SI/Note MAE/69-14
1.9.1969

ZERO-POINT SHIFTING BY CORRECTION WINDINGS

(BOOSTER QUADRUPOLE)

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INTRODUCTION :

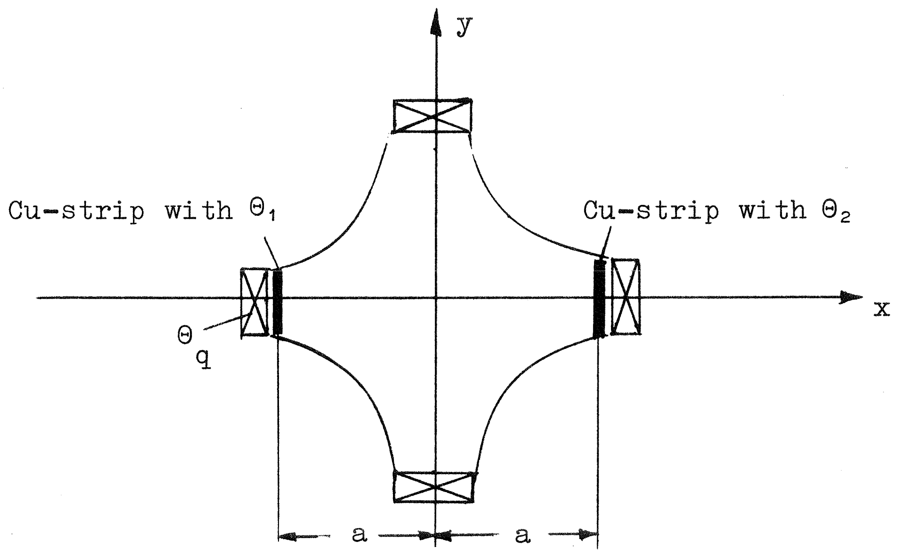
If current-carrying copper strips are inserted into the gaps of a quadrupole, its magnetic field will be modified. This modification consists in a shift of the magnetic zero point and a disturbance of the magnetic gradient. These effects have been calculated, computed and measured, and the results are summarized in this note.

This study has been made in view of possible Booster quadrupole magnetic centre and field gradient corrections which may eventually be required due to the mechanical tolerances and the inherent different reluctances of the individual poles.

1. Analytic approximation

1.1 Geometry

We regard two copper strips situated in the quadrupole as shown in the sketch below.



We distinguish two cases :

Case 1 : The ampere-turns Θ_1 and Θ_2 are in both strips of the same magnitude Θ, but of opposite sign.

Case 2 : $\Theta_1 = \Theta$, $\Theta_2 = 0$ (only one strip).

The superposition of quadrupole field and strip field yields in

case 1 : $$B_{s1} = - mx + \Theta 2ab \frac{1}{a^2 - x^2} , \qquad (1)$$

case 2 : $$B_{s2} = - mx + \Theta\, ab \frac{1}{a + x} . \qquad (2)$$

b = constant, coming from the strip field $\left(\approx \mu_0 \, (2 + \frac{\pi}{2})^{-1} \right)$,

m = quadrupole-field gradient.

1.2 Zero point shifting

Condition for co-ordinate of zero-point : $B_s = 0$.
So we get :

$$\underline{\text{Case 1}}\ (\text{two strips}) : x^3 - a^2x + 2ab\,\frac{\Theta}{m} = 0 \qquad (3)$$

It is allowed to develop (3) into a Mac-Laurin progression, because x is not very much greater than 0 . First order approximation :

$$x \cong \frac{2b}{a} \cdot \frac{\Theta}{m} \quad . \qquad (4a)$$

With $$m = \frac{2\mu_0}{r^2} \cdot \Theta_q \quad , \text{ we get} \qquad (5)$$

$$\boxed{x \cong \frac{0.28r^2}{a} \cdot \frac{\Theta}{\Theta_q}} \ ^{*)} \qquad (4)$$

Herein is x the deviation of the magnetic zero-point, caused by Θ of the strips,

r the quadrupole radius,

a the distance of the strip from geometrical zero-point,

Θ_q the quadrupole ampere-turns,

Θ the correction-strip ampere-turns.

Example : Booster quadrupole ($w_q = 2$, $w_s = 1$, $r = 6$ cm, $a = 14$ cm, $I_q = 625$ A).

*) Remark : For final constant factor in (4) see chapter 2, p. 8.

We have $$x \cong 5.8 \cdot 10^{-3} \cdot I \quad (\mathrm{mm},\ \mathrm{A}) \quad ^{*)} \tag{4b}$$

with I as the strip current.

$$\underline{\text{Case 2}}\ (\text{one strip}) :\quad x^2 + ax - b\,\frac{\Theta}{m} = 0 \quad . \tag{6}$$

In the same way as above we get :

$$x \cong \frac{b}{a} \cdot \frac{\Theta}{m} \quad , \tag{7a}$$

$$\text{or} \quad \boxed{x \cong \frac{0.14 r^2}{a} \cdot \frac{\Theta}{\Theta_q} \ .} \quad ^{*)} \tag{7}$$

Both cases are similar but, as expected, case 2 (one strip) has only half of the efficiency of case 1 (two strips) concerning the shifting of the zero-point. Eq. (4) and (7) are shown in diagram 1, calculated for quadrupole geometry with the strips situated as shown in chapter 2.

1.3 The gradient of B

Derivation of eq. (1) resp. (2) yields in

$$\underline{\text{case 1}} :\quad \frac{dB_s}{dx} = m_1' = - m + 4ab\Theta\,\frac{x}{(a^2-x^2)^2} \quad . \tag{8}$$

$$\underline{\text{Case 2}} :\quad \frac{dB_s}{dx} = m_2' = - m - b\Theta\,\frac{1}{(a+x)^2} \quad . \tag{9}$$

*) Remark : For final constant factors in (4b, 7) see chapter 2, p. 8.

We now define the error of the gradient, caused by the strips :

$$f = 1 + \frac{m'}{m} \quad . \tag{10}$$

<u>Case 1</u> (two strips) :

From (8) and (10) : $$f_1 = 4ab \, \frac{\Theta}{m} \, \frac{x}{(a^2-x^2)^2} \quad . \tag{11}$$

We see : 1. At zero-point, there is no error.

2. For negative x the error is negative and for positive x it is positive.

3. The error increases very much for $x \to a$.

In order to get a more handsome expression for f_1, we develop (11) into a progression (Mac-Laurin) :

$$f_1 \cong \frac{4b}{a^3} \cdot \frac{\Theta}{m} \cdot x \quad . \tag{12a}$$

With replacing b by the already corrected numerical value (see chapter 2) and with (5) :

$$\boxed{f_1 \cong \frac{0.78r^2}{a^3} \cdot \frac{\Theta}{\Theta_q} \cdot x} \quad . \tag{12}$$

Example : Booster quadrupole (as before : $\Theta_q = 1300$ A)

$$f_1 \cong 7.1 \cdot 10^{-3} \cdot I \cdot x \quad (‰, \text{A}, \text{mm}) \qquad (12b)$$

with I as the strip current.

Case 2 (one strip) :

We get with (10) and (9) :

$$f_2 = -\frac{\Theta}{m} \; \frac{b}{(a+x)^2} \quad . \qquad (13)$$

We see : 1. There is also an error for $x = 0$.

2. The error increases for decreasing x.

The first order approximation yields

$$f_2 \cong \frac{2b}{a^2} \cdot \frac{\Theta}{m} \cdot \left(\frac{x}{a} - \frac{1}{2}\right) , \qquad (14a)$$

or

$$f_2 \cong \frac{0.356 r^2}{a^2} \cdot \frac{\Theta}{\Theta_q} \cdot \left(\frac{x}{a} - \frac{1}{2}\right) . \qquad (14)$$

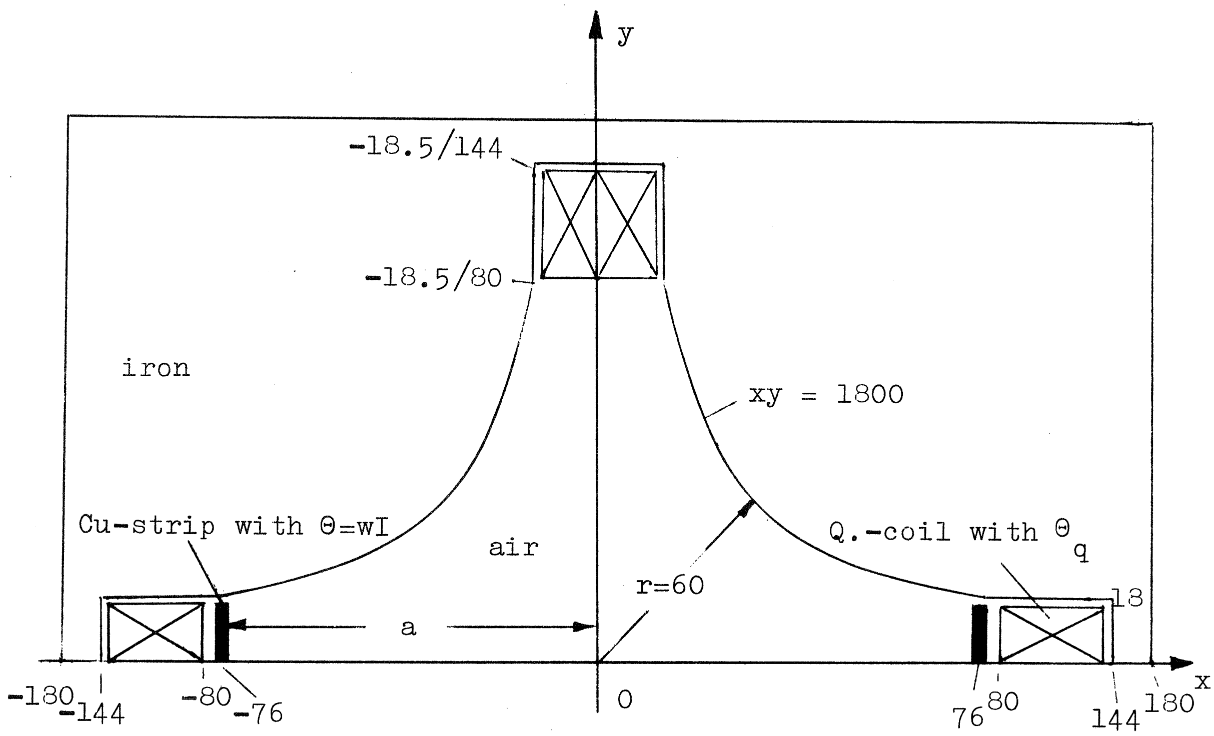
Example : Booster quadrupole (as before : $\Theta_q = 1300$ A, $x = 0$)

$$f_2|0| \cong 2.56 \cdot 10^{-2} \cdot I \quad (‰, \text{A}) \; . \qquad (14b)$$

f_1 and f_2 are plotted in diagram 2 for Booster quadrupole geometry, as shown in chapter 2.

2. Numerical computation

The computation was done with the MAGNET program [1]. The sketch below shows the details.



As a function of I, there was determined :

- the zero-point shifting for case 2,
- the gradient error in x direction for case 2,
- the gradient error in y direction for case 2.

Diagram 1 shows computed zero shifting. A divergence of $\sim$ 28 % from analytic approximation (4) must be stated. That leads to a correction of the analytic approximation as follows :

In (4) take 0.358 instead of 0.28,

in (4b) take 7.4 instead of 5.8,

in (7) take 0.18 instead of 0.14.

(As these constants are approximate values, their correction is admissible).

For the gradient error f_1 according to (10), we need the undisturbed gradient m and the disturbed gradient m' .

In x-direction : $$m = \frac{\partial B_y}{\partial x_1} \qquad (15a)$$

and $$m' = \frac{\partial B'_y}{\partial x} \; . \qquad (15b)$$

Diagram 2 shows the **resulting** f_1 for case 2 and a good agreement of computation and analytic approximation can be stated.

In r-direction (first and third bisecting line) :

$$m = \frac{\partial B_r}{\partial r} \; . \qquad (16a)$$

In an undisturbed quadrupole field is $|B_x| = |B_y|$, so that

$$\vec{B}_r = \vec{B} \quad \text{with} \quad |B| = \sqrt{B_x^{\,2} + B_y^{\,2}} \; .$$

In a disturbed field is $|B'_x| \neq |B'_y|$ and so $\vec{B'_r} \neq \vec{B'}$. For $r \gtrsim \frac{r_0}{2}$ we can set $|B'_r| \approx |B'|$ and get

$$m' \approx \frac{\sqrt{\left(B'_x\,(x+a,\ y+a)\right)^2 + \left(B'_y\,(x+a,\ y+a)\right)^2} - \sqrt{\left(B'_x\,(x,\ y)\right)^2 + \left(B'_y\,(x,\ y)\right)^2}}{\sqrt{2}a} \qquad (16b)$$

For $r \lesssim \frac{r_0}{2}$ (16b) is no more applicable and we have to determine the r-component of B'.

Diagram 3 shows f_1 for both directions and one can see that the strip-caused error is nearly the same for $x < 10$ mm, and for $x > 10$ mm the error is slightly greater in r-direction.

3. Measurements

The measurements were done with a CPS beam transport quadrupole [2].

Main parameters of that quadrupole :

	r	=	10 cm
Max. gradient	m	=	1.1 kG/cm
Length	ℓ	=	1 m
Turns per pole	w	=	79
Exciting current	I	=	700 A.

As connecting elements 4 copper strips had been installed on the right and on the left side of the inscribed circle of the quadrupole, situated at the inner side of the main coils (see sketch in chapter 2). Quadrupole and strips were fed by two separate generators.

3.1 Zero point shifting

The shifting was measured with two different devices :

1. Hall probe
2. Cotton-Mouton effect.

3.1.1 Hall probe

The measuring device is shown in fig. 4 [3]. Results can be seen in diagram 5 for case 1 and case 2. Therein is also plotted the analytic approximation and good agreement can be stated.

3.1.2 Cotton-Mouton effect (CM)

As the CM effect [4] was used before at CERN to measure out the zero point of quadrupoles [5], and as it is treated in another document [6], we restrict ourselves here to some remarks. The device is shown in fig.6, which is self-explanatory. Fig. 7 shows a photograph of the CM pattern.

1. Accuracy

The typical uncertainity of our set-up, which was a preliminary one, lays by ± 0.1 mm

An accuracy better than ± 0.05 mm has already been achieved [5] and other authors [6, 7] speak of an accuracy of the centre determination of ± 0.001 inch or ± 0.025 mm

We have made tests by replacing the normal light source by a laser. We stated an increase of contrast of the pattern. This, together with an improvement of the magnetic active solution and the optical apparatus, seems to make it possible to achieve an accuracy of about ± 0.01 mm.

2. Handling of the set-up

This is not critical and easy [7]. As further advantages we may list that :

- the CM set-up shows direct the actual position of the magnetic zero-point;
- no adjustment of the set-up is necessary;
- the indication of the zero-point depends on no other parameters.

3. Conclusion

This method seems to be a convenient one to determine the magnetic zero-point of multipoles and to align this multipoles.

4. Results

For cases 1 and 2 measured values of zero-point shifting are shown in diagram 5. Comparison with Hall probe measurements shows satisfactory agreement. The agreement with the formula is also good.

3.2 Gradient variations

They have been measured with Hall probes. The results are shown in diagram 8 for case 1 and in diagram 9 for case 2. A satisfactory agreement is observed between theory and measurement. It should be mentioned, however, that the gradient measurements have only been of average accuracy. Therefore, in diagram 9 some measured points have been plotted.

4. Summary

1. Concerning shifting of the magnetic zero-point of a quadrupole, the two-strip correction is twice as efficient as the one-strip correction.

2. The shifting direction is the axis of symmetry (x-axis).

3. Concerning the variation of the gradient, the two-strip correction brings no change of the original gradient at zero-point and a change of the sign of the caused error left and right of the zero-point.

   The one-strip correction brings an error of the gradient along the x-axis, whose sign is for all co-ordinates the same.

4. All results are at least inversely proportional to the distance of the strip from the geometrical zero-point (=a).

5. The CM effect has shown to be a convenient method to observe and measure zero-point shifting.

Acknowledgment

The author is grateful to K.D. Lohmann for the discussions concerning Hall probe measurements and to A. Fluhmann and A. Swift for the excellent execution of the measurements.

The author also thanks Mr. A. Ašner for his helpful interest.

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Distribution : (open)

Y. Baconnier , P. Bossard, C. Bovet, G. Brianti,
M. Giesch, H.G. Hereward, C. Iselin/SI, K.D. Lohmann,
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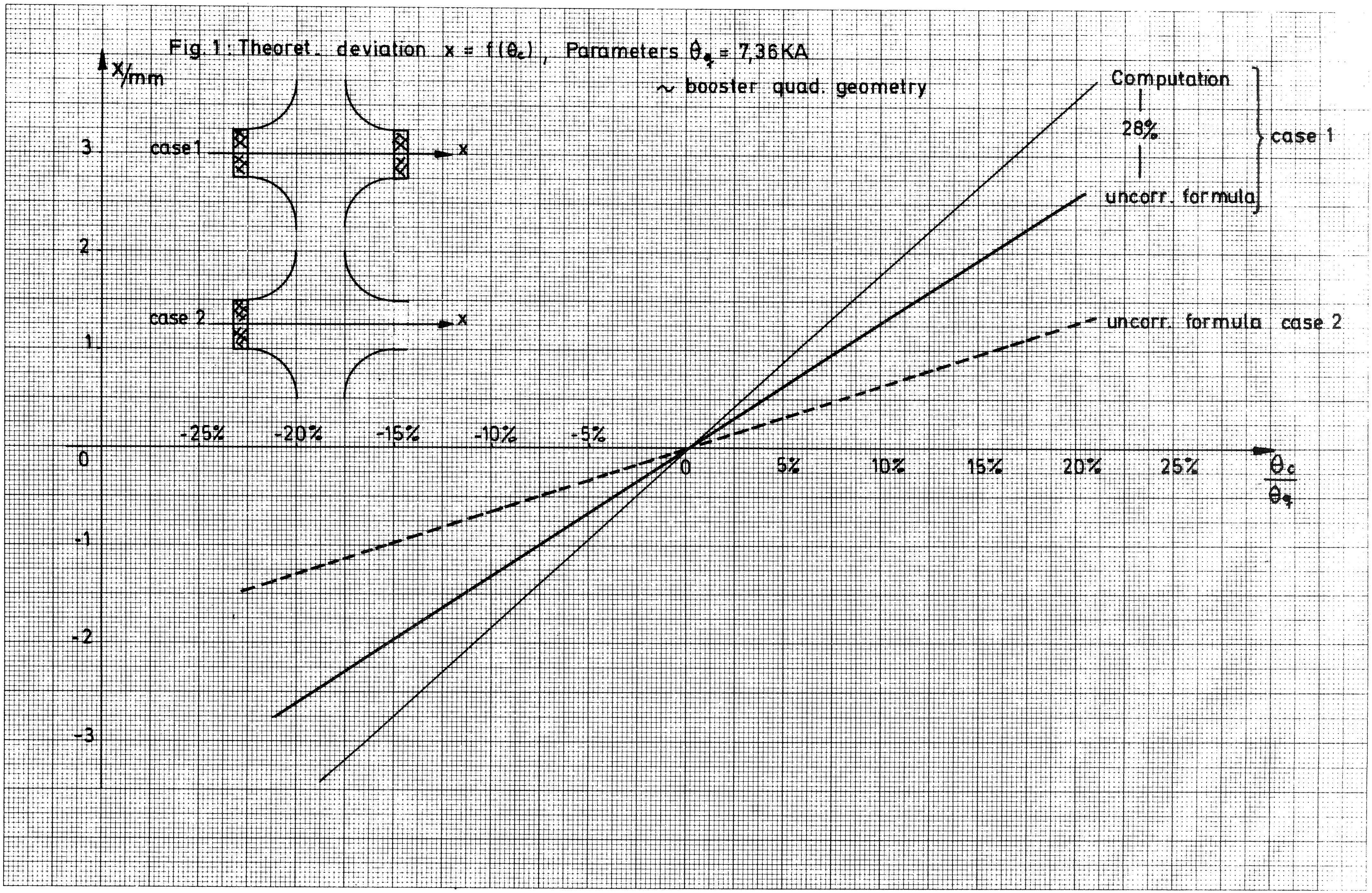


Fig. 1: Theoret. deviation x = f(θc), Parameters θq = 7,36KA
~ booster quad. geometry
x/mm
3
2
1
0
-1
-2
-3
case 1
case 2
x
x
-25%
-20%
-15%
-10%
-5%
0
5%
10%
15%
20%
25%
θc/θq
Computation
28%
uncorr. formula
case 1
uncorr. formula case 2

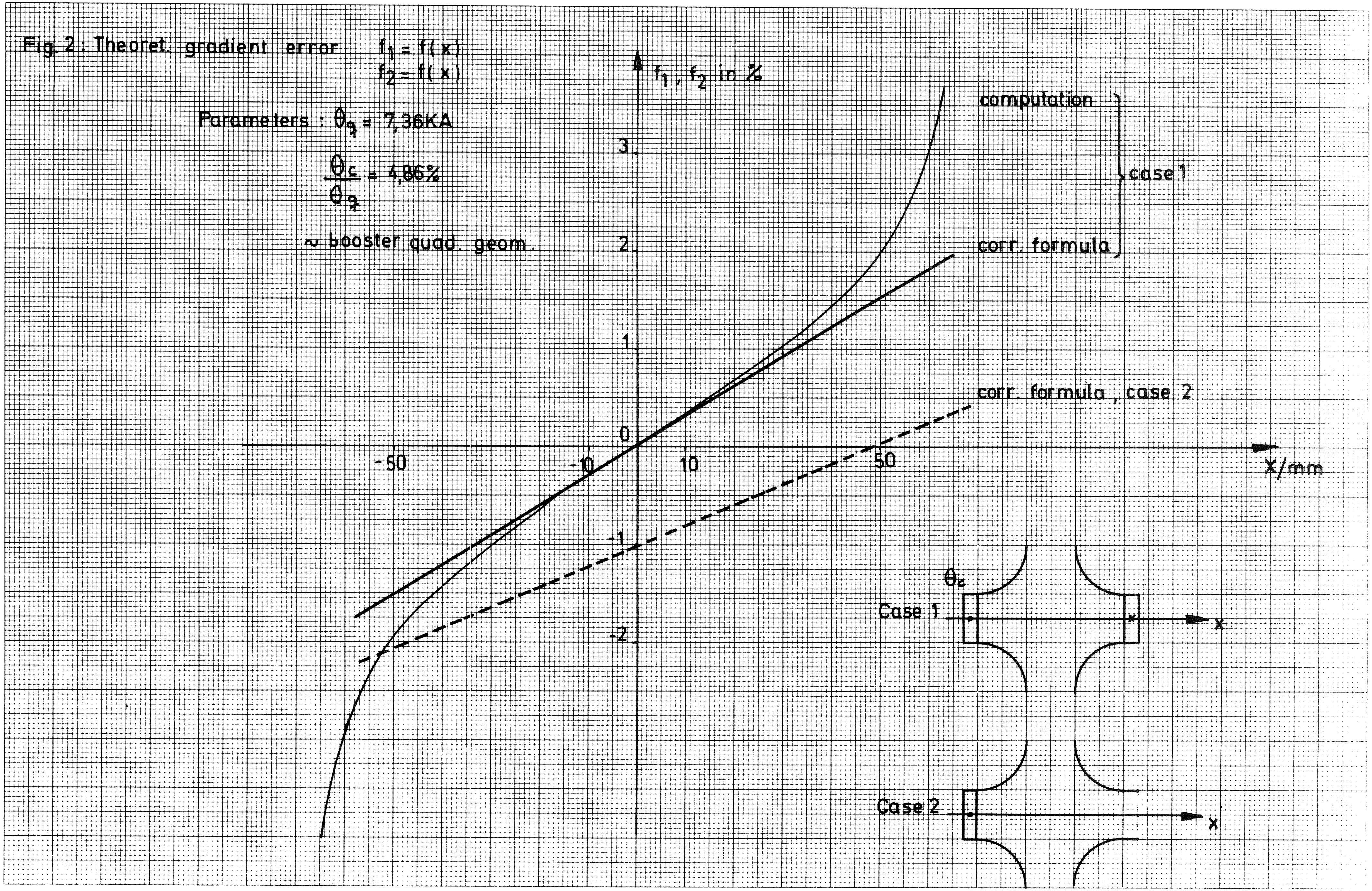
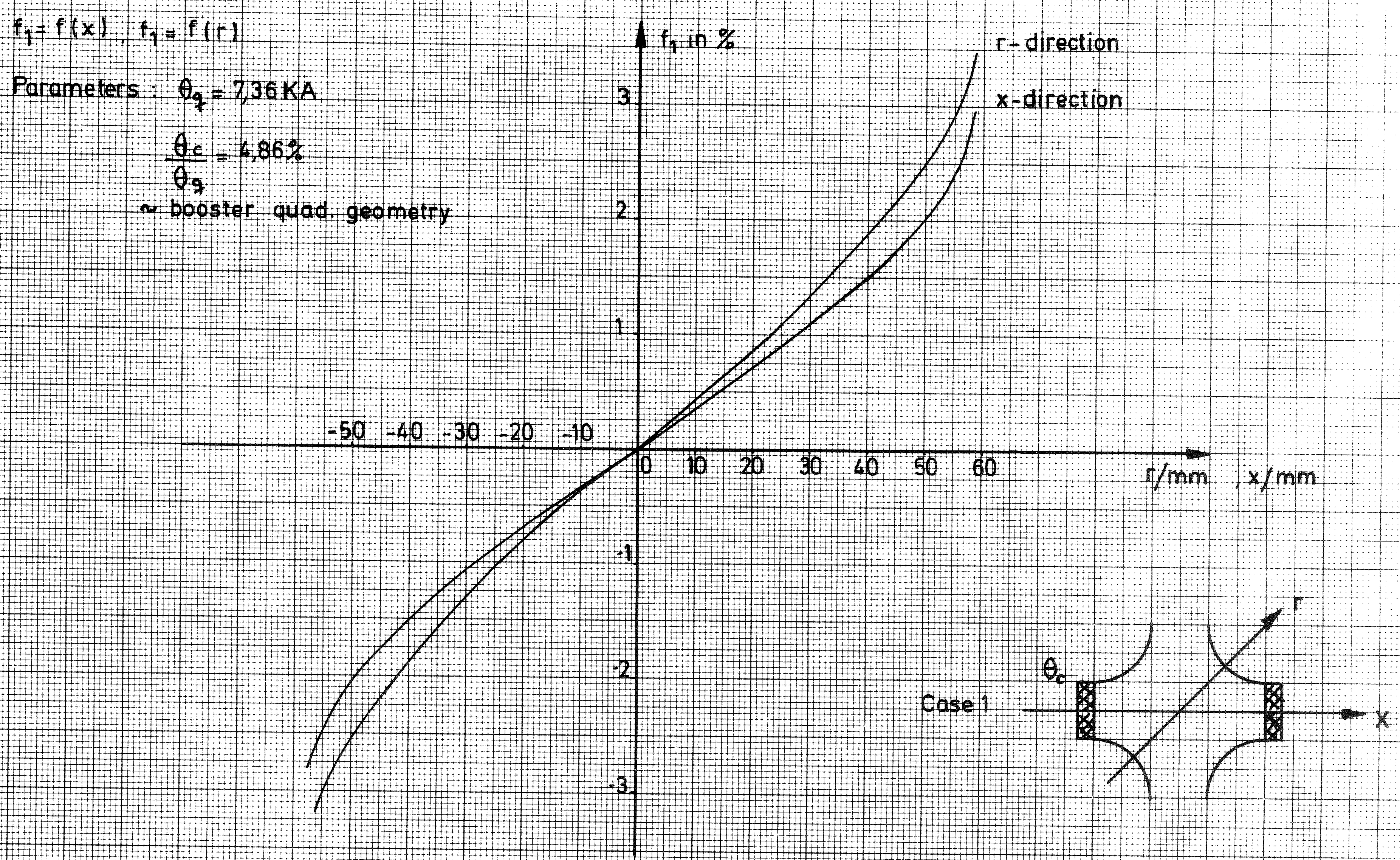
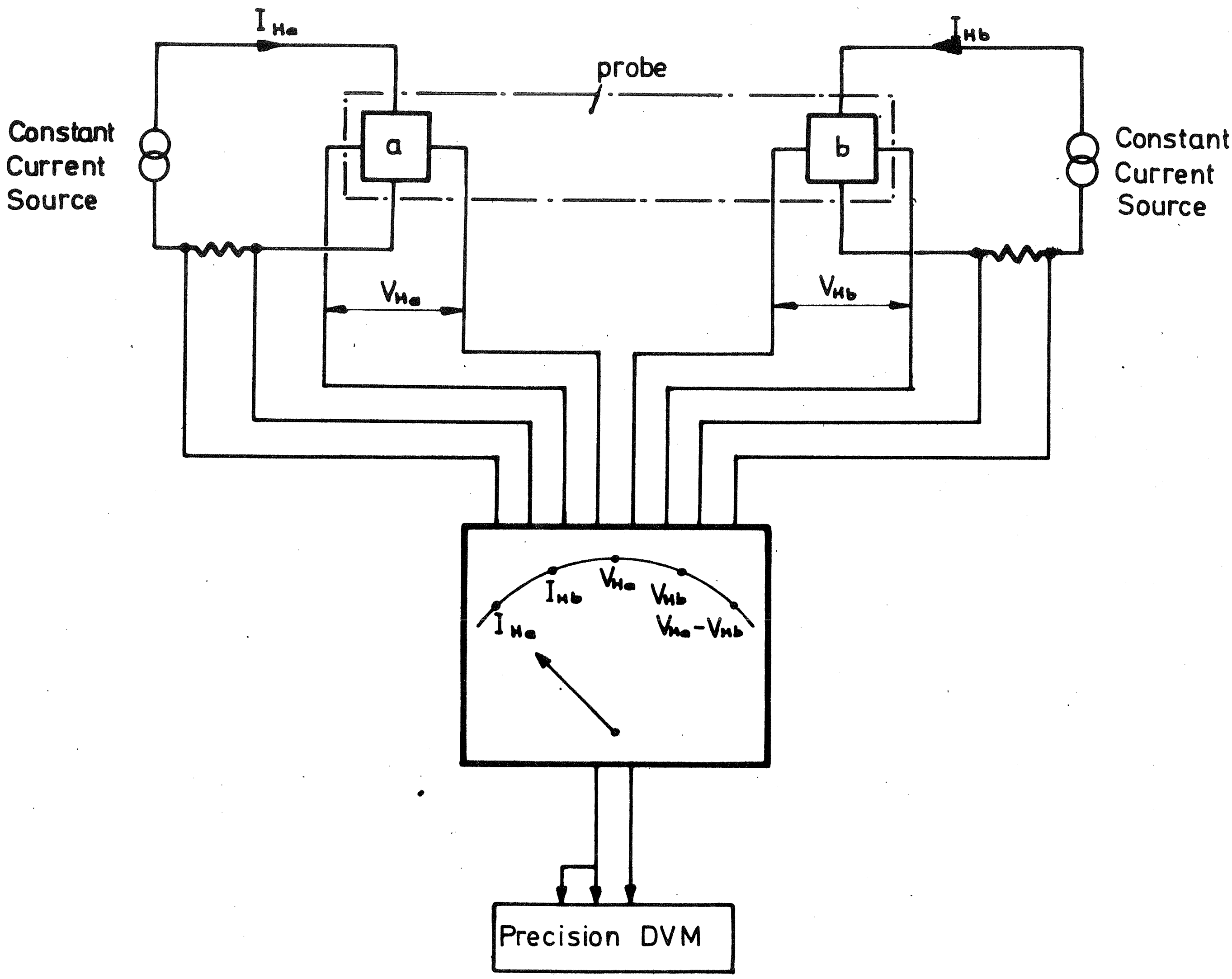


Fig. 2 : Theoret. gradient error $f_1 = f(x)$ $f_2 = f(x)$
Parameters : θ_q = 7,36KA
$\frac{\theta_c}{\theta_q}$ = 4,86%
~ booster quad. geom.
f_1, f_2 in %
computation
case 1
corr. formula
corr. formula, case 2
X/mm
-50
-10
10
50
0
1
2
3
-1
-2
θ_c
Case 1
Case 2
x

Fig. 3 Computed gradient error for 2 different directions





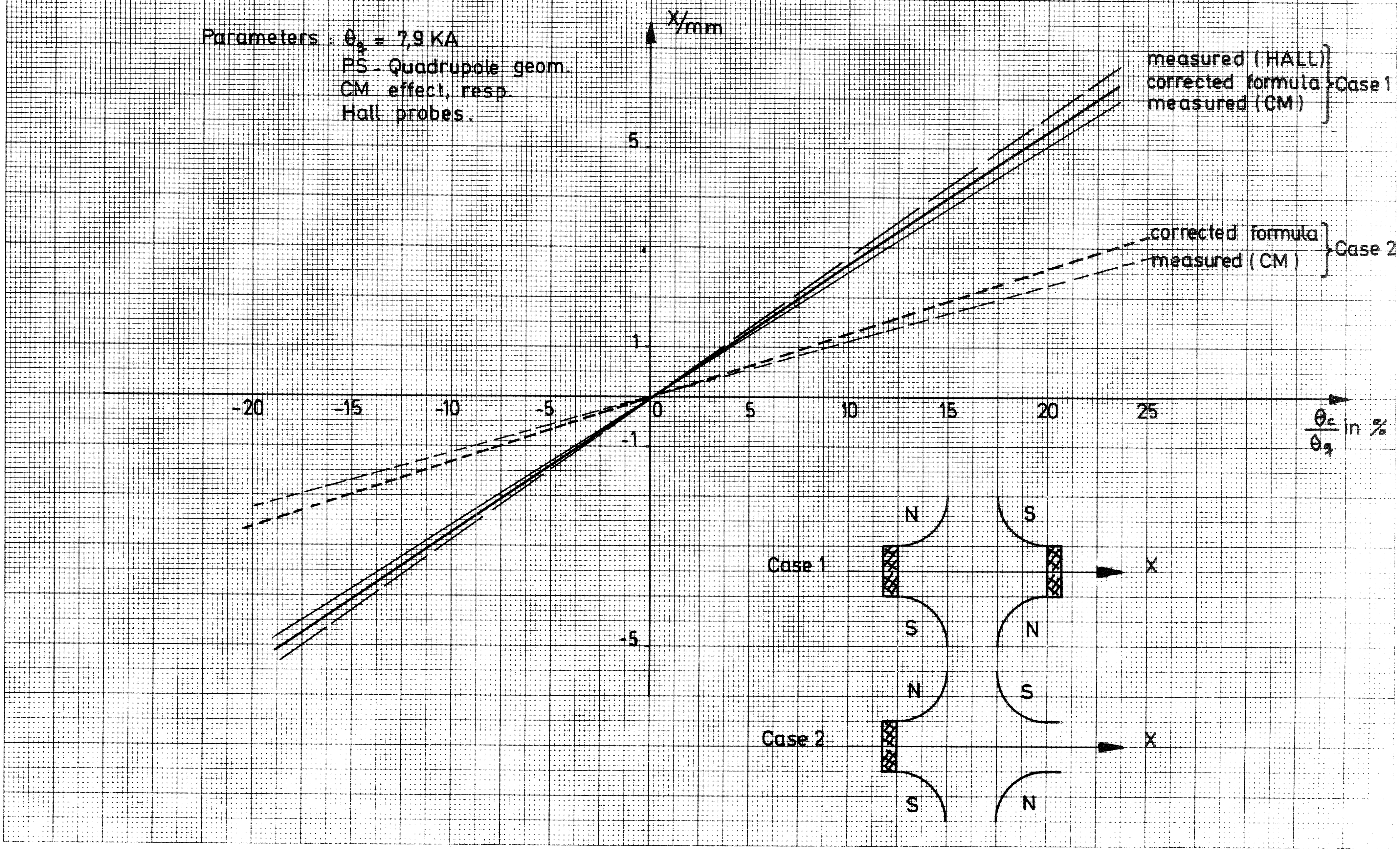
1. a and b are temperature controlled Hall plates, situated a known distance apart.

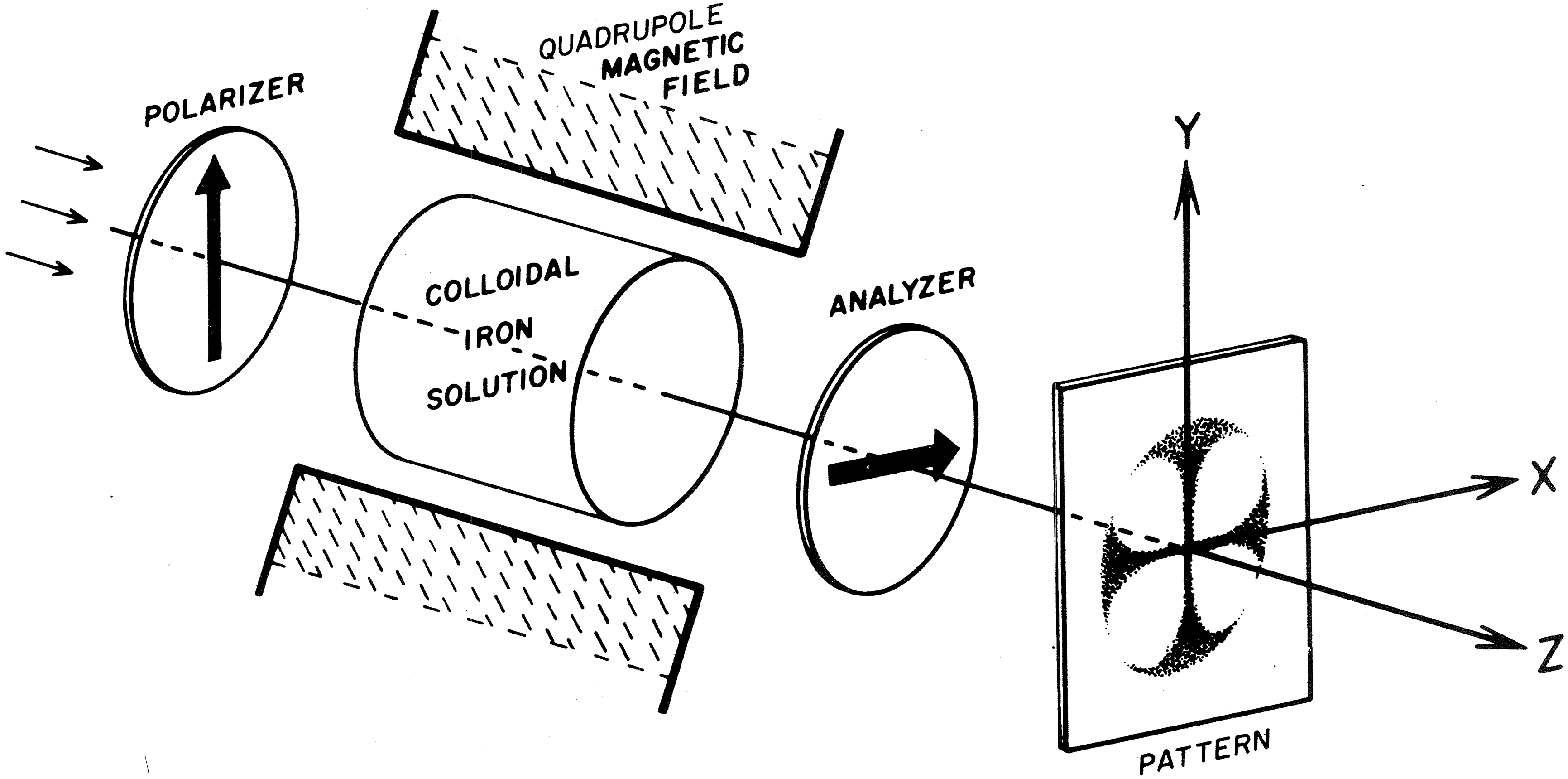
2. Hall currents I_{Ha} and I_{Hb} are measured by precision shunts.

3. Switch selects Hall currents, Hall voltages and difference of Hall voltages to be read on D.V.M.

Fig. 4 : Block diagram of measuring set-up with Hall probe [3]

Fig. 5 Measured deviation $x = f(\theta_c)$





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·EXPERIMENTAL SETUP FOR MAGNETIC CENTER LOCATION IN QUADRUPOLE MAGNETIC FIELD.

Fig. 6

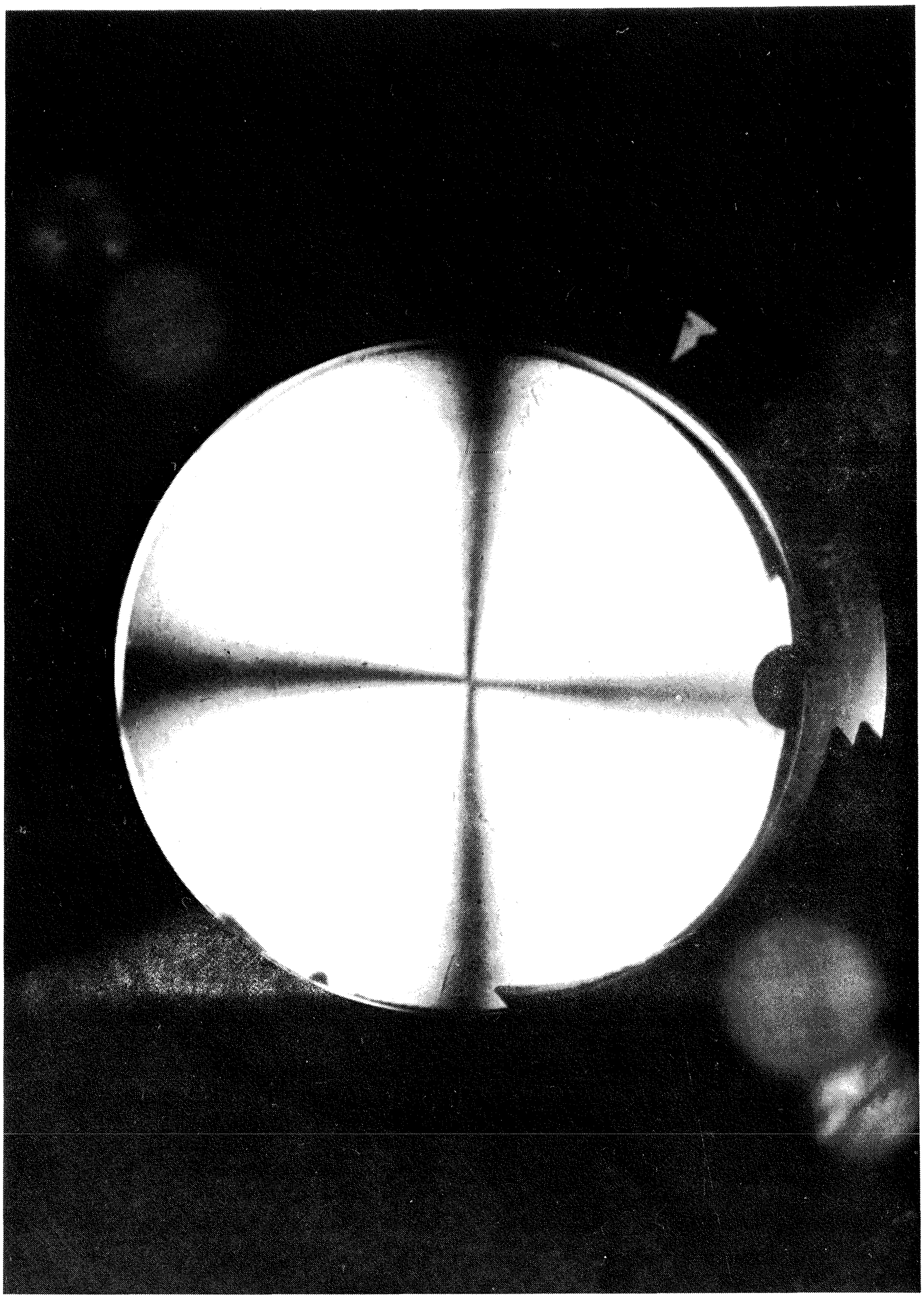

Fig. 7 : Photograph of the pattern in a quadrupole, caused by the Cotton-Mouton effect.

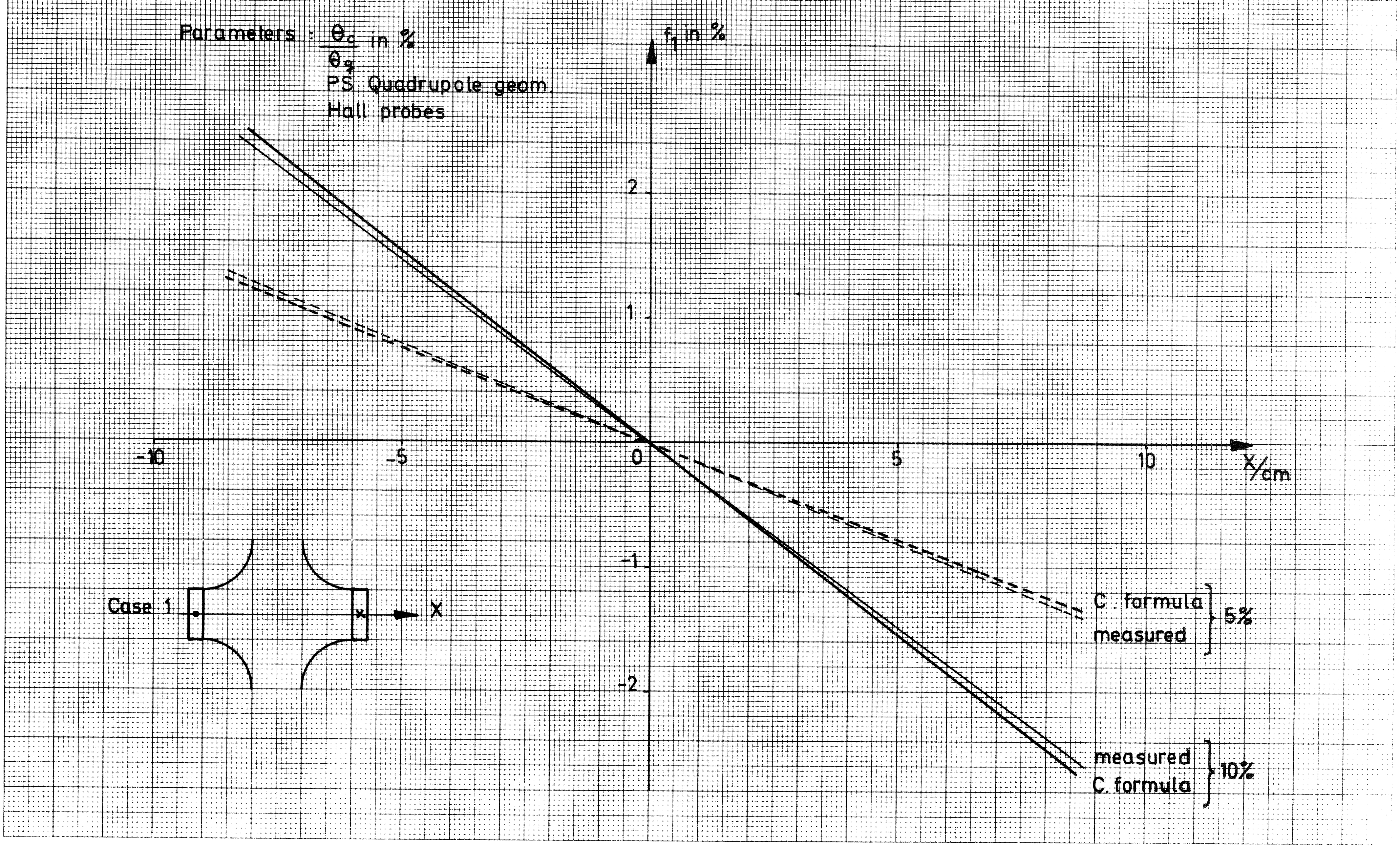

Fig. 8 Gradient error $f_1 = f(x)$

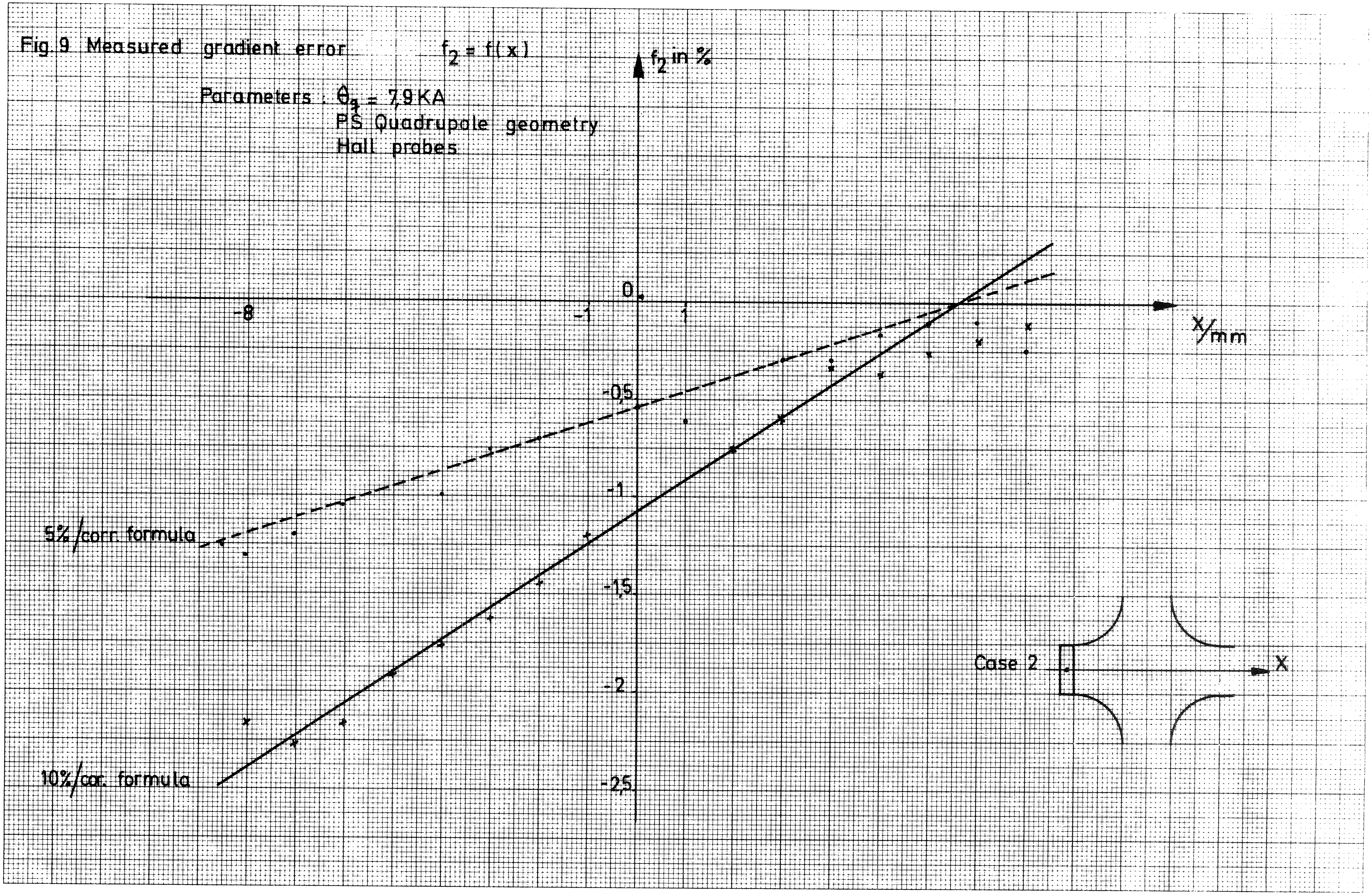

Fig. 9 Measured gradient error f2 = f(x)
f2 in %
Parameters : Θ1 = 7,9 KA
PS Quadrupole geometry
Hall probes
0
-8
-1
1
X/mm
-0,5
-1
-1,5
-2
-2,5
5%/corr. formula
10%/cor. formula
Case 2
X