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SOME TEMPERATURE PROBLEMS
IN MAGNETS WITH CYLINDRICAL SYMMETRY

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1. Summary

In this note the solution of two temperature distribution problems for magnets with cylindrical symmetry is given. In the first case the temperature distribution in space and time in a superconducting magnet, whose surface is initially exposed to a (cold) temperature step Θ is computed.

In the second case the final temperature distribution in a pulsed cylindrical multipole is calculated under the assumption that at $t = 0$ a heat pulse leads to a uniform excitation winding temperature increase Θ followed by a cooling down notably through the outer concentric iron shield during the cycle duration τ.

The mathematical approach which may be useful for treating similar problems is also given.

2. Mathematical approach

Let us assume an (infinitely long) cylinder with radii $\rho = R_1 \dots R_n$ representing i.e. concentric windings, iron screens, insulation or metallic tubes of a magnet. The uniform (if only one cylinder is considered) or for our problems the average heat conduction coefficient be a^2 $[m^2\ s^{-1}]$.

If at time $t = 0$ the cylindrical configuration is exposed to $\Theta = T_a - T_o$ at some boundary(ies) ρ, or if the winding is excited by current pulses which can be considered infinitely short compared to the time constants of the materials involved, such that a uniform temperature increase Θ at $t = 0$ in the winding part can again be assumed, the temperature distribution in space and time can in fair approximation be calculated as follows :

The starting partial differential equation is :

$$\frac{\partial^2 \Theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Theta}{\partial \rho} - \frac{1}{a^2} \frac{\partial \Theta}{\partial t} = 0 . \qquad (1)$$

Writing the solution in the form :

$$\Theta = f(t) \cdot g(\rho) \qquad (2)$$

and assuming that for $\rho = 0$ the temperature increase is finite, one obtains for a particular solution :

$$\Theta_p = A_i \cdot J_o (\lambda_i \rho) \cdot e^{-\lambda_i^2 a^2 t} \qquad (3)$$

and for the general solution :

$$\Theta = \sum_{i=1}^{\infty} A_i \cdot J_o (\lambda_i \rho) \cdot e^{-\lambda_i^2 a^2 t} \qquad (4)$$

$J_o (\lambda_i \rho)$ being the Bessel function of the first kind.

The coefficient A_i ($i = 1..\infty$) can be determined from the orthogonality property of Bessel functions. Writing

$$\varepsilon(\rho) = \sum_{i=1}^{\infty} A_i \, J_o(\lambda_i \rho) \tag{5}$$

$$\int_0^1 \rho \cdot \varepsilon(\rho) \cdot J_o(\lambda_j \rho) \, d\rho = \int_0^1 \rho \sum_{i=1}^{\infty} A_i \, J_o(\lambda_i \rho) \cdot J_o(\lambda_j \rho) \, d\rho$$

$$= \frac{1}{2} \, A_i \left[J_1(\lambda_i \rho) \right]^2 \tag{6}$$

with

$$J_1 \, (\lambda_i \rho) = J_o{}'(\lambda_i \rho) \tag{7}$$

From (6) the coefficient A_i is found to :

$$A_i = \frac{2}{[\, J_1(\lambda_i \rho)]^2} \underbrace{\int_0^1 \rho \cdot \varepsilon(\rho) \cdot J_o(\lambda_i \rho) \, d\rho}_{M(\rho)} \tag{8}$$

$\varepsilon(\rho)$ being the assumed step (or any other) function applied in spatial co-ordinates between 0 and 1.

To evaluate the integral $M(\rho)$ the following relation can be applied :

$$\int_0^z t^{\mu} \, J_{\nu}(t) \, dt = \frac{z^{\mu} \Gamma \left(\frac{\nu + \mu + 1}{2} \right)}{\Gamma \, \frac{\nu - \mu + 1}{2}} \cdot \sum_{k=0}^{\infty} \frac{(\nu + 2k + 1) \; \Gamma \left(\frac{\nu - \mu + 1}{2} + k \right)}{\Gamma \left(\frac{\nu + \mu + 3}{2} + k \right)} \, J_{\nu + 2k + 1} \, (z) \tag{9}$$

With (14) the temperature rise after the first pulse is equal to :

$$\Theta_1 = T_1 - T_o = \sum_{i=1}^{\infty} \left\{ \frac{2(T_a - T_o) \cdot \frac{R_2}{R_3}}{\alpha_i \, J_1\left(\alpha_i \frac{R_2}{R_3}\right)} - \frac{2(T_a - T_o)\, \frac{R_1}{R_3}}{\alpha_i \, J_1\left(\alpha_i \frac{R_1}{R_3}\right)} \right\} \cdot J_o\left(\frac{\alpha_i}{R_3} \cdot \rho\right) \cdot e^{-\frac{\alpha_i^2}{R_3^2} a^2 t} \qquad (15)$$

where the α_i's are found from $J_o\left(\alpha_i \cdot \frac{\rho = R_3}{R_3}\right) = 0$; $\alpha_i = 2.4, 5.5,$ 8.6 etc.

The final temperature repartition within the multipole after n-pulses, $\Theta_n(\rho)$ is given by :

$$\Theta_n = T_n - T_o = \sum_{i=1}^{\infty} \left\{ \frac{2(T_a - T_o)\, \frac{R_2}{R_3}}{\alpha_i \, J_1\left(\alpha_i \frac{R_2}{R_3}\right)} - \frac{2(T_a - T_o)\, \frac{R_1}{R_3}}{\alpha_i \, J_1\left(\alpha_i \frac{R_1}{R_3}\right)} \right\} \cdot J_o\left(\frac{\alpha_i}{R_3} \cdot \rho\right) \cdot \frac{e^{-\frac{\alpha_i^2}{R_3^2} \cdot a^2 \tau}}{1 - e^{-\frac{\alpha_i^2}{R_3^2} \cdot a^2 \tau}} \qquad (16)$$

As a numerical example let us calculate the temperature distribution in the cylindrical correction sextupole for the PS.

The parameters are : $R_1 = 0.08$ m, $R_2 = 0.118$ m, $R_3 = 0.14$ m, $L = 0.1$ m, 1.8 kW x 0.1s corresponding to $T_a - T_o = 0.025\ ^oC$ for $j = 1 \frac{A}{mm^2}$.

With $a^2 = 16.7 \cdot 10^{-6} [m^2 \; s^{-1}]$ the average heat conduction coefficient for iron and insulated copper winding - for iron alone a similar value of $16 \cdot 10^{-6}$ is found - $\Theta_n(\rho)$ has been computed and shown in fig. 1.

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Literature :

M. Abramowitz and Irene A. Segun — Handbook of Mathematical Functions, Dover Publications Inc., New York

Distribution : (open)

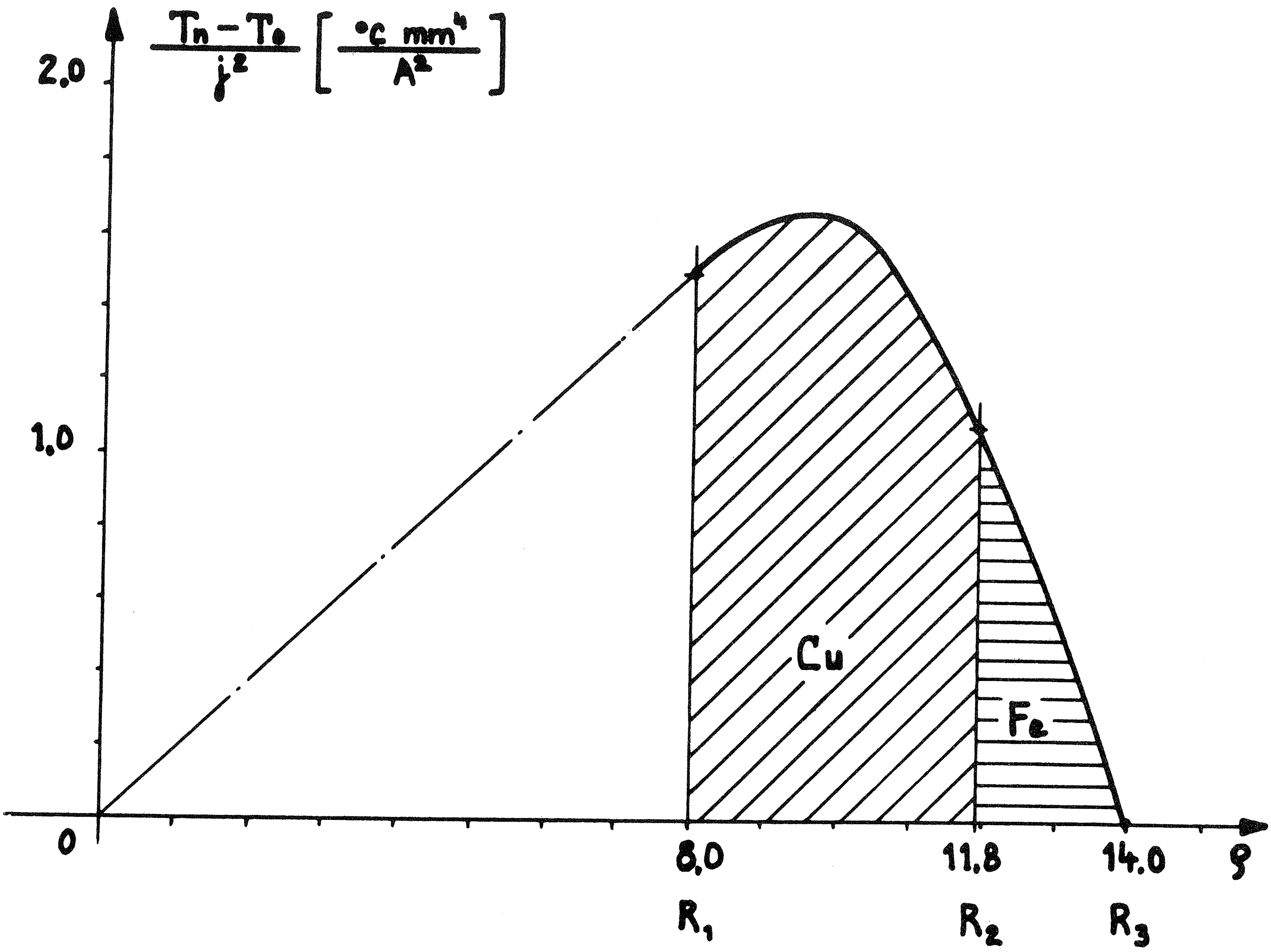
$\frac{T_n - T_0}{j^2}$ $\left[\frac{°C\ mm^4}{A^2}\right]$
2,0
1,0
0
Cu
Fe
8,0
11,8
14.0
ρ
R_1
R_2
R_3

Fig.1