CERN-ACC-2017-0089
Date: 2017-10-31
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Study of Power Converter Topologies with Energy Recovery and grid power flow control[1]

Part C: a review of alternative structures

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Summary

In the framework of a Transfer line (TT2) Consolidation Programme, a number of studies on Energy cycling have been commissioned. Part of this work involves the study of different power electronic system topologies for magnet energy recovery. The key objective of the study is to find topologies and control strategies that result in the control of the peak power required from the power network as well as to recover the magnet energy into capacitor banks with controlled voltage fluctuation.

In previous reports, the use of a boost front-end converter and two-quadrant (2Q) converter supplying DC link of a four-quadrant magnet supply were analyzed. From this analysis, different features and figures of merit were considered in order to compare the structures, which allowed to define some considerations that a proper topology should have. Consequently, these considerations were used to define some novel topologies. The previous topologies are briefly summarized, as a starting point to develop some novel structures. The different topologies are evaluated considering the figures of merit and aspects related to the implementation.

Keywords: Transfer Lines, Energy recovery, Magnet supply, Control strategy, Grid power limitation, Energy management.

[1] This study is carried out through a Research contract between CERN and Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), via a group of researchers of LIC, in the framework of TT2 Consolidation Programme.

Contents

1 Introduction

The European Organization for Nuclear Research (CERN) operates more than 5000 power converters supplying power to magnets and other equipment in the accelerator complex. Some of them have been in operation for over 40 years hence a consolidation programme has been put in place to ensure high performance reliable operation. These converters, which are usually based on thyristor rectifiers, operate in DC current and are permanently taking energy from the public mains network. Since these converters are reaching their expected lifetime, CERN plans to replace them with modern switching power converters. These new topologies are intended to work in cycling operation leading to an inherent reduction in the power consumption. Then, the required magnet current cycle will define the characteristics required for the converter stage. Figure 1 shows an example of the required magnet current cycle, where its important parameters can be seen: rise time, t_r, fall time, t_f, flat-top time, t_{ft}, cycle repetition period, T_p and flat-top current, I_{REF}, while Table 1 shows the magnet and cycle parameters for the particular case of a power supply used at CERN.[1] Additionally, the topologies to be analyzed must allow the control of the active power flow as well as the recovery of the magnet energy on a cycle-by-cycle basis [1–5].

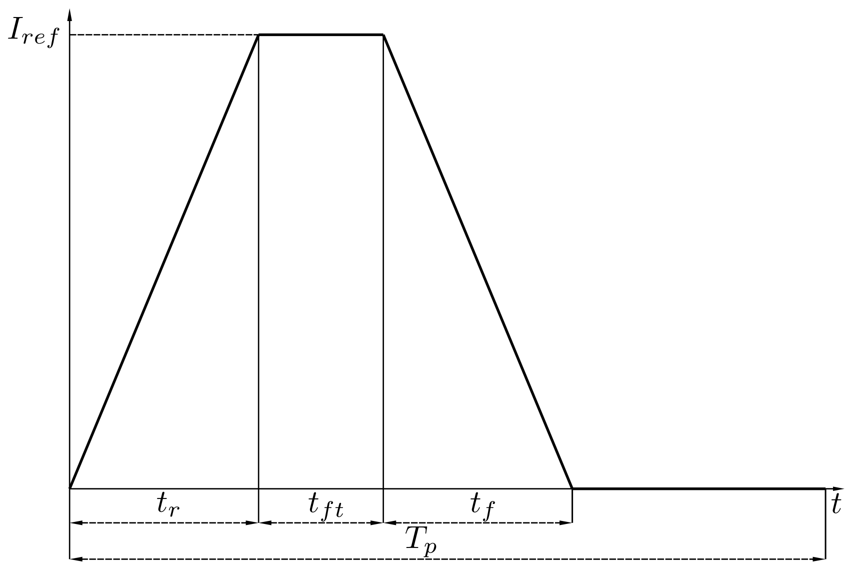

Figure 1: Typical current cycle on the magnet.

Table 1: Load and cycle parameters.

Cycle parameters				Load parameters	
I_{REF}	450 A	t_{ft}	50 ms	L_m	260 mH
t_r	340 ms	T_p	1.2 s	R_m	400 mΩ
t_f	220 ms				

From the analysis of the previous topologies, i.e. the boost based and the two-quadrant (2Q) based, the following objectives are set for the design of new candidate topologies:

- Input current control: this feature allows to keep bounded the input current when transient responses occur (for instance, a change in the grid voltage).

[1] The application demands the generation of a 900 A flat-top current on a $L_m = 130\,\text{mH}$ load, which implies a reactive energy of $E_m = 52650\,\text{J}$, by using two modules in parallel. Hence, each module must handle 450 A, i.e. 26325 J. In order to design the module and to obtain the right values of each element of the converter, an equivalent impedance ($L_m = 2 \cdot 130\,\text{mH} = 260\,\text{mH}$, $R_m = 2 \cdot 200\,\text{m}\Omega = 400\,\text{m}\Omega$) was used.

- Possibility of higher discharge of the energy storage element: a better use of the installed energy is obtained (in theory, a 100% of utilization could be obtained if the storage element is completely discharged). The figure of merit associated to this concept is $\Delta E_{st}/E_{stmax}$.
- Minimizing the energy stored: this feature allows to decrease the size associated to the energy storage element. It is mainly obtained by minimizing the maximum voltage in the energy storage element. The figure of merit associated to this concept is E_{stmax}/E_m.

The boost-based topology defines a variable-voltage dc-link voltage of the H-bridge, which leads to a change in the H-bridge gain. Due to this characteristic, which is not critical, a common practice is to include the voltage variation by feed-forward the dc-link voltage into the control loops of the H-bridge. Furthermore, the useful range of the H-bridge duty cycle determines that there will be a minimum dc-link voltage required to apply the necessary voltage on the load. On the other hand, the 2Q based converter operates with constant dc-link voltage, which allows mitigating these restrictions.

Previous studies [6, 7] highlighted the benefits of better utilization of the capacitive energy storage in 2-quadrant converters. The boost regulator on the other hand demonstrated better ability in controlling the grid current flow and regulating the dc-link voltage. This report will present two new converter structures with the aim to combine the advantages of the previously studied topologies.

The organization of this report is as follows: Section 2 presents a review of the topologies analyzed in previous reports, in order to identify some characteristics that result useful for the generation of new topologies. Two new topologies with their corresponding control strategies and their figures of merit are presented in Section 3. Section 4 shows the evaluation of the figures of merit and main features of the proposed topologies from the specifications of an application case, while simulation results are provided in Section 5. Finally, the conclusions of this report are presented in Section 6.

2 Review of previously analyzed topologies

This section presents a review of the topologies presented in previous reports [6, 7]. The aim of this review is to describe the advantages and drawbacks of each topology, which will be used as a basis for the generation of new structures.

2.1 Boost front-end regulator

The topology based on boost converter is shown in Fig. 2a, where it can be noted the different stages of the circuit: diode rectifier, boost converter, H-bridge, and load.

The stage given by the H-bridge is used to generate the output current cycling with the required

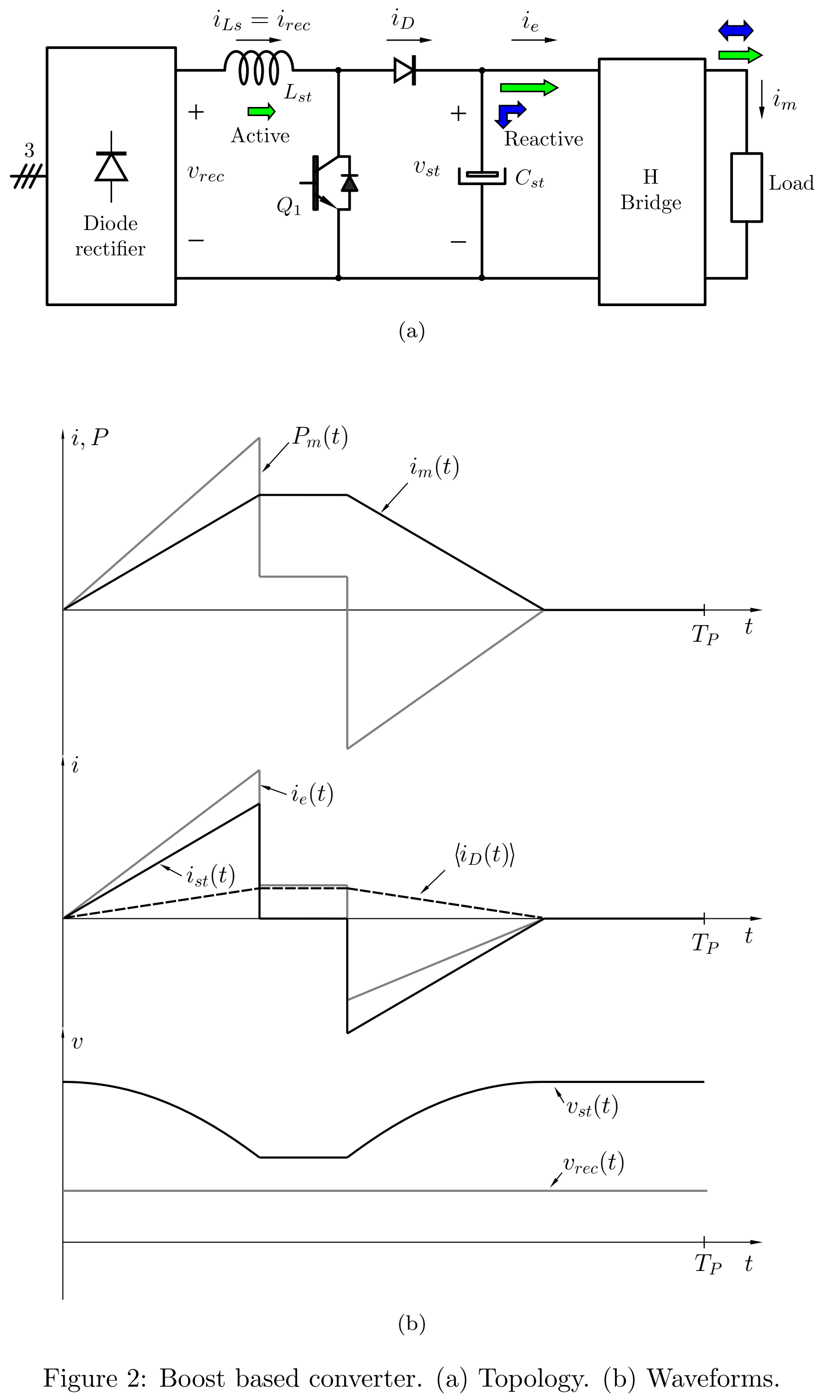

Figure 2: Boost based converter. (a) Topology. (b) Waveforms.

specifications. The stage given by the boost converter has to control the energy flow between the electrical network, the magnet and energy storage capacitor, C_{st}, which is also used as dc-link for the H-bridge [7].

Fig. 2b shows the main waveforms of this topology. The waveforms correspond to the energy flow strategy where the boost converter must provide the instantaneous active power. The magnet current and the corresponding power profile are shown in the image on top. The current demanded by the H-bridge must be supplied by the boost (through the diode D) and the energy storage element (through current i_{st}), as can be seen in the image in the middle. Note that the flat-top power is only supported by the boost, i.e. in this strategy the electrical network must provide the magnet power in the flat-top. Different strategies can be devised, where the division of the power supplied by the electrical network and the storage element changes. Finally, the lower image shows the variation of the energy storage voltage in order to supply the reactive energy. In this structure, the depth of discharge is limited by the grid voltage and the minimum voltage required in the dc-link due to control aspects of the H-bridge.

It must be noticed that in this topology the storage element voltage corresponds to the dc-link of the H-bridge converter, which must produce the current cycle in the magnet. Since the capacitor must deliver/receive energy, its voltage must vary accordingly to the energy flow. One of the effects of this voltage variation in the dc-link of the H-bridge is the change in the open-loop gain of this converter. This effect should not represent a problem, since it can be compensated by a feedforward term proportional to $v_{dc}(t)$ in the regulation loops of the H-bridge control system. Although this effect can be compensated, the minimum voltage in C_{st} must be bounded by two main factors. The first one is that the boost output voltage should be always greater than the electrical network voltage. The second one is that the minimum boost output voltage must always allow to apply a given voltage in the magnet. This dependency of the storage element voltage with the conditions of operation of the H-bridge implies a poor utilisation of the energy installed ($\Delta E_{st}/E_{stmax}$) and a high maximum installed energy (E_{stmax}/E_m).

The control system associated to this structure is formed by an inner loop that regulates the boost inductor current and an outer voltage loop for the voltage in C_{st}. The inner current loop controls the current from the electrical network using a reference generated by the outer loop, while the outer loop regulates the voltage in the storage element in order to control the energy supplied to the magnet. The control of the inductor current gives an inherent robustness against voltage perturbations in the electrical network, i.e. the duty cycle can be adjusted to maintain the control of the inductor current.

This scheme of two-regulation loops allows implementing two different strategies: energy balance and grid peak power limitation. In the case of energy balance strategy, the reference of the outer loop is function of the magnet current considering that the variation of voltage in the storage element is only given by the reactive energy in the magnet. This kind of control limits the power supplied by the electrical network to the instantaneous active power. The possible uncertainties related to this operation mode are associated to differences in the values of L_m and C_{st}. Additionally, although the energy balance strategy limits the energy supplied by the grid to the active energy, the maximum grid current in steady-state will depend on the grid voltage. Then, the worst case for the grid current will correspond to the minimum grid voltage. Although this strategy can be implemented easily, it requires the knowledge of the magnet current.

The lack of information of the magnet current has motivated the generation of grid peak power limitation strategy. In this strategy, the reference of the voltage in the storage element is defined as constant and equal to its initial voltage. The aim of this strategy is to achieve that the electrical network delivers the current demanded by the magnet up to a maximum value defined by the control, which is limited by the use of an antiwindup stage in the outer voltage loop. The limit of the current is defined so as to obtain that the electrical network provides the average active power during the cycle period, T_p. It must be noted that both strategies present the same simplicity in their implementation.

It is worth noting that this topology allows to easily implement a strategy of grid peak power limitation, since the direct control of the rectifier current and its constant voltage characteristic imply a direct control of the power taken from the electrical network. Additionally, in case of using grid peak power limitation strategy, the ratings of the grid connected converter and boost converter can be reduced up to the average active power.

Advantages

- Input current control (direct input power control)
- Robustness against grid voltage perturbations

- Simplicity/robustness in the regulation loops
- Input converter ratings associated to the average active power

To be considered

- Voltage variation in the storage element conditioned to H-bridge operation
- High energy installed
- Variable voltage in the H-bridge dc-link

2.2 Topology based on two-quadrant (2Q) converter

Figure 3 shows a simplified diagram of the topology based on 2Q converter [6]. Like in the boost based structure, in this topology the magnet current cycling is generated with a four-quadrants structure (H-bridge), which has the capacitor C_{dc} as dc-link capacitor.

The H-bridge receives energy from two parallel connected structures: an uncontrolled-rectifier, which takes the energy from the electrical network, and the 2-quadrant (2Q) converter, which takes the energy from a storage capacitor, C_{st}. The operational principle using this scheme is to achieve that the uncontrolled rectifier provides the active power and that the remaining energy is provided by the 2Q converter.

Fig. 4 shows the main waveforms of the 2Q topology. Like in the boost case, the presented waveforms correspond to the energy flow strategy where the input converter must provide the instantaneous active power. In this case, the current demanded by the H-bridge ($i_e(t)$) must be supplied by the 2Q (through the current $i_{rec}(t)$) and the energy storage element (through current in Q_2). Note that, unlike the boost case, the voltage discharge depth is limited only by practical aspects and not by circuit limitations.

An important difference respect to the boost front-end converter is that the H-bridge dc-link is not longer formed by the storage capacitor, C_{st}, but for the capacitor C_{dc}, whose voltage is intended to be regulated constant. Consequently, the voltage in C_{st} can vary from values as low as possible up to a maximum given by the voltage in C_{dc}. This feature allows improving the utilisation factor of the installed energy ($\Delta E_{st}/E_{stmax}$) and reducing the maximum installed energy (E_{stmax}/E_m), with respect to the topology based on en boost front-end converter. The minimum capacitance value depends on the control

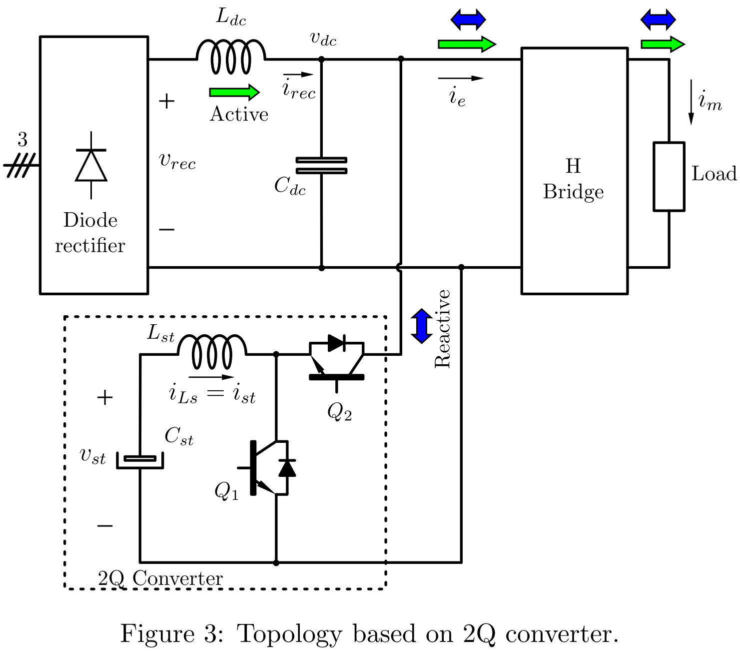

Figure 3: Topology based on 2Q converter.

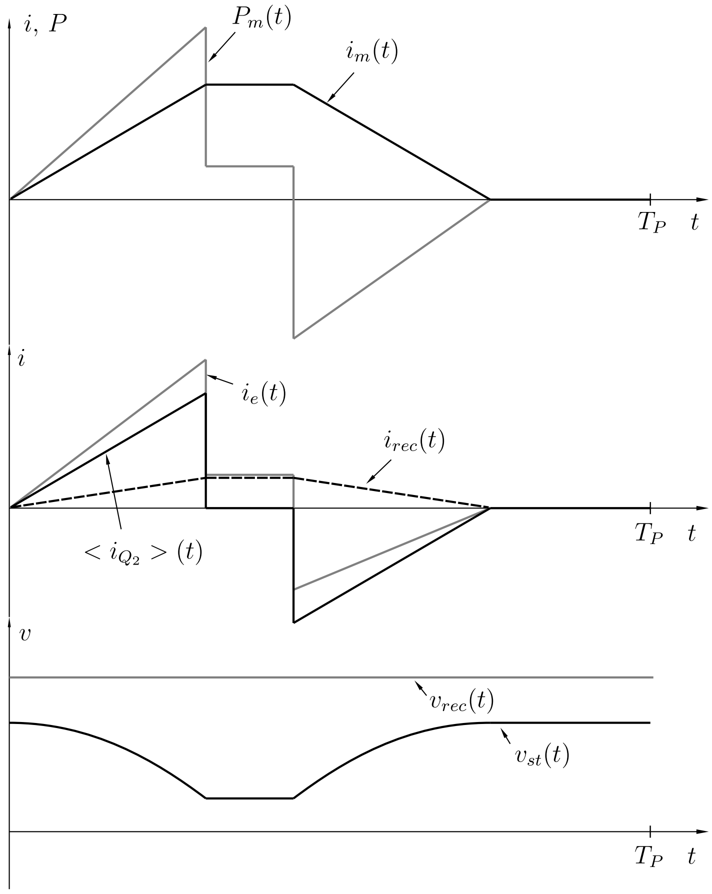

Figure 4: Topology based on 2Q converter. Main waveforms.

strategy (how much energy comes from the electrical network when the load demands energy), on the load and on the depth of discharge of the energy capacitor:

$$C_{st\,\min} = \frac{I_{REF}^2 L_m}{v_{stmax}^2 - v_{stmin}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g[t_r+t_{ft}]}}{\frac{1}{2}L_m\, I_{REF}^2}\right]$$

Assumming the same load and control strategy, the capacitor size will depend on the allowed discharge, which in turns depends on the circuit. Consequently, for the same initial voltage, the boost capacitor will be larger than the 2Q one, due to the possibility of higher discharge of the latter. As an example, Fig.5 shows the capacitance as a function of the voltage depth, for a fixed initial voltage of 700 V. The limits in the voltage discharge are different for each structure: the one of the boost, set at 200 V, is defined considering the minimum achievable voltage in the dc-link, while the one of the 2Q is set at 450 V due to a different constrain (for instance, a maximum value for the storage element current). Note that, if the same discharge depth is selected for boost and 2Q (lower than 200 V in the presented figure), the capacitance value is the same for both structures.

In the report presented in [6], several strategies for controlling the energy flow using this topology are analyzed. Among them, two methodologies can be highlighted. The first one is the already commented above, named energy balance, where the storage voltage is function of the magnet current. The second strategy, named Indirect Grid Current Control by dc-link voltage regulation, belongs to the group of strategies for grid peak power limitation and is a variation of the strategy proposed for the boost front-end converter. The aim of this control is to regulate, by means of the 2Q converter, the dc-link voltage so as to control two aspects:

1. the current supplied by the uncontrolled rectifier (energy coming from the electrical network), and

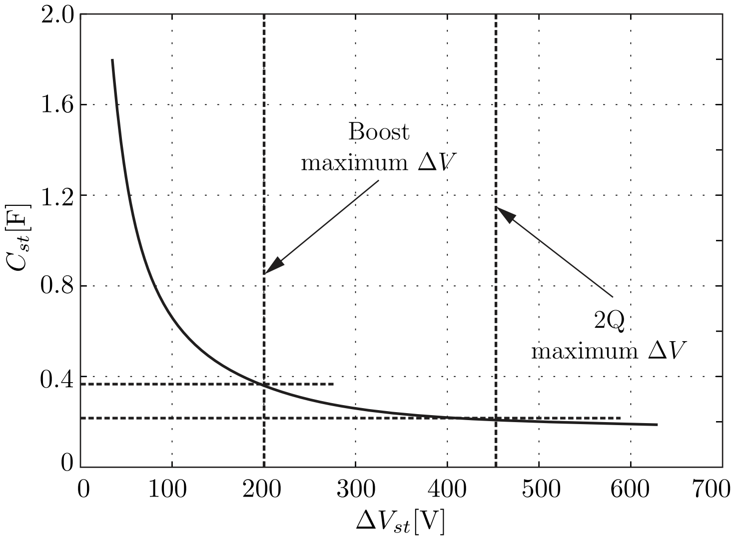

Figure 5: Energy storage capacitor C_{st} as a function of discharge depth ΔV. Fixed $v_{st\,\max}$

2. the current supplied by the 2Q converter to the H-bridge.

Concerning the control of the grid current, the input filter (L_{dc}-C_{dc}) has to be designed to achieve Discontinuous Current Mode (DCM) in the inductor. This operation mode allows changing the amount of grid current averaged in a ripple period of the diode rectifier as a function of the difference between $v_{dc}(t)$ and rectified voltage $v_{rec}(t)$, which leads to a voltage-controlled current source without dynamics behaviour for L_{dc}. In order to achieve this, $v_{dc}(t)$ must be within the ripple of the rectified voltage and the average grid voltage.

Additionally, the control system regulates the voltage $v_{dc}(t)$ so as to not produce variations on $v_{dc}(t)$ due to the current demanded by the H-bridge. Hence, the current supplied by the 2Q converter must follow the difference between the rectifier current $i_{rec}(t)$ and the H-bridge demanded current, $i_e(t)$. Regarding the rectifier current, the goal is to achieve that the energy supplied by the electrical network is equal to the average active power.

The problem with this regulation scheme is that it can not be known how much energy is coming from the grid and how much energy is demanded by the H-bridge, only its difference is known. Then, it can not be known if the electrical network is supplying more energy than the required one (i.e., more thant the active energy). In case of excess of energy from the grid, the voltage in the dc-link rises at the end of the cycle. This increase of $v_{dc}(t)$ blocks the uncontrolled rectifier and avoids the income of energy from the grid at the beginning of the next cycle, during a time interval that ends when the voltage in C_{dc} is low enough to allow the conduction of the diodes in the input stage. In summary, this behaviour finally produces a stable, repetitive condition in which the system adjusts the time interval in which the grid delivers power. Notice that this situation is obtained when the magnet current is repetitive; if there is a change in any of the parameters of the cycle, there will be a transient response [6].

Additionally, it can be observed that there is no direct control of the instantaneous grid current; then, sudden variations in the grid voltage will produce transient peak currents in the electrical network and an increase of the energy supplied to the system. Finally, it can be seen that, in spite of the topological advantages (better figures of merit), the operation strongly depends on the operational conditions (input voltage, cycle parameters, repetitivity of the cycle, etc), leading to a lack of robustness.

Positive

- Wider variation in the storage element voltage.
- Lower maximum installed energy.
- Constant dc-link voltage.

- Peak power of input converter associated to the average active power

To be considered

- Lack of a direct power control of the grid connected converter.
- Two power structures will impose conditions on the same variable.
- Restrictive operational conditions.
- Peak power of storage converter associated to the difference between maximum magnet power and average active power. Then, during magnet energy recovery (negative magnet power), the instantaneous power of the storage converter is higher than the instantaneous magnet power.

3 Proposed topologies

This section presents three new topologies that are expected to fulfill the requirements of control of input power, minimum installed energy and better utilisation of the energy storage elements. Taking into account these objectives, the proposed topologies will have some aspects in common, like:

- Use of a structure for the control of the input power: this structure must control the power supplied from the electrical network in order to allow the bounding of the incoming energy by means of a control strategy. Moreover, the input power should not be affected by disturbances in the grid voltage.
- Separation of the H-bridge dc-link and the storage element: in order to obtain a constant dc-link voltage for the H-bridge, it is noted that its capacitor should not be the one used as energy storage element. Then, there will be a structure that connects both capacitors. Notice that, since the energy storage element is not located in the dc-link, a lower maximum voltage and higher voltage swing could be used, leading to an improvement in both the utilization of installed energy and maximum installed energy. The structure that connect both capacitors should be a converter of (at least) 2 quadrants.

In the following paragraphs, two topologies are presented. In order to compare the different structures, some figures of merit are defined in the Appendix A. These figures of merit are focused on the evaluation of the energy storage size and the current in the input stage. The value of the figures of merit depends on the regulation strategy used; in this case, the analysis will be restricted to Grid Peak Power Limitation.

3.1 Cascaded structure configuration

A topology that fulfils the objectives set in the introduction is presented, Fig. 6.

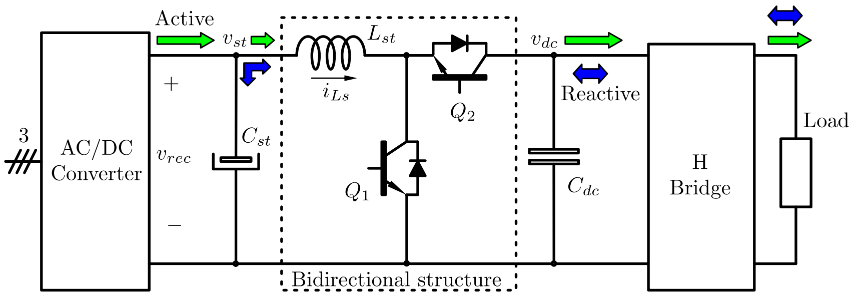

Figure 6: Topology with cascaded structure configuration.

The input stage is composed by a controlled AC/DC converter that is described generally as a capacitor charger, whose main function is to supply a controlled current to a capacitive load (C_{st}) that varies its voltage between 0 V and a given maximum voltage. This stage should supply a controlled current until the storage voltage reaches the value defined from a reference. Notice that this stage could be implemented with an off the shelf equipment. Considering that the rms grid voltage is nearly constant, the limitation of the input power has to be performed by limiting the rms input current. Consequently, the input AC rating of this topology shall be hinted to a predefined value (e.g. 32 Arms). It has to be taken into account that the input current of the capacitor charger will depend on the output current of the charger, input voltage and output voltage (input power equal to output power). The voltages associated to the capacitor charger will be imposed by the system, (the input voltage is the grid voltage and the output voltage is the energy storage element voltage); hence, the bounding of the input current must be obtained from the control of such current or the output current of the capacitor charger. Then, considering the variable to be controlled (input current or output current), systems with particular characteristics will be obtained. For instance, a topology with an Active Front End (AFE) as grid tied converter is proposed in [5]. The AFE converter is connected to the storage capacitor, which in turns constitutes the dc-link of the H-bridge converter. In this case, the AFE can be seen like a capacitor charger, where input current is controlled using a dq transform. The implementation of an antiwindup stage allows limiting the input current and to obtain an input module with constant power. However, this converter presents a step-up voltage conversion ratio, which implies that the operation is comparable to a boost converter with the capability of sending back energy to the electrical network. As a consequence, a more complex structure could be required to obtain an output voltage between 0 V and a given maximum voltage. In this regard, a line-commutated converter could be a simple and valid alternative for controlling such voltage.

The capacitor charger is connected to the energy storage element, which is in turns connected to the dc-link voltage through a bidirectional structure, named 2Q converter. The later is in charge of managing the energy flow between the energy storage element and the H-bridge. The separation between C_{st} and C_{dc} allows a wider voltage variation in the storage element and a constant voltage in the dc-link. To allow a higher discharge and to have a higher degree of freedom for defining the maximum storage voltage determines an improvement in the maximum installed energy and utilisation factor.

Figure 6 also shows the energy flow among the different stages. In the case of the ac/dc converter, it is in charge of recovering the energy in the capacitor by supplying the active energy that is consumed during the cycle period. Then, the power ratings of the input stage will be related to the active energy and the time to recover this energy. Regarding the 2Q converter, its has to handle the full magnet power, since it has to supply the active energy from the input rectifier and the reactive energy that the storage element has to exchange with the magnet. Consequently, the 2Q converter must be sized with the maximum magnet power rating. However, it has to be noticed that the requirements of conduction and switching losses can be lowered down by the implementation of an interleaved scheme in the 2Q converter.

3.1.1 Conceptual regulation system

The block diagram of the control system, which is form by two regulation loops (each one with one actuator and one specific task) is shown in Fig. 7. The first regulation loop, whose actuator is the 2Q converter, regulates the dc-link to a constant voltage. In this regulation loop, the voltage $v_{dc}(t)$ is compared with v_{dcref} so as to generate a voltage error that feeds the controller block, G_{cdc}. The output of this controller is the reference of a transfer function named TLC_v, which regulates the storage element voltage $v_{st}(t)$. This transfer function comprise an inner state-feedback loop and outer voltage loop and is in depth analyzed in [6]. The current that the 2Q converter takes from this node, $i_{Ls}(t)$, is formed

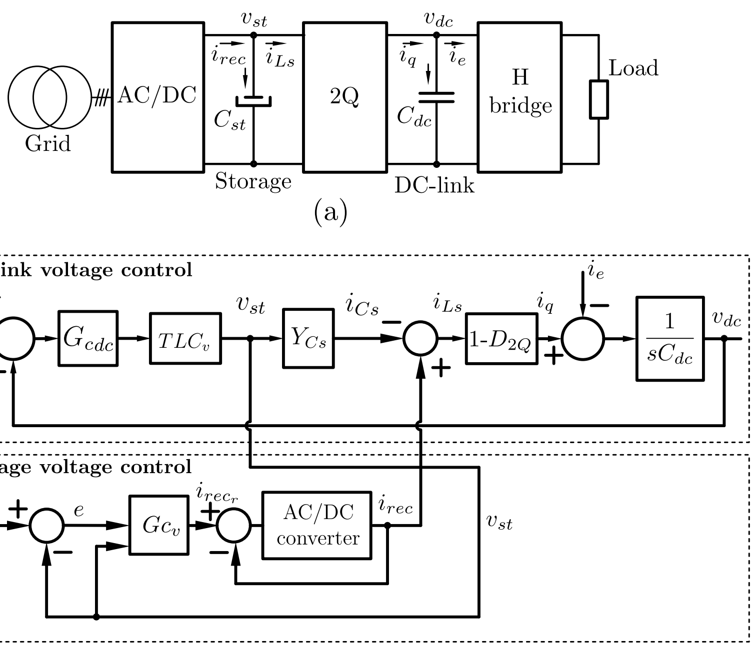

Figure 7: Topology with cascaded structure configuration. (a) Topology scheme. (b) Conceptual regulation system.

with the storage element current and the AC/DC output current. Then, $i_{Ls}(t)$ affected by the duty cycle of the 2Q converter defines the output current of such structure, $i_q(t)$. The difference between $i_q(t)$ and the current demanded by the H-bridge, $i_e(t)$, is equal to the current in the dc-link capacitor, i.e. this difference of current defines the voltage $v_{dc}(t)$. Consequently, it can be seen that to control $v_{dc}(t)$ to a constant value implies that the 2Q converter must deliver a current equal to the current demanded by the H-bridge.

The second regulation loop is defined for recovering the power losses from the control of voltage $v_{st}(t)$, by using the difference between this voltage and a voltage reference to determine the output current of the rectifier module, $i_{rec}(t)$. Then, the AC/DC converter output current is regulated by closed-loop action, and its reference is determined by the storage element voltage. The value of such reference will depend on the strategy selected for the power taken from the grid, i.e. the AC/DC module behaves as a controlled current source with the power characteristics previously defined. The difference between this current and the current in the storage element, $i_{Cs}(t)$, is equal to the current demanded by the 2Q converter at its input, $i_{Ls}(t)$.

It can be observed that the use of a controlled converter in the input stage allows to regulate the power taken from the electrical network, which allows to maintain the control even during grid voltage disturbances. On the other hand, the use of different converters to perform the required tasks (2Q converter used for dc-link voltage control and ac/dc converter used for storage voltage control) allows implementing regulation loops to control different variables, which implies a higher simplicity in the definition of such regulation loops.

The qualitative voltage and current waveforms that corresponds to the proposed topology operated with constant $i_{rec}(t)$ are shown in Fig. 8. The figure shows the currents in the node corresponding to the dc-link, where it can be seen that the output current of the 2Q converter, $i_q(t)$, is equal to the current demanded by the H-bridge, $i_e(t)$, which in turns determines that $v_{dc}(t)$ remains constant. Additionally, the figure shows the currents in the node of storage voltage, where it can be seen that the current demanded by the 2Q converter, $i_{Ls}(t)$, is divided in the current supplied by the electrical network, $i_{rec}(t)$, and the current supplied by the storage element, $i_{Cs}(t)$. The variation in $i_{Cs}(t)$ corresponds to the voltage variation in C_{st}, as it is shown in the figure, where it can be seen that after the fall time the voltage in C_{st} increases linearly until it reaches the value given by the reference. Such charge is generated by the constant current provided by the input stage, $i_{rec}(t)$, which goes to zero once the storage voltage reaches the value given by the reference.

3.1.2 Figures of merit

This section presents the figures of merit corresponding to the cascaded structure configuration.

Metrics for input stage converter:

Regardless the strategy used for controlling the input stage, if unitary efficiency is assumed, the condition of input power equal to output power must be accomplished:

$$I_{in}\,V_{in} = I_{out}\,V_{out} = i_{rec}\,v_{st}$$

where V_{in} and I_{in} are the input voltage and current of the ac/dc module, and $V_{out} = v_{st}$ and $I_{out} = i_{rec}$ are the output voltage and current of the ac/dc module. Then, since V_{in} and $V_{out} = v_{st}$ are defined by the operation of the system, the maximum power supplied by the input stage can be regulated by controlling either I_{in} or i_{rec}. The figures of merit will depend on the current to be controlled; hence, the expressions for each case will be analyzed.

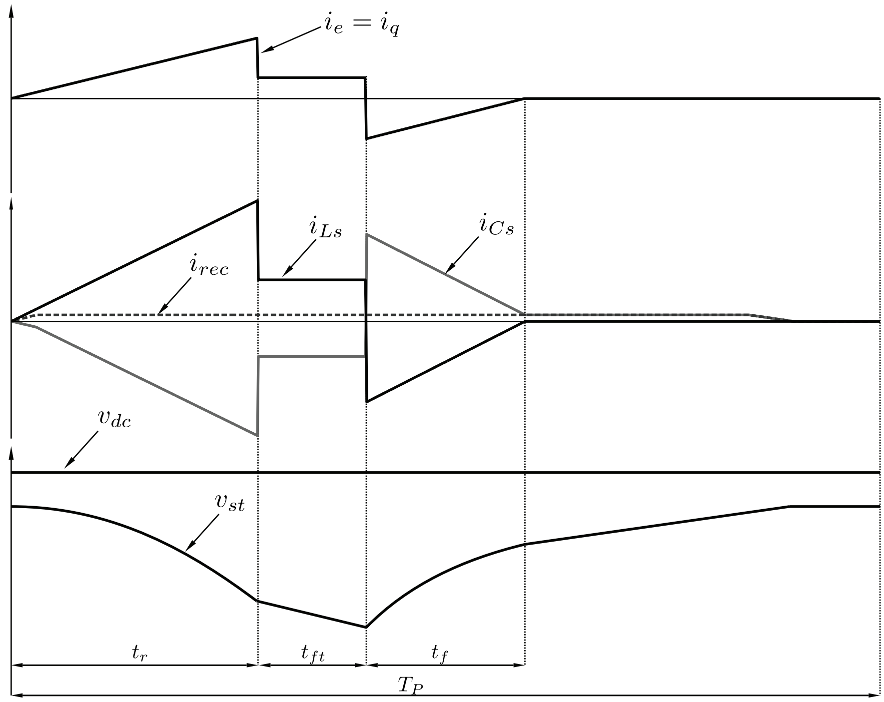

Figure 8: Conceptual waveforms of topology with cascaded structure configuration.

Variable grid power (constant i_{rec}):

If the current $i_{rec}(t)$ is controlled to a constant value, $I_{rec\,\max}$, the power of the input stage module, $P_{rec}(t)$, will have its maximum and minimum values related to the maximum and minimum of $v_{st}(t)$:

$$\begin{aligned} P_{rec\,\max} &= I_{rec\,\max} V_{st\,\max} \\ P_{rec\,\min} &= I_{rec\,\max} V_{st\,\min} \end{aligned} \tag{1}$$

As shown in (1), the value of current used as a limit defines the peak power that the grid must supply. In general, the time interval in which the grid delivers energy, T_g, is related to the active energy and the grid power waveform. Assumming a constant grid current equal to $I_{rec\,\max}$ during T_g and that the energy provided must be equal to the active energy of the system, the following condition can be obtained:

$$\begin{aligned} \Delta E_{g\,[T_p]} &= \int_0^{T_g} i_{rec}(t)\, v_{st}(t)\, dt = \Delta E_{act\,[T_p]} \\ I_{rec\,\max}\, T_g\, \overline{V_{st}}_{[T_g]} &= I_{REF}^2\, R_m \left(t_{ft} + \frac{t_r + t_f}{3} \right) \end{aligned} \tag{2}$$

where $\overline{V_{st}}_{[T_g]}$ is the storage element voltage averaged in the time interval when the grid supplies current, T_g. Notice that a constant output current in this structure implies that the average voltage at its output will be function of such current, storage capacitor and power demanded by the magnet. Therefore, the average value of $v_{st}(t)$ will be function of $I_{rec_{max}}$, i.e. they can not be defined independently (see Appendix B). Then, the maximum average current in the input stage is given by:

$$\boxed{I_{rec\,\max} = \frac{I_{REF}^2\, R_m}{\overline{V_{st}}_{[T_g]}} \frac{t_{ft} + \frac{t_r + t_f}{3}}{T_g}} \tag{3}$$

Notice that, in order to be able to supply the power losses, the grid peak current must increase as the time T_g decreases. The figure of merit that relates the maximum current with the normalization current is given by:

$$\boxed{\frac{I_{rec\max}}{I_{rec_N}} = \frac{\overline{V_{st}}\, T_p}{\overline{V_{st}}_{[T_g]}\, T_g}} \tag{4}$$

where $\overline{V_{st}}$ is storage element voltage averaged in the cycle period T_p and I_{rec_N} is calculated as explained in Appendix A.

For the particular case in which the grid current is adopted to achieve a constant current in the input stage during the cycle period T_p, this current is given by:

$$\boxed{I_{rec\max} = \frac{I_{REF}^2\, R_m}{\overline{V_{st}}}\, \frac{t_{ft} + \frac{t_r+t_f}{3}}{T_p}} \tag{5}$$

Hence, the relationship $\dfrac{I_{rec\max}}{I_{rec_N}}$ is given by:

$$\boxed{\frac{I_{rec\max}}{I_{rec_N}} = 1} \tag{6}$$

As can be seen in (5), this operational mode allows obtaining a peak current in the output of the grid connected rectifier as low as possible. However, the peak power drawn from the electrical network will vary from $P_{rec\max}$ to $P_{rec\min}$. The average power of the input module, $P_{rec_{avg}}$, is given by the ratio between the active power and the cycle period, i.e. $P_{rec_{avg}} = \Delta E_{act[T_p]}/T_p$. Then, operating with (1) and (5):

$$\begin{aligned}
P_{rec\max} &= I_{rec\max}\, V_{st\max} = \frac{\Delta E_{act[T_p]}}{\overline{V_{st}}\, T_p}\, V_{st\max} \\
P_{rec\min} &= I_{rec\max}\, V_{st\min} = \frac{\Delta E_{act[T_p]}}{\overline{V_{st}}\, T_p}\, V_{st\min} \\
P_{rec_{avg}} &= \frac{\Delta E_{act[T_p]}}{T_p}
\end{aligned} \tag{7}$$

From (7), the ratio between the maximum and average power of the input module power is given by:

$$\frac{P_{rec\max}}{P_{rec_{avg}}} = \frac{V_{st\max}}{\overline{V_{st}}} \tag{8}$$

Notice that the limiting of the output current is also implemented in the topologies based on 2Q and boost converters [6, 7]. Unlike those structures, in this case the grid power is variable. Moreover, in this case the cycle timing is not necessary, which is similar to the case of boost converter. In addition, if the value $I_{rec\max}$ is calculated for the most demanding case, i.e. maximum active power, a cycle-by-cycle modulation both in amplitude and time could be implemented without problem. Notice that if the limit $I_{rec\max}$ is kept constant regardless the parameters of the next cycle to be generated, the system will take from the grid a power higher than necessary, which will rise $v_{st}(t)$ above the value set from the reference when the system recovers energy. Despite this increase of the voltage, the system will operate without problems as $v_{st}(t)$ can be kept bounded to a maximum.

Constant Grid Power (constant P_{rec}):

In this case, the current $i_{rec}(t)$ will be function of the storage voltage in order to achieve a constant power in the grid connected converter:

$$i_{rec(t)}\, v_{st}(t) = P_{rec}(t) = P_{rec_{avg}} \Longrightarrow i_{rec}(t) = \frac{P_{rec_{avg}}}{v_{st}(t)}$$

Then, the energy taken from the electrical network and the power of the input stage converter are given by:

$$\Delta E_{g[T_p]} = \Delta E_{act[T_p]}$$

$$\int_0^{T_p} i_{rec}(t)\, v_{st}(t)\, dt = P_{rec_{avg}}\, T_p = I^2_{REF}\, R_m \left(t_{ft} + \frac{t_r + t_f}{3} \right)$$

$$P_{rec_{avg}} = \frac{\Delta E_{act[T_p]}}{T_p} = I^2_{REF}\, R_m \frac{\left(t_{ft} + \frac{t_r + t_f}{3} \right)}{T_p}$$

The output current of the grid connected converter will vary between $I_{rec\,\max}$ and $I_{rec\,\min}$, accordingly to the storage element voltage:

$$I_{rec\,\max} = \frac{P_{rec_{avg}}}{V_{st\,\min}}$$

$$I_{rec\,\min} = \frac{P_{rec_{avg}}}{V_{st\,\max}}$$

Then, the figure of merit that relates $I_{rec\,\max}$ to the normalization current I_{rec_N} is given by:

$$\boxed{\frac{I_{rec\,\max}}{I_{rec_N}} = \frac{\overline{V_{st}}}{V_{st\,\min}}}$$

Energy storage metrics:

With respect to the energy storage sizing, it is defined by the minimum voltage that the storage element can hold without compromising the operation of the system. With the limited input power strategies, the minimum voltage at C_{st} is obtained at the end of flat-top. Between the beginning of the cycle, $t_0 = 0$, and the end of flat-top, $t_r + t_{ft}$, the energy provided by C_{st} and the energy provided by the grid has been used to rise up the magnet current from 0 to I_{REF} and to supply the magnet losses:

$$\Delta E_{st[t_r+t_{ft}]} + \Delta E_{g[t_r+t_{ft}]} = \Delta E_{m[t_r+t_{ft}]} + \Delta E_{act[t_r+t_{ft}]} \tag{9}$$

where:

$$\Delta E_{st[t_r+t_{ft}]} = E_{st}(0) - E_{st}(t_{ft}) = \frac{1}{2} C_{st} \left[v^2_{st}(0) - v^2_{st}(t_{ft}) \right] = \frac{1}{2} C_{st} \left[{V_{st\,\max}}^2 - {V_{st\,\min}}^2 \right]$$

$$\Delta E_{g[t_r+t_{ft}]} = \int_0^{t_r+t_{ft}} i_{rec}(t)\, v_{st}(t)\, dt$$

$$\Delta E_{m[t_r+t_{ft}]} = E_m(t_{ft}) - E_m(0) = \frac{1}{2} L_m I^2_{REF} = E_m$$

$$\Delta E_{act[t_r+t_{ft}]} = \int_0^{t_r+t_{ft}} i^2_m(t) R_m\, dt = I^2_{REF} R_m \left(t_{ft} + \frac{t_r}{3} \right) \tag{10}$$

From (9), the maximum energy variation in the storage element normalized to the reactive energy in the magnet can be obtained:

$$\boxed{\frac{\Delta E_{st[t_r+t_{ft}]}}{E_m} = 1 + \frac{\Delta E_{act[t_r+t_{ft}]} - \Delta E_{g[t_r+t_{ft}]}}{E_m}} \tag{11}$$

It is worth noticing that (11) is the same figure of merit that the one obtained with the 2Q converter based topology. Furthermore, this expression shows that, with this strategy, the energy variation changes with the amount of energy taken from the grid during t_r and t_{ft}. Figure 9 shows the ratio $\Delta E_{st}/E_m$ given by (11), where it can be noticed that the slope of the energy variation changes with the amount of energy drawn from the grid during the time t_r and t_{ft}.

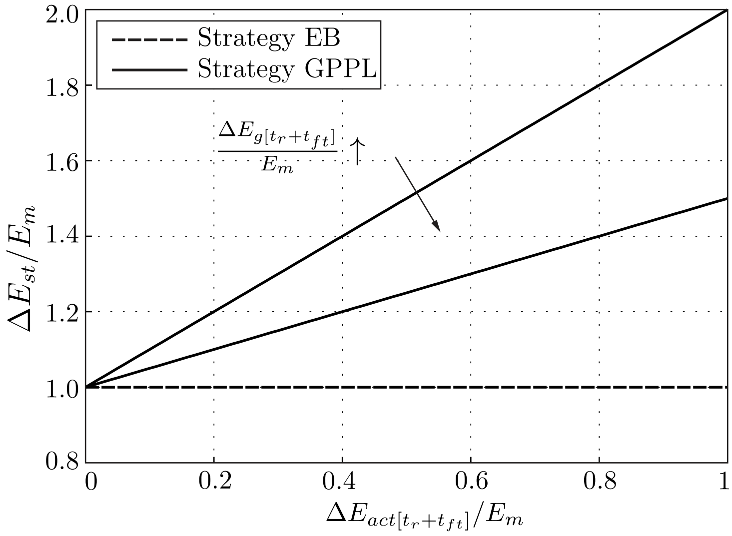

Figure 9: Maximum energy variation in the storage element as a function of the active energy, normalized to the maximum reactive energy in the magnet (parameter: energy supplied by the grid). Strategy Grid Peak Power Limitation.

Another important aspect is the minimum voltage at C_{st}, which will be related to the normal operation of the 2Q converter stage, both in terms of duty cycle and maximum current in its switches (notice that the lower the voltage in C_{st}, the higher the current i_{Ls}). From (9) and (10), the minimum voltage at C_{st} is obtained:

$$V_{st\,\min} = \sqrt{{V_{st\,\max}}^2 - \frac{L_m}{C_{st}} I_{REF}^2 \left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g[t_r+t_{ft}]}}{\frac{1}{2} L_m\, I_{REF}^2}\right]} \tag{12}$$

Then, the maximum voltage swing is the difference between $V_{st\,\max}$ and $V_{st\,\min}$:

$$\Delta v_{st} = V_{st\,\max} - V_{st\,\min} \tag{13}$$

Then, the minimum value of C_{st} that allows obtaining a certain minimum voltage in the storage element $V_{st\,\min}$ is given by:

$$\boxed{C_{st\,\min} = \frac{I_{REF}^2 L_m}{{V_{st\,\max}}^2 - {V_{st\,\min}}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g[t_r+t_{ft}]}}{\frac{1}{2} L_m\, I_{REF}^2}\right]} \tag{14}$$

Regarding the utilisation factor of the installed energy, the corresponding figure of merit is given by:

$$\frac{\Delta E_{st}}{E_{stmax}} = \frac{(1/2)\, C_{st}\, \left({V_{st\,\max}}^2 - {V_{st\,\min}}^2\right)}{(1/2)\, C_{st}\, {V_{st\,\max}}^2} = 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 \tag{15}$$

It can be noticed that the utilisation factor improves when the discharge of C_{st} is higher. Since the maximum and minimum voltages in the storage element can be adopted equal to the topology based on 2Q converter reviewed in Section 2.1, the value of the figure of merit will be the same. Regarding the maximum installed energy, it can be obtained from (11) and (15):

$$\frac{E_{stmax}}{E_m} = \frac{(1/2)\, C_{st}\, {V_{st\,\max}}^2}{E_m} = \frac{\Delta E_{st}}{E_m}\,\frac{E_{stmax}}{\Delta E_{st}} = \frac{1 + \dfrac{\Delta E_{act[t_r+t_{ft}]} - \Delta E_{g[t_r+t_{ft}]}}{E_m}}{1 - \left(\dfrac{V_{st\,\min}}{V_{st\,\max}}\right)^2} \tag{16}$$

From (16) can be observed that the maximum installed energy decreases when both the energy from the electrical network during $t_r + t_{ft}$ and the utilisation factor increase.

The figures of merit related to energy storage will depend on the operation (Constant/Variable Grid Power). The main difference is associated to the amount of energy coming from the electrical network during $t_r + t_{ft}$, which is given by:

$$\Delta E_{g_{[t_r+t_{ft}]}} = \int_0^{t_r+t_{ft}} i_{rec}(t)\, v_{st}(t)\, dt$$

Variable grid power:

$$\Delta E_{g_{[t_r+t_{ft}]}} = I_{rec\,\max}\, \overline{V_{st}}_{[t_r+t_{ft}]}(t_r + t_{ft}) = \Delta E_{act} \frac{(t_r + t_{ft})}{T_p} \frac{\overline{V_{st}}_{[t_r+t_{ft}]}}{\overline{V_{st}}}$$

Constant grid power:

$$\Delta E_{g_{[t_r+t_{ft}]}} = P_{rec_{avg}}\,(t_r + t_{ft}) = \Delta E_{act} \frac{(t_r + t_{ft})}{T_p}$$

Then, the value of the figures of merit has to be calculated by evaluating expressions (11), (14), (15) and (16) with the expression of $\Delta E_{g_{[t_r+t_{ft}]}}$ that corresponds to the operation mode used.

3.2 Parallel structure configuration

Figure 10 shows the proposed topology based on the parallel connection of two controlled structures, where can be noticed the flow of energy in each stage. This topology uses the concept presented in the topology based on 2Q converter, where the grid connected stage is used for supplying the average active power and the storage converter is used for supplying the difference between the magnet power and the average active power.

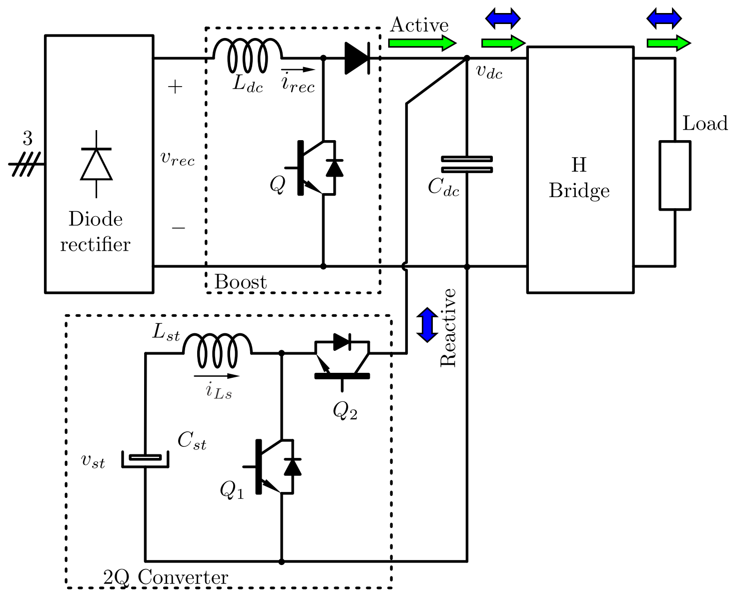

Figure 10: Topology with parallel structure configuration.

The input stage is formed by a boost converter, whose main task is to supply a controlled current (function of the active power) to the dc-link formed by C_{dc}. The control of this current allows to have a high rejection to disturbances in the electrical network voltage. The 2Q converter is used as coupling stage between the dc-link and the storage element. This converter is bidirectional in current, in order to supply energy from C_{st} to the H-bridge (during rise time and fall time) and to receive energy during the fall time and the time interval when the magnet current is zero. If C_{st} is considered the input of the 2Q converter, it can be seen that this converter behaves like a boost. Hence, the voltage in C_{dc} must be always higher than the voltage in C_{st}. This feature implies that the voltage can vary from values as low as it is practically possible up to the voltage in the dc-link. As it was previously mentioned, this feature is advantageous since it allows to decrease the installed energy and to improve its utilisation. Regarding the voltage in C_{dc}, it is controlled to a constant value in order to avoid variations in the gain of the H-bridge and to ensure that the load can receive a voltage waveform according to the magnet current cycle requirements.

3.2.1 Conceptual regulation system

The control system of this topology is divided in two parts with different functions: the control of dc-link voltage and the control of storage element voltage. Figure 11 shows a circuital scheme of the topology with the different variables involved in the control and the block diagram of the control system with the above mentioned division. It can be noted that, although each part control different variables, there is a coupling between them due to the circuit interconnection between the two structures (boost and 2Q converter) and by the variables that regulates each structure.

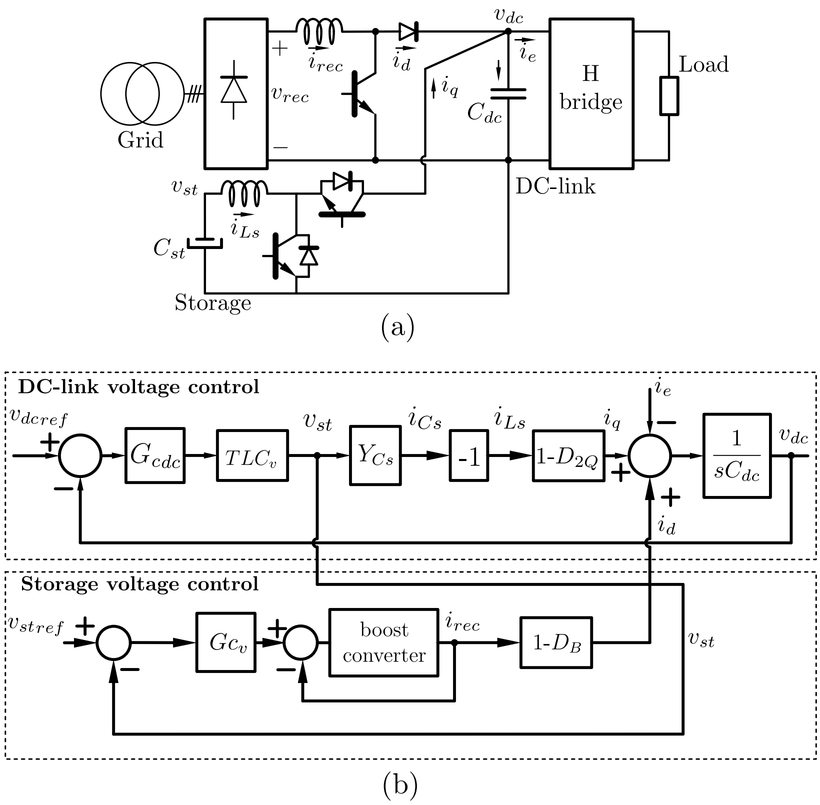

Figure 11: Topology with parallel structure configuration. (a) Topology scheme. (b) Conceptual regulation system.

With respect to the control of dc-link voltage, it is performed by means of the 2Q converter. The objective of this control is to regulate the voltage $v_{dc}(t)$ to a constant value, which determines that $i_q(t)$ is equal to the difference between $i_e(t)$ and $i_d(t)$. In the block diagram of the control system can be seen that the regulation of $i_q(t)$ is performed indirectly with the control of $v_{st}(t)$, which is given by the transfer function $TLC_v(s)$ [6] as in the cascaded topology.

The control of the storage element voltage is performed with the boost converter. This control is represented in the block diagram with a regulation loop that controls $v_{st}(t)$ to a constant value. The aim of this control is to achieve that the boost supplies a current $i_d(t)$, which is bounded to a maximum value related to the active energy, during the time interval in which the voltage v_{st} is lower than the value set with the reference. The current $i_d(t)$ is indirectly controlled from the inner current control of $i_{rec}(t)$ and both are bounded by incorporating an antiwindup stage in the controller of the boost inductor current control.

Then, having into account the discharge of C_{st}, the boost converter is going to supply current during the cycle period T_p. The current $i_d(t)$ that is supplied after the fall time flows between the boost converter and 2Q converter, in order to restore the charge in C_{st}. In a qualitative way, since $i_e = 0$ after the fall time, the current $i_d(t)$ charges C_{dc}, which increases the dc-link voltage. Then, the control system that regulates the dc-link voltage generates a current $i_q(t)$ that tries to decrease $v_{dc}(t)$ up to the reference value. Consequently, $i_q(t)$ will be equal to the current $i_d(t)$. The current $i_q(t)$ starts charging C_{st}, until its voltage results equal to the reference value. When this condition is met, the control loop imposes $i_d(t) = 0$. Finally, the energy in C_{st} is restored indirectly and involves the operation of the two control systems.

The qualitative voltage and current waveforms related to this topology are shown in Fig. 12. It can be seen the currents associated to the dc-link node, where it can be noted that the boost current is constant and it lasts beyond the fall time. Moreover, the figure shows the currents in the storage element node, where it can be noted the currents that generate the charge and discharge of C_{st}. Unlike the cascaded structure configuration topology, in the Parallel Structure Configuration the dc-link node determines that $i_e = i_d + i_q$. Then, the current supplied by the 2Q converter will be $i_q = i_e - i_d$. In the figure can be observed that when i_e is positive, the current i_q supplies only a part of the energy to the magnet, due to the current i_d; hence, both structures (2Q and boost) supplies the magnet energy. However, it can be seen that when the magnet is giving back energy ($i_e(t) < 0$) the current $i_q(t)$ results higher than $i_e(t)$. Consequently, in this condition the 2Q converter must support an instantaneous power higher than the magnet power. From the previously mentioned, it can be concluded that the maximum positive power in the 2Q converter is lower than the maximum positive magnet power, while the negative power will be higher than the maximum negative power in the magnet.

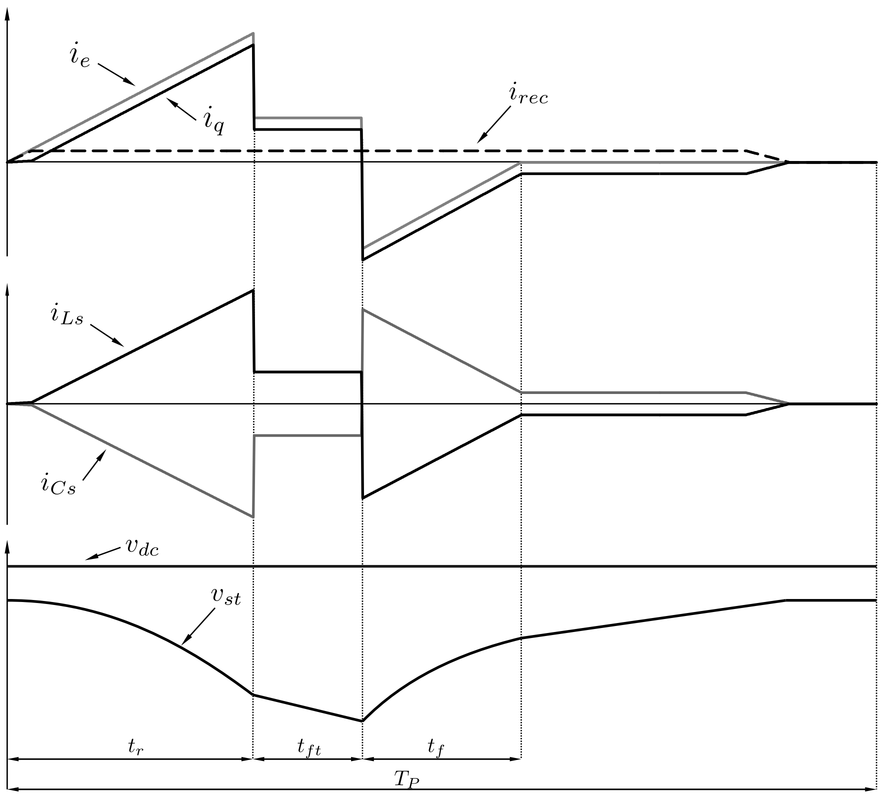

Figure 12: Waveforms of topology with parallel structure configuration.

3.2.2 Figures of merit

Metrics for input stage converter:

In this case, the power of the grid tied converter can be controlled directly, since its output voltage is constant and its output current is equal to the current in the boost inductor. Then, considering that the boost inductor current will be controlled to a constant value, $I_{rec\,\max}$:

$$I_{out} = i_{rec} = I_{rec\,\max} \qquad V_{out} = V_g \qquad \Longrightarrow P_{rec} = I_{rec\,\max}\, V_g$$

Then, the maximum output current in the grid connected converter is obtained from (3), considering that in this case its output voltage is constant ($\overline{v_{rec}}$):

$$\boxed{I_{rec\,\max} = \frac{I_{REF}^2\, R_m}{\overline{v_{rec}}}\, \frac{t_{ft} + \frac{t_r + t_f}{3}}{T_g}} \tag{17}$$

The figure of merit that relates the maximum current to the normalization current is given by:

$$\boxed{\frac{I_{rec\,\max}}{I_{rec_N}} = \frac{T_p}{T_g}} \tag{18}$$

where $I_{rec_N} = \frac{I_{REF}^2\, R_m}{V_g}\, \frac{t_{ft} + \frac{t_r + t_f}{3}}{T_p}$. For the particular case when the current is constant during the cycle period T_p, i.e. $T_g = T_p$, the following condition is obtained:

$$\boxed{I_{rec\,\max} = \frac{I_{REF}^2\, R_m}{\overline{v_{rec}}}\, \frac{t_{ft} + \frac{t_r + t_f}{3}}{T_p}} \qquad \text{Grid power spread during } T_p \tag{19}$$

Hence, the relationship $\dfrac{I_{rec\,\max}}{I_{rec_N}}$ is given by:

$$\boxed{\frac{I_{rec\,\max}}{I_{rec_N}} = 1} \tag{20}$$

It is worth noticing that the figure of merit that corresponds to output current of the grid tied converter results the same that the one obtained with the cascaded structure configuration with Variable Grid Power (constant $i_{rec}(t)$), as shown in expression (6).

Energy storage metrics:

With the limited input power strategies, the minimum voltage at C_{st} (i.e. its minimum energy stored) is obtained at the end of flat-top. Between the beginning of the cycle, $t_0 = 0$, and the end of flat-top, $t_r + t_{ft}$, the energy provided by C_{st} and the energy provided by the grid has been used to rise up the magnet current from 0 to I_{REF} and to supply the magnet losses:

$$\Delta E_{st\,[t_r+t_{ft}]} + \Delta E_{g\,[t_r+t_{ft}]} = \Delta E_{m\,[t_r+t_{ft}]} + \Delta E_{act\,[t_r+t_{ft}]} \tag{21}$$

where:

$$\begin{aligned}
\Delta E_{st\,[t_r+t_{ft}]} &= E_{st}(0) - E_{st}(t_{ft}) = \frac{1}{2} C_{st} \left[v_{st}^2(0) - v_{st}^2(t_{ft}) \right] = \frac{1}{2} C_{st} \left[{V_{st\,\max}}^2 - {V_{st\,\min}}^2 \right] \\
\Delta E_{g\,[t_r+t_{ft}]} &= \int_0^{t_r+t_{ft}} i_{rec}(t)\, v_g(t)\, dt = I_{rec\,\max}\, \overline{v_{rec}}\, (t_r + t_{ft}) \\
\Delta E_{m\,[t_r+t_{ft}]} &= E_m(t_{ft}) - E_m(0) = \frac{1}{2} L_m I_{REF}^2 = E_m \\
\Delta E_{act\,[t_r+t_{ft}]} &= \int_0^{t_r+t_{ft}} i_m^2(t) R_m\, dt = I_{REF}^2 R_m \left(t_{ft} + \frac{t_r}{3} \right)
\end{aligned} \tag{22}$$

From (21), the maximum energy variation in the storage element normalized to the reactive energy in the magnet can be obtained:

$$\boxed{\frac{\Delta E_{st[t_r+t_{ft}]}}{E_m} = 1 + \frac{\Delta E_{act[t_r+t_{ft}]} - \Delta E_{g[t_r+t_{ft}]}}{E_m}} \tag{23}$$

It is worth noticing that (23) is the same figure of merit that the one obtained with the 2Q converter based topology and with the previous proposed structure (cascaded structure configuration). As a result, the expressions and conclusions for $V_{st\,\min}$, C_{st}, $\frac{\Delta E_{st}}{E_{stmax}}$ and $\frac{E_{stmax}}{E_m}$, which are repeated below for the sake of clarity, will be also the same:

$$V_{st\,\min} = \sqrt{{V_{st\,\max}}^2 - \frac{L_m}{C_{st}} I_{REF}^2 \left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g[t_r+t_{ft}]}}{\frac{1}{2} L_m\, I_{REF}^2}\right]} \tag{24}$$

$$C_{st\,\min} = \frac{I_{REF}^2 L_m}{{V_{st\,\max}}^2 - {V_{st\,\min}}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g[t_r+t_{ft}]}}{\frac{1}{2} L_m\, I_{REF}^2}\right] \tag{25}$$

$$\frac{\Delta E_{st}}{E_{stmax}} = 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 \tag{26}$$

$$\frac{E_{stmax}}{E_m} = \frac{(1/2)\, C_{st}\, {V_{st\,\max}}^2}{E_m} = \frac{\Delta E_{st}}{E_m}\,\frac{E_{stmax}}{\Delta E_{st}} = \frac{1 + \dfrac{\Delta E_{act[t_r+t_{ft}]} - \Delta E_{g[t_r+t_{ft}]}}{E_m}}{1 - \left(\dfrac{V_{st\,\min}}{V_{st\,\max}}\right)^2} \tag{27}$$

Considering that the maximum and minimum voltages for the storage converter stage can be set equal than in the original 2Q converter and proposed cascaded/parallel structure configuration, the value of the figures of merit related to energy storage element will be exactly the same.

3.3 Parallel connection of H-bridges

During the research process, a proposal for a topology employing parallel-series connection of lower voltage H-bridges that would allow a lower dc-link capacitor voltage in operation was introduced. The topology is illustrated in Fig. 13.

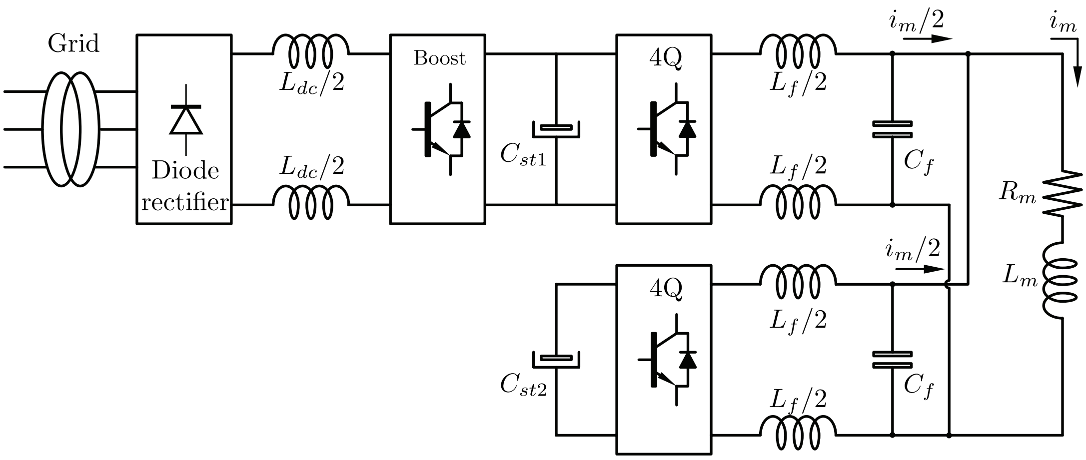

Figure 13: Topology with parallel connection of H-bridges.

Since only 1 brick among 2 is connected to the grid (brick 1) this means that the recharge of the other bricks has to be made in an "indirect" way. It can be assumed, to start with, that the dc bus of the upper and lower bridges employ the same capacity (storage). Over a basic cycle time the upper and lower bridges energy equals the "active"+"reactive" power by the magnet.

The magnet current can be measured and compared with the reference. The upper bridge and lower bridge provide half current each. In addition, the bottom bridge shall employ a slow voltage balancing control which takes care of its "average" capacitor voltage with long time constant. The "error" generated by this loop can be added to the reference of the upper bridge current and subtracted by the reference of the lower bridge.

Assuming that the energy variation in the capacitors necessary for proper operation (ΔE_{st}) is equally supplied for each capacitor (ΔE_{st1}, ΔE_{st2}) and that the initial voltage ($V_{st\,\max}$) and final voltage ($V_{st\,\min}$) are the same for both capacitors, the following expression can be written:

$$\Delta E_{st} = \Delta E_{st1} + \Delta E_{st1} = \frac{1}{2} C_{st1} \left(V_{st\,\max}^2 - V_{st\,\min}^2 \right) + \frac{1}{2} C_{st2} \left(V_{st\,\max}^2 - V_{st\,\min}^2 \right) \tag{28}$$

$$\Delta E_{st} = \frac{1}{2} \left(C_{st1} + C_{st2} \right) \left(V_{st\,\max}^2 - V_{st\,\min}^2 \right) \tag{29}$$

If the proposed structure is compared to the boost originally analyzed, assuming again that:

- (a) the energy supplied for the storage capacitor is equal to the energy variation required for proper operation, and that
- (b) the storage element has initial voltage $V_{st\,\max}$ and final voltage $V_{st\,\min}$,

the following expression can be obtained:

$$\Delta E_{st} = \frac{1}{2} C_{st} \left(V_{st\,\max}^2 - V_{st\,\min}^2 \right) \tag{30}$$

By inspection of (29) and (30), it can be noticed that $C_{st} = C_{st1} + C_{st2}$. Consequently, the proposed structure acts as a boost that has a capacitor equal to $C_{st1} + C_{st2}$. The difference between the proposed structure and the original boost is that the storage capacitor was split in 2 and located in different converters. In spite of this, the proposed structure has to store the same amount of energy than the original boost and has the same problems regarding minimum voltage. Consequently, the figures of merit $\Delta E_{st}/E_{st\,\max}$ and $E_{st\,\max}/E_m$ will be exactly the same than those of the original boost. Summarizing:

- the proposed structure will present the same drawbacks than the original boost (storage element discharge bounded by the minimum voltage in the dc-link of the H-bridge and, consequently, higher maximum installed energy). In fact, there will be no differences in any of the figures of merit.
- Taking into account that the system acts as if C_{st1} was in parallel to C_{st2}, it results easier to connect both capacitors directly in parallel. This solves the indirect charge of C_{st2} and leads to the boost structure originally analyzed.
- Additionally, it could result interesting to use 2 H-bridge in parallel, since this feature allows alleviating the semiconductor devices operation of the stage used for magnet current generation.

According to the abovementioned, this topology was not analyzed in depth in this report.

4 Evaluation of topologies

This section presents the evaluation of different topologies using the expressions developed in the previous sections. Table 2 shows the main parameters of the system that are used for defining the operational conditions and the converter elements.

Table 2: System parameters for proposed topologies.

Pulse parameters				Load parameters	
I_{REF}	450 A	v_m	±450 V	L_m	260 mH
t_r	340 ms	T_p	1.2 s	R_m	400 mΩ
t_{ft}	50 ms				
t_f	220 ms				

In the following, the figures of merit presented in Appendix A, capacitor value and output current of the grid connected converter are evaluated for each topology (boost, 2Q, cascaded structure configuration and parallel structure configuration) assuming grid peak power limitation.

4.1 Figures of merit

Within the topologies analyzed, there is a group that presents a constant voltage in the output of the grid tied converter (boost, 2Q, parallel structure configuration). This feature allows to implement a control of constant power from the control of the output current to a constant value (Constant Grid Power). In the case of the remaining topology (cascaded structure configuration), its output voltage is not constant, since it corresponds to the storage element voltage. Then, for this structure two operation modes have been proposed. In the first mode, the output current of the grid tied converter is controlled to be constant, which leads to a variable power module (Variable Grid Power). In the second mode, the input or output current of this module is controlled in order to obtain a constant power (Constant Grid Power), which determines that the grid tied converter presents the same characteristics in its power like in the cases analyzed for boost, 2Q and Parallel Structure Configuration topologies.

As can be seen from the expressions developed in Appendix A, the figures of merit depend on how the energy is taken from the electrical network, the involved voltages (which in turn depend on the topology used) or both of them. Among the different figures of merit, the one that corresponds to the variation of energy stored with respect to the maximum reactive energy in the magnet only depends on the consumed energy and energy taken during the time interval in which the storage element discharges. Then, the value of such figure of merit will be the same for all the analyzed topologies that operates under the same operational condition (Constant Grid Power/Variable Grid Power):

$$\frac{\Delta E_{st[t_r+t_{ft}]}}{E_m} = 1 + \frac{\Delta E_{act[t_r+t_{ft}]}}{E_m} - \frac{\Delta E_{g[t_r+t_{ft}]}}{E_m} \tag{31}$$

In the same way, the efficiency in the installed energy depends only on the maximum and minimum voltages in the storage element, regardless the kind of strategy used for managing the energy:

$$\frac{\Delta E_{st}}{E_{st max}} = 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 \tag{32}$$

Notice that this utilisation factor corresponds to the depth of discharge of the energy storage element. In this sense, this figure of merit will impact on the capacitor technology (electrolytic, metalized film, etc) and in the capacitor bank structure to get the required energy (parallel/series combination of capacitor units). Consequently, it will impact both in price and volume.

Finally, the remaining figures of merit will depend on both the handling of the incoming energy and the voltages in the storage element:

$$\frac{E_{st\,max}}{E_m} = \frac{\Delta E_{st}}{E_m}\,\frac{E_{st\,max}}{\Delta E_{st}} \tag{33}$$

$$C_{st\,\min} = \frac{I_{REF}^2 L_m}{v_{st\,max}^2 - v_{st\,min}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g\,[t_r+t_{ft}]}}{\frac{1}{2}L_m\,I_{REF}^2}\right] \tag{34}$$

$$\frac{I_{rec\,\max}}{I_{rec_N}} \qquad \text{with} \qquad I_{rec_N} = \frac{\Delta E_{act_{[T_p]}}}{\overline{v_{rec}}T_p} \tag{35}$$

$$\frac{P_{rec\,\max}}{P_{rec_{avg}}} \qquad \text{with} \qquad P_{rec_{avg}} = \frac{\Delta E_{act_{[T_p]}}}{T_p} \tag{36}$$

The evaluation of the figures of merit for each structure is presented in the following paragraphs. Some figures of merit depend on the voltages involved in each topology. Hence, in order to evaluate the structures in similar conditions, it will be considered that the maximum voltage in the dc-link is $v_{dc} = 700\,\mathrm{V}$.

4.1.1 Boost front-end regulator

The topology to be evaluated corresponds to the one presented in Fig. 2a. With this structure, the maximum storage voltage will be equal to the maximum dc-link voltage (700 V), while the minimum voltage will be related to the maximum voltage to be applied in the load (450 V), which occurs when $v_{st}(t)$ has its minimum value. Additionally, due to the operation of the boost converter, the rectified voltage must be higher than $V_{st\,\min}$. Then, the following values are adopted:

$$V_{st\,\max} = 700\,\mathrm{V}$$
$$V_{st\,\min} = 500\,\mathrm{V}$$
$$\overline{v_{rec}} = 400\,\mathrm{V}$$

The use of a constant input power strategy determines that the energy supplied by the electrical network during the time that C_{st} discharges, $\Delta E_{g_{[t_r+t_{ft}]}}$, and the input module power, P_{rec}, are given by:

$$\Delta E_{g_{[t_r+t_{ft}]}} = P_{rec_{avg}}\,(t_r + t_{ft}) = 6.23\,\mathrm{kJ} \qquad \text{with} \qquad P_{rec\,\max} = P_{rec_{avg}} = \frac{\Delta E_{act\,[T_p]}}{T_p} = 15.975\,\mathrm{kW}$$

Then, according to expression (31), the figure of merit that relates the energy variation of the storage element to the maximum reactive energy is given by:

$$\frac{\Delta E_{st}}{E_m} = 1 + \frac{\Delta E_{act\,[t_r+t_{ft}]}}{E_m} - \frac{\Delta E_{g\,[t_r+t_{ft}]}}{E_m} = 1.266$$

where:

$$E_m = \frac{1}{2}L_m I_{REF}^2 = 26.33\,\mathrm{kJ}$$
$$\Delta E_{act\,[t_r+t_{ft}]} = I_{REF}^2 R_m\left(t_{ft} + \frac{t_r}{3}\right) = 13.23\,\mathrm{kJ}$$

which means that, assumming that the main task of the capacitor is to supply the reactive energy to the magnet, it is required an 26.6% of extra energy to supply the power losses using this strategy. Notice that this figure of merit is the same regardless the topology analyzed, since the energy from the electrical network during $(t_r + t_{ft})$ is the same, i.e. it is a feature of this kind of strategy (constant power during T_p). The efficiency in the installed energy is given by:

$$\frac{\Delta E_{st}}{E_{st\,max}} = 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 = 1 - \left(\frac{500\,\mathrm{V}}{700\,\mathrm{V}}\right)^2 = 0.49 \tag{37}$$

which means that a 49% of the installed energy is already used. From (31), the minimum capacitor can be obtained:

$$C_{st\,\min} = 277.7\,\mathrm{mF}$$

The next figure of merit relates the maximum energy stored compared to the maximum reactive energy in the magnet, which means how much energy is necessary to store for feeding a given magnet:

$$\frac{E_{stmax}}{E_m} = \frac{\Delta E_{st}}{E_m}\,\frac{E_{stmax}}{\Delta E_{st}} = 2.58$$

which means that it is necessary to store almost 2.6 times the magnet reactive energy. Finally, the normalization current, I_{rec_N}, and the output current of the grid connected rectifier, $I_{rec\,\max}$, are given by:

$$I_{rec_N} = \frac{\Delta E_{act_{[T_p]}}}{\overline{v_{rec}}\,T_p} = 40\,\mathrm{A} \qquad \Longrightarrow I_{rec\,\max} = 40\,\mathrm{A}$$

4.1.2 Topology based on 2Q converter

The topology to be evaluated corresponds to the one presented in Fig. 3. With this structure, the maximum storage voltage could be as high as the dc-link voltage, while the minimum voltage will be defined by aspects related to duty cycle/maximum switch current in the 2Q structure (the lower $v_{st}(t)$, the higher $i_{Ls}(t)$). The rectified voltage is equal to the dc-link voltage. Then, the following values are adopted:

$$V_{st\,\max} = 650\,\mathrm{V}$$
$$V_{st\,\min} = 200\,\mathrm{V}$$
$$\overline{v_{rec}} = 700\,\mathrm{V}$$

Like in the case of boost converter, the strategy used with this topology corresponds to Constant Grid Power, implemented with the Indirect Grid Current Control by dc-link voltage regulation control scheme [6]. Then, the energy coming from the electrical network during $(t_r + t_{ft})$ and power of the grid tied converter, which are repeated here for the sake of clarity, are the same than in the case of the boost:

$$\Delta E_{g_{[t_r+t_{ft}]}} = P_{rec_{avg}}\,(t_r + t_{ft}) = 6.23\,\mathrm{kJ} \qquad \text{with} \qquad P_{rec\,\max} = P_{rec_{avg}} = \frac{\Delta E_{act[T_p]}}{T_p} = 15.975\,\mathrm{kW}$$

Then, using in (31), (32), (33) and (34) the value of $\Delta E_{g_{[t_r+t_{ft}]}}$ previously obtained, and the values of $V_{st\,\max}$ and $V_{st\,\min}$ adopted for this structure, the following values are obtained:

$$\frac{\Delta E_{st}}{E_m} = 1.266$$
$$\frac{\Delta E_{st}}{E_{stmax}} = 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 = 0.905$$
$$\frac{E_{stmax}}{E_m} = \frac{\Delta E_{st}}{E_m}\,\frac{E_{stmax}}{\Delta E_{st}} = 1.4$$
$$C_{st\,\min} = \frac{I_{REF}^2 L_m}{v_{stmax}^2 - v_{stmin}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g_{[t_r+t_{ft}]}}}{\frac{1}{2} L_m\, I_{REF}^2}\right] = 174.2\,\mathrm{mF}$$

From these values, it can be noticed that the amount of extra energy to be stored results equal than in the case of the boost (26.6%), since the strategy adopted is the same. Moreover, a 90.5% of the installed energy is used. Additionally, the maximum installed energy results only a 40% higher than the magnet peak reactive energy, while the capacitor value results lower than in the case of the boost converter. Finally, from (35), the normalization current, I_{rec_N}, and the output current of the grid connected rectifier, $I_{rec\,\max}$, are given by:

$$I_{rec_N} = \frac{\Delta E_{act_{[T_p]}}}{\overline{v_{rec}}\,T_p} = 22.8\,\mathrm{A} \qquad \Longrightarrow I_{rec\,\max} = 22.8\,\mathrm{A}$$

4.1.3 Parallel Structure Configuration

The topology to be evaluated corresponds to the one presented in Fig. 10. With this structure, the maximum storage voltage could be as high as the dc-link voltage, while the minimum voltage will be defined by aspects related to duty cycle/maximum switch current in the 2Q structure (the lower $v_{st}(t)$, the higher $i_{Ls}(t)$). Additionally, the rectified voltage will be limited by the operation of the boost converter ($\overline{v_{rec}} < v_{dc}$). Then, the following values are adopted:

$$\begin{aligned} V_{st\,\max} &= 650\,\mathrm{V} \\ V_{st\,\min} &= 200\,\mathrm{V} \\ \overline{v_{rec}} &= 400\,\mathrm{V} \end{aligned}$$

Like in the case of boost and 2Q converter, the strategy used with this topology corresponds to Constant Grid Power. Then, the energy coming from the electrical network during $(t_r + t_{ft})$ and power of the grid tied converter, which are repeated here for the sake of clarity, are the same than in the case of the boost and 2Q converter:

$$\Delta E_{g_{[t_r+t_{ft}]}} = P_{rec_{avg}}\,(t_r + t_{ft}) = 6.23\,\mathrm{kJ} \qquad \text{with} \qquad P_{rec\,\max} = P_{rec_{avg}} = \frac{\Delta E_{act\,[T_p]}}{T_p} = 15.975\,\mathrm{kW}$$

Then, using in (31), (32), (33) and (34) the value of $\Delta E_{g_{[t_r+t_{ft}]}}$ previously obtained, and the values of $V_{st\,\max}$ and $V_{st\,\min}$ adopted for this structure, the following values are obtained:

$$\begin{aligned} \frac{\Delta E_{st}}{E_m} &= 1.266 \\ \frac{\Delta E_{st}}{E_{stmax}} &= 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 = 0.905 \\ \frac{E_{stmax}}{E_m} &= \frac{\Delta E_{st}}{E_m}\,\frac{E_{stmax}}{\Delta E_{st}} = 1.4 \\ C_{st\,\min} &= \frac{I_{REF}^2 L_m}{v_{stmax}^2 - v_{stmin}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g_{[t_r+t_{ft}]}}}{\frac{1}{2} L_m\, I_{REF}^2}\right] = 174.2\,\mathrm{mF} \end{aligned}$$

In this case, the figures of merit and capacitor value results equal than in the case of 2Q converter. Finally, from (35), the normalization current, I_{rec_N}, and the output current of the grid connected rectifier, $I_{rec\,\max}$, are given by:

$$I_{rec_N} = \frac{\Delta E_{act_{[T_p]}}}{\overline{v_{rec}}\,T_p} = 40\,\mathrm{A} \qquad \Longrightarrow I_{rec\,\max} = 40\,\mathrm{A}$$

4.1.4 Cascaded Structure Configuration

The topology to be evaluated corresponds to the one presented in Fig. 6. With this structure, the maximum storage voltage should be as high as the dc-link voltage, while the minimum voltage will be defined by aspects related to duty cycle/maximum switch current in the 2Q structure (the lower $v_{st}(t)$, the higher $i_{Ls}(t)$). Regarding the grid voltage, it will depend on the kind of structure used as grid tied converter. Then, the following values are adopted:

$$\begin{aligned} V_{st\,\max} &= 650\,\mathrm{V} \\ V_{st\,\min} &= 200\,\mathrm{V} \\ \overline{v_{rec}}_{\max} &= 650\,\mathrm{V} \\ \overline{v_{rec}}_{\min} &= 200\,\mathrm{V} \end{aligned}$$

The efficiency in the installed energy is given by:

$$\frac{\Delta E_{st}}{E_{stmax}} = 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 = 1 - \left(\frac{200\,\mathrm{V}}{650\,\mathrm{V}}\right)^2 = 0.905 \tag{38}$$

which means that a 90.5% of the installed energy is used, like in the boost, 2Q and Parallel structure configuration converters. Since the remaining figures of merit depend on the energy strategy used, they are evaluated for the two operation modes proposed.

Constant grid power (constant P_{rec}):

In order to obtain a constant power in the grid tied converter, the input or output current of such module, $I_{in}(t)$ and $I_{out}(t)$ respectively, must be controlled for obtaining a constant instantaneous power, $P_{rec}(t)$:

$$
\begin{aligned}
P_{rec}(t) = P_{rec\,\max} = V_{in}(t)\, I_{in}(t) = v_{st}(t)\, i_{rec}(t) \\
= \frac{\Delta E_{act[T_p]}}{T_p} \qquad \text{with} \qquad P_{rec\,\max} = \frac{\Delta E_{act[T_p]}}{T_p} = 15.975\,\text{kW}
\end{aligned} \tag{39}
$$

Assuming that the input power is constant, the energy supplied by the electrical network during $t_r + t_{ft}$ results:

$$
\Delta E_{g[t_r+t_{ft}]} = \int_0^{t_r+t_{ft}} i_{rec}(t)\, v_{st}(t)\, dt = \frac{\Delta E_{act[T_p]}}{T_p}\,(t_r + t_{ft}) = 6.23\,\text{kJ}
$$

which is equal to the energy obtained in the boost, 2Q and parallel topologies under Constant Grid Power. Then, using in (31), (32), (33) and (34) the value of $\Delta E_{g[t_r+t_{ft}]}$ previously obtained, and the values of $V_{st\,\max}$ and $V_{st\,\min}$ adopted for this structure, the following values are obtained:

$$
\begin{aligned}
\frac{\Delta E_{st}}{E_m} &= 1.266 \\
\frac{\Delta E_{st}}{E_{stmax}} &= 1 - \left(\frac{V_{st\,\min}}{V_{st\,\max}}\right)^2 = 0.905 \\
\frac{E_{stmax}}{E_m} &= \frac{\Delta E_{st}}{E_m}\,\frac{E_{stmax}}{\Delta E_{st}} = 1.4 \\
C_{st\,\min} &= \frac{I_{REF}^2 L_m}{v_{stmax}^2 - v_{stmin}^2}\left[1 + 2\frac{R_m}{L_m}\left(t_{ft} + \frac{t_r}{3}\right) - \frac{\Delta E_{g[t_r+t_{ft}]}}{\frac{1}{2} L_m\, I_{REF}^2}\right] = 174.2\,\text{mF}
\end{aligned}
$$

Consequently, the figures of merit related to energy result the same than the ones using 2Q or Parallel Structure Configuration topologies. Regarding the output current and power of grid connected converter, they result:

$$
\begin{aligned}
P_{rec\,\max} = P_{rec\,\min} = P_{rec_{avg}} &= \frac{\Delta E_{act[T_p]}}{T_p} = 15.975\,\text{kW} \\
I_{rec\,\min} &= \frac{P_{rec_{avg}}}{V_{st\,\max}} = 24.58\,\text{A} \\
I_{rec\,\max} &= \frac{P_{rec_{avg}}}{V_{st\,\min}} = 79.88\,\text{A}
\end{aligned}
$$

In this condition, the average voltage in C_{st} results $\overline{v_{st}} = 549.85\,\text{V}$. Then, the normalization current and figure of merit related to the output current result:

$$
I_{rec_N} = \frac{\Delta E_{act[T_p]}}{\overline{v_{st}}\, T_p} = 29.06\,\text{A} \Longrightarrow \frac{I_{rec\,\max}}{I_{rec_N}} = 2.75
$$

Notice that the use of a module with constant power implies that the figures of merit related to energy parameters will be equal that the ones obtained with boost, 2Q and Parallel Structure Configuration for Constant Grid Power strategy. On the other hand, the figure of merit related to the output current of the grid tied converter changes, since in the Cascaded Structure Configuration such current is not constant.

Variable grid power (constant i_{rec}):

In this case, the use of a constant current, $I_{rec\,\max}$, determines an input power that varies between $P_{rec\,\max}$ and $P_{rec\,\min}$, as it was previously mentioned. As shown in (5), the value of $I_{rec\,\max}$ depends on the average voltage in the storage element. Additionally, such voltage will depend on a non linear way of $I_{rec\,\max}$, C_{st} and the required $V_{st\,\min}$ required. Then, for a given pair of values of C_{st} and $V_{st\,\max}$, there will be only one value of $I_{rec\,\max}$ that produces the required value of $V_{st\,\min}$, which in turn automatically defines $\overline{v_{st}}$.

Due to the non linear relationship between $I_{rec\,\max}$ and $v_{st}(t)$, the value of $I_{rec\,\max}$ must be obtained by numerically solving the nonlinear equation until the condition $V_{st\,\min} = 200\,\text{V}$ is met, as it is shown in the Appendix B. In this case, this voltage is obtained for $C_{st\,\min} = 175.5\,\text{mF}$, and the values of average voltage in storage element, averaged in the time intervals T_p and $t_r + t_{ft}$, result $\overline{v_{st}} = 547.14\,\text{V}$ and $\overline{v_{st}}_{[t_r+t_{ft}]} = 525.64\,\text{V}$, respectively. Then, the current $I_{rec\,\max}$ will be given by (5), which is repeated here for the sake of clarity:

$$I_{rec\,\max} = \frac{\Delta E_{act_{[T_p]}}}{\overline{v_{st}}\,T_p} = 29.2\,\text{A} \tag{40}$$

while the energy from the electrical network during $t_r + t_{ft}$ results:

$$\Delta E_{g\,[t_r+t_{ft}]} = \int_0^{t_r+t_{ft}} i_{rec}(t)\,v_{st}(t)\,dt = I_{rec\,\max}\,\overline{v_{st}}_{[t_r+t_{ft}]}\,(t_r + t_{ft}) = 5.99\,\text{kJ}$$

Then, the maximum energy variation is:

$$\frac{\Delta E_{st[t_r+t_{ft}]}}{E_m} = 1.275$$

The next figure of merit relates the maximum energy stored compared to the maximum reactive energy in the magnet, which means how much energy is necessary to store for feeding a given magnet:

$$\frac{E_{st\,max}}{E_m} = \frac{\Delta E_{st}}{E_m}\,\frac{E_{st\,max}}{\Delta E_{st}} = 1.41$$

which means that it is necessary to store an additional 41% of the magnet reactive energy. The maximum, minimum and average power are given by:

$$\begin{aligned}
P_{rec\,\max} &= I_{rec\,\max}\,V_{st_{max}} = 29.2\,\text{A}\cdot 650\,\text{V} = 18.98\,\text{kW}\\
P_{rec\,\min} &= I_{rec\,\max}\,V_{st_{min}} = 29.2\,\text{A}\cdot 200\,\text{V} = 5.84\,\text{kW}\\
P_{rec_{avg}} &= \frac{\Delta E_{act[T_p]}}{T_p} = 15.975\,\text{kW}
\end{aligned}$$

Finally, the ratio between the maximum power of the input module and its average power is given by:

$$\frac{P_{rec\,\max}}{P_{rec_{avg}}} = \frac{V_{st_{max}}}{\overline{V_{st}}} = 1.19$$

4.2 Summary

Table 3 shows the figures of merit and element values for each strategy. Notice that most of the parameters strongly depend on the cycle timing and power losses. Hence, they are valid for Table 2 parameters. Additionally, Table 4 shows the most relevant features of each topology. The relationship between the maximum power of the storage converter, $P_{st\,\max}$, and the maximum magnet power, $P_{m\,\max}$ has been presented in Table 3. In the case of Cascaded Structure Configuration, the power of the storage converter is equal to the power of the magnet. In the case of 2Q and parallel structures, the magnet power is equal to storage converter power plus the input converter power. Notice that the magnet power could be positive (during rise time and flat-top time) or negative (during fall time), while the power in the input converter is always positive. Consequently, in 2Q/Parallel structures the maximum positive power is lower than the maximum positive magnet power, while the negative power will be higher than the maximum negative power in the magnet. Then, the expressions for calculating this figure of merit for each topology are given by:

$$\frac{P_{st\max}}{P_{m\max}} = \begin{cases} \dfrac{P_{m\max} - P_{recavg}}{P_{m\max}} & \text{2Q, Parallel} \\ 1 & \text{Cascaded} \end{cases}$$

where $P_{m\max}$ is the maximum power in the magnet, which is coincident with the maximum negative power.

Table 3: Figures of merit and capacitor value for each strategy (valid for Table 2 parameters).

	$\frac{\Delta E_{st}}{E_m}$	$\frac{\Delta E_{st}}{E_{st\max}}$	$\frac{E_{st\max}}{E_m}$	$\frac{I_{rec\max}}{I_{rec_N}}$	$\frac{P_{rec\max}}{P_{recavg}}$	$\frac{P_{st\max}}{P_{m\max}}$	$I_{rec\max}$	C_{st}
Constant grid power								
Boost front-end	1.266	0.49	2.58	1	1	–	40 A	277.7 mF
2Q based topology	1.266	0.91	1.4	1	1	1.08	22.8 A	174.2 mF
Cascaded	1.266	0.91	1.4	2.78	1	1	79.88 A	174.2 mF
Parallel	1.266	0.91	1.4	1	1	1.08	40 A	174.2 mF
Variable grid power								
Cascaded	1.275	0.91	1.41	1	1.19	1	29.2 A	175.5 mF

Table 4: Features of the analyzed topologies.

Topology	**Input power control**	**Control complexity**	**Cycle timing**	**Controller robustness**
Boost	Yes	Low	NO	High
2Q based	NO	High	YES	Low
Cascaded	Yes	Medium	NO	High
Parallel	Yes	Medium	NO	High

As can be seen from Tables 3 and 4, the boost topology presents the most disadvantageous metrics, while it allows a control on the input power, robustness and low complexity on the control side. On the other hand, the 2Q topology presents the most advantageous metrics; however, its control complexity, low robustness to changes in the operating point and need of knowing the cycle timing[2] makes it an impractical option. Note that the improvement in the energy indicators is obtained by increasing the discharge depth, which impacts in the capacitor technology. Then, film capacitors should be used instead of electrolytic, with the consequence of a drastic increase of price and volume. Concerning the proposed topologies, it can be seen that both presents the advantages associated to the boost (direct power control, robustness) and to the 2Q converter (acceptable figures of merit).

[2]Cycle timing refers to the information of start and end time of a current cycle.

5 Simulations

Simulations were conducted to evaluate the proposed structures under Grid Peak Power Limitation strategy. The parameters of the cycle, load and of each structure are the ones defined in Section 4. Consequently, the elements and parameters of each structure is defined to have a maximum and minimum storage voltage $V_{st\,\max} = 650\,\mathrm{V}$ and $V_{st\,\min} = 200\,\mathrm{V}$. It is worth noting that the aim of the simulations is to qualitatively show the current and voltage waveforms in the different stages of each topology.

Figure 14 shows the current and voltage waveforms of the Cascaded Structure Configuration topology for the case in which the grid tied converter is operated in order to obtain constant input power (Constant Grid Power operation). In the upper graphic, a filtered version of the currents in the dc-link node (i_q and i_e) is shown. It can be seen that the 2Q converter supplies the full current demanded by the H-bridge, which denotes that the 2Q must manage the full magnet power.

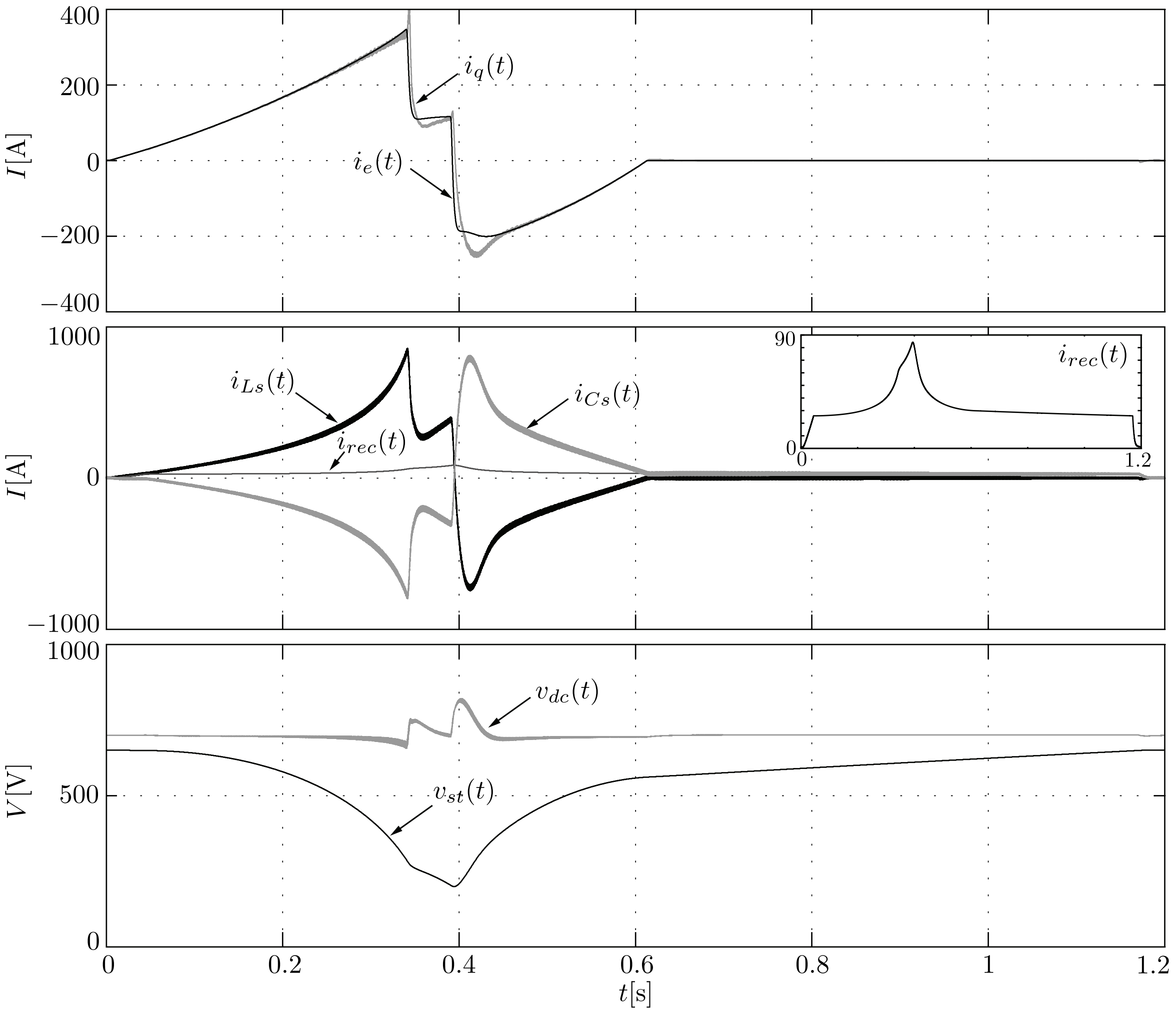

Figure 14: Simulation. Cascaded structure configuration (constant grid power). Upper: H-bridge average current $i_e(t)$, average output current of 2Q converter $i_q(t)$. Middle: output rectifier current $i_{rec}(t)$, current in inductor of storage converter $i_{Ls}(t)$, current in capacitor of storage converter $i_{Cs}(t)$. Bottom: dc-link voltage $v_{dc}(t)$, storage capacitor voltage $v_{st}(t)$.

In the center graphic are shown the currents of the storage capacitor node. It can be observed that, for the particular case of the generated magnet current cycle, the input current of the 2Q converter (i_{Ls}) is mainly given by the discharge of the storage capacitor, while the current supplied by the grid tied converter is considerably lower. Additionally, it can be seen that constant power feature of the grid tied converter produces an output current i_{rec} that varies accordingly to the changes in the storage capacitor voltage. The maximum and minimum values of i_{rec} (25.8 A, 84 A) are similar to the ones obtained using the equations provided in Section 4. In the bottom graphic is shown the dc-link voltage and storage capacitor voltage. Moreover, it can be observed that the voltage in C_{dc} is constant and that the transient responses are generated when there are differences between the current i_e and i_q. Regarding v_{st}, it can be observed that its minimum voltage is produced at the end of flat-top and that it recovers part of its energy during the fall time. After the fall time, the voltage v_{st} increases due to the charge given by i_{rec}.

Figure 15 shows the current and voltage waveforms for the topology Cascade, considering that the grid tied converter is controlled to produce a constant output current (Variable Grid Power operation). Concerning currents in the dc-lilnk node, it can be seen that there are no differences with respect to the previous case (the 2Q converter must supply all the current required by the H-bridge).

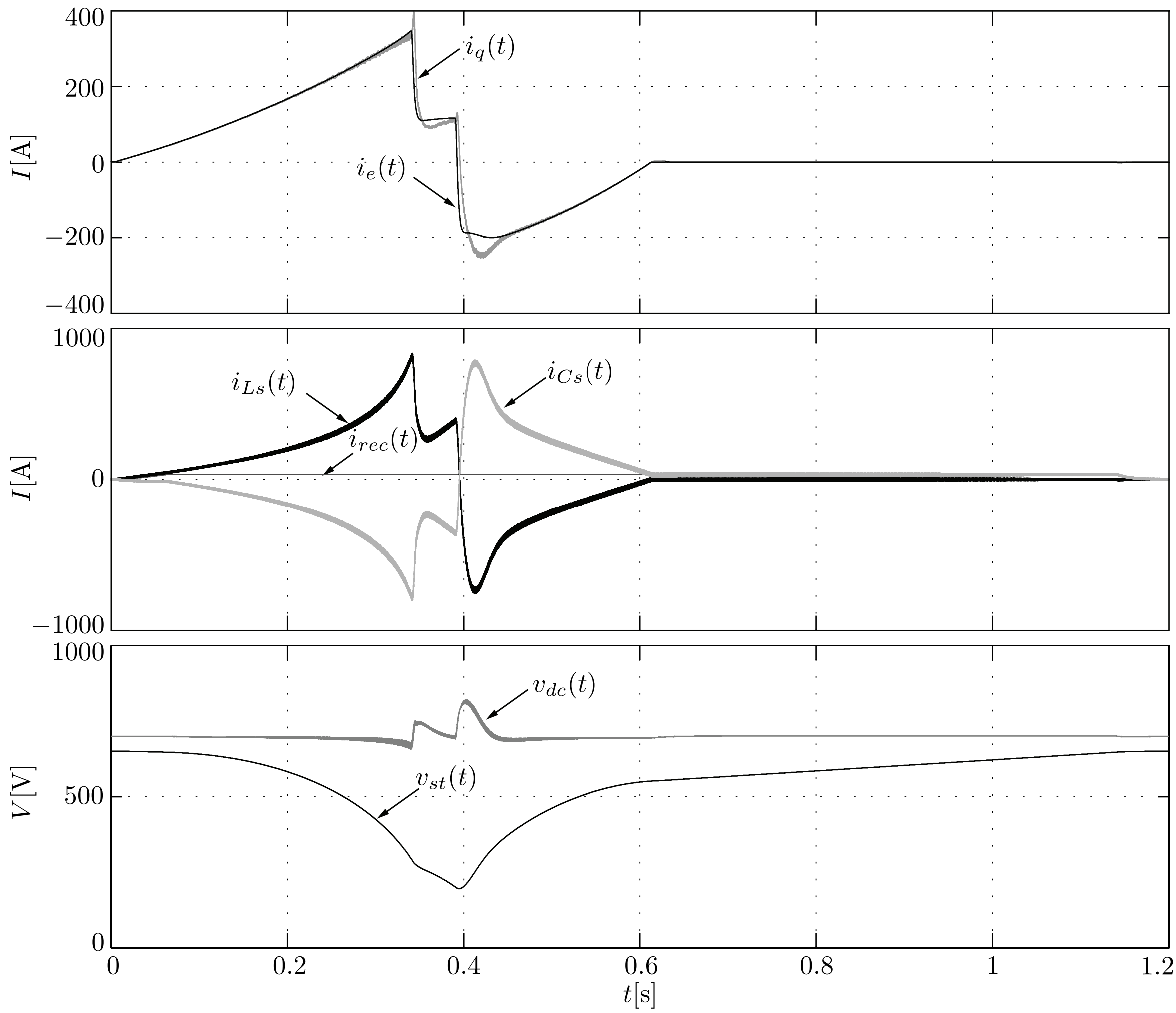

Figure 15: Simulation. Cascaded structure configuration (variable grid power). Upper: H-bridge current $i_e(t)$, average output current of 2Q converter $i_q(t)$. Middle: output rectifier current $i_{rec}(t)$, current in inductor of storage converter $i_{Ls}(t)$, current in capacitor of storage converter $i_{Cs}(t)$. Bottom: dc-link voltage $v_{dc}(t)$, storage capacitor voltage $v_{st}(t)$.

Analyzing the storage capacitor node can be observed that, like in the previous case, the current that the 2Q converter must handle is mainly given by the storage capacitor current. The main difference with the Constant Grid Power operation previously analyzed is that the output current of the grid connected converter is constant during the cycle time T_p. Concerning the voltages v_{dc} and v_{st}, it can be seen that there are no remarkable differences with the previous case.

Figure 16 shows the current and voltage waveforms of the parallel structure configuration operating with constant i_{rec} (Constant Grid Power). In the upper graphic are shown the filtered version of the currents in the dc-link node. In this case, the H-bridge current is supplied by the 2Q converter and the boost converter through the currents i_q and i_d, respectively. Hence, the current in the 2Q converter is given by $i_e - i_d$. This feature is shown in the figure, where it can be seen that in the case in which the current i_e is positive, the current i_q is lower than i_e. On the other hand, when i_e is negative, the current i_q is higher than i_e.

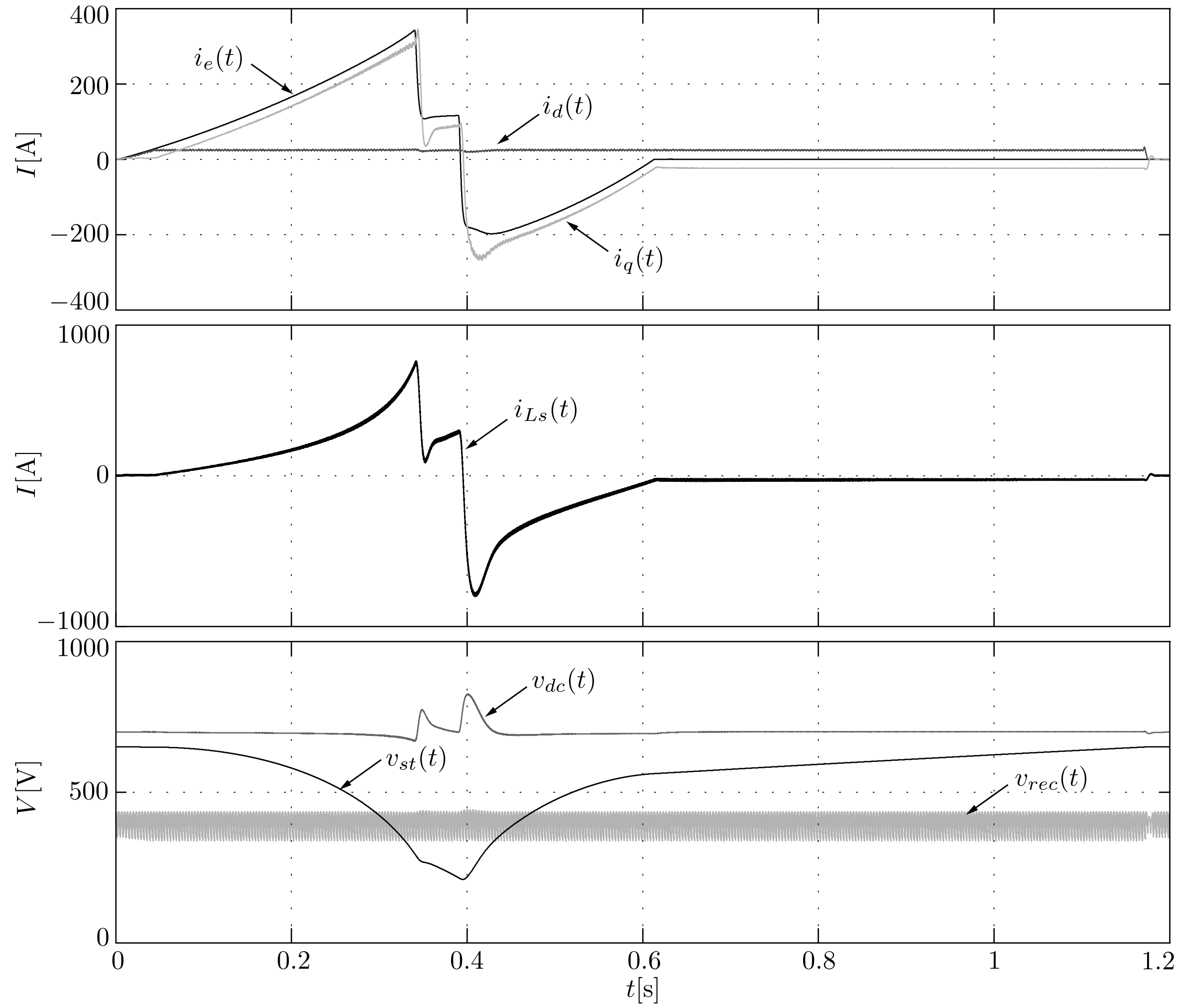

Figure 16: Simulation. Parallel structure configuration. Upper: H-bridge current $i_e(t)$, average current in boost diode $i_d(t)$, average output current of 2Q converter $i_q(t)$. Middle: current in inductor of storage converter $i_{Ls}(t)$. Bottom: dc-link voltage $v_{dc}(t)$, output rectifier voltage $v_{rec}(t)$, storage capacitor voltage $v_{st}(t)$.

In the center graphic is shown the current in the inductor of the 2Q converter, which in this case is defined only by the discharge of the storage capacitor. Finally, in the bottom graphic can be seen the voltages $v_{st}(t)$, $v_{dc}(t)$ and $v_{rec}(t)$. It can be observed that the voltage v_{dc} is always higher than both v_{st} and v_{rec}, which is a condition to be met due to the operation of the 2Q and boost converter. Additionally, it can be noticed that v_{st} presents a variation similar to the cases previously shown (Cascaded structure configuration operating with Constant/Variable Grid Power).

Figure 17 shows the magnet power P_m, 2Q converter power P_{2Q} and grid connected converter power P_{rec} for the different topologies. In the case of Parallel Structure configuration can be seen that the power in the input converter is constant (16.8 kW). Moreover, the power in the 2Q converter is lower than the magnet power when the magnet is absorbing power, while it is higher than the magnet power when the magnet is giving back energy. This feature is related to the previous comment on the currents in the dc-link node for this topology.

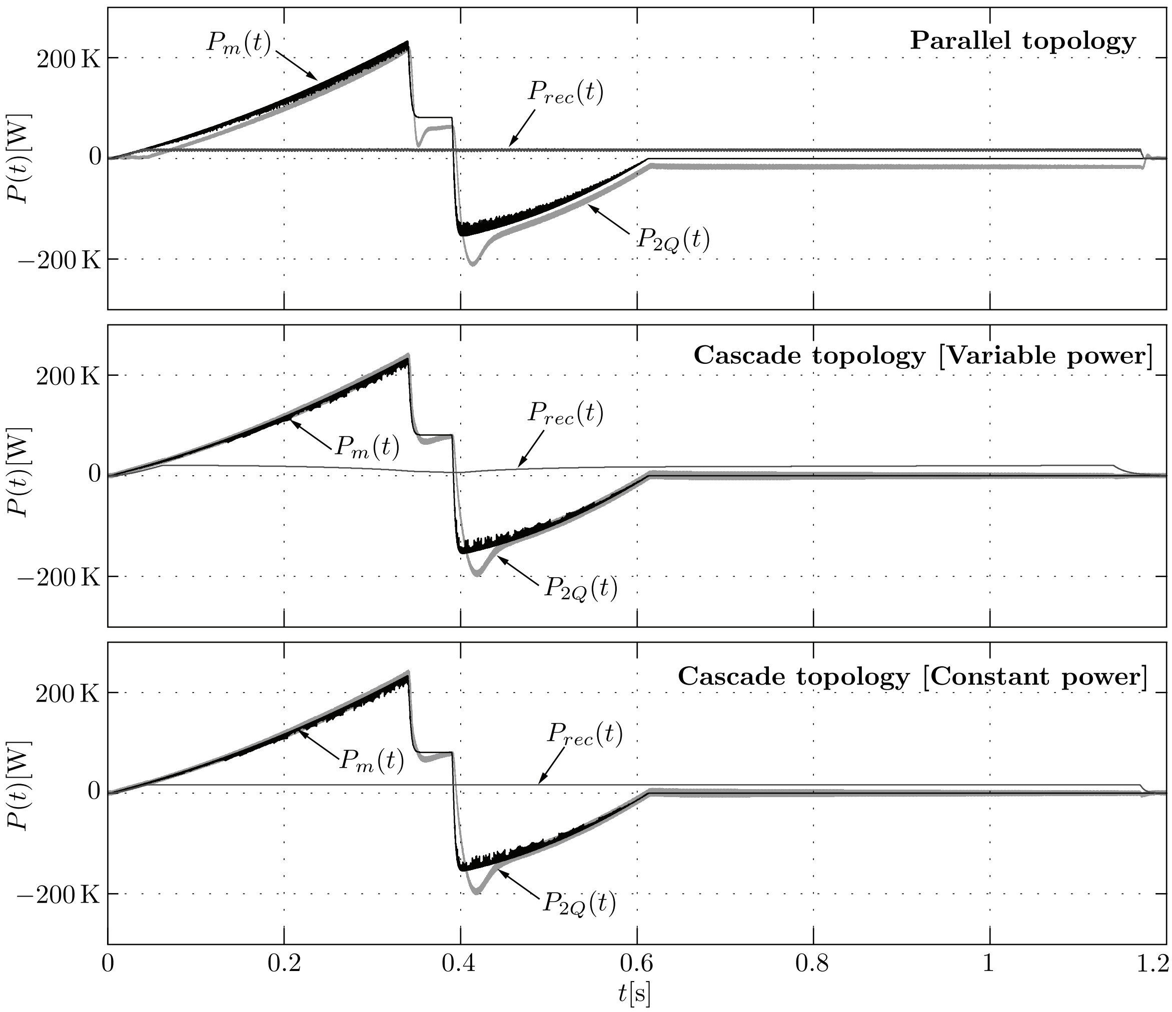

Figure 17: Simulation. Magnet power $P_m(t)$, input rectifier power $P_{rec}(t)$, storage converter power $P_{st}(t)$. Upper: Cascaded configuration (Constant power). Middle: Cascaded configuration (Variable power). Bottom: Parallel configuration.

In the center graphic can be seen the power involved in the Cascaded Structure Configuration operating with Variable Grid Power, i.e. the output current in the grid tied converter is constant. This characteristic can be noticed in the input power P_{rec}, where can be seen that the power is minimum at the end of flat-top, where the voltage in the storage converter is minimum, and is maximum when the storage voltage is maximum ($P_{rec\,\max} = 20\,\mathrm{kW}$, $P_{rec\,\min} = 6\,\mathrm{kW}$). Additionally, it can be seen that the power that the 2Q converter must handle is close to the magnet power. Finally, in the bottom graphic is shown the case of the Cascaded Structure Configuration topology operating with constant input power, where the only difference with the previous case is that the input power P_{rec} remains constant during the cycle time T_p (16.8 kW).

6 Conclusions

This section outlines the conclusions of this report. First, a brief introduction including the key aspects considered in the study is presented in Section 6.1. Then, a summary of the main points of the two proposed topologies is described in Section 6.2. Finally, a more detailed description of each one is presented in Section 6.3.

6.1 Objectives

The analysis of topologies 2Q [6] and boost [7] has allowed to define some important features that should be accomplished, leading to the development of new topologies:

- Input current control: this feature allows to keep bounded the input current when transient responses occur (for instance, a change in the grid voltage).
- Possibility of higher discharge of the energy storage element: in this way, a better use of the installed energy is obtained (in theory, a 100% of utilization could be obtained if the storage element is completely discharged). The figure of merit associated to this concept is $\Delta E_{st}/E_{stmax}$.
- Minimizing the energy stored: this feature allows to decrease the size associated to the energy storage element. It is mainly obtained by minimizing the maximum voltage in the energy storage element. The figure of merit associated to this concept is E_{stmax}/E_m.

In addition, the constant voltage in the H-bridge dc-link determines that is not necessary a feed-forward term of this voltage into the control loops of the H-bridge to account for its gain variation. In general, the useful range of the H-bridge duty cycle determines that there will be a minimum dc-link voltage required to apply the necessary voltage on the load. These issues, which are not critical, are mitigated in the 2Q converter configuration due to its constant dc-link voltage feature.

From these characteristics, the new topologies must satisfy the following common aspects:

- Use of a structure for controlling the input current: the aim is that the current supplied by the electrical network can be bounded in a controlled way, and to have a high rejection to grid voltage disturbances.
- Separation between the H-bridge dc-link capacitor and energy storage capacitor: in order to improve the energy storage figures of merit, the voltage depth of the storage element must not be limited for another part of the circuit. The easiest way to do this is to separate the energy storage element from the dc-link of the H-bridge. Notice that, since the energy storage element is not located in the dc-link, a lower maximum voltage and higher voltage swing could be used, leading to an improvement in both the utilization of installed energy and maximum installed energy. Then, there will be a structure that connects both capacitors, which should be (at least) a 2 quadrant converter.

6.2 Summary

The proposed topologies present the following common features:

- Direct power control of the grid connected converter.
- Wider variation in the storage element voltage.
- Lower maximum installed energy.

The differences between the proposals are summarized as follows:

Cascaded structure topology

- Peak power of the grid connected converter for
  - Variable Grid Power (Constant i_{rec}): equal to the average active power multiplied by the factor $\frac{V_{st\,\max}}{V_{st}}$.

– Constant Grid Power (Constant P_{rec}): equal to the average active power.

- Peak power of the storage converter equal to the maximum magnet power.
- Each regulation loop is related to one converter and one task (2Q converter: dc-link voltage control; ac/dc converter: storage voltage control.)

Parallel structure topology

- Peak power of grid connected converter: average active power.
- Peak power of storage converter: difference between peak magnet power and average active power. Then, during magnet energy recovery (negative magnet power), the instantaneous power of the storage converter is higher than the instantaneous magnet power.
- Two regulation loops for two tasks (dc-link voltage control/storage voltage control). However, while the control of dc-link voltage is related to only one converter (2Q), the charge of the storage element is performed through two converters (boost+2Q).

6.3 Proposed topologies features

Cascaded structure topology

- A topology named cascaded structure topology was presented. This topology arises from the separation between the dc-link (input voltage for the H-bridge) and the storage element. In such sense, a bidirectional current converter (2Q converter) between the dc-link and the storage element was used to control the energy flow between them. Additionally, the storage capacitor was charged through an AC/DC converter, having into account two premises: 1) to charge the capacitor in a controlled way from 0 V up to a maximum value, and 2) to keep bounded the current supplied by the electrical network.
- A characteristic of this topology is that the 2Q converter must handle the full load power. In case of high currents, an interleaved scheme can be devised for the 2Q converter.
- The output voltage of the grid tied converter is the storage element voltage. Then, if the output current of such module is controlled to a constant value, the power drawn from the grid will vary. On the other hand, if the control of the grid tied converter is defined to obtain a constant grid power, the output current of this converter will vary.
- Regarding the control system, the possibility of having two controlled converters (AC/DC and 2Q) has allowed the implementation of two regulation loops with two different and well defined functions. The aim of the first regulation loop, which controls the 2Q converter, is to regulate the dc-link to a constant value. Due to the involved voltages, and considering as input the storage capacitor and as output the dc-link capacitor, the structure of the 2Q converter results in a boost with bidirectional current capability. The second regulation loop, which controls the AC/DC converter, is employed to restore the energy in the storage capacitor from the electrical network. In such sense, the AC/DC converter supplies a controlled current so as to achieve that the storage voltage reaches the value set from the reference before the arrive of the next cycle. The energy restored to the capacitor implies that the AC/DC converter must supply the energy consumed during the magnet **cycle** generation. An advantageous feature of this control scheme is that the two variables ($v_{st}(t)$ and $v_{dc}(t)$) can be regulated in an easy and independent way.

Parallel structure topology

- Other topology developed is the named Parallel structure topology. This structure arises as a variation of the topology 2Q analyzed in [6]. The main difference is the inclusion of a controlled converter as input stage, which allows controlling the current supplied by the electrical network. The proposed scheme is composed by two converters connected in parallel to the dc-link (input voltage of the H-bridge), so as to supply each one a part of the energy required by the magnet. The input stage, formed by a diode bridge and boost converter, supplies a controlled current to the

H-bridge that is function of the active energy. The control of this current allows a high rejection to disturbances in the grid voltage. The 2Q converter is used as a coupling stage between the dc-link and the storage capacitor. This converter is bidirectional in current, which allows to deliver energy from the storage capacitor during rise and flat-top times and to receive energy during fall time and time between cycles. Like in the cascade structure configuration, in this topology the dc-link capacitor and the storage capacitor are not the same point, which allows a constant voltage in the dc-link and a wide variation in the storage voltage (in theory, from 0 V up to the maximum voltage in the dc-link capacitor).

- A characteristic of this structure is that, if the *energy balance* strategy is used, the requirements of power could be divided. Then, the input stage could be sized for the instantaneous active power (maximum power: peak active power), while the 2Q could be sized for the instantaneous reactive power (maximum power: peak reactive power). On the other hand, if *limited input power* strategies are implemented, the ratings of the grid tied converter are decreased (maximum power: average active power), while the ratings of the 2Q converter are worst (maximum power: peak magnet power+average active power). In case of high currents, an interleaved scheme can be devised for the 2Q converter.
- The constant output voltage feature of the grid tied converter, together with a control of the output current of such converter to a constant value, allows taking a constant power from the electrical network.
- With respect to the control system, the use of two controlled converters (2Q and boost) has allowed the implementation of two regulation loops with different functions, likewise in the cascade structure configuration. The first regulation loop, which controls the 2Q converter, maintains the dc-link voltage to a constant value, which determines that the 2Q output current plus the boost output current is equal to the current demanded by the H-bridge. The aim of the the second regulation loop, which controls the boost converter, is to regulate the voltage in the storage capacitor to a constant value. The regulation loop must restore the energy in C_{st} before the arrive of the next cycle. This feedback loop produces the recharge of the storage capacitor in an indirect way, since the current that charges the capacitor circulates between the boost and 2Q converters. Unlike the cascade structure configuraton, the implemented control scheme produces an interaction between the two regulation loops that becomes more complex the adjustment of the controllers.

6.4 Results

The following are the results obtained from the design example in Section 4. Note that the values obtained are valid only for the parameters presented in such section, and they can vary according to the cycle and/or load parameters.

- The boost capacitor is 59% higher than the one in the remaining structures.
- Theoretically, it will be possible to use almost 90% of installed energy; in the boost case, this number decreases up to 50%.
- In general, the maximum energy to be stored is aproximately 40% higher than the maximum magnet energy, except in the case of the boost, where reaches 160%.
- The improvement in the energy indicators is obtained by increasing the discharge depth, which impacts in the capacitor technology. Then, film capacitors should be used instead of electrolytic, with the consequence of a drastic increase of price and volume.
- The input power can be limited to the power losses.
- Both the 2Q based and the proposed parallel structure must withstand a 10% more power than the negative magnet power.

Acknowledgment

This work has been performed under a research contract between the European Organization for Nuclear Research (CERN) Switzerland and the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), with the support of the European Particle Physics Latin American Network (EPLANET), the Universidad Nacional de Mar del Plata (UNMDP), the Ministerio de Ciencia, Tecnología e Innovación Productiva (MINCYT), and the Agencia Nacional de Promoción Científica y Tecnológica (ANPCYT).

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A Figures of merit

In order to make a fair comparative between the developed strategies, some important figures of merit for each strategy are defined. One of these parameters is the variation of energy in the storage element. This parameter plays a key role in the selection of C_{st}, since the energy gives an idea of the storage element sizing. Associated to this, the minimum voltage in the storage element and some particular values of the voltage v_{st} are calculated, as they are important for both operative aspects (for example, the minimum v_{st} must be higher than the maximum grid voltage) and elements sizing (for example, the maximum voltage in the capacitors define the voltage that must withstand the semiconductors). Another important parameter that was evaluated is the maximum current of the input stage, as it defines the sizing of the diode bridge rectifier and the rating of the connection on the distribution network.

In the following, an analysis based on energy flow between the grid, the storage element and the magnet is devised. In order to perform such analysis, the following nomenclature is used:

$$\Delta E_{[t_b - t_a]} = \int_{t_a}^{t_b} i(t) v(t)\, dt \tag{41}$$

where $\Delta E_{[t_b - t_a]}$ represents the energy variation in the time interval between t_a and t_b. Notice that, regardless the strategy, the energy supplied by the grid in a period T_p must equal the energy losses of the system in such period, which are given by the efficiency of the power converters and the dissipated power on resistive components. In this analysis, only magnet losses were considered.

A.1 Figures of merit

A.1.1 Maximum energy variation in the storage element normalized to the maximum reactive energy in the magnet

$$\frac{\Delta E_{st}}{E_m} = \frac{(1/2)\, C_{st}\, \left(v_{st}{}^2_{max} - v_{st}{}^2_{min}\right)}{E_m} \tag{42}$$

where $E_m = (1/2) L_m\, I_{REF}^2$.

A.1.2 Maximum energy variation in the storage element normalized to its maximum energy

$$\frac{\Delta E_{st}}{E_{st\,max}} = \frac{(1/2)\, C_{st}\, \left(v_{st}{}^2_{max} - v_{st}{}^2_{min}\right)}{(1/2)\, C_{st}\, v_{st}{}^2_{max}} = 1 - \left(\frac{v_{st\,min}}{v_{st\,max}}\right)^2 \tag{43}$$

A.1.3 Maximum energy in the storage element normalized to the maximum reactive energy in the magnet

$$\frac{E_{st\,max}}{E_m} = \frac{(1/2)\, C_{st}\, v_{st}{}^2_{max}}{E_m} = \frac{\Delta E_{st}}{E_m} + \frac{(1/2)\, C_{st}\, v_{st}{}^2_{min}}{E_m} = \frac{\Delta E_{st}}{E_m}\, \frac{E_{st\,max}}{\Delta E_{st}} \tag{44}$$

A.1.4 Maximum input current of the converter normalized to constant rectifier output current

$$\frac{I_{rec\,max}}{I_{rec_N}} \tag{45}$$

The normalization current, I_{rec_N}, is the minimum current in the output of the grid connected converter that allows recovering the active energy. Consequently, this current is considered constant during the cycle, T_p. Then, since the power from the grid must equal the active power:

$$\boxed{I_{rec_N} = \frac{\Delta E_{act\,[T_p]}}{\overline{V}_{orec}\, T_p} = \frac{I_{REF}^2 R_m \left(t_{ft} + \frac{t_r + t_f}{3}\right)}{\overline{V}_{orec}\, T_p}} \tag{46}$$

where $\overline{V_{orec}}$ is the output voltage of the input stage converter averaged in the cycle period T_p.

Notice that the parameter $\frac{\Delta E_{st}}{E_{st\,max}}$ is given by the maximum and minimum voltages in the storage element; hence, this parameter does not depend on the strategy adopted, but on the selected voltages. Moreover, the information given by the parameter $\frac{E_{st\,max}}{E_m}$ could be obtained from $\frac{\Delta E_{st}}{E_m}$, as shown in (44).

B Numerical solution for operational conditions of Cascaded Structure Configuration

In the following appendix is analyzed how to determine the value of output current of the grid connected rectifier, I_{rec}, in the topology Cascaded Structure Configuration, when this current is constant. In this conditon, considering that the output voltage of the grid connected converter is the storage element voltage, the power of such module will vary between $P_{rec\,\max}$ and $P_{rec\,\min}$. These values will depend on the maximum and minimum storage voltage, $v_{st\,\max}$ and $v_{st\,\min}$, respectively, and on the current $I_{rec\,\max}$. As it is observed in (47), the time evolution of the storage voltage (and hence, its maximum and minimum value) depend on I_{recmax} in a non-linear way:

$$I_{rec\,\max} = C_{st}\,\frac{d\,v_{st}(t)}{dt} + \frac{p_m(t)}{v_{st}(t)} \tag{47}$$

where $v_{st}(t)$ is the storage voltage, C_{st} is the storage element and $p_m(t)$ is the magnet power. Then, since $p_m(t)$ is imposed by the application, the values of C_{st} and $I_{rec\,\max}$ must be defined in order to satisfy the following conditions:

- $v_{st}(t)$ must be the same at the beginning and at the end of the cycle, i.e. $v_{st}(0) = v_{st}(T_p)$, and
- $v_{st}(t)$ must have a voltage swing between $v_{st\,\max}$ and $v_{st\,\min}$

Consequently, the use of these two conditions on (47) will lead to two equations with two unknowns, i.e. there is only a pair ($I_{rec\,\max}$, C_{st}) that solves (47). Moreover, the achievement of these conditions must be performed by solving the non-linear expression given by (47). In such sense, the value of current must be obtained by numerically solving such expression. In this case, it is proposed to solve a discrete approximation of (47), by using the following expression for the derivative term:

$$\frac{dv_{st}(t)}{dt} \approx \frac{v_{st}[k+1] - v_{st[k]}}{T_s} \tag{48}$$

where T_s is the discretization time. Then, the discrete expression of the storage element voltage results:

$$v_{st}[k+1] = \frac{T_s}{C_{st}}\left[I_{rec\,\max} - \frac{p_m[k]}{v_{st}[k]}\right] + v_{st}[k] \tag{49}$$

If the condition $v_{st(t=T_p)} = v_{st(t=0)}$ is met, the value of C_{st} imposes a value for $I_{rec\,\max}$, which in turns defines how much energy is supplied from the electrical network during $t_r + t_{ft}$ and the value of minimum voltage obtained, $v_{st\,\min}$. Then, expression (49) is solved by changing the value of C_{st} until obtaining a current I_{recmax} that produces a minimum voltage as required ($v_{st\,\min} = 200\,\mathrm{V}$, in this case).

C Review history

Table C5: Review history.

Rev.	Chapter(s)	Reviewer(s)	Date
0.0	all	SM, RGR	$2015 - 07 - 15$
0.1	(3): Figures of control system. Figure topology proposed by CERN	SM, RGR	$2015 - 12 - 01$
0.2	all	SM	2017-02-16
1.0	all	KP	2017-05-29
2.0	all	KP	2017-09-04