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PSB STOP-BAND WIDTHS COMPUTED FROM MEASURED MAGNETIC FIELDS

C. Bovet, K.H. Schindl

1. INTRODUCTION

Formulae giving stop-band widths for resonances up to order five have been worked out by G. Guignard[1] in 1970. At that time only educated guesses or recommendations could be made on the amount of non-linear fields in the PSB magnet.

We are now in a situation where field maps have been measured for each bending magnet and harmonic analysis has been performed for each quadrupole[2]. The measurements made with straight long coils on the bending magnet proved to be adequate for further analytic developments[3]. Therefore, we cannot resist the temptation to compute the stop-band widths that correspond to the distribution of all known magnetic fields in the machine.

In section 2 magnetic field measurements will be normalized to a unique system of reference tangential to the beam and expanded in the transverse plane with harmonic polynomials. Then azimuthal harmonics are derived in section 3 and stop-band widths computed. Compensation of most dangerous stop-bands is reviewed in section 4.

2. EVALUATION OF MAGNET IMPERFECTIONS

In order to compute stop-band widths for all resonances up to order 4, we need the azimuthal distribution of the quadrupole, sextupole and octupole component, both normal and skew, around the ring. These coefficients are defined as :

$$B_z^{(k-1)}(\theta) = \frac{\partial^{(k-1)} B_z(\theta)}{\partial x^{(k-1)}} \Bigg|_{\substack{x=0\\ z=0}} ,$$

$$B_x^{(k-1)}(\theta) = \frac{\partial^{(k-1)} B_x(\theta)}{\partial x^{(k-1)}} \Bigg|_{\substack{x=0\\ z=0}} \tag{1}$$

with 2k being the number of poles of multipole, B_x, B_z the transverse magnetic field components. They are obtained by fitting the harmonic function series

$$\Delta B_z(x,z) \cong \sum_{k=2}^{N} \left\{ \frac{1}{(k-1)!} B_z^{(k-1)} \, \mathrm{Re}\left[(x+iz)^{k-1}\right] - \frac{1}{(k-1)!} B_x^{(k-1)} \, \mathrm{Jm}\left[(x+iz)^{k-1}\right] \right\} \tag{2}$$

to the measured field map $\Delta B_z(x,z)$ of each magnetic element.

For each of the 128 bending magnet gaps installed in the Booster ring, the function

$$\Delta B_z(x,z) = \frac{\int_{\ell_m} B_z(x,z)\, d\ell - B_o \ell_m}{\ell_m} \tag{3}$$

(B_o nominal field, ℓ_m magnetic length) has been measured[2] in 21 points of the transverse (x,z)-plane. The coefficients $B_z^{(k-1)}$ and $B_x^{(k-1)}$ are obtained by least square fit [4] (equ. 2) with N chosen in such a way that the approximation error is comparable to the measurement error.

All relevant measurements have been performed at transfer energy 800 MeV (B_o = .59 T), but conclusions will be drawn for injection (more critical from the stop-band point of view) by scaling down the imperfections field. This procedure is justified since it has been established [2] that the field imperfections are not due to remanence or saturation.

For each of the 192 quadrupole gaps, field inperfections have been explored by means of harmonic coil measurements, where the various multipole strengths are evaluated by Fourier analysis [5] of the measured data, yielding α_k, β_k. The coefficients α_k,(β_k) are the ratios between the field due to the normal (or skew) multipole of order k and the main quadrupole field, taken at the bore radius (60 mm). Harmonic coil studies were performed on a measuring bench where quadrupoles were positioned with :

i) Bus bar connection opposite telescope side,

ii) South pole in positive quadrant.

Each multipole component is evaluated in the measurement frame and has to be transformed to the coordinates seen by the beam [6]. Depending on the position of the bus bar connection, we have the four quadrupole types QF1, QDU, QDD, QF2.*) The relationship between $B_{z,x}^{(k-1)}$ and α_k, β_k for these types and for multipole components up to order 4 is given in Table 1. g_o is the mean gradient, computed for each ring with the formula

$$g_o = \frac{\sum_{i=1}^{48} g_i \ \ell_i}{\sum \ell_i} \qquad (4)$$

with ℓ_i the length of quadrupole in position i along the synchrotron ring, g_i the measured gradient.

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*) QF1 and QDD have bus bar connection downstream (with respect to the beam), QF2 and QDU upstream.

Order k	Multipole Component $(T/m^{(k-1)})$	Multipole	$\lvert B_{z,x}^{(k-1)} \rvert$ expressed in α_k, β_k	Sign $(B_{z,x}^{(k-1)})$ for			
				QF1	QDU	QDD	QF2
2	$B_z^{(1)}$	Norm. Quadr.	$g_i - g_o$	−	+	+	−
2	$B_x^{(1)}$	Skew Quadr.	cannot be measured with harm. coil				
3	$B_z^{(2)}$	Norm. Sext.	$2 \dfrac{3{,}83\ \alpha_3}{0{,}06}$	−	−	+	+
3	$B_x^{(2)}$	Skew Sext.	$2 \dfrac{3{,}83\ \beta_3}{0{,}06}$	−	+	+	−
4	$B_z^{(3)}$	Norm. Oct.	$6 \dfrac{3{,}83\ \alpha_4}{(0{,}06)^2}$	−	+	+	−
4	$B_x^{(3)}$	Skew Oct.	$6 \dfrac{3{,}83\ \beta_4}{(0{,}06)^2}$	−	−	+	+

Table I : Relation between α_k, β_k as measured with harmonic coil, and multipole components as defined in equ. (1)

3. COMPUTATION OF AZIMUTHAL HARMONICS AND STOP-BAND WIDTHS

Taking into account

i) the ring position of each bending magnet and quadrupole,

ii) the multipole strength of each element,

we may derive the functions $B_z^{(k-1)}(\theta)$ and $B_x^{(k-1)}(\theta)$ for each multipole of order k. Note that these functions are considered constant along one element, because long coil measurement do not give more information.

A few definitions are needed for computing stop-bands[1)] :

i) Resonances may occur for

$$\lvert n_1 Q_H + n_2\ Q_V - p \rvert \leq \left| \frac{k\ \Delta Q}{2} \right| \qquad (5)$$

with $k = n_1 + n_2$ the order of resonance excited by a multipole of order k with azimuthal harmonic p.

ii) ΔQ is the stop-band width.

iii) Furthermore, the following parameters are used :
$R = 25$ m (mean machine radius), $B_o = .1253$ T (main field at injection), $\rho = 8.24$ m (magnetic bending radius), $\varepsilon_x = 130$ and $\varepsilon_z = 40\pi$ mrad.mm the emittances.

The azimuthal harmonic of a normal (dp) or skew (fp) multipole is given by equ. (100) of Ref. 1 (after correction of a sign error : the exponential should read $e^{+ip\theta}$ instead of $e^{-ip\theta}$). We put this formula into real and obtain, for the cos. and sine components, respectively :

$$\left\{\begin{matrix} dp \begin{pmatrix}\cos\\ \sin\end{pmatrix} \\ \\ fp \begin{pmatrix}\cos\\ \sin\end{pmatrix}\end{matrix}\right\} = \frac{1}{2\pi}\int_o^{2\pi} \beta_x^{\frac{n_1}{2}} \beta_z^{\frac{n_2}{2}} \left\{\begin{matrix} B_z^{(k-1)}(\theta) \\ \\ B_x^{(k-1)}(\theta)\end{matrix}\right\} \begin{matrix}\cos\\ \\ \sin\end{matrix}\left[n_1\psi_x(\theta) + n_2\psi_z(\theta)\right] d\theta \text{ for } \left\{\begin{matrix} n_2 \text{ even} \\ \\ n_2 \text{ odd}\end{matrix}\right\} \tag{6}$$

with

$$\psi_{\substack{x\\z}}(\theta) = \int_o^\theta \frac{R\, d\theta^*}{\beta_{\substack{x\\z}}(\theta^*)} \tag{7}$$

The betatron functions β_x, β_z are strongly varying along one element, therefore Simpson's integration was employed for numerical computation of (6) along one element. For example, the resulting formula is given for dp (cos), with i the element number, U = upstream edge, C = centre, D = downstream edge

$$\begin{aligned} dp\ (\cos) \cong \frac{1}{2\pi R} \sum_{i=1}^{I_{EL.}} B_z^{(k-1)}\Big|_i \frac{\ell_i}{6} \Bigg\{ & \left[\beta_x^{\frac{n_1}{2}} \beta_z^{\frac{n_2}{2}} \cos\,(n_1\psi_x + n_2\psi_z)\right]_U + \\ & + 4\left[\beta_x^{\frac{n_1}{2}} \beta_z^{\frac{n_2}{2}} \cos\,(n_1\psi_x + n_2\psi_z)\right]_C + \\ & + \left[\beta_x^{\frac{n_1}{2}} \beta_z^{\frac{n_2}{2}} \cos\,(n_1\psi_x + n_2\psi_z)\right]_D \Bigg\} \end{aligned} \tag{8}$$

Combining equ. (99), (51), (32), (67) of Ref. 1, stop-band widths are computed as to

$$\Delta Q = \frac{n_1}{k.2^{k-2}} \; \frac{R}{B_o \rho} \; \frac{\varepsilon_x^{(\frac{k}{2}-1)}}{n_1!\; n_2!} \left(\frac{\varepsilon_z}{\varepsilon_x}\right)^{\frac{n_2}{2}} \left(n_1 + \frac{n_2^2}{n_1}\frac{\varepsilon_z}{\varepsilon_x}\right) \begin{Bmatrix} |dp| \\ |fp| \end{Bmatrix} \text{ for } \begin{Bmatrix} n_2 \text{ even} \\ n_2 \text{ odd} \end{Bmatrix}, \; n_1 \neq 0 \tag{9a}$$

$$\Delta Q = \frac{n_2}{k.2^{k-2}} \; \frac{R}{B_o \rho} \; \frac{\varepsilon_z^{(\frac{k}{2}-1)}}{n_1!\; n_2!} \left(\frac{\varepsilon_x}{\varepsilon_z}\right)^{\frac{n_1}{2}} \left(n_2 + \frac{n_1^2}{n_2}\frac{\varepsilon_x}{\varepsilon_z}\right) \begin{Bmatrix} |dp| \\ |fp| \end{Bmatrix} \text{ for } \begin{Bmatrix} n_2 \text{ even} \\ n_2 \text{ odd} \end{Bmatrix}, \; n_2 \neq 0 \tag{9b}$$

Note that for both $n_1 \neq 0$ and $n_2 \neq 0$, (9a) and (9b) are equivalent.

Azimuthal harmonic multipole strenghts dp, fp as well as stop-band widths ΔQ have been computed for each resonance up to order 4 and are given in Table II (only the most critical of the four rings is considered). A detailed analysis of all results shows that, from the resonance point of view, gap 4 is the most critical one.

REMARKS ON ACCURACY OF RESULTS

i) The bending magnets contribute more to the azimuthal harmonics than the quadrupoles,

ii) B.M. field imperfections have been measured with an accuracy of about 10^{-4} of the main field,

iii) varying N in the least square fit (equ. (2)) shows virtually no changes for the quadrupole component (order 2), but serious changes for order 4. Putting these facts together, we estimate the precision of dp, fp as to ± 10% for order 2, ± 20% for order 3, ± 40% for order 4.

Type of effect	Measured properties		Compensation										
Resonances	$\|dp\|, \|fp\|$ $T/m^{1-k/2}$	ΔQ 10^{-4}	Correcting lenses per ring	Position	Control parameters	Number of controls	Polarity change	Harmonic strength	$\|dp\|, \|fp\|$ $T/m^{1-k/2}$	Power Supplies	Current A	Voltage V
$2Q_H = 9$ $2Q_V = 9$	$2\ 10^{-4}$ $6\ 10^{-4}$	12 30	2 quad. (cos H) 2 quad. (sin H) 2 quad. (cos H) 2 quad. (sin H)	3L1 - 11L1 8L1 - 16L1 4L3 - 12L3 8L3 - 16L3	ρ_H ϕ_H ρ_V ϕ_V	4 (1 per ring)	yes	.10 T	$33\ 10^{-4}$ $76\ 10^{-4}$	16	80	25
$Q_H + Q_V = 9$			2 skew quad.(cos) 2 skew quad.(sin)	2L3 - 10L3 6L3 - 14L3	ρ ϕ	4	yes	.05 T	$25\ 10^{-4}$	8	40	25
$Q_H - Q_V = 0$			8 skew quadrupoles	1L3...15L3	I_{sq}	4	yes	.20 T		4	40	25
$3Q_H = 14$ $Q_H + 2Q_V = 14$	.012 .012	1.8 2.5	2 sext. (cos) 2 sext. (sin)	3L1 + 11L1 8L1 + 16L1	ρ ϕ	4	yes	1.0 T/m	.076 .057	8	60	25
$2Q_H + Q_V = 14$ $3Q_V = 14$	.022 .037	4.2 3.0	no correcting elements (but space reserved)									
Landau damping			16 sextupoles	1L3...16L3	I_{sext}	1	no	38.4 T/m		1	300	380
$4Q_H = 19$ $2Q_H + 2Q_V = 19$ $4Q_V = 19$	6.3 4.2 3.5	1.8 2.3 0.3	2 oct. (cos) 2 oct. (sin)	3L1 - 11L1 8L1 - 16L1	ρ ϕ	4	yes	34 T/m^2	5.9 4.5 3.4	8	60	25
$3Q_H + Q_V = 19$ $Q_H + 3Q_V = 19$	2.0 2.3	0.9 0.8	no correcting elements (but space reserved)									
$2Q_H - 2Q_V = 0$			16 octupoles	1L3...16L3	I_{oct}	1	no	1360T/m^2		1	300	380

Table II : PSB resonances up to order four : azimuthal harmonics, stop-band widths at injection and correcting elements.

4. STOP-BAND COMPENSATION

Correcting elements have been proposed[7],[8] to compensate for field imperfections and reduce stop-band widths.

Table II gives a summary of installed correcting lenses, their azimuthal position with polarity to create either odd or even harmonics (amplitude ρ, phase ϕ), control parameters that allow to act in coupled mode on several elements[9]. The number of independent controls is also listed, together with the computer control of polarity. The harmonic strengths and harmonic coefficients $|dp|$or $|fp|$ of the correcting lenses are evaluated and can be compared to the corresponding measured values of the magnet system.

5. CONCLUSIONS

From Table II it is clear that enough strength has been provided for any type of compensation foreseen. It was thought in advance that sextupole harmonic coefficients would be greater than skew sextupole ones. Field error measurements show the contrary so that we would be in a better position with skew sextupoles as correcting lenses. But in any event the stop-band widths involved are pretty small and no third order resonance has been observed till now. For octupoles the choice made is optimum.

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