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# ON THE TYPE OF COOPER PAIRING IN THE HIGH-TEMPERATURE SUPERCONDUCTIVITY

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ABSTRACT.

The Hubbard model with repulsive interaction is considered as a toy model for the High-Temperature SuperConductivity (HTSC). An application the temperature Green's functions method demonstrates the existence of the antiferromagnetic state in the case of the half-filled band. It is shown also that the Cooper pairing with a non-zero angular momentum l does exist in both three-dimensional (3D) and two-dimensional (2D) Hubbard models in the low-density limit. The type of Cooper pairing turns out to be completely different for these two cases, namely the p-pairing in the 3D-model and the specific type of pairing with l=2 in the 2D-model. Formulae for the critical temperature $T_c$ in the low-density limit are obtained for both 3D- and 2D-models. Arguments are presented that the obtained types of Cooper pairings should exist not only in the low-density limit but also for the real HTSC.

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## Introduction.

The Hubbard model was considered by many authors (see [1]-[3]) as the most favourite candidate to become a model for the High Temperature SuperConductivity (HTSC). At any case the HTSC-model must explain the transition from the antiferromagnetic behaviour of the system at small values of the doping coefficient to the superconducting behaviour at sufficiently large values of this coefficient. The Hubbard model with repulsive interaction indeed behaves as an antiferromagnetic for the case of the half-filled band. It is a well known fact. But a possibility of superconductivity in this model is a problem. The point is that the repulsive interaction makes the ordinary Cooper pairing with the zero angular momentum (s-pairing) impossible.

Many authors (see[4]-[8]) try to find an explanation for the HTSC-phenomenon in the framework of the topologically nontrivial effective action containing the Chern-Simons terms.

In this preprint another opportunity namely that corresponding to the Cooper pairing with a nonzero angular momentum l in the repulsive Hubbard model is investigated. We consider both three-dimensional (3D) and two-dimensional (2D) Hubbard models with repulsive interaction. It turns out that such a pairing does really exist for both cases (2D and 3D) and in any case in the so-called low-density limit when the fermion density becomes small due to an appropriate choice of the doping coefficient. It is clear that all the results are valid also in the symmetric case when the fermion density is close to its upper limit (we have to go from fermions to holes in this case). It is interesting that the form of the Cooper pairing is completely different for 3D- and 2D-models. Namely in the 3D-model we obtain the possibility for the p-pairing and we come to the system like the superfluid He in this case. As to the 2D-model we show the possibility of pairing with l=2. We call such a pairing the quasi-d-pairing because it is different also from the ordinary d-pairing.

In order to explain this term it should be said that we have obtained the effective interaction potential $V(\vec{k},\vec{k}')$ between two fermions near the Fermi surfase (Fermi line in the 2D-case). In the low-density limit such a surface (line) is a sphere (circle) $|\vec{k}|=k_F$ ($k_F<<1$).In the 2D-case we have $\vec{k}=k_F\vec{n}$, $\vec{k}'=k_F\vec{n}'$, where n,n' are unite vectors of the form

$$\vec{n} = (\cos\varphi, \sin\varphi) \quad , \quad \vec{n}' = (\cos\varphi', \sin\varphi') .$$

For a system with a rotational symmetry the potential $V(\vec{k},\vec{k}')$ may depend only on $(\vec{n},\vec{n}') = \cos(\varphi - \varphi')$ in the vicinity of the Fermi line. In particular the potential V which leads to the d-pairing has a form

$$V_d = - g \cos(2\varphi-2\varphi') = - g \cos2\varphi \cos2\varphi' - g \sin2\varphi \sin2\varphi'.$$

But in our case we get the potential $V(\vec{k},\vec{k}')$ which contains several terms. The term which leads to the pairing with l=2 has a for with $\cos(2\varphi+2\varphi')$ instead of $\cos(2\varphi-2\varphi')$. It is interesting that this quasi-d-potential leads to pairing for any sign of the coefficient g because it has coefficients of opposite signs before $\cos2\varphi \cos2\varphi'$ and $\sin2\varphi \sin2\varphi'$.

$$V_{quasi-d} = -g \cos(2\varphi+2\varphi') = g \cos2\varphi \cos2\varphi' + g \sin2\varphi \sin2\varphi'.$$

The further organization of this preprint is as follows. In the first section we apply the temperature Green's function method to the Hubbard model and derive the perturbation diagram techniques with anomalous Green's functions which allows us to describe the antiferromagnetic state for the model with an arbitrary dimension D in the case when the average number of fermions is close to the number of lattice points. If the renormalized doping coefficient becomes sufficiently large this antiferromagnetic state disappears and our system goes into the normal state. But in this case small denominators of Green's functions near the Fermi surface may lead to singularities of the scattering amplitudes and further to superconductivity.

The investigation such scattering amplitudes is done in the second section where an equation of Bethe-Salpeter type for the scattering amplitudes with particle momenta near the Fermi surface is derived. The solution of this equation for the 3D-model shows that singularities arise indeed in the channel of the p-wave scattering, and the critical temperature $T_c$ corresponding to this singularity is obtained. At $T<T_c$ the system should behave like the superfluid He and the Cooper pairing is the p-pairing in this case.

The 2D-model is considered in the third section. It is shown that the first approximation for the irreducible part of the scattering amplitude contributes only to the s-channel and does not lead to any singularities of the scattering amplitudes. The second approximation for the irreducible part of the scattering amplitude leads to singularities of the scattering amplitude in the channel with l=2, and the corresponding critical temperature is also obtained. The very cumbersome calculations for obtaining the abovementioned second approximation are given in the Appendix.

## 1.The temperature Green's function method in the Hubbard model.

The main object of our work is the Hubbard model with the following Hamiltonian

$$\hat{H} = -t \sum_{\langle i,j \rangle} a^{+}_{is} a_{js} - \lambda \sum_{i,s} \hat{n}_{i} + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}, \tag{1.1}$$

where

$$\hat{n}_{is} = a^{+}_{is} a_{is} \tag{1.2}$$

are Fermi creation and annihilation operators, s= ↑ or ↓ is a spin index, i means a point of the D-dimensional lattice with periodical boundary conditions, means a summation over the nearest neighbours pairs <i,j>.

Properties of the model (1.1) do not depend on the sign of $t$ but depend drastically on the sign of U. The value $\lambda$ is a chemical potential parameter.This parameter plays a role of a "doping" coefficient. Namely we can change an average number of fermions in the model by changing .

Using the Fourier expansion

$$a_{is} = N^{-1/2} \sum_{k} a_{ks} e^{i(\vec{i},\vec{k})} , \qquad a^{+}_{is} = N^{-1/2} \sum_{k} a^{+}_{ks} e^{-i(\vec{i},\vec{k})} , \tag{1.3}$$

where $a_{ks}$ , $a^{+}_{ks}$ are Fermi operators in the momentum representation we can rewrite the Hubbard Hamiltonian (1.1) in the form

$$\hat{H} = \sum_{k,s} ( \mathcal{E}_0(k) - \lambda) a^{+}_{ks} a_{ks} + (U/N) \sum_{k_1+k_2=k_3+k_4} a^{+}_{k_1\uparrow} a^{+}_{k_2\downarrow} a_{k_4\downarrow} a_{k_3\uparrow} , \tag{1.4}$$

where

$$\mathcal{E}_0(k) = -2t \sum_{i=1}^{D} \cos k_i \tag{1.5}$$

has a meaning of a quasiparticle energy in the momentum space ($|k_i| < \pi$ ,i=1,...,D).

We are going now to apply the Matsubara-Abrikosov-Gor'kov-Dzyaloshinskii temperature diagram perturbation theory [9] to the Hubbard Hamiltonian (1.4). The simplest way to derive it is to use the functional integral approach. The functional integral has a form

$$D\mu\, e^{S} \tag{1.6}$$

where $D\mu$ is the integration measure, S is an action functional, corresponding to the Hamiltonian of the system in question.



The S-functional, corresponding to the Hubbard Hamiltonian (1.4) looks like

$$S = \sum_{p,s} (i\omega - \mathcal{E}_0(k) + \lambda)\, a^{*}_{s}(p)\, a_{s}(p) - (U/\beta N) \sum_{p_1+p_2=p_3+p_4} a^{*}_{\uparrow}(p_1)\, a^{*}_{\downarrow}(p_2)\, a_{\downarrow}(p_4)\, a_{\uparrow}(p_3) . \tag{1.7}$$

Here $a_s(p), a^{*}_s(p)$ are Grassmann variables of the functional integration which anticommute with each other, p means a four-momentum

$$p = (\omega, \vec{k}) \quad , \quad \omega = (2\pi/\beta)(n + 1/2) , \qquad k_i = 2\pi n_i / L \quad , \tag{1.8}$$

where $n, n_i$ are integers (i=1,...,D, $1 < n_i < L$, $L^{D} = N$ is a number of lattice points) and $\omega = (2\pi / \beta)(n + 1/2)$ is the so-called Matsubara frequence. The parameter $\beta$ is an inverse temperature $T^{-1}$.

The measure $D\mu$ in the functional integral (1.6) may be written as

$$D\mu = \prod_{p,s} da(p)\, da(p) \tag{1.9}$$

for the Hubbard model with the action functional (1.7). It is not difficult to build up the diagram techniques if we split S into the sum

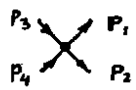
$$S = S_0 + S_1 , \tag{1.10}$$

$$S_0 = \sum_{p,s} (i\omega - \mathcal{E}_0(k) + \lambda)\, a^{*}_{s}(p)\, a_{s}(p) , \tag{1.11}$$

$$S = -(U/\beta N) \sum_{p_1+p_2=p_3+p_4} a^{*}_{\uparrow}(p_1)\, a^{*}_{\downarrow}(p_2)\, a_{\downarrow}(p_4)\, a_{\uparrow}(p_3) \tag{1.12}$$

and consider $S_0$ as a "bare" action and $S_1$ as a perturbation. It will be a perturbation theory with the following elements


$$G(p) = (i\omega - \mathcal{E}_0(k) + \lambda)$$




U . (1.13)

In order to get an expression corresponding to a given diagram we have to take a product of expressions corresponding to each of its elements and to sum over all possible four-momenta of inner lines. Then we have to multiply the result by

$$(-1/\beta N)^{\ell - v} (-1)^{F} \tag{1.14}$$

for the case of a vacuum diagram and by

$$(-1/\beta N)^{\ell - v - 1} (-1)^{F} \tag{1.15}$$

for the diagram contributing into the two-point (one-particle) Green's function. We denote here by l the number of lines, v the number of vertices and F the number of independent closed Fermi cycles in the diagram in question.

It is not difficult also to build up different modifications of the above-described perturbation theory. In applications to the Hubbard model the modified perturbative scheme with anomalous Green's functions will be especially useful. Such modifications are adopted especally in order to take possible broken symmetries in the system into account.

The Hubbard model (1.1) with repulsive interaction goes into the antiferromagnetic state at sufficiently low temperatures and for an appropriate choice of the chemical potential parameter $\lambda$. This state is characterized by the nonvanishing Neel spin vector and by the corresponding order parameter.

In order to describe such a state we need both normal and anomalous Green's functions

$$G_s(p) = -< a_s(p)\, a_s^*(p) > \quad , \quad G_{sa}(p) = -< a_s(p)\, a_s^*(p_\pi) > , \tag{1.16}$$

where

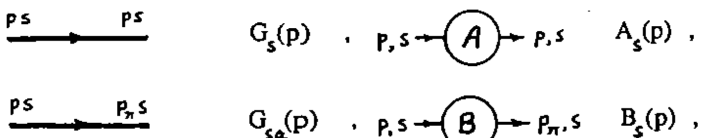
$$p = (\omega, \vec{k}) \quad , \quad p_\pi = (\omega, \vec{k} + \vec{\pi}) \tag{1.17}$$

and $\vec{\pi}$ is a vector, each component of which is equal to $\pi$.

$$\vec{\pi} = \underbrace{(\pi, \pi, \ldots, \pi)}_{D \text{ times}} \tag{1.18}$$

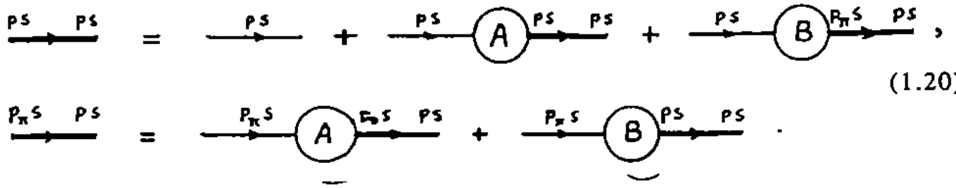
Namely this type of anomalous Green's functions (the second one in (1.16)) corresponds to an opportunity to have different spin averages for different sublattices of the lattice. In (1.16) we suppose $G_{sa}(p)$ to be diagonal with respect to s (there are no averages $<a_\uparrow(p)\, a_\downarrow^*(p_\pi)>$). It means that we suppose the Neel spin vector to be parallel to the z-axis.

In diagram language we have two types of lines and two types of the self-energy parts



(1.19)

which obey the coupled system of equations



(1.20)



These equations are analogous to the famous Gor'kov equations in superconductivity theory. Its analytical form is as follows

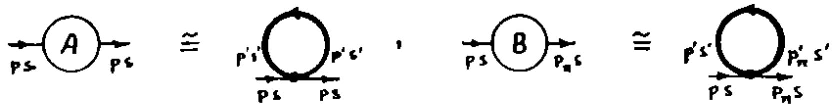
$$\begin{aligned} G_s(p) &= G_o(p) + G_o(p)\, A_s(p)\, G_s(p) + G_o(p)\, B_s(p)\, G_{sa}(p_\pi) , \\ G_{sa}(p_\pi) &= G_o(p_\pi)\, A_s(p_\pi)\, G_{sa}(p_\pi) + G_o(p_\pi)\, B_s(p_\pi)\, G_s(p) . \end{aligned} \tag{1.21}$$

It is a linear system of two equations for $G_s(p)$, $G_{sa}(p_\pi)$. Its solution may be written as

$$G_s(p) = (G_o^{-1}(p_\pi) - A_s(p_\pi))/[(G_o^{-1}(p) - A_s(p))(G_o^{-1}(p_\pi) - A_s(p_\pi)) - B_s(p)\, B_s(p_\pi)],$$

$$G_{sa}(p_\pi) = B_s(p_\pi)/[(G_o^{-1}(p) - A_s(p))(G_o^{-1}(p_\pi) - A_s(p_\pi)) - B_s(p)\, B_s(p_\pi)] \; . \tag{1.22}$$

Now we have to choose some approximations for the self energy parts $A_s(p)$, $B_s(p)$. It is natural to use approximations according to the following diagram equalities:



(1.23)

Here s' means the spin index opposite to s, so if s= ↑ then s'= ↓ and vice versa. The expressions corresponding to the RHS-s of (1.23) do not depend on p. Let us suppose that the normal Green's function $G_s(p)$ and the corresponding normal self energy part $A_s(p)$ do not depend on s and the anomalous Green's function and the anomalous self-energy part $B_s(p)$ are proportional to sign s. So we can write down analytical formulae corresponding to the diagram equalities (1.23) as

$$A_s(p) = A \quad , \quad B_s(p) = B_s , \tag{1.24}$$

where A, $B_s$ are constants (they do not depend on p). We may denote

$$A = \lambda - x \;, \; B_\uparrow = \Delta_\uparrow , \; B_\downarrow = \Delta_\downarrow = -\Delta_\uparrow \;, \; |\Delta_\uparrow| = |\Delta_\downarrow| = \Delta \; . \tag{1.25}$$

We have also

$$G_o^{-1}(p) = i\omega - \varepsilon_o(\vec{k}) + \lambda \quad , \quad G_o^{-1}(p_\pi) = i\omega + \varepsilon_o(\vec{k}) + \lambda \quad , \tag{1.26}$$

because $\varepsilon_o(\vec{k}+\vec{\pi}) = -\varepsilon_o(\vec{k})$ according to (1.5). Substituting (1.24)-(1.26) into (1.22) we get

$$G(p) = (i\omega + x + \varepsilon_o(k))/[(i\omega+x)^2 - \varepsilon_o^2(k) - \Delta^2] , \quad G_{sa}(p) = \Delta_s/[(i\omega + x)^2 - \varepsilon_o^2(k) - \Delta^2] \; . \tag{1.27}$$

Now we can substitute these expressions into the RHS-s of (1.23) where thick lines are just the normal and anomalous Green's functions. So we get

$$\lambda - x = \lim_{\varepsilon \to +0} (U/\beta N) \sum_p e^{i\omega\varepsilon} G(p) , \qquad \Delta_s = (U/\beta N) \sum_p G_{s\alpha}(p) . \quad (1.28)$$

We have to use the prescription $\lim_{\varepsilon \to +0}(...e^{i\omega\varepsilon}...)$ in the RHS of the first equation in (1.28) because the sum over frequences $\omega$ does not converge absolutely. The corresponding sum in the second equation (1.28) converges absolutely and here we do not need such a prescription.

Substituting (1.27) into (1.28) and evaluating sums over frequencies , we get

$$\begin{aligned}\lambda - x &= (U/\beta N) \sum_k \lim_{\varepsilon \to +0} \sum_\omega e^{i\omega\varepsilon}(i\omega + x + \varepsilon_0(k))/[(i\omega + x)^2 - \varepsilon_0^2(k) - \Delta^2] \\ &= (U/2N) \sum_k [(1 + \varepsilon_0(k)(\varepsilon_0^2(k) + \Delta^2)^{-1/2})(\exp\beta(-x + (\varepsilon_0^2(k) + \Delta^2)^{1/2}) + 1) \\ &\quad + (1 - \varepsilon_0(k)(\varepsilon_0^2(k) + \Delta^2)^{-1/2})(\exp\beta(-x - (\varepsilon_0^2(k) + \Delta^2)^{1/2}) + 1)] \\ &= (U/2N) \sum_k [1 + \mathrm{sh}\beta x / (\mathrm{ch}\beta x + \mathrm{ch}\beta(\varepsilon_0^2(k) + \Delta^2)^{1/2})] , \end{aligned} \quad (1.29)$$

$$\begin{aligned}\Delta_s &= (U/N) \sum_k \beta^{-1} \sum_\omega \Delta_{s'}/[(i\omega + x)^2 - \varepsilon_0^2(k) - \Delta^2] \\ &= (U\Delta_{s'}/2N) \sum_k (\varepsilon_0^2(k) + \Delta^2)^{-1/2}[(\exp\beta(-x + (\varepsilon_0^2(k) + \Delta^2)^{1/2}) + 1) - \exp\beta(-x - (\varepsilon_0^2(k) + \Delta^2)^{1/2} + 1)] \\ &= -(U/2N) \sum_k \Delta_{s'}(\varepsilon_0^2(k) + \Delta^2)^{-1/2} \mathrm{sh}\beta(\varepsilon_0^2(k) + \Delta^2)^{1/2}/[\mathrm{ch}\beta x + \mathrm{ch}\beta(\varepsilon_0^2(k) + \Delta^2)^{1/2}] .\end{aligned}$$

We can rewrite these equations in the equivalent form

$$\begin{aligned}\lambda - U/2 &= x + (U/2N) \sum_k \mathrm{sh}\beta x / [\mathrm{ch}\beta x + \mathrm{ch}\beta(\varepsilon_0^2(k) + \Delta^2)^{1/2}] , \\ \Delta &= (U/2N) \sum_k \Delta(\varepsilon_0^2(k) + \Delta^2)^{-1/2} \mathrm{sh}\beta x / [\mathrm{ch}\beta x + \mathrm{ch}\beta(\varepsilon_0^2(k) + \Delta^2)^{1/2}] .\end{aligned} \quad (1.30)$$

From the first equation we can, in principle, find $x$ as a function of $\lambda$ . But we can also consider $x$ as an independent parameter. Then the first equation (1.30) may be regarded as a definition of $\lambda$ (the chemical potential).

The second equation in (1.30) is a gap equation for the order parameter . If we divide it by $\Delta$ and then put $\Delta = 0$ , we get the equation for $\beta_c = T_c^{-1}$, where $T_c$ is the critical temperature, which looks as follows

$$1 = (U/2N) \sum_k (\mathrm{sh}\beta_c \varepsilon_0(k)) \, \varepsilon_0^{-1}(k) \, (\mathrm{ch}\beta_c x + \mathrm{ch}\beta_c \varepsilon_0(k)) . \quad (1.31)$$

This equation shows that the minimal possible value of $\beta_c$ (or the maximum of the critical temperature $T_c$ ) as achieved for $x = 0$. It should be mentioned that $x$ can be called a renormalized doping coefficient.

Eqs.(1.30) have a more simple form at $x = 0$ :



$$\lambda - U/2 = 0 , \quad \Delta = (U/2N) \sum_k \Delta(\varepsilon_0^2(k) + \Delta^2)^{-1/2} \mathrm{th}(\beta(\varepsilon_0^2(k) + \Delta^2)^{1/2}/2) . \quad (1.32)$$

Eq.(1.31) at $x = 0$ takes a form

$$1 = (U/2N) \sum_k \varepsilon_0^{-1}(k) \, \mathrm{th}(\beta \varepsilon_0(k)/2) . \quad (1.33)$$

Using an inequality $\mathrm{th}|x| < |x|$ we get the following estimate for the critical temperature T from (1.33) :

$$T_c \leqslant U/4. \quad (1.34)$$

We can also consider the limit $T \to 0$ ($\beta \to \infty$) in Eqs (1.30),(1.32). We get

$$\begin{aligned}\lambda - U/2 &= x + (U \,\mathrm{sgn}\, x /2N) \sum_{\varepsilon_0^2(k) + \Delta^2 < x^2} 1 , \\ \Delta &= (U/2N) \sum_k \Delta(\varepsilon_0^2(k) + \Delta^2)^{-1/2} .\end{aligned} \quad (1.35)$$

( $\mathrm{sgn}\, x = +1$ if $x>0$ and $\mathrm{sgn}\, x = -1$ if $x<0$ ) instead of (1.30) and

$$1 = (U/2N) \sum_{|\varepsilon_0(k)| > |x_c|} |\varepsilon_0(k)|^{-1} \quad (1.36)$$

instead of (1.31). The last equation defines a critical value of $x$ (at T=0) such that antiferromagnetism in the system is impossible for $|x| > |x_c|$. Such a critical value $x$ exists for all $T < T_{c\,max}$ ($T_{c\,max}$ is defined by (1.33)). The point is that Eq.(1.31) may be regarded also as an equation for $x = x_c$ for a given $\beta = T^{-1}$. So if $|x| > |x_c|$ and $\Delta = 0$ we have more simple formulae for Green's functions instead of (1.27). Namely we have

$$G(p) = (i\omega - \varepsilon_0(k) + x)^{-1} , \qquad G_{s\alpha}(p) = 0 , \quad (1.37)$$

where $x$ is defined by the equation

$$\lambda - U/2 = x + (U/2N) \sum_k \mathrm{sh}\beta x / (\mathrm{ch}\beta x + \mathrm{ch}\beta \varepsilon_0(k)) . \quad (1..38)$$

At T= 0 ( $\beta = \infty$ ) we have

$$\lambda - U/2 = x + (U \,\mathrm{sgn}\, x /2N) \sum_{|\varepsilon_0(k)| < |x|} 1 \quad (1.39)$$

instead of (1.37).

The obtained formulae show that our system indeed behaves as an antiferromagnetic at the sufficiently low temperatures and sufficiently small values of the renormaized "doping" coefficient x. If we increase T or increase |x| at the fixed T antiferromagnetism disappears at some $T_c$ (or at some $x_c$ at the fixed T).

At $|x|>|x_c|$ our Green's function (1.37) differs from the bare Green's function (1.13) only by the change $\lambda \rightarrow x$, where $x$ is defined by (1.38). But the point is that now the denominator $(i\omega - \varepsilon_0(k) + x)$ may vanish at $\omega = 0$ and some k. The corresponding equation

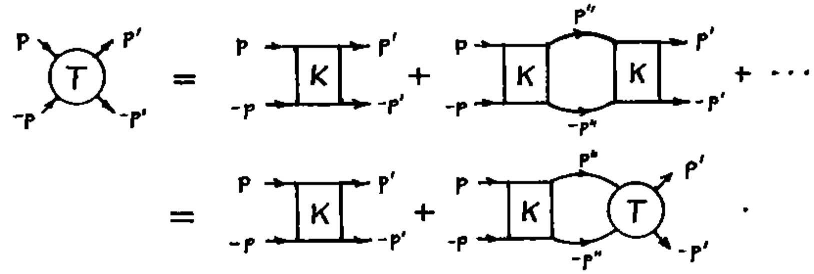
$$\varepsilon_0(k) = x \tag{1.40}$$

gives us a Fermi surface for the 3D-system and a Fermi line at the 2D-case. Such vanishing of the Green's function denominator may lead to singularities of the particle scatterring amplitudes with particle momenta near the Fermi surface (line). As it will be shown at the next section such singularities occur indeed in some channels with nonzero angular momenta and it leads to superconductivity with a nontrivial Cooper pairing. It should be mentioned that the possibility of a superfluid phase transition in a slightly nonideal Fermi gas with repulsion was considered by M.Yu. Kagan and A.V. Chubukov [10].



## 2. Singularities of the scattering amplitudes with nonzero angular momenta and nontrivial Cooper pairing.

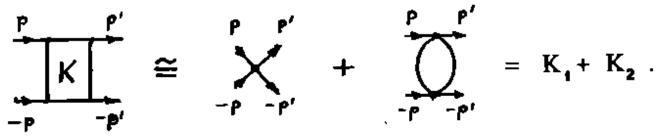
As it was noted at the end of the previous section, singularities of the Green's function on the Fermi surface may lead to singularities of the scattering amplitudes for particles with momenta near the Fermi surface. Let us consider the following diagram equation




(2.1)

Here T is a scattering amplitude and K is its irreducible part. Irreducibility means that we cannot separate the incoming part of a given diagram for K with incoming arrows p,-p from its outgoing part containing outgoing arrows p',-p' by cutting two diagram lines.

If we want to solve Eq.(2.1) we have to choose an approximation for K. Let us take the simplest possible nontrivial approximation as follows




(2.2)

We take the two simplest diagrams into account. The first one K is of first order and the second K is of second order. There exists one more diagram of second order


, (2.3)

but it is reducible and does not contribute to K.

Should U be negative it was sufficient to take only the first diagram K=U into account in order to describe the s-pairing effects. But in our case U is positive. It makes the s-pairing impossible and we need higher terms in K if we try to obtain fermion pairing with a nonzero angular momentum.

It will be shown below that the p-pairing is preferable in the three-dimensional (3D) Hubbard model and we have a system with properties resembling those of the superfluid He. It is wonderful that the situation is different in the two-dimensional (2D)-case where the pairing with the angular momentum l=2 takes place.

First of all we need expressions for $K_1$ and $K_2$. According to (1.13)

$$K_1 = U . \qquad (2.4)$$

The expression for $K_2$ may be written as

$$K_2 = K_2(q) = (U^2/\beta N) \sum_{p_1} G(p_1)\, G(p_1+q) \ , \qquad (2.5)$$

where $q=p-p'=(\omega,\vec{q})$. If we take $G(p_1)$, $G(p_1+q)$ according to (1.37) and perform the summation over frequencies $\omega_1$ we get

$$K_2 = (U^2/N) \sum_{K_1} (n(k_1) - n(k_2))/( \varepsilon_0(k_1) - \varepsilon_0(k_2) + i\ \ ) , \qquad (2.6)$$

where $\vec{k}_2=\vec{k}_1+\vec{q}$ and

$$n(k) = ( \exp\beta( \varepsilon_0(k) - x) + 1 ) . \qquad (2.7)$$

Here we consider only the case where the fermion density is small and the opposite case when almost all fermion states are filled. Both cases are symmetric to each other and it is sufficient to consider only one of them, namely the case of small fermion density. Here the parameter x is close to the minimum of (k) (1.5). This minimum corresponds to all $k_i=0$, and the value of this minimum is equal to -2Dt (D is a dimension of the model). So let us take

$$x = -2D\,t + t\,k_F^2, \qquad (2.8)$$

where $k_F$ is small ($k_F$ <<1). In the vicinity of this minimum we have

$$\cos k_i = 1 - (1/2)\, k_i^2 + (1/24)\, k_i^4 - ... \qquad (2.9)$$

It is sufficient to take only the first two terms in the RHS of (2.9) for the 3D-system. But in the 2D-case we need all three terms written down in the RHS of (2.9).

Let us begin with the 3D-case. Here we can put

$$\varepsilon(k) = \varepsilon_0(k) - x - t(k^2\ \ k_F^2) \qquad (2.10)$$

according to (1.5), (2.8), (2.9). We shall consider the case of low temperatures where

$$n(k) \cong \theta( x - \varepsilon_0(k)) = \theta( k_F^2 - k^2 ) \qquad (2.11)$$



( $\theta(x)=0$ for x<0 and $\theta(x)=1$ for x>0).

Now we can try to solve Eq (2.1). But first of all it is suitable to modify it slightly. The reason for such a modification is as follows. The point is that the main contribution into the sum over inner four-momentum $p''=(\omega'',k'')$ is due to the narrow shell near the Fermi surface $k^2=k_F^2$. One can define this shell S by the inequality

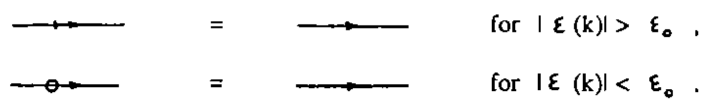
$$S: \qquad |\varepsilon_0(k)| < \varepsilon_0 \quad \text{or} \quad t\,|k^2 - k_F^2| < \varepsilon_0 . \qquad (2.12)$$

The value of $\varepsilon_0$ should obey the inequality

$$\varepsilon_0 << \varepsilon_F = t\,k_F^2 . \qquad (2.13)$$

This inequality only estimates the order of the value of $\varepsilon_0$ and all the physical results must not depend on the exact value of $\varepsilon_0$ .

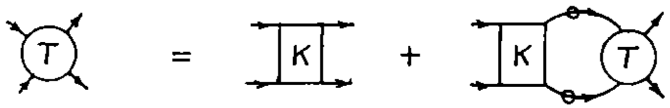
Now we want only the sum over four-momenta p" belonging to S to contribute to Eq (2.1). We can achieve this aim by using another definition of the reducible and irreducible parts of T. Up to now let us define K (the irreducible part of T) as a sum of all diagrams in which it is impossible to separate the incoming part of the diagram from the outgoing one by cutting two lines with energies less than $\varepsilon_0$ .We introduce also the following diagram notations



(2.14)

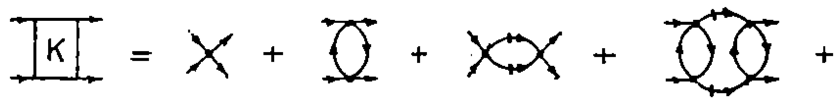
If there is no restriction on the value of the energy in some line of a given diagram we shall use the ordinary notation ——▸—— for such a line.

So we arrive at the following diagram equation



, (2.15)

containing only the inner lines with $|\varepsilon(k'')| < \varepsilon_0$ . Here K is an irreducible part of T according to our new definition. Using this definition we have to add some extra terms to K as compared with (2.2). Now we shall take the following approximation for K :



(2.16)

+ all other diagrams of the third and fourth orders.

As it will be explained below we have to take all the diagrams up to the fourth order into account if we want to obtain an asymptotically exact formula for T (i.e. a formula which is valid not only with a so-called logarithmic accuracy).

Now we can solve Eq.(2.15). Both T and K may depend on frequencies $\omega, \omega'$ and also on $|\vec{k}|$, $|\vec{k}'|$ and $\cos\theta$, where $\theta$ is an angle between vectors $\vec{k}$ and $\vec{k}'$. In the shell region S, where $|\varepsilon(k)| < \varepsilon_o$ and $|\varepsilon(k')| < \varepsilon_o$ we may neglect the $\omega, \omega'$-dependence of T, K and put $\vec{k} = k_F\vec{n}$, $\vec{k}' = k_F\vec{n}'$, where $\vec{n}, \vec{n}'$ are unit vectors. So we can consider T and K as functions of only one variable $\cos\theta = (\vec{n},\vec{n}')$ and use the Legendre series

$$T = \sum_{\ell=0}^{\infty} t_\ell P_\ell((\vec{n},\vec{n}')) \quad , \qquad K = \sum_{\ell=0}^{\infty} k_\ell P_\ell((\vec{n},\vec{n}')) \ , \tag{2.17}$$

where

$$P_\ell(x) = (2^l l!)^{-1} (d^\ell/dx^\ell)(x^2-1)^\ell \tag{2.18}$$

is the l-th Legendre polynomial. Using the well-known orthogonality formula

$$\int_{-1}^{1} P_\ell(x) P_m(x)\, dx = 2(2l+1)^{-1} \delta_{\ell m}, \tag{2.19}$$

we obtain the following formulae for t ,k :

$$t_\ell = ((2l+1)/4\pi) \int T(\cos\theta) P_\ell(\cos\theta)\, d\Omega = (2l+1) < T >_\ell \ ,$$
$$k_\ell = ((2l+1)/4\pi) \int K(\cos\theta) P_\ell(\cos\theta)\, d\Omega = (2l+1) < K >_\ell \ , \tag{2.20}$$

where $d\Omega = \sin\theta\, d\theta\, d\varphi$ and <...> means the angular averaging with a weight factor $P_\ell(\cos\theta)$.

Substituting the Legendre series (2.17) into (2.15) we shall have

$$\sum_{\ell=0}^{\infty} t_\ell P_\ell((\vec{n},\vec{n}')) = \sum_{\ell=0}^{\infty} k_\ell P_\ell((\vec{n},\vec{n}')) - (\beta N)^{-1} \sum_{k''\in S,\omega''} \Big( \sum_{\ell=0}^{\infty} k_\ell P_\ell((\vec{n},\vec{n}''))$$
$$\cdot (i\omega'' - \varepsilon(k''))^{-1} (-i\omega'' - \varepsilon(k''))^{-1} \sum_{\ell'=0}^{\infty} t_{\ell'} P_{\ell'}((\vec{n}'',\vec{n}')) \ . \tag{2.21}$$

We have also

$$\sum_{\omega''} (i\omega'' - \varepsilon(k''))(-i\omega'' - \varepsilon(k'')) = \beta^{-1} \sum_{\omega''} (\omega''^2 + \varepsilon_o^2(k''))^{-1} = (2\,\varepsilon(k''))^{-1} \mathrm{th}(\beta\varepsilon(k'')/2) \ . \tag{2.22}$$

Changing the summation over $k'' \in S$ by the integration



$$N^{-1} \sum_{k''\in S} \rightarrow k_F (2t(2\pi)^3)^{-1} \int_{|\varepsilon''|<\varepsilon_o} d\varepsilon''\, d\Omega'' \ , \tag{2.23}$$

$(\varepsilon'' = t(k''^2 - k_F^2))$, we can use the orthogonality formula

$$\int P_\ell((\vec{n},\vec{n}'')) P((\vec{n}'',\vec{n}'))\, d\Omega'' = 4\pi(2l+1)^{-1} \delta_{\ell\ell'} P_\ell((\vec{n},\vec{n}')) \tag{2.24}$$

and also the formula

$$\int_{|\varepsilon''|<\varepsilon_o} d\varepsilon'' (2\varepsilon'')^{-1} \mathrm{th}(\beta\varepsilon''/2) = \int_0^{\beta\varepsilon_o/2} dx\, x^{-1} \mathrm{th}\, x = \ln(2\beta\gamma\varepsilon_o/\pi) \ , \tag{2.25}$$

which is valid for $\beta\varepsilon_o >> 1$. Here $\ln\gamma = C$ is the Euler constant. Substituting (2.22)-(2.25) into (2.21) we get

$$\sum_{\ell=0}^{\infty} t_\ell P_\ell((\vec{n},\vec{n}')) = \sum_{\ell=0}^{\infty} k_\ell P_\ell((\vec{n},\vec{n}'))$$
$$- \sum_{\ell=0}^{\infty} (k_F \ln(2\beta\gamma\varepsilon_o/\pi)(4\pi^2 t(2l+1))^{-1} k_\ell t_\ell P_\ell((\vec{n},\vec{n}')) \ . \tag{2.26}$$

Equating coefficients before $P_\ell((\vec{n},\vec{n}'))$ on both sides of (2.26) one gets an equation

$$t_\ell = k_\ell - [k_F \ln(2\beta\gamma\varepsilon_o/\pi)/(4\pi^2 t(2l+1))^{-1}]\, k_\ell t_\ell \ , \tag{2.27}$$

which implies the following formula for $t_\ell$ :

$$t_\ell = k_\ell [1 + k_\ell (k_F \ln(2\beta\gamma\varepsilon_o/\pi)/(4\pi^2 t(2l+1))]^{-1} \ . \tag{2.28}$$

Using the notation (2.20) we can change here $k_\ell$ by $(2l+1)<K>_\ell$. Then we can substitute (2.28) into the Legendre series (2.17) for T and arrive at the following solution of the diagram equation (2.15) :

$$T = T((\vec{n},\vec{n}')) = \sum_{\ell=0}^{\infty} (2l+1) < K >_\ell P_\ell((\vec{n},\vec{n}'))$$
$$/[1 + < K >_\ell (k_F/4\pi^2 t) \ln(2\beta\gamma\varepsilon_o/\pi)]. \tag{2.29}$$

This formula shows that T may become singular for sufficiently low temperatures T (i.e. for sufficiently large $\beta$) if $<K>_\ell$ is negative even for one value of l.

We have $k_o>0$ and according to (2.28) for l=0 we get

$$0 < t_o < k_o \ , \tag{2.30}$$

and $t_o$ is not singular at low temperatures. Physically it means that the s-pairing is impossible in our system.

Now let us consider the next term in the RHS of (2.29) namely the term with l=1. This term turns out to be the most interesting for us because it becomes singular (for the 3D-model) at sufficiently low temperatures. The point is that $<K>_1$ is negative. The main contribution into $<K>_1$ is due to the second diagram in the RHS of (2.16) which is just the diagram $K_2$ in (2.2). The expression for $K_2$ is given by (2.6). Now let us calculate $K_2$ for $\omega = 0$, $\varepsilon_o(k_i) = -2Dt + tk_i^2$, $n(k_i) = \Theta(k_F^2 - k_i^2)$. We have the expression

$$K_2 = (U^2/t\,N) \sum_{K_1} ( \Theta(k_F^2 - k_1^2) - \Theta(k_F^2 - k_2^2))/(k_1^2 - k_2^2)$$

$$= (U^2/t\,N) \sum_{K_1} \Theta(k_F^2 - k_1^2)/(k_1^2 - k_2^2)$$

$$= 2U^2/((2\pi)^3 t) \int_0^{K_F} k_1^2\, dk_1 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi\, (k_1^2 - k_1^2 - 2k_1 q \cos\theta - q^2)^{-1}$$

$$= -(U^2/8\pi^2 t)\,[\,q^{-1}(k_F^2 - q^2/4)\ln|(q + 2k_F)/(q - 2k_F)| + k_F\,] , \qquad (2.31)$$

which is proportional to the well-known Lindhardt function. The simplest way to get

$$< K >_1 \cong (4\pi)^{-1} \int K_2 P_1(\cos\theta)\, d\Omega = (1/2) \int_0^{\pi} K_2 \cos\theta \sin\theta\, d\theta \qquad (2.32)$$

is to go to the variable q instead of $\Theta$ according to the following formulae

$$q^2 = k^2 - 2k\,k' \cos\theta + k'^2 = 2\,k_F^2 (1 - \cos\theta) ,$$
$$\cos\theta = 1 - q^2/2k_F^2 , \qquad \sin\theta\, d\theta = q\, dq / k_F^2 . \qquad (2.33)$$

Substituting (2.31), (2 33) into (2.32) we shall have

$$(1/2) \int_0^{2K_F} q\, dq\, k_F^{-2} (1 - q^2/2k_F^2)\, (-U^2/8\pi^2 t)[q^{-1}(k_F^2 - q^2/4) \ln|(q + 2k_F)/(q + 2k_F)| + k_F]$$

$$= -\,(U^2 k_F/8\pi^2 t) \int_0^1 dx\, (1 - 2x^2)\, [\,\ln((1+x)/(1-x)) + 2x\,]$$

$$= -\,(U^2 k_F/20\pi^2 t)\,(2\ln 2 - 1) < 0 . \qquad (2.34)$$

So $<K>_1 < 0$, and the term with l=1 in (2.30), which looks like

$$-\,(\vec{n},\vec{n}')\,(3\,(2\ln 2 - 1)/20\pi^2)\,(U^2 k_F / t)$$
$$[1 - ((2\ln 2 - 1)/80\pi^4)\,(Uk_F/t)^2 \ln(2\beta\gamma\,\varepsilon_o/\pi))]^{-1} \qquad (2.35)$$



becomes infinite at $T = T_c$, where

$$T_c = (2\gamma\varepsilon_o/\pi) \exp(-(80\pi^4/(2\ln 2 - 1))\,(t/Uk_F)^2) . \qquad (2.36)$$

This formula contains $\varepsilon_o$ which is an arbitrary parameter. The $\varepsilon_o$ - dependence arises if we take into account only the diagram $K_2$ confining oneself by the so-called logaritmic accuracy of calculations. It is easy to see that this parameter $\varepsilon_o$ disappears from the formula for $T_c$ if we include also the fourth diagram in the RHS of (2.16) into consideration. Taking both $K_2$ and $K_4$ into account we get the following expression for the term with l=1 in (2.29) instead of (2.35) :

$$3(\vec{n},\vec{n}')\,[\,<K_2>_1 + <K_4>_1\,]\,[1 + (<K_2>_1 + <K_4>_1)\,(k_F/4\pi^2 t)\ln(2\beta\gamma\varepsilon_o/\pi)\,]$$
$$= 3(\vec{n},\vec{n}')\,\{\,(<K_2>_1 + <K_4>_1)^{-1} + (k_F/4\pi^2 t)\ln(2\beta\gamma\varepsilon_o/\pi)\,\} . \qquad (2.37)$$

We have further

$$(<K_2>_1 + <K_4>_1)^{-1} \cong <K_2>_1^{-1} - <K_4>_1 <K_2>_1^{-2} . \qquad (2.38)$$

According to (2.34) the expression

$$<K_2>_1^{-1} = 20\pi^2 t/(2\ln 2 - 1)\,U^2 k_F \qquad (2.39)$$

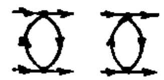
does not depend on $\varepsilon_o$. The $\varepsilon_o$-dependence of $K_4$ is due to the inequality $|\,\varepsilon(k'')| > \varepsilon_o$ imposed on the energy of two inner lines of $K_4$ according to (2.14). Let us divide the region of integration over k" in $K_4$ into two subregions (1) and (2) defined as follows

$$(1) \quad |\,\varepsilon(k'')| > \eta_o \quad , \qquad (2) \quad \varepsilon_o < |\,\varepsilon(k'')| < \eta_o . \qquad (2.40)$$

so

$$K_4 = K_4^{(1)} + K_4^{(2)}, \qquad (2.41)$$

where $\eta_o$ is a new parameter, and its order of magnitude is the same as the order of $\varepsilon_o$. It is clear that $K_4^{(1)}$ does not depend on $\varepsilon_o$. When calculating $K_4^{(2)}$, we can consider both subdiagrams



(2.42)

in $K_4$ depending only on the angular variables. Each of these subdiagrams is just the diagram $K_2$. So we have

$$K_4^{(2)} = -(\beta N)^{-1} \sum_{\substack{k'' \in (2) \\ \omega''}} K_2((n,n'')) K((n'',n'))(\omega''^2 + \varepsilon^2(k''))^{-1} \tag{2.43}$$

$$= -(k_F/2t(2\pi)^3) \int_{\varepsilon_0 < |\varepsilon''| < \eta_0} d\varepsilon'' \left(\beta^{-1} \sum_{\omega''} (\omega''^2 + \varepsilon''^2)^{-1}\right) \int d\Omega'' K_2((\vec{n},\vec{n}'')) K_2((\vec{n}'',\vec{n})) .$$

Further

$$\int d\varepsilon'' \beta^{-1} \sum_{\omega''} (\omega''^2 + \varepsilon''^2)^{-1} = \int_{\varepsilon_0}^{\eta_0} d\varepsilon'' (\varepsilon'')^{-1} \mathrm{th}(\beta\varepsilon''/2) \cong \int_{\varepsilon_0}^{\eta_0} d\varepsilon''/\varepsilon'' = \ln(\eta_0/\varepsilon_0) , \tag{2.44}$$

and after the angular averaging we have

$$< K_4^{(2)} >_1 = -(k_F/4\pi^2 t) < K_2 >_1^2 \ln(\eta_0/\varepsilon_0) , \tag{2.45}$$

because

$$< \int d\Omega'' K_2((\vec{n},\vec{n}'')) K_2((\vec{n}'',\vec{n})) > = 4\pi < K_2 >_1^2 . \tag{2.46}$$

If we substitute (2.38) into (2.37) and take $<K_4>_1 = <K_4^{(1)}>_1 + <K_4^{(2)}>_1$, where $<K_4^{(1)}>_1$ does not depend on $\varepsilon_0$, and $<K_4^{(2)}>_1$ is given by (2.45), we obtain the following expression for the denominator in (2.37)

$$< K_2 >_1^{-1} - < K_4^{(1)} >_1 < K_2 >_1^{-2} - (k_F/4\pi^2 t) \ln(\eta_0/\varepsilon_0) + (k_F/4\pi^2 t) \ln(2\beta\gamma\varepsilon_0/\pi)$$

$$= < K_2 >_1^{-2} - < K_4^{(1)} >_1 < K_2 >_1^{-2} + (k_F/4\pi^2 t) \ln(2\beta\gamma\eta_0/\pi) , \tag{2.47}$$

which does not depend on $\varepsilon_0$. It is clear that $<K_4>_1$ may be written as

$$< K_4 >_1 = -(k_F/4\pi^2 t) < K_2 >_1^2 [\ln(t k_F^2/\varepsilon_0) + a] , \tag{2.48}$$

where $a$ is a dimensionless constant. It implies the following formula for the denominator in (2.37) :

$$< K_2 >_1^{-1} + (k_F/4\pi^2 t) [\ln(2\beta\gamma t k_F^2/\pi) + a] \tag{2.49}$$

$$= (k_F/4\pi^2 t) [(80\pi^4/(2\ln 2 - 1)(t/U k_F)^2 + a + \ln(2\beta\gamma t k_F^2/\pi)] ,$$

which does not contain $\varepsilon_0$. It seems that now we can obtain a more correct formula for $T_c$ without any $\varepsilon_0$ by equating the RHS of (2.49) to zero. But here we have to be careful because all other fourth order diagrams contribute also to (2.49) and give contribution to [...] in (2.49) of the same order as a. Moreover there exist also third order diagrams



(2.50)

contributing into (2.49). Two other third order diagrams

(2.51)

contribute only into $k_0$ but not into the higher $k_l (l>1)$.

So in order to take all the third and fourth order terms into account we have to change

$$a \longrightarrow \tilde{a} + b(t/U k_F) \tag{2.52}$$

in (2.50). Here the coefficient b goes from the third order diagrams (2.50), and both third and fourth order diagrams contribute into a. We have indeed

$$< K >_1^{-1} = (< K_2 >_1 + < \delta K >_1)^{-1} =$$
$$= < K_2 >_1^{-1} - < \delta K >_1 < K_2 >_1^{-2} + < \delta K >_1^2 < K_2 >_1^{-3} + \dots , \tag{2.53}$$

where $< \delta K >_1$ is a contribution of the third and fourth order diagrams. Now it is easy to see that the contribution of the third order diagrams into the linear in $< \delta K >_1$ term in the RHS of (2.53) gives us the term $b(t/Uk_F)$ in (2.52), the fourth order diagrams must be taken into account only in the linear in $< \delta K >_1$ term and they contribute into a, and there are also contributions from the third order diagrams into the last term in the RHS of (2.53) which is quadratic in $<\delta K>_1$ .

Now if we insert (2.53) in the RHS of (2.49) and then put

$$\tilde{a} = \ln(\pi A/2\gamma) , \tag{2.54}$$

we obtain the following equation for $T_c$ :

$$80\pi^4(2\ln 2 - 1)(t/Uk_F)^2 + b(t/Uk_F) + \ln(A\beta_c t k_F^2) = 0 . \tag{2.55}$$

The corresponding formula for $T_c$ :

$$T_c = A t k_F^2 \exp[-(80\pi^4/(2\ln 2 - 1))(t/Uk_F)^2 + b(t/Uk_F)] \tag{2.56}$$

contains two dimensionless constants A and b which are defined by the third and fourth order diagrams in K, as it was explained.

Let us note that the numerical value of the coefficient before $-(t/Uk\ )$ in the exponent in (2.56) is very large, namely

$$80\pi^4/(2\ln 2 - 1) \sim 2\cdot 10^4 , \qquad (2.57)$$

and it makes $T_c$ to be incredibly low. But the point is that we have considered only a "tail" of superconductivity existing for arbitrary small values of k when the fermion density is also very small. The hope arises to obtain some other formula for $T_c$ if $|x|$ (the absolute value of the renormalized doping coefficient) is not close to its limiting nontrivial boundary 2Dt, which will give us a higher value for $T_c$ .

Thus we may consider the formula (2.56) as the result supporting the idea of superconducting states in the Hubbard model with repulsion. Another meaning of this result is that it shows us the type of pairing. Namely it should be the p-pairing in the three-dimensional Hubbard model with repulsive interaction.



## 3. An unusual type of Cooper pairing in the 2D-Hubbard model.

In this section we consider the low-density limit for the two-dimensional (2D) Hubbard model. If we want to solve the diagram equation (2.15) it is natural to suppose that both the scattering amplitude T and its irreducible part K depend only on $\cos\theta = (\vec{n},\vec{n}')$ in the shell domain S around the Fermi line. So we have to use trigonometrical series

$$T = \sum_{m=0}^{\infty} t_m^{(1)} \cos m\theta + \sum_{m=1}^{\infty} t_m^{(2)} \sin m\theta ,$$
$$K = \sum_{m=0}^{\infty} k_m^{(1)} \cos m\theta + \sum_{m=1}^{\infty} k_m^{(2)} \sin m\theta \qquad (3.1)$$

instead of the Legendre series (2.17). The analogue of the formulae (2.36) looks as follows

$$K_2 = (U^2/t N) \sum_{K_1} (\theta(k_F^2 - k_1^2) - \theta(k_F^2 - k_2^2))/(k_1^2 - k_2^2)$$
$$= (2U^2/t N) \sum_{K_1} \theta(k_F^2 - k_1^2) /(k_1^2 - k_2^2)$$
$$= (2U^2/(2\pi)^2 t) \int_0^{K_F} k_1 dk_1 \int_{-\pi}^{\pi} d\theta\, (k_1^2 - k_1^2 - 2k_1 q \cos\theta - q^2)^{-1}$$
$$= -(U^2/2\pi^2 t) \int_0^{K_F} k_1 dk_1 q^{-1} \int_{-\pi}^{\pi} d\theta\, (q + 2k_1 \cos\theta)^{-1}$$
$$= (U^2/4\pi t)(-1 + (1 - 4k_F^2/q^2)^{1/2}\, \theta(1 - 4k_F^2/q^2)) . \qquad (3.2)$$

Here the formula

$$\int_{-\pi}^{\pi} d\theta (q + 2k_1 \cos\theta)^{-1} = 2\pi (q^2 - 4k_1^2)^{-1/2} \qquad (3.3)$$

was used which is valid for $q>2k_1$. If $q<2k_1$ we can change $q+2k_1\cos\theta$ by $q+2k_1\cos\theta + i\varepsilon$ with infinitesimal $\varepsilon$ and then take the real part. The point is that (3.3) is valid also for arbitrary complex q and $k_1$ if $q+2k_1\cos\theta \neq 0$ for all $\theta \in [-\pi, \pi]$. The obtained formula (3.2) shows that

$$K_2 = -U^2/4\pi t = \text{const} \quad \text{for} \quad q^2 \leqslant 4k_F^2 . \qquad (3.4)$$

But when evaluating the Fourier coefficients $k_m^{(1)}, k_m^{(2)}$ in (3.1) we have $q^2 = 2k_F^2(1-\cos\theta) < 4k_F^2$, and $K_2$ does not depend on q. Hence

$$k_m^{(1)} = \pi^{-1} \int_{-\pi}^{\pi} K_2 \cos m\theta \, d\theta = 0 \quad , \quad k_m^{(2)} = \pi^{-1} \int_{-\pi}^{\pi} K_2 \sin m\theta \, d\theta = 0 \qquad (3.5)$$

for all $m > 1$.

We see that the situation in the 2D-case differs drastically from those in the 3D-model where it was $\langle K \rangle_1 < 0$.

Now let us try to take also the third term in the cos-series (2.9) into account. So we shall use

$$\varepsilon(k)/t = (\varepsilon_o(k) - x)/t = k_x^2 + k_y^2 - (k_x^4 + k_y^4)/12 \qquad (3.6)$$

instead of $\varepsilon(k)/t = (\varepsilon_o(k) - x)/t = k_x^2 + k_y^2$. The problem of evaluation the correction terms to (3.4) is not so simple because the integrand $(\varepsilon(k_1) - \varepsilon(k_2))^{-1}$ may become singular in the integration domain. The result of this calculation is as follows

$$K_2 = -(U^2/4\pi t)[1 + q^2(1 - 2 q_x^2 q_y^2 q^{-4})/48 + \varepsilon_F/8t] \quad , \qquad (3.7)$$

where $\vec{q} = \vec{k}_1 - \vec{k}_3$ is the transferred momentum of the diagram $K_2$. In the Appendix one can find the way this result was obtained.

It is interesting that K (3.7) contains a term proportional to

$$q_x^2 q_y^2 q^{-2} = q_x^2 q_y^2 / (q_x^2 + q_y^2) \quad , \qquad (3.8)$$

which is not analytic in the limit $q_x, q_y \to 0$. Namely this term implies an unusual type of the Cooper pairing in the 2D-case.

If $k_1^2, k_3^2$ are close to $k_F^2$, we have

$$\vec{k}_1 = k_F \vec{n}_1 \quad , \quad \vec{k}_3 = k_F \vec{n}_3 \quad , \qquad (3.9)$$

where $\vec{n}_1$, $\vec{n}_3$ are unite vectors ($n_1^2 = n_3^2 = 1$). So we can take

$$n_{1x} = \cos\varphi, \quad n_{1y} = \sin\varphi, \quad n_{3x} = \cos\varphi', \quad n_{3y} = \sin\varphi' \qquad (3.10)$$

and get

$$q^2 = k_F^2((\cos\varphi - \cos\varphi')^2 + (\sin\varphi - \sin\varphi')^2) = 2k_F^2(1 - \cos(\varphi - \varphi')) , \qquad (3.11)$$

$$q_x q_y = k_F^2(\cos\varphi - \cos\varphi')(\sin\varphi - \sin\varphi') = k_F^2 \sin(\varphi + \varphi')(\cos(\varphi - \varphi') - 1)) ,$$

$$q^2 - q_x^2 q_y^2 q^{-2} = 2k_F^2(1 - \cos(\varphi - \varphi')) - k_F^2 \sin^2(\varphi + \varphi')(1 - \cos(\varphi - \varphi'))$$

$$= (k_F^2/4)[6 - 6\cos(\varphi - \varphi') + 2\cos(2\varphi + 2\varphi') - \cos(\varphi + 3\varphi') - \cos(3\varphi + \varphi')] .$$

We arrive therefore at the following formula for $K \cong K_1 + K_2 = U + K_2$:

$$K = K(\varphi, \varphi') = k_o + \sum_{m,n=1}^{3} p_{mn} \cos m\varphi \cos n\varphi' + \sum_{m,n=1}^{3} q_{mn} \sin m\varphi \sin n\varphi' , \qquad (3.12)$$



where

$$k_o = U - U^2/4\pi t - 5 U^2 k_F^2/(128\pi t) \quad , \qquad (3.13)$$

and $p_{mn}$, $q_{mn}$ are elements of the matrices P and Q which look as follows

$$P = A U^2 k_F^2/(768\pi t) \quad , \quad Q = B U^2 k_F^2/(768\pi t) \quad , \qquad (3.14)$$

and A, B are matrices of the form

$$A = \begin{matrix} 6 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{matrix} \quad , \quad B = \begin{matrix} 6 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{matrix} \quad . \qquad (3.15)$$

The obtained form for K (3.11) forces us to look for a solution of the equations (2.15) for $T(\varphi, \varphi')$ as

$$T(\varphi, \varphi') = t_o + \sum_{m,n=1}^{3} t_{mn} \cos m\varphi \cos n\varphi' + \sum_{m,n=1}^{3} u_{mn} \sin m\varphi \sin n\varphi' . \qquad (3.16)$$

The equation for $T(\varphi, \varphi')$ looks like

$$t_o + \sum_{m,n=1}^{3} t_{mn} \cos m\varphi \cos n\varphi' + \sum_{m,n=1}^{3} u_{mn} \sin m\varphi \sin n\varphi' =$$

$$= k_o + \sum_{m,n=1}^{3} p_{mn} \cos m\varphi \cos n\varphi' + \sum_{m,n=1}^{3} q_{mn} \sin m\varphi \sin n\varphi'$$

$$-(\beta N)^{-1} \sum_{\substack{k'' \in S \\ \omega''}} (k_o + \sum_{m,n=1}^{3} p_{mn} \cos m\varphi \cos n\varphi'' + \sum_{m,n=1}^{3} q_{mn} \sin m\varphi \sin n\varphi'')$$

$$(i\omega'' - \varepsilon(k''))^{-1}(-i\omega'' - \varepsilon(k''))^{-1} \qquad (3.17)$$

$$(t_o + \sum_{m',n'=1}^{3} p_{m'n'} \cos m'\varphi'' \cos n'\varphi' + \sum_{m',n'=1}^{3} q_{m'n'} \sin m'\varphi'' \sin n'\varphi') .$$

Here we can use the following analogue of (2.23)

$$N^{-1} \sum_{k'' \in S} \longrightarrow (2t(2\pi)^2)^{-1} \int_{|\varepsilon''| < \varepsilon_o} d\varepsilon'' \, d\varphi'' \qquad (3.18)$$

and also (2.22), (2 25). Equating coefficients on both sides of (3.17) one can obtain equations for $t_o$, $t_{mn}$, $u_{mn}$. These equations may be written down as

$$t_o = k_o - (4\pi t)^{-1} (\ln(2\beta\gamma\varepsilon_o/\pi)) k_o t_o \quad , \qquad (3.19)$$

$$T = P - P\,T(\ln(2\beta\gamma\varepsilon_o/\pi))/8\pi t = AU^2 k_F^2/(768\pi t) - AT(Uk_F/t)^2(\ln(2\beta\gamma\varepsilon_o/\pi)/6144\pi^2 ,$$

$$U = Q - Q\,U(\ln(2\beta\gamma\varepsilon_o/\pi))/8\pi t = BU^2 k_F^2/(768\pi t) - BU(Uk_F/t)(\ln(2\beta\gamma\varepsilon_o/\pi)/6144\pi^2 .$$

Here k is defined by (3.12), and matrices A,B,P,Q are defined by (3.14), (3.15). One can easily find the solutions of (3.19) :

$$t_o = k_o(1 + k_o(4\pi t)^{-1}\ln(2\beta\gamma\varepsilon_o/\pi))^{-1},$$

$$T = (U^2 k_F^2/768\pi t)\, A\, (I + A\,(Uk_F/t)^2\,(\ln(2\beta\gamma\varepsilon_o/\pi))/6144\pi^2) \quad , \qquad (3.20)$$

$$U = (U^2 k_F^2/768\pi t)\, B\, (I + B\,(Uk_F/t)\,(\ln(2\beta\gamma\varepsilon_o/\pi))/6144\pi^2) \quad .$$

We see from (3.12) that $k_o$ is positive for $U<4\pi t(1+5k_F^2/32)^{-1}$. If $k_o$ is positive we have the same inequality $0<t_o<k_o$ (2.30) as in the 3D-case, and $t_o$ cannot become singular at low temperatures.

As to the matrices T and U they can become singular if matrices A and B have negative eigenvalues. The eigenvalues of A and B are as follows

$$-2\ ,\ 3 \pm \sqrt{10}\ , \qquad A\ ,$$
$$2\ ,\ 3 \pm \sqrt{10}\ , \qquad B\ . \qquad (3.21)$$

We see that A has two negative eigenvalues -2 and $3-\sqrt{10}$ and B has only one negative eigenvalue $3-\sqrt{10}$.

The transition temperature $T_c$ is a maximal value of $T=\beta^{-1}$ for which at least one of two matrices T and U becomes singular. It is clear that $T_c$ corresponds to the negative eigenvalue with the maximal absolute value namely to the eigenvalue -2 of the matrix A. So we get the following equation for $T_c=\beta_c^{-1}$:

$$1 - (Uk_F/t)^2(\ln(2\beta\gamma\varepsilon_o/\pi))/3072\pi^2 = 0 \quad , \qquad (3.22)$$

which gives us

$$T_c = (2\gamma\varepsilon_o/\pi)\exp(-3072\pi^2(t/Uk_F)^2)\ . \qquad (3.23)$$

This formula is obtained with a "logarithmic accuracy" and contains $\varepsilon_o$ like Eq.(2.36) for $T_c$ in the 3D-case. A more exact formula analogous to (2.56) may be obtained by taking all the diagrams up to the fourth order for K into account. Both formulae (2.36) and (3.23) have the same structure. The coefficient before $-(t/Uk_F)^2$ in the exponent in (3.23)

$$3072\pi^2 \sim 3\cdot 10^4 \qquad (3.24)$$

is also very large which makes $T_c$ incredibly low, like it was in the 3D-case. But nevertheless the hope arises to obtain other formulae for $T_c$ different from those derived in the low-density limit.

The results obtained up to now confirm the possibility for superconductivity in the Hubbard model with repulsion and show us the type of pairing which turns out to be completely different in the 2D and in



the 3D cases. In the 3D-model we come to a system like the superfluid $^3$He with p-pairing, where the following anomalous averages may exist

$$\langle a_{k\uparrow}a_{-k\downarrow}\rangle \quad , \quad \langle a_{k\uparrow}a_{-k\uparrow}\rangle \quad , \quad \langle a_{k\downarrow}a_{-k\downarrow}\rangle\ . \qquad (3.25)$$

Each of these functions must be an odd function of k in the case of the p-pairing. In the ordinary superconductors only the average $\langle a_{k\uparrow}a_{-k\downarrow}\rangle$ may exist and it must be an even function of k (the case of s-pairing).

The situation in the 2D Hubbard model is completely different from that in the 3D-model and also from the standard superconductivity theory. Here we arrive at a new type of pairing which can be called the quasi-d-type. Such a pairing arises from the term in K proportional to

$$-\cos 2\varphi\cos 2\varphi'\ . \qquad (3.26)$$

This term is a part of the term

$$-\cos(2\varphi + 2\varphi') = -\cos 2\varphi\cos 2\varphi' + \sin 2\varphi\sin 2\varphi' \qquad (3.27)$$

in K. The addend $\sin 2\varphi\sin 2\varphi'$ corresponds not to attraction but repulsion. The "real" d-pairing term should be as follows

$$-\cos(2\varphi - 2\varphi') = -\cos 2\varphi\cos 2\varphi' - \sin 2\varphi\sin 2\varphi' \quad , \qquad (3.28)$$

and here both terms correspond to attraction. If we change a sign before (3.28) both terms in the RHS of (3.28) will describe the repulsive interaction instead of the attractive one. But if we change a sign before (3.27) the terms with sin-functions becomes attractive instead of the term with cos-functions in the RHS of (3.27). So the term (3.27) leads to pairing for any sign of a coefficient before this term.

The quasi-d-pairing corresponds to only one type of "superfluid" anomalous averages

$$\langle a_{k\uparrow}a_{-k\downarrow}\rangle \qquad (3.29)$$

instead of three types (3.25) in the case of p-pairing. It is an even function of k but it has a more complicated angular symmetry as compared with the case of s-pairing.

It should be mentioned that neither the order parameter nor even the type of pairing in HTSC is known for the moment. A method for the pairing type determination from sound and microwave absorbsion experiments is suggested recently in ref.[11]. Such experiments may, in principle, determine what type of Cooper pairing exists in HTSC namely the p-pairing which arises in the 3D Hubbard model or the quasi-d-pairing which is characteristic of the 2D-model.

## Appendix.

Here the formula (3.7) for $K_2$ is derived. The main idea is to use the natural variables

$$u = (2t)^{-1}(\varepsilon(k_1) + \varepsilon(k_2)) \ , \quad v = (2t)^{-1}(\varepsilon(k_1) - \varepsilon(k_2)) \ , \quad w = t^{-1}\varepsilon(k_1) = u + v \qquad (A.1)$$

when calculating

$$K = (U^2/N) \sum_{k_1} (n(k_1) - n(k_2))/(\varepsilon(k_1) - \varepsilon(k_2))$$

$$= (U^2/N) \sum_{k_1} (n(k_1 + q/2) - n(k_1 - q/2))(\varepsilon(k_1 + q/2) - \varepsilon(k_2 - q/2))^{-1}$$

$$= (2U^2/N) \sum_{k} n(k + q/2)/(\varepsilon(k + q/2) - \varepsilon(k - q/2)) = (U^2/tN) \sum_{w<\varepsilon_F/t} v^{-1} =$$

$$(U^2/(2\pi)^2 t) \int_{w<\varepsilon_F/t} dk_x dk_y/v = (U^2/(2\pi)^2 t) \int_{w<\varepsilon_F/t} dv\, dw/v|J| \ , \qquad (A.2)$$

where

$$|J| = \det \begin{pmatrix} u_{k_x} & u_{k_y} \\ v_{k_x} & v_{k_y} \end{pmatrix} \qquad (A.3)$$

is a Jacobian corresponding to the change of variables $(k_x, k_y) \to (u,v)$. The change $(u,v) \to (v,w) = (v, u+v)$ has $|J|=1$.

Let us consider first the simplest case with $\varepsilon(k)/t = k_x^2 + k_y^2$. Here we have

$$u = (2t)^{-1}(\varepsilon(k + q/2) + \varepsilon(k - q/2)) = k^2 + q^2/4 \ ,$$

$$v = (2t)\,(\varepsilon(k + q/2) - \varepsilon(k - q/2)) = (\vec{k},\vec{q}) \ . \qquad (A.4)$$

and

$$J = \det \begin{pmatrix} 2k_x & 2k_y \\ q_x & q_y \end{pmatrix} = 2(k_x q_y - k_y q_x) = 2[\vec{k},\vec{q}]_z \qquad (A.5)$$

is a double z-component of the vector product of two vectors $\vec{k},\vec{q}$. In our case both vectors $\vec{k}$, $\vec{q}$ are two-dimensional so their vector product has only the z-component. Thus we have

$$J^2 = 4\,|[\vec{k},\vec{q}]|^2 = 4\,(k^2 q^2 - (\vec{k},\vec{q})^2) = 4\,q^2(u - q^2/4) - 4\,v^2$$

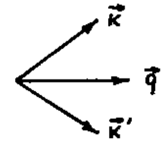


$$= 4\,q^2(w - v - q^2/4) - 4\,v^2 = 4\,w\,q^2 - (2\,v + q^2)^2 \ , \qquad (A.6)$$

and we get the following formula for $K_2$

$$K_2 = (U^2/(2\pi)^2 t) \int_0^{\varepsilon_F/t} dw \int 2(dv/v)\,(4wq^2 - (2v + q^2)^2)^{-1/2}. \qquad (A.7)$$

Here we have to integrate with respect to v (at the fixed w) over all v for which the expression under the square root is positive. The factor 2 in the numerator arises because the pair of variables (u,v) does not determine the vector k by the unique way. The point is that if we know u,v, we know also $k^2 = k_x^2 + k_y^2$ and $(\vec{k},\vec{q}) = k_x q_x + k_y q_y$. But there exist not one but exactly two vectors $\vec{k}$, $\vec{k}'$ with the same $k^2$, $(\vec{k},\vec{q})$ as one can see :



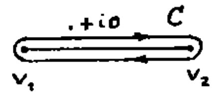

(A.8)

Now we can express the inner integral in (A.7) via the contour integral:

$$\int (2dv/v)(4wq^2 - (2v + q^2)^2)^{-1/2} = \operatorname{Re} \int_{v_1}^{v_2} (2dv/(v + i0))\,(4wq^2 - (2v + q^2)^2)^{-1/2}$$

$$= \oint dv\,(v - i0)^{-1}(4wq^2 - (2v + q^2)^2)^{-1/2}, \qquad (A.9)$$

where the contour C encloses the cut $(v_1, v_2)$ ($v_1, v_2$ are points, where J=0) as follows



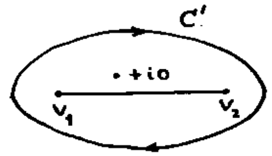

(A.10)

We have further

$$\oint_C dv(v - i0)^{-1}(4wq^2 - (2v + q^2)^2)^{-1/2} \qquad (A.11)$$

$$= 2\pi i \operatorname{res} (v - i0)^{-1}(4wq^2 - (2v + q^2)^2)^{-1/2}|_{v=+i0} + \oint_{C'} (dv/v)(4wq^2 - (2v + q^2)^2)^{-1/2},$$

where the countour C' encloses both the pole v= i0 and the cut $(v_1, v_2)$ :




(A.12)

It is easy to see that $\oint_{C'} = 0$, because we can now enlarge C' up to infinity. We have also

$$\text{res}\ (v - i0)^{-1}(4wq^2 - (2v + q^2)^2)^{-1/2}|_{v=+i0} = (4wq^2 - q^4 - i0)^{-1/2}, \tag{A.13}$$

which implies the following expression for K

$$K_2 = (U^2/(2\pi)^2 t)\ \text{Re}\ (2\pi i \int_0^{\varepsilon_F/t} dw\ (4wq^2 - q^4 - i0)^{-1/2}) = (U^2/(2\pi)^2 t)\ \text{Re}\pi i\ q^{-2}(4wq^2 - q^4 - i0)^{1/2}|_{w=0}^{w=\varepsilon_F/t}$$

$$= (U^2/4\pi t)\ (-1 + (1 - 4\varepsilon_F/tq^2)^{1/2}\ \Theta(1 - 4\varepsilon_F/t\ q^2)\ )\ , \tag{A.14}$$

which coincides with (3.2) for $\varepsilon_F = t\ k_F^2$ .

Now we want to take the correction terms in (3.6) into account. We have

$$u = (1/2)\ ((k_x + q_x/2)^2 + (k_y + q_y/2)^2 + (k_x - q_x/2)^2 + (k_y - q_y/2)^2)$$

$$- (1/24)\ ((k_x + q_x/2)^4 + (k_y + q_y/2)^4 + (k_x - q_x/2)^4 + (k_y - q_y/2)^4)$$

$$= k_x^2 + k_y^2 + (1/4)\ (q_x^2 + q_y^2) - (1/12)(k_x^4 + k_y^4) - (1/8)(k_x^2 q_x^2 + k_y^2 q_y^2) - (1/192)(q_x^4 + q_y^4),$$

$$v = (1/2)\ ((k_x + q_x/2)^2 + (k_y + q_y/2)^2 - (k_x - q_x/2)^2 - (k_y - q_y/2)^2)$$

$$- (1/24)\ ((k_x + q_x/2)^4 + (k_y + q_y/2)^4 - (k_x - q_x/2)^4 - (k_y - q_y/2)^4)$$

$$= k_x q_x + k_y q_y - (1/6)\ (k_x^3 q_x + k_y^3 q_y) - (1/24)\ (k_x q_x^3 + k_y q_y^3)\ . \tag{A.15}$$

Then we get

$$J = \det \begin{pmatrix} u_{k_x} & u_{k_y} \\ v_{k_x} & v_{k_y} \end{pmatrix} = \det \begin{pmatrix} 2k_x - (1/3)\ k_x^3 - (1/4)\ k_x q_x^2, & 2k_y - (1/3)\ k_y^3 - (1/4)\ k_y q_y^2 \\ q_x - (1/24)\ q_x^3 - (1/2)\ q_x k_x^2, & q_y - (1/24)\ q_y^3 - (1/2)\ q_y k_y^2 \end{pmatrix}$$

$$= 2(k_x q_y - k_y q_x) + (k_x q_x - k_y q_y)\ (k_x k_y - (1/4)\ q_x q_y)$$

$$+ (1/3)(q_x k_y^3 - q_y k_x^3) + (1/12)\ (k_y q_x^3 - k_x q_y^3)\ . \tag{A.16}$$

Using identities

$$q_x k_y^3 - q_y k_x^3 = -(k_x q_y - q_x k_y)\ (k_x^2 + k_y^2) - k_x k_y (k_x q_x - k_y q_y)\ ,$$

$$k_y q_x^3 - k_x q_y^3 = -(k_x q_y - q_x k_y)\ (q_x^2 + q_y^2) + q_x q_y (k_x q_x - k_y q_y)\ , \tag{A.17}$$

we can rewrite (A.16) as

$$J = 2(k_x q_y - k_y q_x) - (1/3)\ (k_x q_y - k_y q_x)\ (k_x^2 + k_y^2 + (1/4)\ (q_x^2 + q_y^2))$$

$$+ (2/3)\ (k_x q_x - k_y q_y)\ (k_x k_y - (1/4)\ q_x q_y\ )\ . \tag{A.18}$$



Then we can get the expression (which vanishes if we do not take corrections into account)

$$J^2 + 4v^2 - q^2(4u - q^2) =$$

$$= -(k^4 + 2k_x^2 k_y^2)\ q^2 + 4\ k^2 k_x^2 k_y^2 q_x q_y + (1/6)\ q^2 (\vec{k},\vec{q})^2$$

$$- (1/3)\ q^2 [\vec{k},\vec{q}]^2 - (1/3)\ k^2 q_x^2 q_y^2 + (1/48)\ q^2 (q_x^4 + q_y^4)\ . \tag{A.19}$$

We may rewrite the RHS of this formula as follows

$$- k^4 q^2 (1 + 2q_x^2 q_y^2 q^{-4}) - 2q^{-2}(k^2 q_x q_y - q^2 k_x k_y)^2 + (1/6)\ q^2 (\vec{k},\vec{q})^2$$

$$- (1/3)\ q^2 [\vec{k},\vec{q}]^2 - (1/3)\ k^2 q_x^2 q_y^2 + (1/48)\ q^2 (q_x^4 + q_y^4)\ . \tag{A.20}$$

We have further

$$k^2 q_x q_y - q^2 k_x k_y = (k_x q_y - k_y q_x)\ (k_x q_x - k_y q_y) \cong (J/2)\ (k_x q_x - k_y q_y)\ . \tag{A.21}$$

Then we get $k_x, k_y$, from the first approximation equations

$$k_y q_x + k_y q_y = v \quad , \quad k_x q_y - k_y q_x = J/2\ . \tag{A.22}$$

So we obtain

$$k_x = (v\ q_x + J\ q_y/2)\ q^{-2} \quad , \quad k_y = (v\ q_y - J\ q_x/2)\ q^{-2}\ ,$$

$$k_x q_x - k_y q_y = (v(q_x^2 - q_y^2) + J\ q_x q_y)\ q^{-2}\ . \tag{A.23}$$

Using also the relations

$$(\vec{q},\vec{k})^2 = v^2\ , \quad [\vec{q},\vec{k}]^2 = J^2/4 \quad , \quad k^2 q^2 = v^2 + J^2/4 \quad , \tag{A.24}$$

which are valid also as the first approximation, we arrive at the following expression for the RHS of (A.19)

$$-(v^2 + J^2/4)\ q^{-2}(1 + 2q_x^2 q_y^2 q^{-4}) - J^2 (2q^6)^{-1} (v(q_x^2 - q_y^2) + J\ q_x q_y)^2 +$$

$$q^2 v^2/6 - q^2 J^2/12 - (v^2 + J^2/4)\ q_x^2 q_y^2 / 3q^2 - q^2 (q_x^4 + q_y^4)/48\ . \tag{A.25}$$

Substituting (A.25) into the RHS of (A.19) we can rewrite this equation in the equivalent form as follows

$$J\ (1 + 2a + b(v,J)) = 4w\ q^2 - (2v + q^2)^2 + c(v)\ , \tag{A.26}$$

where

$$a = q^2(1 + q_x^2 q_y^2 q^{-4})/24 \quad ,$$

$$b(v,J) = v^2 q^{-2}(1 - q_x^2 q_y^2 q^{-4}) + v\,J\,q^{-6} q_x q_y (q_x^2 - q_y^2) + J^2(1 + 10 q_x^2 q_y^2 q^{-4})/16q^2 \,,$$

$$c(v) = -v^4 q^{-2}(1 + 2q_x^2 q_y^2 q^{-4}) + v^2 q^2(1 - 2q_x^2 q_y^2 q^{-4})/6 + q^2(q_x^4 + q_y^4)/48 \qquad (A.27)$$

(we change $u \rightarrow w-v$ according to (A.1)).
Values of $a$, $b$, $c$, are small as compared with unity and we have

$$|J|^{-1} \cong (1 + a + (1/2)\,b(v,J))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2}. \qquad (A.28)$$

Thus we arrive at the following formula for $K_2$:

$$K_2 = (U^2/(2\pi)^2 t) \int_0^{\varepsilon_F/t} dw \int (dv/v)\,(2(1 + a) + (1/2)(b(v,J) + b(v,-J)))$$

$$\cdot\,(4wq^2 - (2v + q^2)^2 + c(v))^{-1/2} \qquad (A.29)$$

$$= (U^2/(2\pi)^2 t) \int_0^{\varepsilon_F/t} dw \int (dv/v)\,2(1 + a + f(v^2,J^2))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2},$$

where

$$2f(v^2,J^2) = (1/2)(b(v,J) + b(v,-J)) = v^2 q^{-2}(1 - q_x^2 q_y^2 q^{-4}) + (J^2/16q^2)(1 + 10 q_x^2 q_y^2 q^{-4}) \qquad (A.30)$$

The inner integral in the RHS of (A.29) is taken from $v_1$ to $v_2$, where $v_1$, $v_2$ are just zeros of the expression under the square root. Now we can use the method of evaluating such integrals elaborated above. Namely we have

$$\int (dv/v)\,2\,(1 + a + f(v^2,J^2))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2}$$

$$= \mathrm{Re} \oint_C (dv/(v - i0))\,(1 + a + f(v^2,J^2))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2} \qquad (A.31)$$

$$= \mathrm{Re}\,[2\pi i \;\mathrm{res}\;(v - i0)^{-1}(1 + a + f(v^2,J^2))(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2}|_{v=+i0}]$$

$$+ \;\mathrm{Re} \oint_{C'} (dv/v)\,(1 + a + f(v^2,J^2))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2},$$

where the contours C, C' are the same as before (see (A.10), (A.12)). Thus

$$\mathrm{res}\;(v - i0)^{-1}(1 + a + f(v^2,J^2))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2}|_{v=+i0}$$

$$= (1 + a + f(0,J^2))\,(4w\,q^2 - q^4 + c(0) - i0)^{-1/2} \qquad (A.32)$$



$$= (1 + a)\,(4w\,q^2 - q^4 + c(0) - i0)^{-1/2} + (1/32q^2)\,(1 + 10 q_x^2 q_y^2 q^{-4})\,(4w\,q^2 - q^4 - i0)^{1/2}.$$

The contribution of (A.32) into $K_2$ is equal to

$$(U^2/2\pi t)\;\mathrm{Re} \int_0^{\varepsilon_F/t} dw\;[\,i(1 + a)\,(4w\,q^2 - q^4 - c(0) - i0)^{-1/2}$$

$$+ (i/32q^2)\,(1 + 10 q_x^2 q_y^2 q^{-4})\,(4w\,q^2 - q^4 - i0)^{1/2}\,]$$

$$= (U^2/2\pi t)\;\mathrm{Re}\,[i(1 + a)(2q^2)^{-1}(4w\,q^2 - q^4 - c(0) - i0)^{1/2}$$

$$+ (i/192q^4)(1 + 10 q_x^2 q_y^2 q^{-4})(4wq^2 - q^4 - i0)^{3/2}\,|_{w=0}^{w=\varepsilon_F/t} \qquad (A.33)$$

$$= -(U^2/4\pi t)[(1 + a)(1 - c(0)\,q^{-4}) + (q^2/96)(1 + 10 q_x^2 q_y^2 q^{-4})]$$

for the case $q^2 < 4\varepsilon_F/t$ (namely this condition we shall have). Further we have (note that $\oint_{C'}$ is the clock-wise integral!).

$$\mathrm{Re} \oint_{C'} (dv/v)\,(1 + a\,f(v^2,J^2))\,(4w\,q^2 - (2v + q^2)^2 + c(v))^{-1/2} =$$

$$\mathrm{Re} \oint_{C'} (dv/v)\,\{-(1/2)\,c(v)\,(1 + a)\,(4w\,q^2 - (2v + q^2)^2)^{-3/2}$$

$$+ \;(4w\,q^2 - (2v + q^2)^2)^{-1/2}[(v^2/2q^2)(1 - q_x^2 q_y^2 q^{-4}) + (J^2/32q^2)(1 + 10 q_x^2 q_y^2 q^{-4})\,]\,\}$$

$$\mathrm{Re} \oint_{C'} (dv/v)\,\{-(1/2)(1 + a)[-v^4 q^{-2}(1 + 2q_x^2 q_y^2 q^{-4}) + (1/6)\,v^2 q^2(1 - 2q_x^2 q_y^2 q^{-4})$$

$$+ (q^2/48)(q_x^4 + q_y^4)](4w\,q^2 - (2v + q^2)^2)^{-3/2} + (v^2/2q^2)(1 - q_x^2 q_y^2 q^{-4})(4w\,q^2 - (2v + q^2)^2)^{-1/2}$$

$$+ (1/32q^2)(1 + 10 q_x^2 q_y^2 q^{-4})(4w\,q^2 - (2v + q^2)^2)^{1/2}\,\}$$

$$= \mathrm{Re}\,[-i(1 + a)(2q^2)^{-1}\,(1 + 2\,q_x^2 q_y^2 q^{-4}) \oint_{C'} (v/(2v + q^2))^3\,dv$$

$$+ (i/2q^2)(1 - q_x^2 q_y^2 q^{-4}) \oint_{C'} (v/(v + q^2))dv - (i/32q)(1 + 10\,q_x^2 q_y^2 q^{-4}) \oint_{C'} ((2v + q^2)/v)\,dv]$$

$$= (\pi/16)\,[3(1 + a)(1 + 2\,q_x^2 q_y^2 q^{-4}) - 4(1 - q_x^2 q_y^2 q^{-4}) - (1 + 10\,q_x^2 q_y^2 q^{-4})\,]$$

$$= -\pi/8 + (3\pi/16)\,a\,(1 + 2\,q_x^2 q_y^2 q^{-4}) = -\pi/8 \quad , \qquad (A.34)$$

because here we can neglect the term with a<<1. The contribution of (A.34) into $K_2$ is equal to

$$- (U^2/32\pi t) \int_0^{\varepsilon_F/t} dw \quad = \quad - (U^2 \varepsilon_F/32\pi t^2) . \qquad (A.35)$$

So we have obtained the following expression for K :

$$K \ = \ - (U^2/4\pi t)[(1 + a)(1 - c(0)\, q^{-4})^{1/2} - (q^2/96)(1 + 10\, q_x^2 q_y^2 q^{-4}) + \varepsilon_F/8t \,]$$

$$= \ - (U^2/4\pi t)[1 + a - c(0)/2q^4 - (q^2/96)(1 + 10\, q_x^2 q_y^2 q^{-4}) + \varepsilon_F/8t \,]$$

$$= \ - (U^2/4\pi t)\, [1 + (q^2/48)(1 - 2\, q_x^2 q_y^2 q^{-4}) + \varepsilon_F/8t \,] \ , \qquad (A.36)$$

which coincides with (3.7).



## References.


[1]. Anderson P.W., Science, 1987, v.235, n.4793, p.1196-1198.

[2]. Emery V.J., Phys. Rev. Lett., 1987, v.58, n.26, p.2794-2797.

[3]. Anderson P.W., Bascaran G., Zou Z., Hsu T., Phys. Rev. Lett., 1987, v.58, p.2790-2793.

[4]. Dzyaloshinskii I.E., Polyakov A.M., Wiegmann P.B., Phys. Lett., 1988, A127, n.2,p.112-114.

[5]. Wiegmann P.B., Phys. Rev. Lett., 1988, v.60, n.9, p.821-824.

[6]. Polyakov A.M., Mod.Phys. Lett., 1988, A3, n.3, p.325-328.

[7]. Mavromatos N.E., Ruiz-Altaba M., Phys. Lett.,1989, A142, n.6-7, p.419-425.

[8]. Randjabar-Daemi S., Abdus Salam, Strathdee J. Trieste Preprint IC/89/283, 1989.

[9]. Abrikosov A.A., Gor'kov L.P., Dzyaloshinskii I.E. Methods of Quantum Field Ttheory in Statistical Physics. 1962. Fizmatgiz, Moscow (in Russian). English translation: Ed. by Silverman, Richard A., New York: Dover, 1975, 352 p.

[10]. Kagan M.Yu., Chubukov A.V. JETP Letters, 1988, v.47 (10), p.614-617.

[11]. Brusov P.N., Nasten'ka M.Y., Filatova-Novoselova T.V. Phys. Lett., 1989, A142, n.2,3, p.179-180.