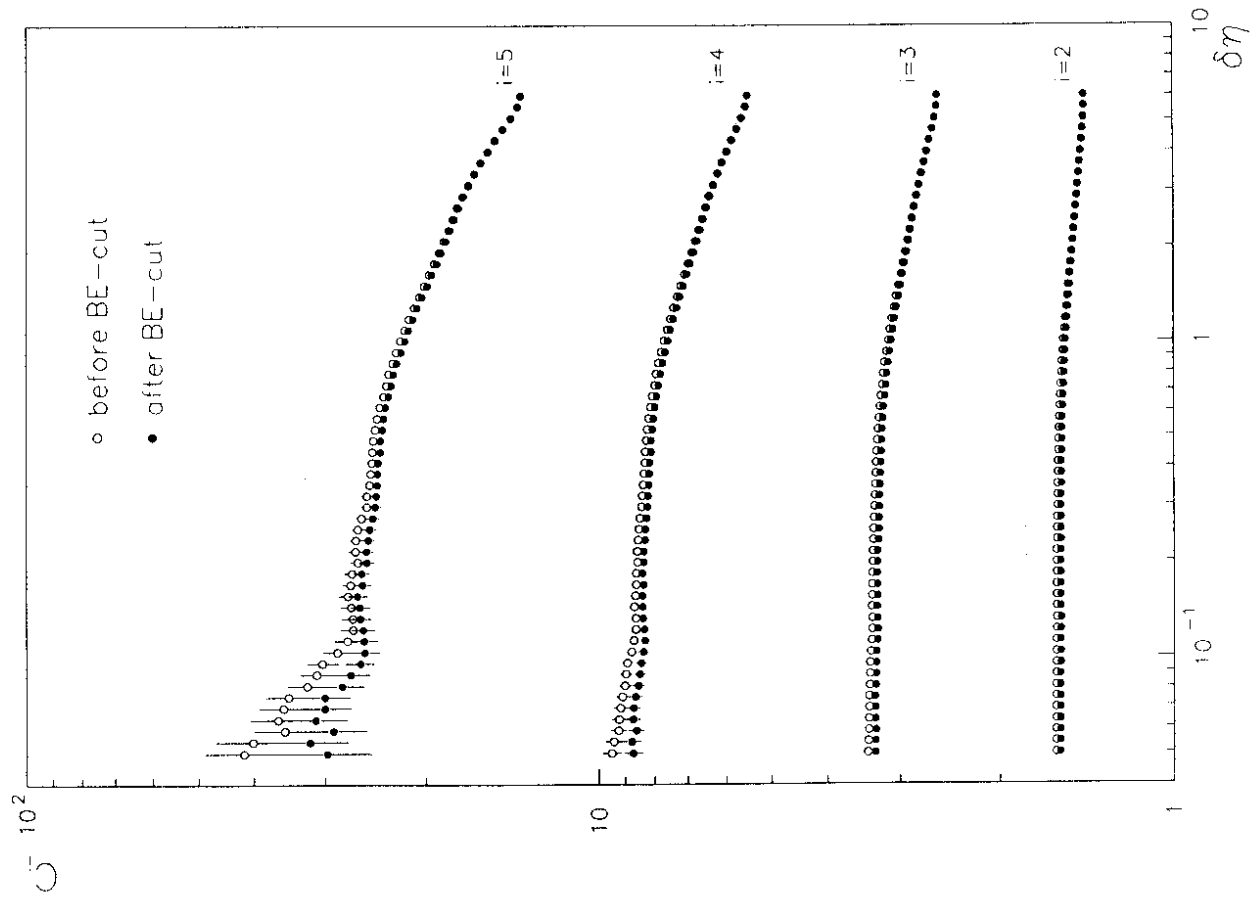


Fig. 3c

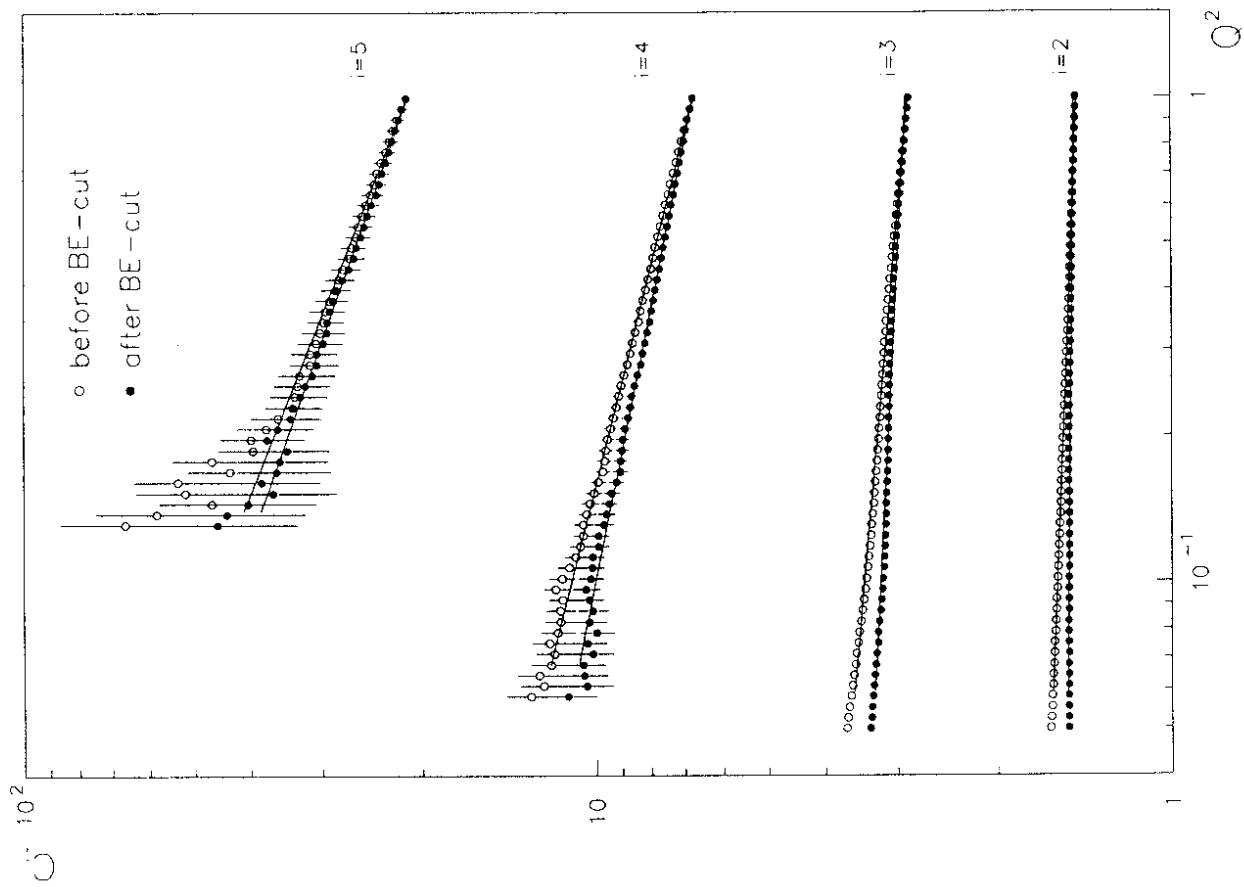


Fig. 3d

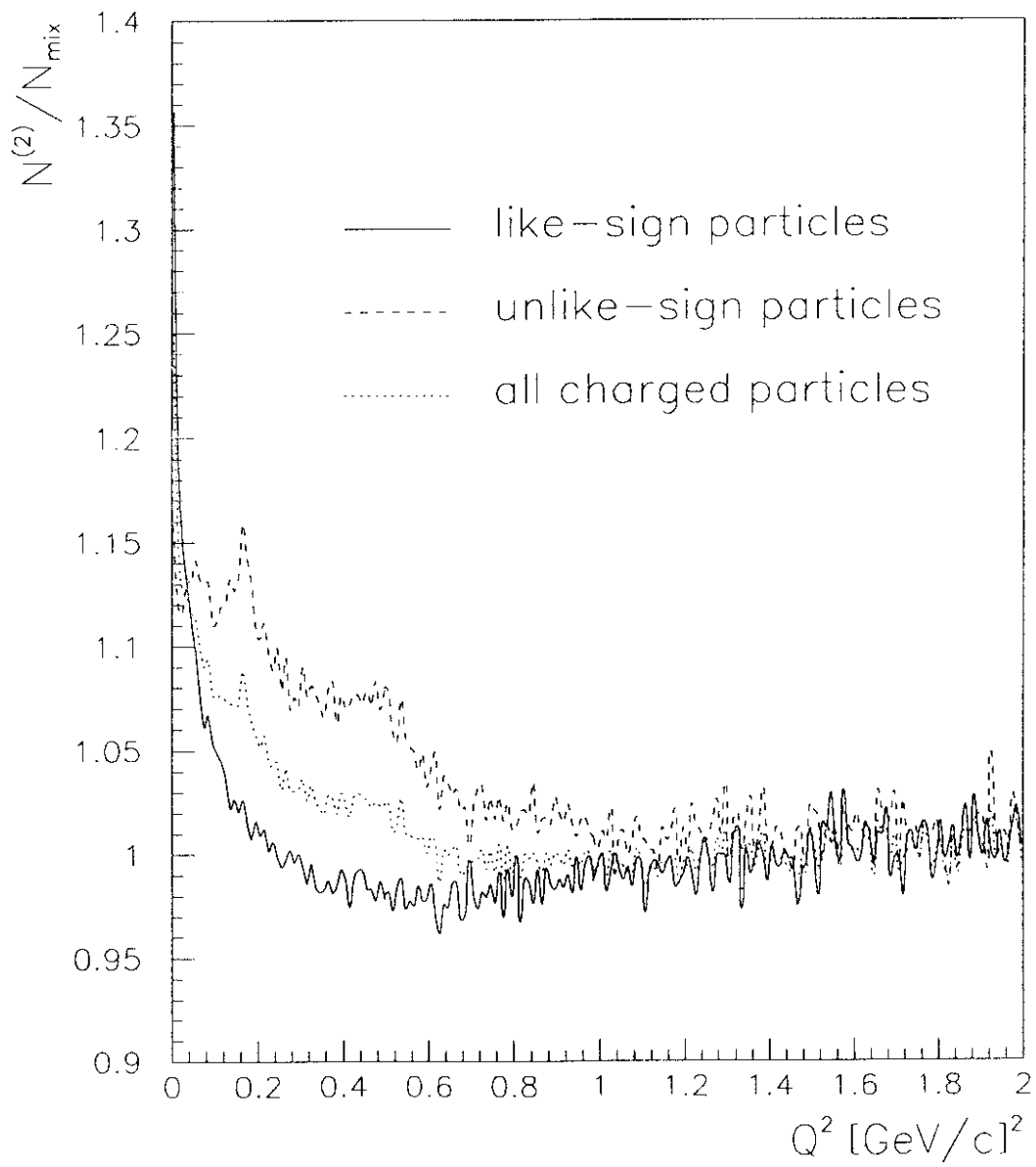


Fig. 4

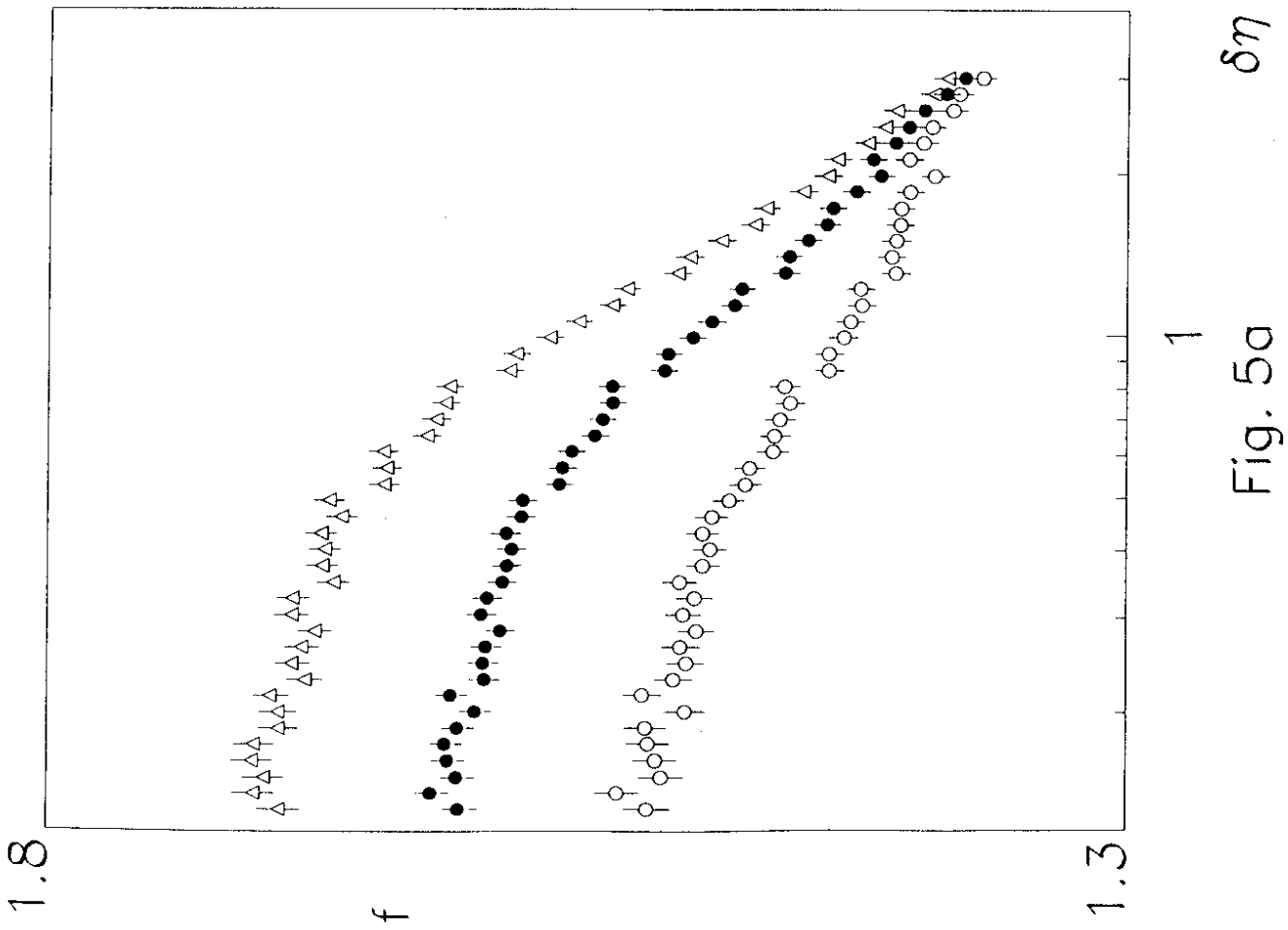


Fig. 5a

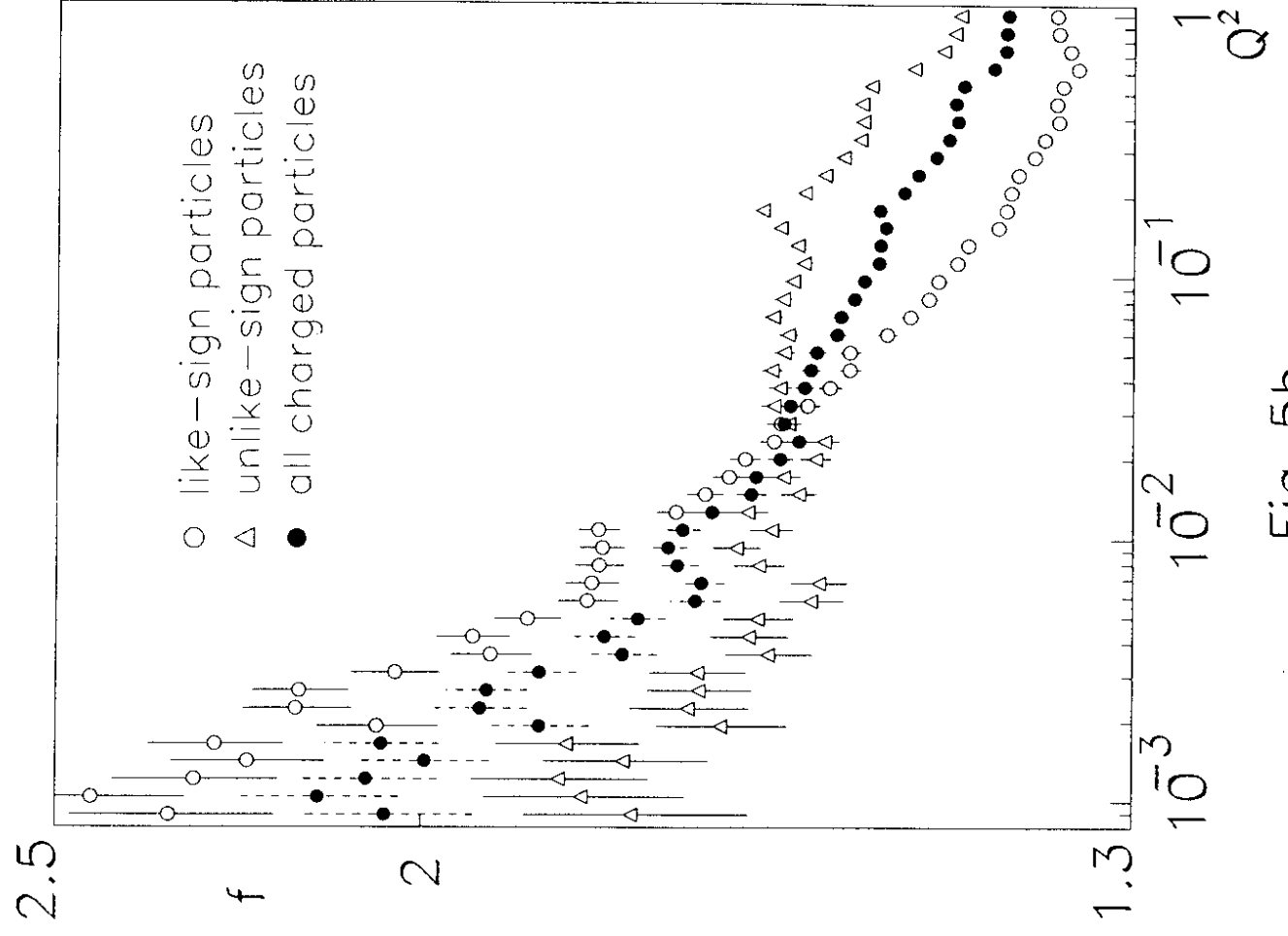


Fig. 5b

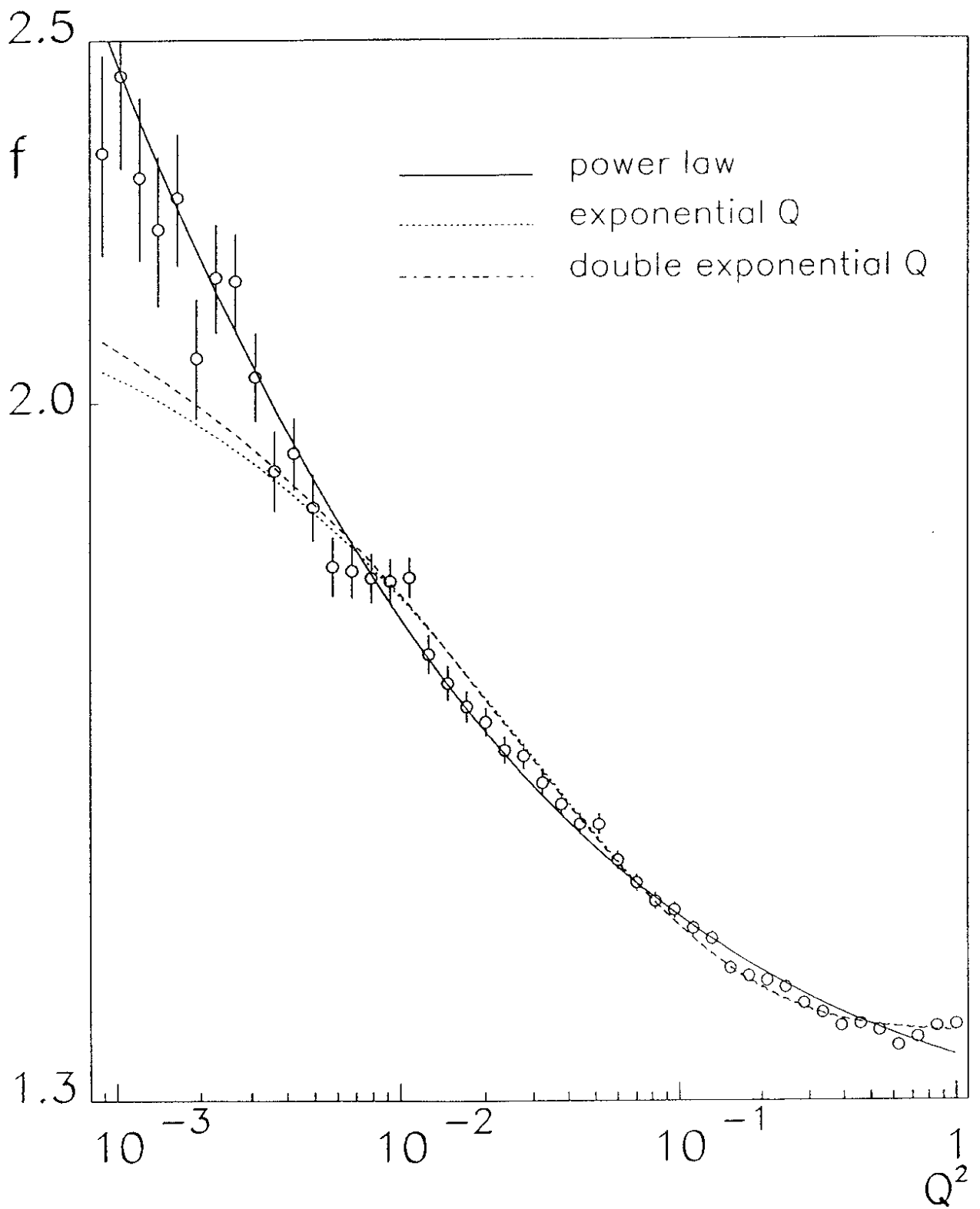


Fig. 6