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MANY-BODY PROCESSES AT HIGH ENERGY

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During the last few years a very large amount of information concerning two-body processes was accumulated, and adequate techniques to analyse high energy interactions in terms of two-body processes were developed. The case of many-body processes does not look so clean and nice. Our knowledge of what is really happening, when, in the result of collision of two particles at accelerator energy, several secondary particles are produced, is very limited.

The aim of this report is to present a few examples of the approach to the problem of the analysis of high multiplicity events, based mainly on the results of the study of pion-proton interaction in the range 4 to 10 GeV/c.

The first question is : how big is the cross section for many-body final states ? At present no-one can answer this question. Some people believe, and may have reasons for believing, that the big majority of the observed processes are two-body reactions, where two particles decay in many pions, thus producing many pions in the final state. This problem is far from being settled. On the other hand there is no sharp distinction between low multiplicity and high multiplicity events, there being some sort of continuum which extends from elastic and quasi-elastic scattering at low multiplicity, to isotropically produced pions in c.m. at the highest multiplicities observed. Definition of high multiplicity, where many-body processes can dominate, is therefore rather arbitrary. Let us define, for present accelerator energies, high multiplicity as five or more pions in the final state. The typical cross-sections for many-pion reactions which one can analyse in bubble chamber are not very large, since one is restricted to events with no more than one neutral particle. For example at 8 GeV/c the π^+ proton interaction cross section for fitted six-prong events(1) is $\sigma_6^{fit} = 1.65 \pm 0.15$ mb, and for 8-prong events at the same energy(2) $\sigma_8^{fit} = 0.21 \pm 0.03$ mb. In order to calculate the complete contribution of many-pion final states to the total cross section one has to take into account all the unobservable processes with two or more neutral particles. The real multiplicity spectrum - by real multiplicity I mean the number of pions produced independently of their charge - can be calculated under the assumption that for a given multiplicity of pions, the probability of a given charge configuration is given by the statistical model(3). The validity of this assumption can be checked as follows : starting from the cross section for an observable charge configuration, e.g. $p3\pi^+2\pi^-$, one calculates, using statistical weights, the cross section for the production of n pions, in our example five pions. One repeats this for all multiplicities. Now the sum of the cross sections for all multiplicites is equal to the total cross section for pion production, known from independent measurement. The results of the calculation are as follows(4) : at 4 GeV/c, where for π^+ proton interaction the cross section for pion production is $\sigma_\pi = 20.1$ mb, the calculated value is $\sigma_\pi^{cal} = 20.35$ mb ; at 8 GeV/c, $\sigma_\pi = 17.9$ mb, $\sigma_\pi^{cal} = 18.29$ mb. Since σ_π^{cal} was calculated starting from only about 30% of the total cross section, fraction corresponding to fitted channels, the agreement is quite satisfactory. Resulting multiplicity distributions(4) for 4 GeV/c π^+p, 8 GeV/c π^+p and 10 GeV/c π^-p interactions are presented in Fig. 1. It can be seen from Fig. 1 that at 8 GeV/c the cross section for high multiplicity processes ($\geqslant 5\pi$) corresponds to more than half of pion production cross section. It is obvious, that one must try to understand processes responsible for about half of the inelastic cross section.

Let us proceed to single-particle distributions. The most typical for high energy interaction is the baryon angular distribution. Fig. 2 presents some examples of nucleon angular distributions in the c.m. system for π-proton interaction.(5,6,7,8,9). The common features of all this distributions are: a pronounced backward peak and a more or less isotropic continuum. Shaded area of histograms corresponds to symmetric part of angular distribution(*). The relative amount of this symmetric component increases when energy decreases and multiplicity increases. This symmetric component may be related to "central" collisions if one believes that what one observes is a mixture of peripheral and central collisions. By central we will mean collisions which can be described by the statistical model. It is a purely phenomenological definition based on the shape of

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(*) Obtained by the reflection of the forward part of the distribution by $\cos\vartheta^* = 0$ into backward part.

angular distributions, of spectra, etc. for secondary particles. Whether such a distinction between periphal processes and central or statistical processes has deeper physical meaning is certainly a problem. It is possible that the isotropic component is only a tail of peripheral process. If, on the other hand, high multiplicity is dominated by the processes that we call statistical, which give symmetric or even isotropic angular distribution, one can try to estimate the importance of such process quantitatively. The result of such an estimate of the cross section for statistical collisions$^{(4)}$ at several primary energies is presented in Fig. 3. The amount of central collisions drops when the c.m. energy increases, roughly as $1/E_{c.m.}$

There exists also a second, independent possibility of the determination of the cross section in question. One can start from the data on large angle π-p scattering$^{(10,11,12)}$ and assuming the validity of the statistical model calculate the cross section for the statistical collisions. The lower lying points on Fig. 3 are calculated according to this method.

There is a big discrepancy between results obtained by the two methods. It seems that the first method gives an upper limit of the σ_c because this estimation contains a tail of peripheral processes. Too low values of σ_c as determined from the elastic scattering may be related to too high value of the probability of the decay of compound system into two primary particles, calculated without fully taking into account resonance production. Both methods give the same type of energy dependence of the cross section, in agreement with the prediction of Fast, Hagedorn and Jones$^{(13)}$. The shaded area in Fig. 1 corresponds to the central cross section for given multiplicity of pions in final state.

The next problem I would like to discuss is the behaviour of the transverse momentum of particles. A few years ago, at the Geneva Conference, Bigi et al.$^{(14)}$ presented evidence for a mass-dependence of average transverse momentum. How does transverse momentum depend on the multiplicity of the produced particles ? No such a dependence seems to be present in Fig. 4, where is plotted the average value of the transverse momentum vs the number of charged secondary particles for 24 GeV/c p-p collision(15). The same graph contains also the average values of the c.m. particle momentum. The $<p_t>$ corresponding to the highest multiplicity is lower than the other values but this is only the reflection of the fact that p^* average is, in this case, also very low. It seems therefore that there is no multiplicity dependence of $<p_t>$ in proton-proton interactions. One has to bear in mind, however, that the result for a given charge mutliplicity is averaged over several channels, if one takes into account neutral particles. Fig. 5 presents results of the Aachen-Berlin-CERN collaboration(26) on the transverse momentum in several low-multiplicity channels of π^+p interaction at 8 GeV/c. The situation is very complicated - the particles in almost every channel have their own set of $<p_t>$ values, different from those for other channels. At higher multiplicities the situation becomes simpler. Fig. 6 contains, together with data from Fig. 5, the results of Bardadin et al (1) and Bartke et al.(2) for the high multiplicity transverse momentum of pions. Once one is beyond the complicated region of low multiplicity, one observes in pion-proton interaction a monotonic decrease of $<p_t>$ of pions when multiplicity increase. Fig. 7 contains data on nucleon transverse momenta. The effect, if there is one, is much feebler, i.e. the transverse momentum of nucleons has very feeble dependence on multiplicity. This effect is illustrated better in Fig. 8, where the ratio $<p_t^\pi> / < p_t^N>$ is plotted against multiplicity. At low multiplicity this ratio is about one, decreasing for 8 pions in final state to the value of about 0.65. Maybe this is the explanation why in some experiments one finds strong mass-dependence of transverse momentum and in others not - the effect is multiplicity dependent. In very peripheral, low multiplicity interactions there seems to be no mass-dependence of transverse momentum, while the effect is strongest at highest multiplicity. It is plausible therefore, that this effect is of statistical nature.

The Fig. 9 presents the ratio of the average c.m. momentum of pions to the average transverse momentum for different multiplicities. There is a distinct difference between the very low multiplicity channels, where $<p^*> >> <p_t>$, and the higher multiplicity channels, where this ratio slowly approaches the limit, predicted for isotropic angular distribution and for a secondary spectrum independent of the angle of emission, this limit being equal to $4/\pi$.

Proceeding to a more detailed analysis of the experimental data let us discuss now the correlations between transverse momentum and longitudinal component of c.m. momentum of particles. A strong correlation of this type was found by Bardadin et al.$^{(16)}$ in the reaction $\pi^- p \to p 2\pi^+ 3\pi^-$ at 10 GeV/c primary momentum. Average value of p_t against p_l for pions and protons in this experiment is presented in Fig. 10. For pions there is a distinct drop in the average p_t for the vicinity of $p_l^* = 0$. Fig. 11 shows similar graph for the reaction $\pi^+ p \to p 2\pi^+ \pi^-$ at 8 GeV/c$^{(26)}$. This reaction is dominated by two-body processes, and there is a complicated structure in p_t vs p_l^*. Apart from the dip at $p_l^* = 0$ for pions there is an enhancement in p_t for forward protons and for backward positive pions. Fig. 12 presents the results for pions of the Dubna and Bucharest collaboration, Belyakov et al.$^{(17)}$, on four-prong π^--proton interactions at 7.5 GeV/c. The dip in the middle is very pronounced. It was shown by Bardadin et al.$^{(16)}$ that such a dip can be explained in terms of a geometric effect, depending on the shape of spectrum of particles, mainly. An example of how this effect can be created is presented in Fig. 13. Assume a spectrum of pions of the shape of Fig. 13a - just two lines, one at very low momentum, p_1 , the second at high momentum, p_2. The situation on the p_t vs p_l^* plot is shown in Fig. 13b, under the assumption of isotropic angular distribution. For $|p_l^*|$ bigger than p_1 transverse momentum is given by the circle of radius p_2 on the plot (Fig. 13c), for $|p_l^*| < p_1$ the main contribution comes from the low momentum line, making p_t much smaller. The spectrum of pions, calculated from covariant phase-space produces this effect, the spectrum of protons does not. This can be seen in Fig. 10, where the results of calculations using covariant phase-space, and using phase-space corrected to include peripheral effects, are also plotted.

The main interest of looking for effects like the dip in p_t for small p_l^* is connected with the question of whether high multiplicity events are dominated by two-body processes, both bodies decaying into several particles. Such processes, very peripheral, are easily detectable at low multiplicity. A good example is the reaction : $\pi^+ p \rightarrow N^{*++} \omega^o$. Hunting for such final states, if the spectrum of masses of "fireballs" is broad and if they decay into many particles, is very difficult. In the vicinity of 10 GeV/c, decay products of both intermediate bodies will mix together. There is some evidence that high multiplicity events produced in π-p collision are not dominated by such processes. The analysis performed by the Warsaw group(8,18) on six-prong events from 8 GeV/c $\pi^+ p$ and 10 GeV/c $\pi^- p$ interactions, was made as follows. One arranges all particles of a given jet according to their c.m. longitudinal momentum (Fig. 14a) and one makes a projection of all momenta on the plane perpendicular to the primary direction (Fig. 14b). All transverse momenta are vectors on this plane. Now one produces a vector sum of transverse momenta of particles number one and number two, one and two and three, and so on. The length of this vector sum has some dependence on longitudinal momentum (Fig. 14c). One repeats this analysis for all events, and one produces the distribution of the average vector sum of transverse momenta vs longitudinal momentum. This type of distribution is very sensitive to the presence of two-body processes. The result of the analysis was that the experimental distribution is incompatible with the hypothesis of an intermediate two-body state, but only if the angle of emission of fireballs in c.m. system is smaller than about 25°, and if their mass spectrum is not very broad and flat. So the problem is not fully settled.

Once one assumes that there are two-body processes at high multiplicity, one must try to separate the individual event into two parts, corresponding to two bodies, in order to study their properties. It seems that there is no unique prescription on how to do this at present accelerator energies. A few years ago Brandt et al.(19) proposed the so-called "principal axis" method. In this method one chooses as the intermediate bodies, two groups of particles in such a way that the c.m. momentum of each body is the largest possible. Studying properties of 10 GeV/c $\pi^- p$ interactions with production of strange particles, these authors found some evidence in favor of the two-body hypothesis. Dremin et al.(20)

propose a method consisting in the study of Δ^2 distribution between different groups of particles within an event, in order to distinguish between different types of interactions (statistical, excited nucleons, fireball production) in proton-proton interaction. Another method, based on the search for maximum "u" (four-momentum transfer in crossed channel) was proposed by French(21). This method works well for low multiplicity events from 8 GeV/c π^+p interaction(22). Before one applies one of these methods to high multiplicity events, where an important mixing between particles from two bodies must occur, one should undertake a detailed Monte Carlo study of the problem.

There exists one simple method of at least diminishing the number of particles in final state, if one cannot reduce it easily to two-body process: one has to look for known resonances. Fig. 15 presents the results of the Aachen-Berlin-Bonn-Hamburg-Munich collaboration(23), obtained in the study of six-prong interactions of 4 GeV/c π^+ mesons with protons. The reaction is $\pi^+p \rightarrow p3\pi^+2\pi^-\pi^0$. The π^+p effective mass distribution shows abundant production of N^{*++} and a large amount of ω^0 is present in the $\pi^+\pi^-\pi^0$ effective mass distribution. These two resonances reduce considerably the number of particles in the final state: 50% of the process in question is in fact $\pi^+p \rightarrow N^{*++}\omega^0\pi^+\pi^-$. Another example is ρ^0 production in six-prong interactions of 8 GeV/c π^+ mesons with protons(1), Fig. 16.

The last part of this report is related to angular correlations between pions. A few years ago Goldhaber et al.(24) have presented evidence for a certain type of correlations in the c.m. angle between two pions produced in many-pion proton antiproton annihilation. The average angle between pions of like charge (++ or --) is very significantly smaller than the average angle between pions of different (+-). This effect is also present in many-pion final states in 8 GeV/c π^+-proton interaction.(2) Fig. 17 presents the distributions of the cosine of $\pi\pi$ angle, $\alpha_{\pi\pi}$, for the process: $\pi^+p \rightarrow p4\pi^+3\pi^-$. The distribution for $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs is very different from the distribution for $\pi^+\pi^-$ pairs. As a measure of the effect Goldhaber et al. introduced $\gamma = \frac{B}{F}$, B = number of pairs with $\alpha_{\pi\pi} > 90^\circ$, F = number of pairs with $\alpha_{\pi\pi} < 90^{\circ}$. Table I contains the results for different channels, together with γ-coefficients for 7π annihilation$^{(25)}$. Different behaviour of like-charge and unlike-charge pairs was explained by Goldhaber et al. in terms of the statistical model by the effect of Bose-Einstein statistics of pions. The essential assumptions being the representation of the constant matrix element of the ordinary statistical model in the form

$$\int_{V} d^{3n}r \left| e^{i\Sigma p_i r_i} \right|^{2}$$

and subsequent symmetrization of the exponential over permutations of identical particles. The presence of this effect in many-body final states in π-p collisions is a strong argument in favour of the statistical nature of such processes. It is a very interesting question, whether this effect is present also at lower multiplicities. Investigating such and similar correlations will possibly lead us to a better understanding of high multiplicity processes.

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TABLE I

Ratio of the number of pion pairs with $\alpha_{\pi\pi} < 90^\circ$ to the number of pion pairs with $\alpha_{\pi\pi} > 90^\circ$ for like charge ($\pi^+\pi^+$, $\pi^-\pi^-$) and unlike charge ($\pi^+\pi^-$, $\pi^+\pi^0$, $\pi^-\pi^0$) pion pairs for different channels.

Final State	Pairs	γ^{like}	γ^{unlike}
$\pi^+ p \to p4\pi^+3\pi^-$	$\pi^+\pi^+$ $\pi^-\pi^-$	1.11 ± 0.04	-
	$\pi^+\pi^-$	-	1.46 ± 0.05
$\to p4\pi^+3\pi^-\pi^0$	$\pi^+\pi^+$ $\pi^-\pi^-$	1.09 ± 0.04	-
	$\pi^+\pi^-$	-	1.40 ± 0.04
	$\pi^+\pi^0$ $\pi^-\pi^0$	-	1.20 ± 0.04
$\to n5\pi^+3\pi^-$	$\pi^+\pi^+$ $\pi^-\pi^-$	1.15 ± 0.05	-
	$\pi^+\pi^-$	-	1.25 ± 0.05
ANNIHILATION : $\bar{p}p \to 3\pi^+3\pi^-\pi^0$ at 3 GeV/c (25)	$\pi^+\pi^+$ $\pi^-\pi^-$	1.25 ± 0.03	-
	$\pi^+\pi^-$	-	1.73 ± 0.03

Figure Captions

Fig. 1 Multiplicity distributions for π-p interactions. Shaded area corresponds to "central" collisions.

n = number of pions in final state.
$\bar{n}$ = average number of pions.
$\bar{n}^c$ = " " " " in "central" collision.

Fig. 2 Examples of c.m.s. angular distributions of nucleons from π-proton interactions$^{(5,6,7,8,9)}$.

Fig. 3 Fraction of "central" collisions in function of incident momentum, for π-p interactions.

o calculated from the symmetric part of nucleon angular distribution.

x,Δ calculated from large angle elastic π-p scattering data (ref. 10,11,12).

Fig. 4 Average value of $p_\perp$ for pions produced in 24.5 GeV/c p-p collisions vs the number of prongs$^{(15)}$.

Fig. 5 Average transverse momentum in low multiplicity channels 8 GeV/c π^+p interaction $^{(26)}$.

Fig. 6 Average transverse momentum of pions vs multiplicity for the following channels :

$$\left.\begin{array}{ll} \pi^+p \rightarrow & p\pi^+\pi^0 \\ & n\pi^+\pi^+ \\ & p\pi^+\pi^+\pi^- \\ & p\pi^+\pi^+\pi^-\pi^0 \\ & n\pi^+\pi^+\pi^+\pi^- \end{array}\right\} (26) \qquad \begin{array}{ll} \pi^+p \rightarrow & \left.\begin{array}{l} p3\pi^+2\pi^- \\ p3\pi^+2\pi^-\pi^0 \end{array}\right\} (1) \\ & \left.\begin{array}{l} p4\pi^+3\pi^- \\ p4\pi^+3\pi^-\pi^0 \\ n5\pi^+3\pi^- \end{array}\right\} (2) \end{array}$$

Fig. 7 Average transverse momentum of nucleons vs multiplicity.

Fig. 8 Ratio of the average value of $p_\perp$ for pions to the average value of $p_\perp$ for nucleons vs multiplicity.

Fig. 9 Ratio of the average c.m. momentum of pions to the average transverse momentum of pions vs multiplicity.

Fig. 10 Average value of transverse momentum vs c.m. longitudinal momentum for pions and protons from the reaction $\pi^-p \rightarrow p2\pi^+3\pi^-$ at 10 GeV/c$^{(16)}$.

Fig. 11	Average value of transverse momentum vs c.m. longitudinal momentum for pions and protons from the reaction $\pi^+p \to p2\pi^+\pi^-$ at 8 GeV/c(26).
Fig. 12	Average value of transverse momentum vs. c.m. longitudinal momentum for pions from four-prong interactions of π^- with protons at 7.5 GeV/c(17).
Fig. 13	Example of the creation of the dip in $p_\perp$ vs p_L^* distribution.
Fig. 14	Analysis of a jet in terms of the dependence of $\Sigma\hat{p}_\perp$ on p_L^*.
Fig. 15	Effective mass distributions for (proton π^+) and ($\pi^+\pi^-\pi^0$) combinations for the process: $\pi^+p \to p3\pi^+2\pi^-\pi^0$ at 4 GeV/c (23).
Fig. 16	Effective mass distributions for ($\pi^+\pi^-$) pairs from the reactions: $\pi^+p \to p3\pi^+2\pi^-$ $\to p3\pi^+2\pi^-\pi^0$ at 8 GeV/c(1).
Fig. 17	Distribution of the cosine of the π-π c.m. angle for like charge and unlike charge pion pairs in the reaction : $\pi^+p \to p4\pi^+3\pi^-$ at 8 GeV/c(2).

References

1. M. Bardadin-Otwinowska, M. Danysz, T. Hofmokl, S. Otwinowski, M. Piotrowska, R. Sosnowski, M. Szeptycka and A. Wróblewski, "Six prong interactions of 8 GeV/c π^+ in hydrogen", Report submitted to the XIIIth International Conference on High Energy Physics, Berkeley, 1966.
2. J. Bartke, O. Czyżewski, J. Danysz, A. Eskreys, J. Łoskiewicz, P. Malecki, K. Eskreys, K. Juszczak, D. Kisielewska and W. Zielinski, "Some aspects of many-pion production by 8 GeV/c π^+", Report submitted to the XIIIth International Conference on High Energy Physics, Berkeley, 1966.
3. J. Bartke, Institute of Nuclear Research Report 646/VI/PH/1965/1, to be published in Nucl. Physics.
4. J. Bartke and O. Czyżewski, "Cross sections for 'central' collisions and multiplicity distributions in pion production in π-p collisions at 4 - 10 GeV/c", Report submitted to the XIIIth International Conference on High Energy Physics, Berkeley, 1966.
5. Aachen-Berlin-Birmingham-Bonn-Hamburg-London(I.C.)-München Collaboration, Phys. Rev. 138, B.897 (1965).
6. Private Communication from Prof. M. Deutschmann.
7. Aachen-Berlin-CERN Coll., Private Communication from Dr. D.R.C. Morrison.
8. M. Bardadin, L. Michejda, S. Otwinowski, R. Sosnowski, Institute of Nuclear Research Report 511/VI/1964.
9. Cracow Bubble Chamber Group, preliminary data.
10. M.L. Perl, Y.Y. Lee and E. Marquit, Phys. Rev. 138 B, 707 (1965)
11. C.T. Coffin, N. Dikman, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger and D. Williams, Phys. Rev. Letters 15, 838 (1965)
12. J. Orear, R. Rubinstein, D.B. Scarl, D.H. White, A.D. Krisch, W.B. Frisken, A.L. Read and H. Rudermann, Phys. Rev. Letters 15, 309 (1965).
13. G. Fast, R. Hagedorn and L.W. Jones, Nuovo Cimento 27, 856 (1963).

14. A. Bigi, S. Brandt, R. Carrara, W.A. Cooper, A. de Marco, G.R. Macleod, Ch. Peyrou, R. Sosnowski and A. Wróblewski, Proceedings of the 1962 International Conference on High Energy Physics at CERN, p.237.

15. D.R.O. Morrison, "The High Energy Physics of Strong Interactions up to 30 GeV", CERN/TC/PHYSICS 63-1 (1963).

16. M. Bardadin-Otwinowska, L. Michejda, S. Otwinowski, R. Sosnowski, Phys. Letters **21**, 351 (1966).

17. V.A. Beljakov, E.N. Kladnitskaja, E.S. Kuznetsova, E. Balea, O. Balea, A. Mihul, M. Sabău, "Four prong π^-p interactions at 7.5 GeV", Part I, Report submitted to the XIII International Conference on High Energy Physics, Berkeley, 1966.

18. Private Communication from Dr. R. Sosnowski.

19. S. Brandt, Ch. Peyrou, R. Sosnowski, A. Wróblewski, Phys. Letters **12**, 57 (1964).

20. I.M. Dremin, G.B. Zhdonov, M.I. Tretyakova, M.M. Chernavski, "New Method of Nuclear Interaction Mechanism Investigations", Report submitted to the XIII International Conference on High Energy Physics, Berkeley, 1966.

21. Private Communication from Dr. B. French.

22. Private Communication from Dr. D.R.O. Morrison.

23. Aachen-Berlin-Bonn-Hamburg-München Collaboration, "π^+p Interactions at 4 GeV/c : Six Prong Events", DESY preprint 66/11 (1966).

24. G. Goldhaber, S. Goldhaber, W. Lee and A. Pais, Phys. Rev. **120**, 300 (1960).

25. J.A. Danysz and A. Shapira, CERN/TC/PHYSICS 65/8 (1965).

26. Aachen-Berlin-CERN Collaboration, Private Communication from Dr. J. Łoskiewicz.

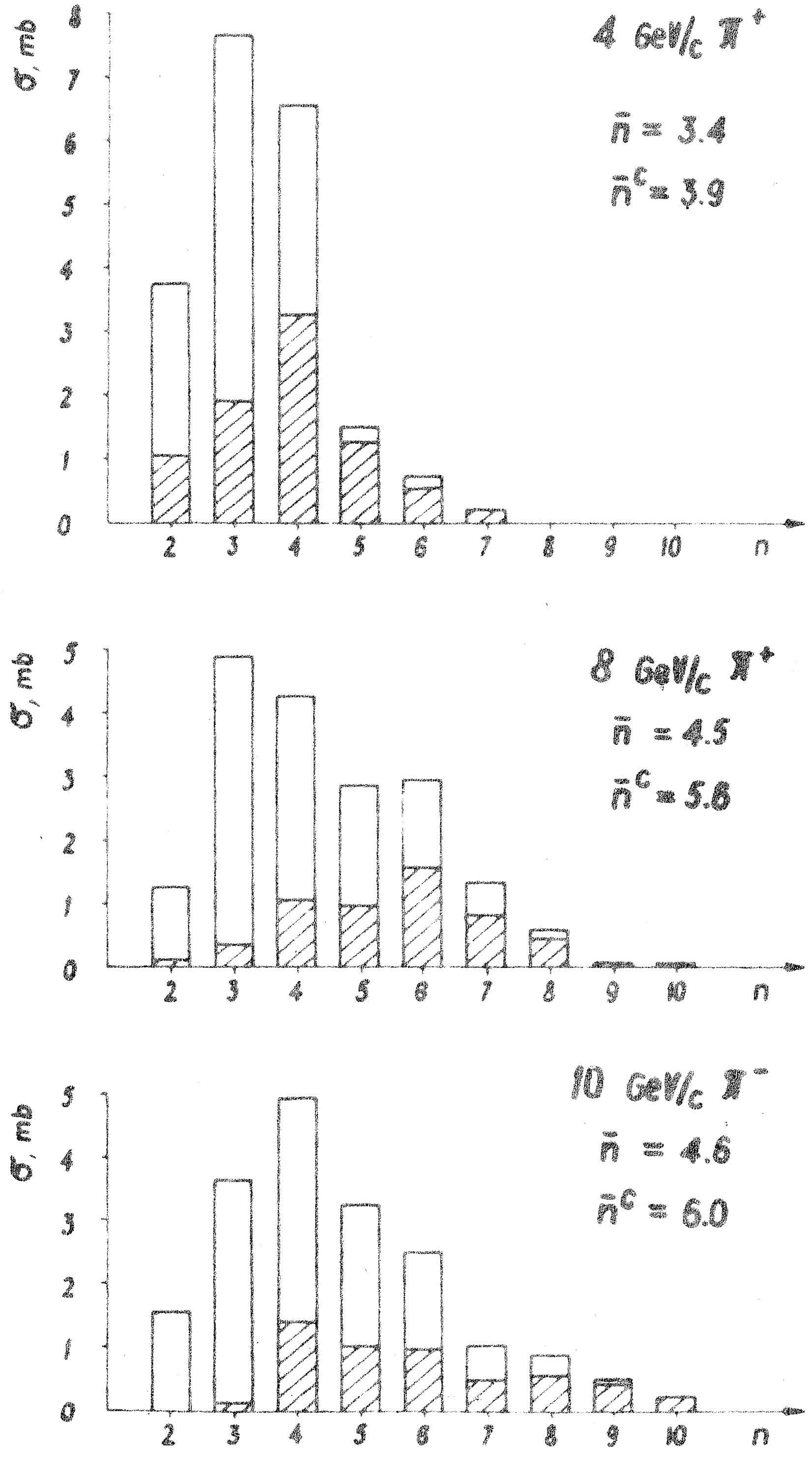


Fig. 1 — Multiplicity distributions.

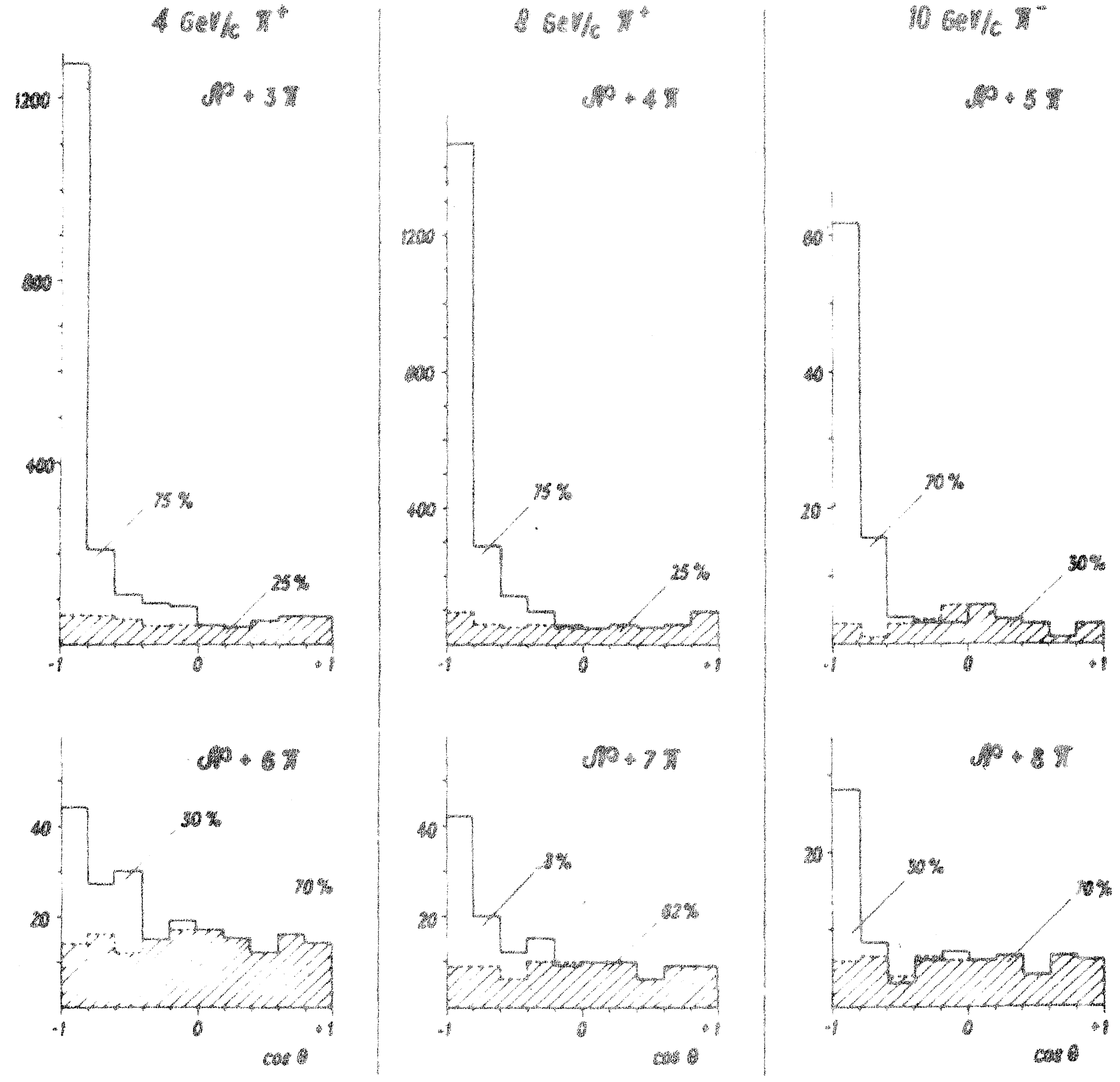


Fig. 2 — C.m.s. angular distributions of baryons.

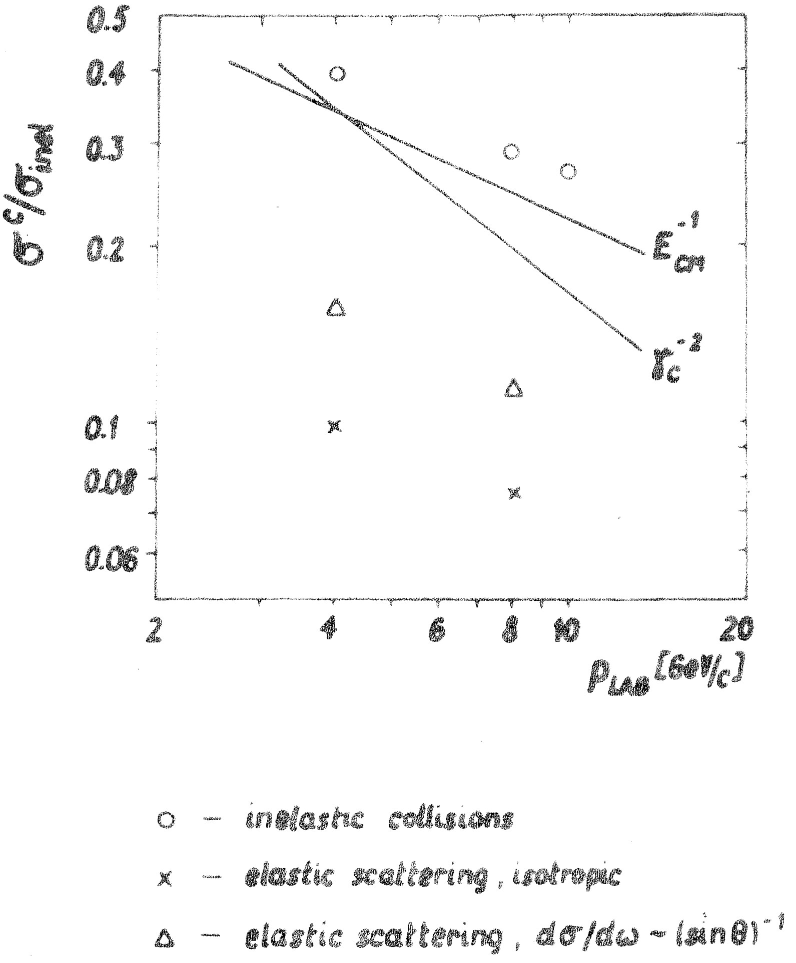


Fig. 3 – Fraction of "central" collisions vs. incident momentum.

VARIATION OF TRANSVERSE AND C.M. TOTAL MOMENTUM WITH NUMBER OF CHARGED PRONGS IN 24.5 GeV/c p–p INTERACTION

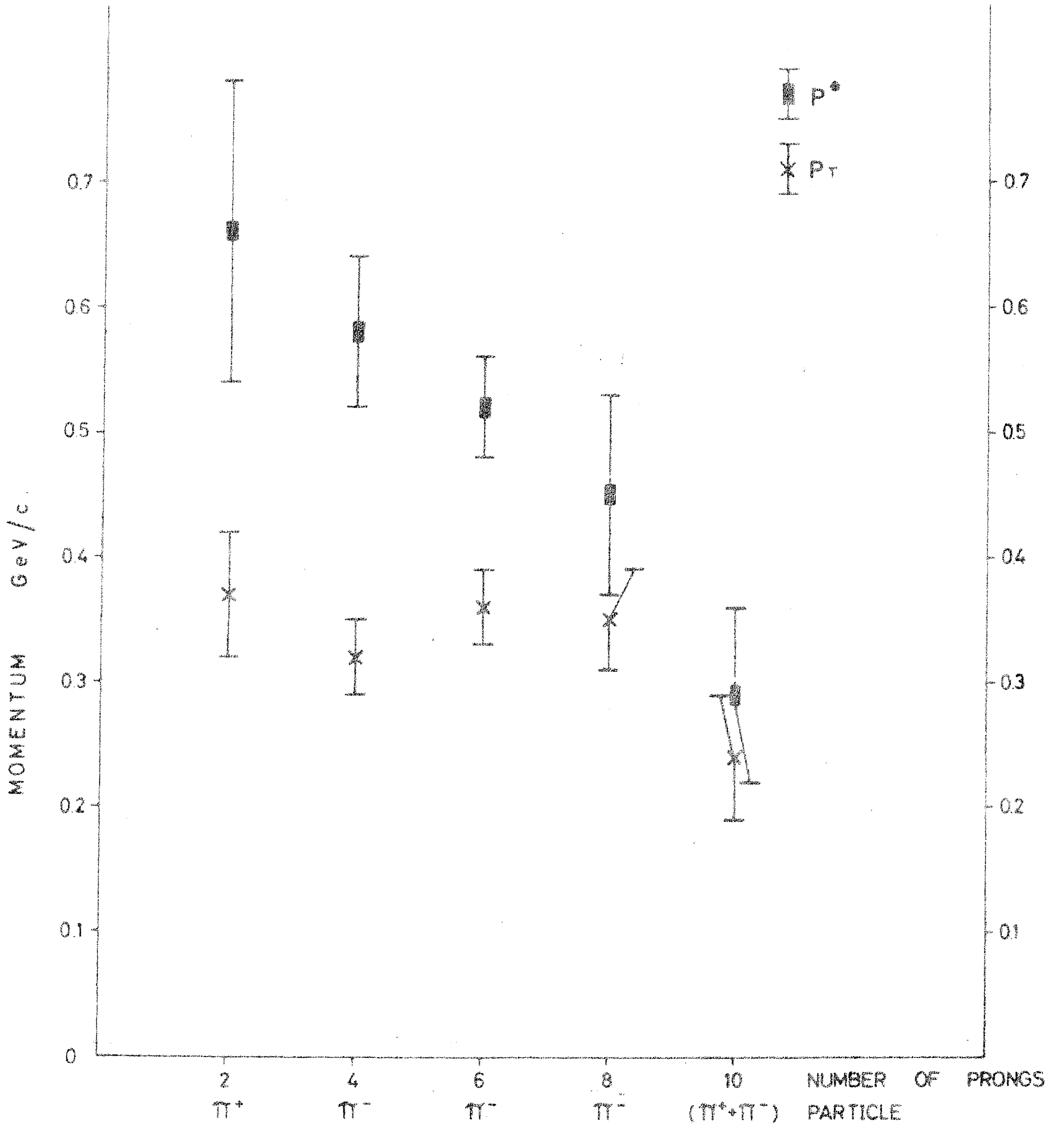


Fig. 4

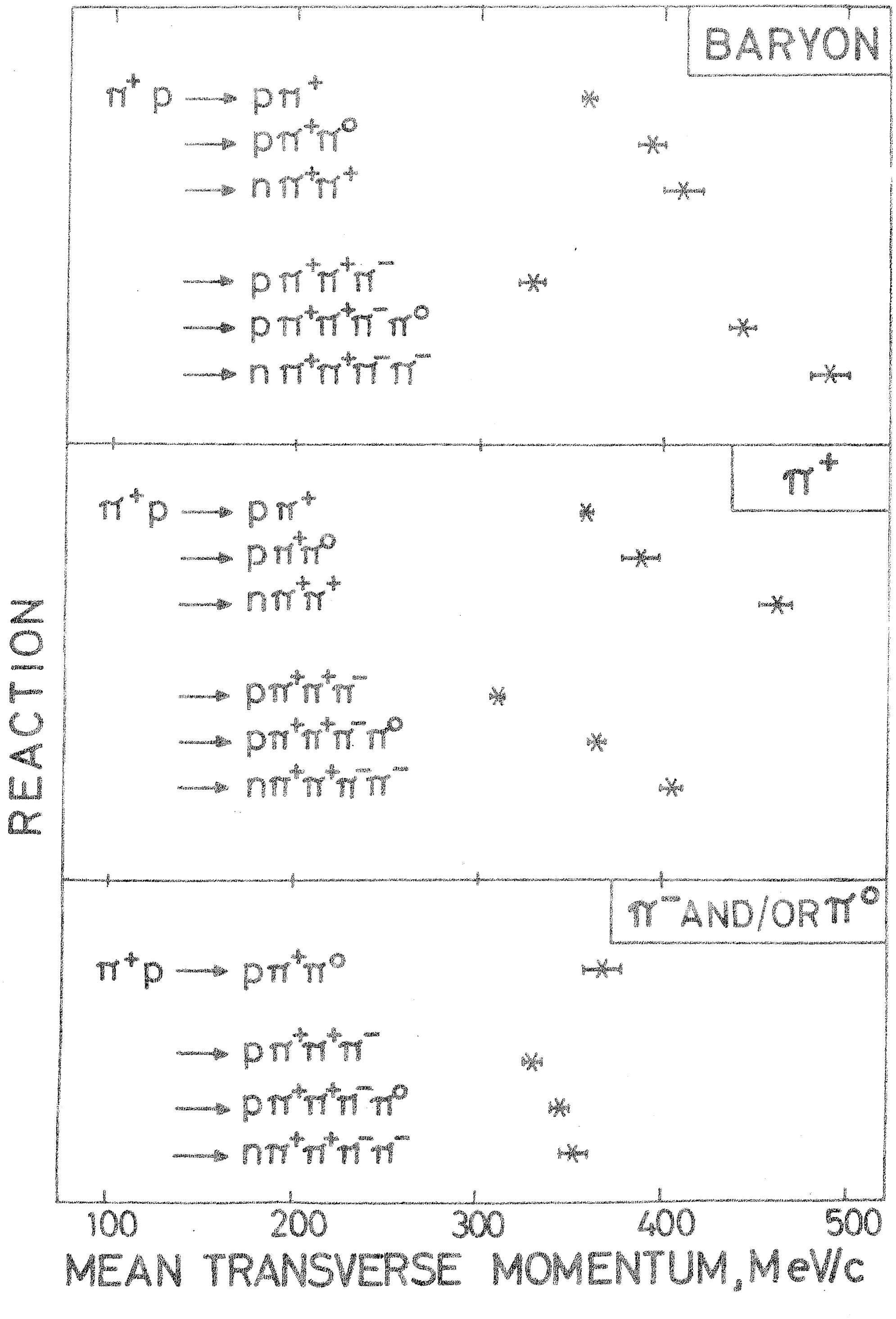

BARYON
$\pi^+ p \rightarrow p\pi^+$
$\rightarrow p\pi^+\pi^0$
$\rightarrow n\pi^+\pi^+$
$\rightarrow p\pi^+\pi^+\pi^-$
$\rightarrow p\pi^+\pi^+\pi^-\pi^0$
$\rightarrow n\pi^+\pi^+\pi^+\pi^-$
π^+
$\pi^+ p \rightarrow p\pi^+$
$\rightarrow p\pi^+\pi^0$
$\rightarrow n\pi^+\pi^+$
$\rightarrow p\pi^+\pi^+\pi^-$
$\rightarrow p\pi^+\pi^+\pi^-\pi^0$
$\rightarrow n\pi^+\pi^+\pi^+\pi^-$
π^- AND/OR π^0
$\pi^+ p \rightarrow p\pi^+\pi^0$
$\rightarrow p\pi^+\pi^+\pi^-$
$\rightarrow p\pi^+\pi^+\pi^-\pi^0$
$\rightarrow n\pi^+\pi^+\pi^+\pi^-$
REACTION
100
200
300
400
500
MEAN TRANSVERSE MOMENTUM, MeV/c

Fig. 5

PS/5681

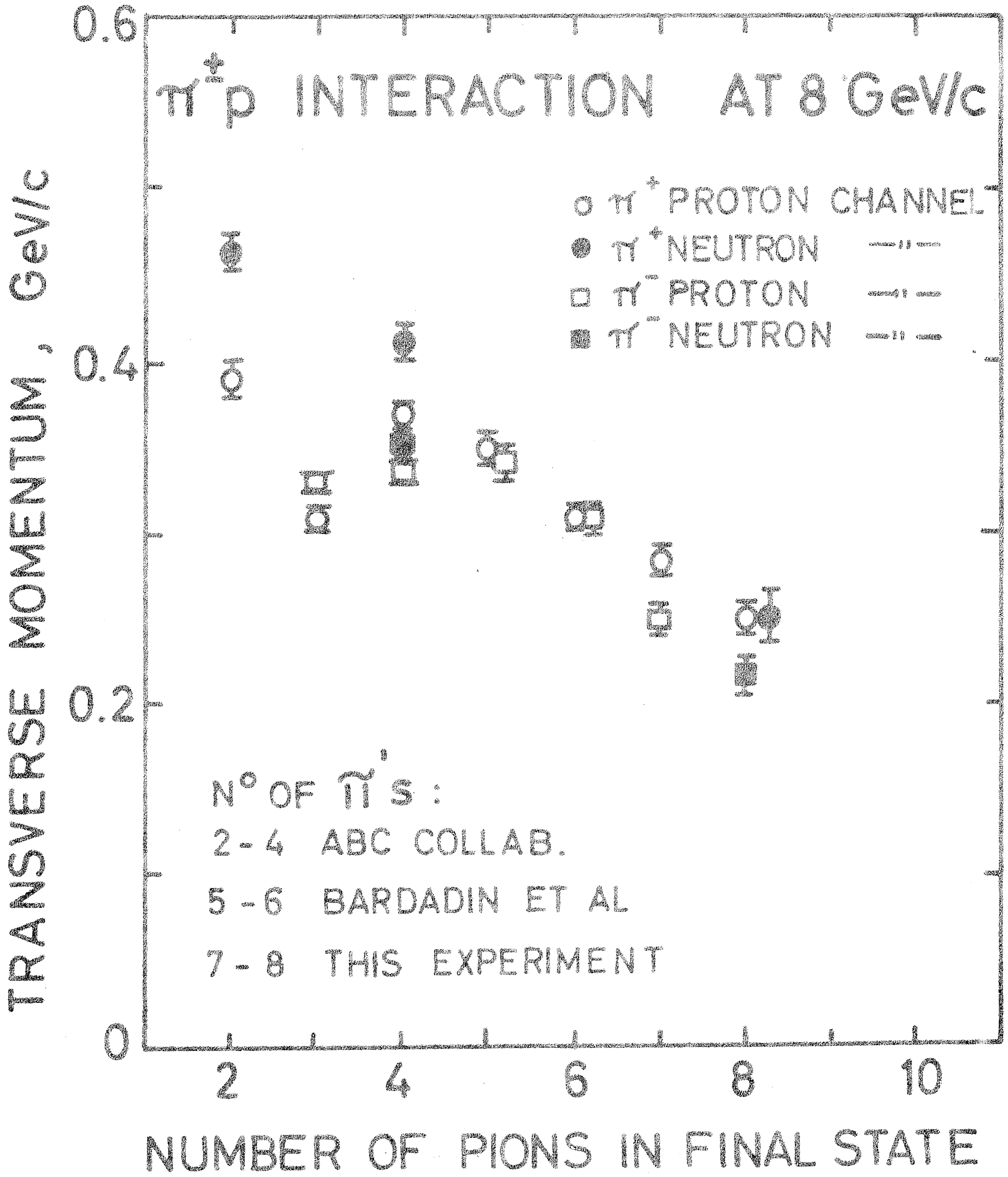

π±p INTERACTION AT 8 GeV/c
○ π+ PROTON CHANNEL
● π+ NEUTRON –"–
□ π− PROTON –"–
■ π− NEUTRON –"–
N° OF π'S :
2-4 ABC COLLAB.
5-6 BARDADIN ET AL
7-8 THIS EXPERIMENT
TRANSVERSE MOMENTUM, GeV/c
0.6
0.4
0.2
0
2
4
6
8
10
NUMBER OF PIONS IN FINAL STATE

Fig. 6

PS/ 5681

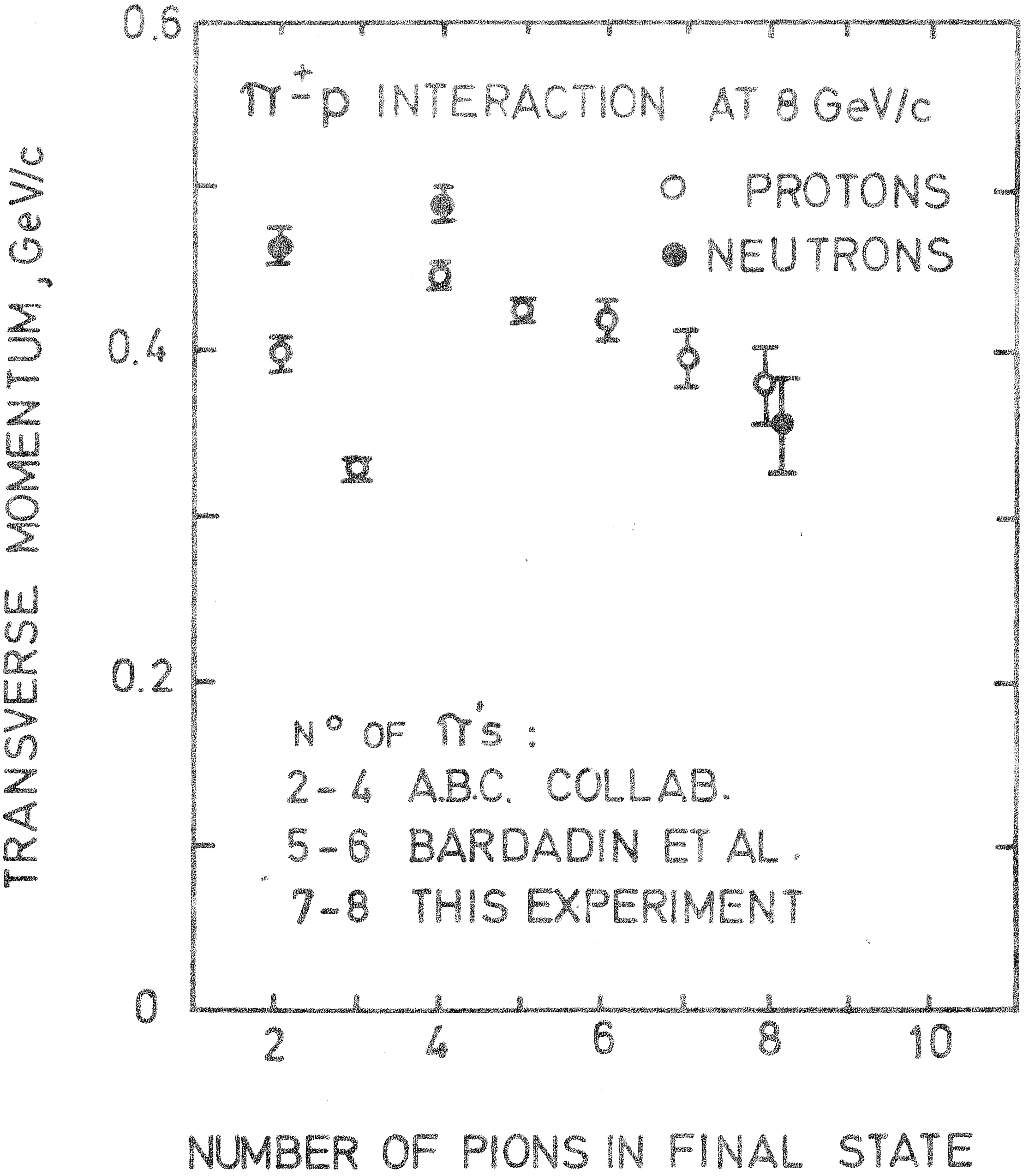

$\pi^{\pm}p$ INTERACTION AT 8 GeV/c
○ PROTONS
● NEUTRONS
N° OF π's :
2-4 A.B.C. COLLAB.
5-6 BARDADIN ET AL.
7-8 THIS EXPERIMENT
TRANSVERSE MOMENTUM, GeV/c
0.6
0.4
0.2
0
2
4
6
8
10
NUMBER OF PIONS IN FINAL STATE

Fig. 7

PS/5681

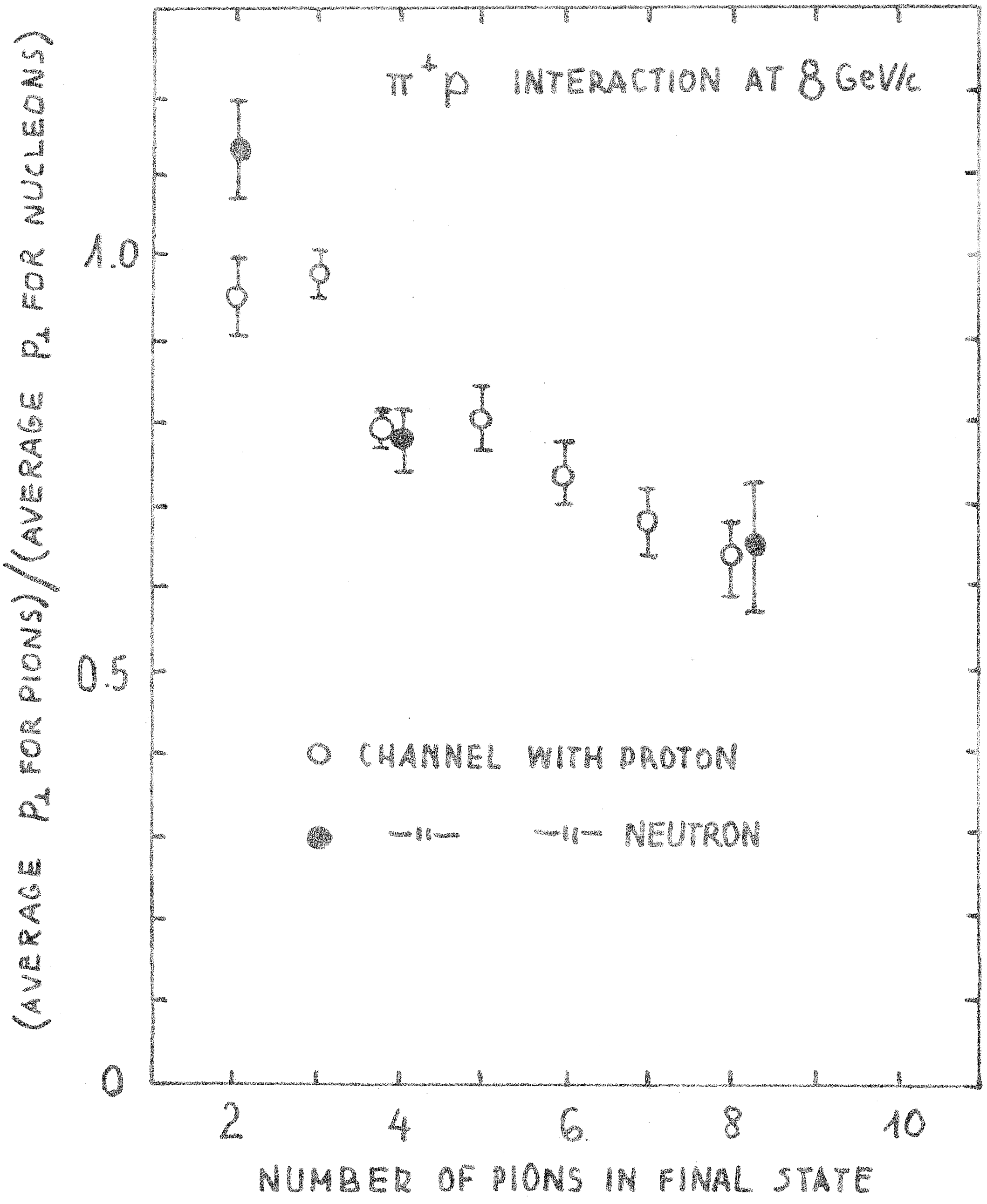

π^+p INTERACTION AT 8 GeV/c
(AVERAGE $p_\perp$ FOR PIONS)/(AVERAGE $p_\perp$ FOR NUCLEONS)
1.0
0.5
0
2
4
6
8
10
NUMBER OF PIONS IN FINAL STATE
○ CHANNEL WITH PROTON
● —"— —"— NEUTRON

Fig. 8

PS/5681

PS/5681

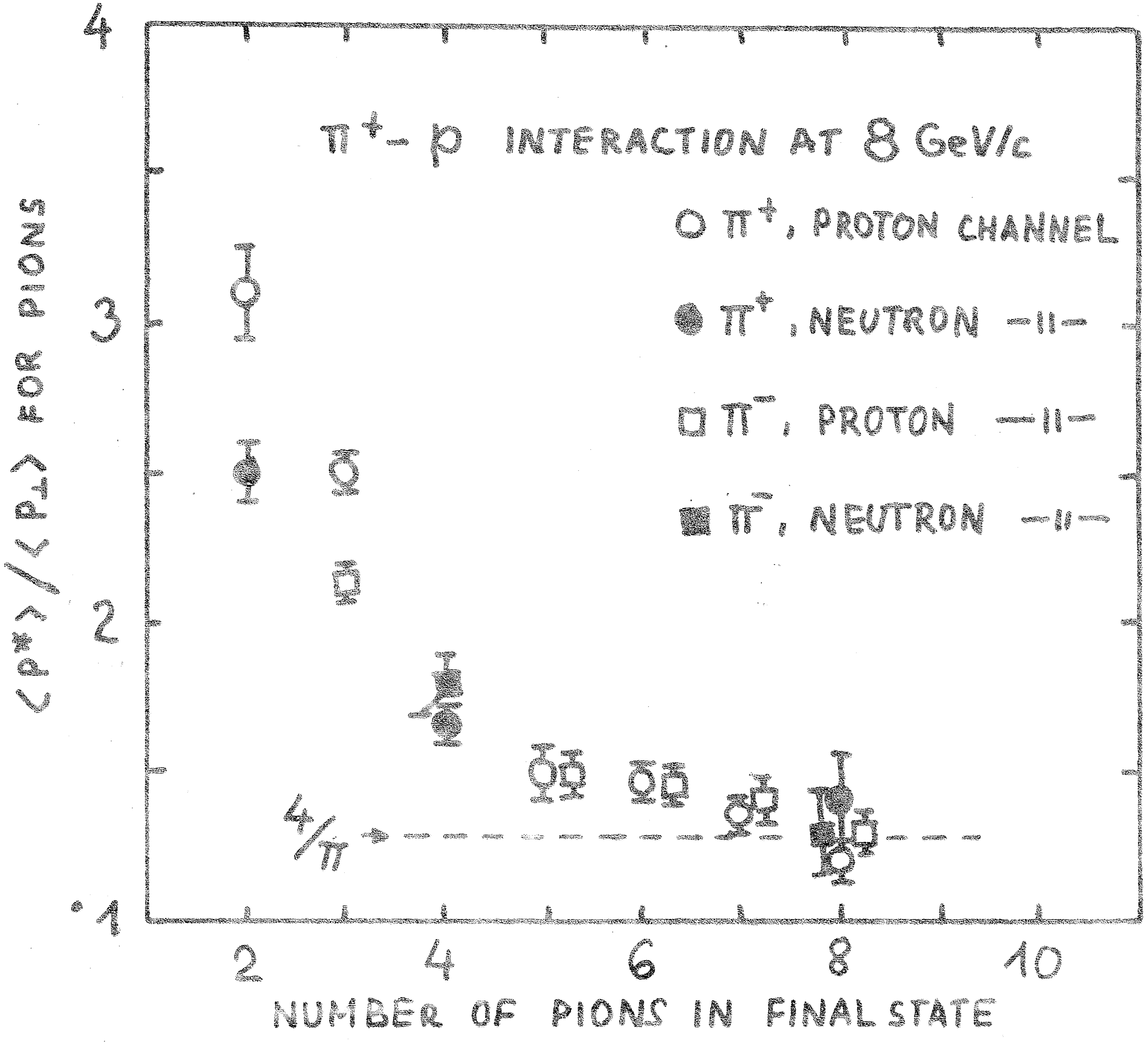

π⁺- p INTERACTION AT 8 GeV/c
○ π⁺, PROTON CHANNEL
● π⁺, NEUTRON -"-
□ π⁻, PROTON -"-
■ π⁻, NEUTRON -"-
4/π
⟨p*⟩/⟨p⊥⟩ FOR PIONS
NUMBER OF PIONS IN FINALSTATE
4
3
2
1
2
4
6
8
10

Fig. 9

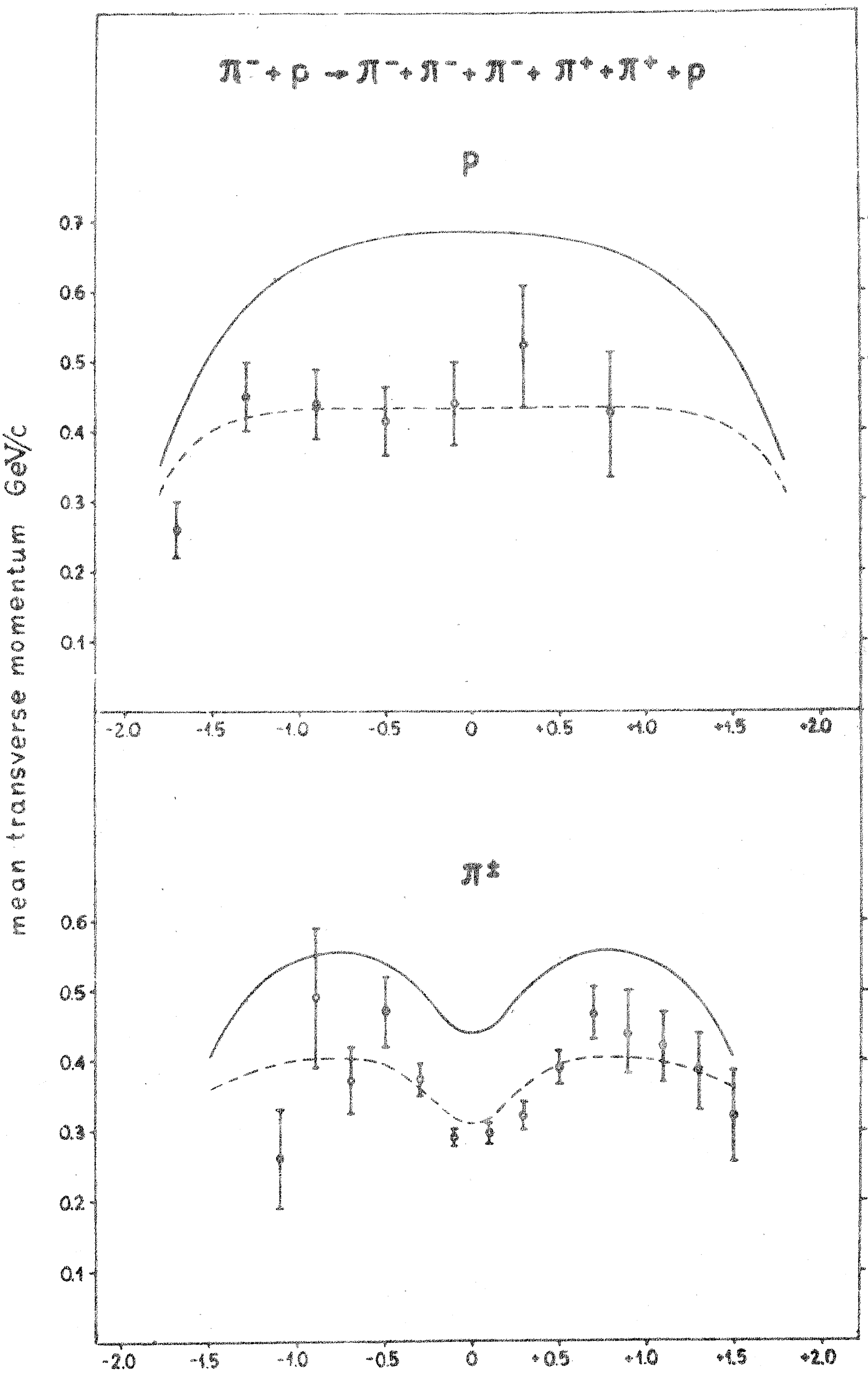
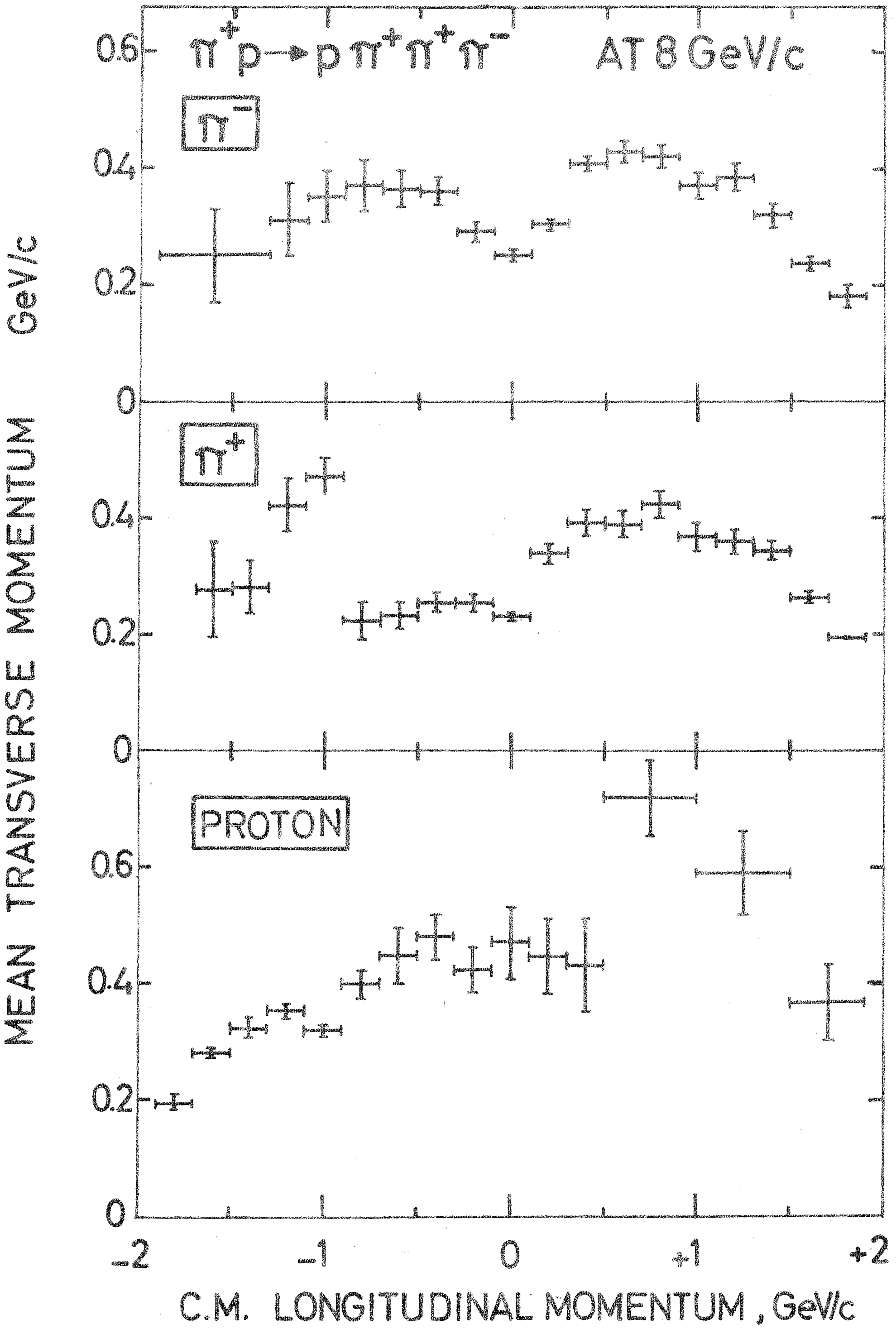

$\pi^- + p \rightarrow \pi^- + \pi^- + \pi^- + \pi^+ + \pi^+ + p$
p
$\pi^\pm$
mean transverse momentum GeV/c
0.1
0.2
0.3
0.4
0.5
0.6
0.7
-2.0
-1.5
-1.0
-0.5
0
+0.5
+1.0
+1.5
+2.0

Fig. 10



$\pi^+p \to p\pi^+\pi^+\pi^-$
AT 8 GeV/c
π^-
π^+
PROTON
0.6
0.4
0.2
0
MEAN TRANSVERSE MOMENTUM GeV/c
-2
-1
0
+1
+2
C.M. LONGITUDINAL MOMENTUM , GeV/c

Fig.11

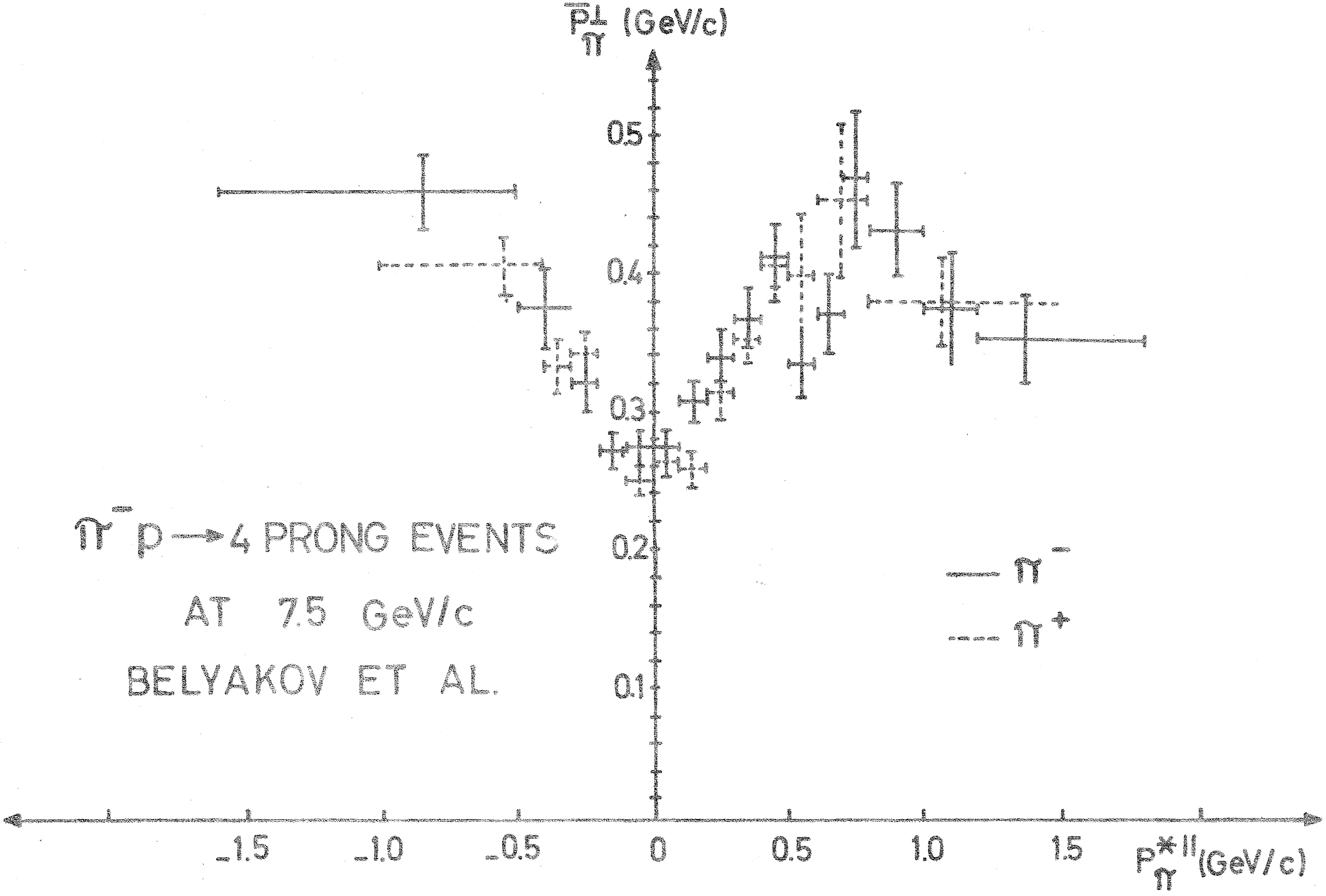

$\overline{P}^{\perp}_{\pi}$ (GeV/c)
0.5
0.4
0.3
0.2
0.1
$\pi^- p \rightarrow 4$ PRONG EVENTS
AT 7.5 GeV/c
BELYAKOV ET AL.
— π^-
--- π^+
-1.5
-1.0
-0.5
0
0.5
1.0
1.5
$P^{*\parallel}_{\pi}$ (GeV/c)

Fig. 12

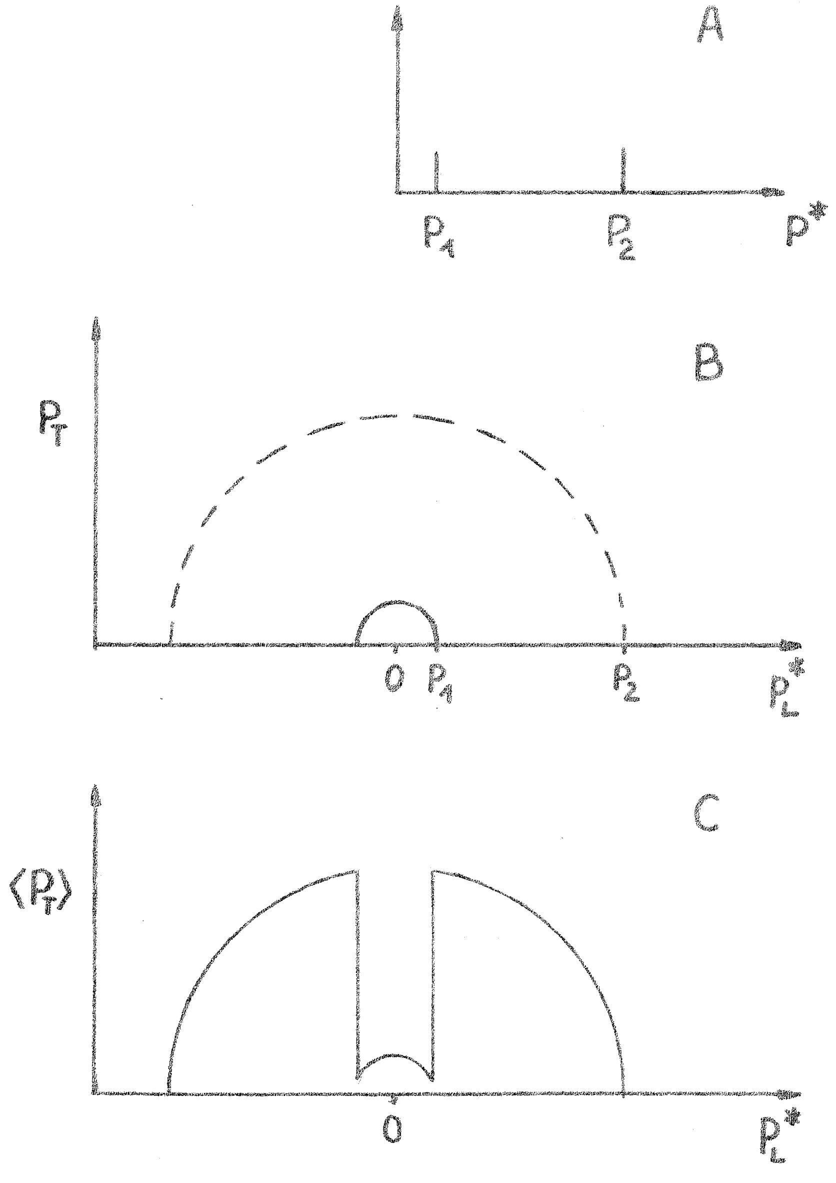


Fig. 13

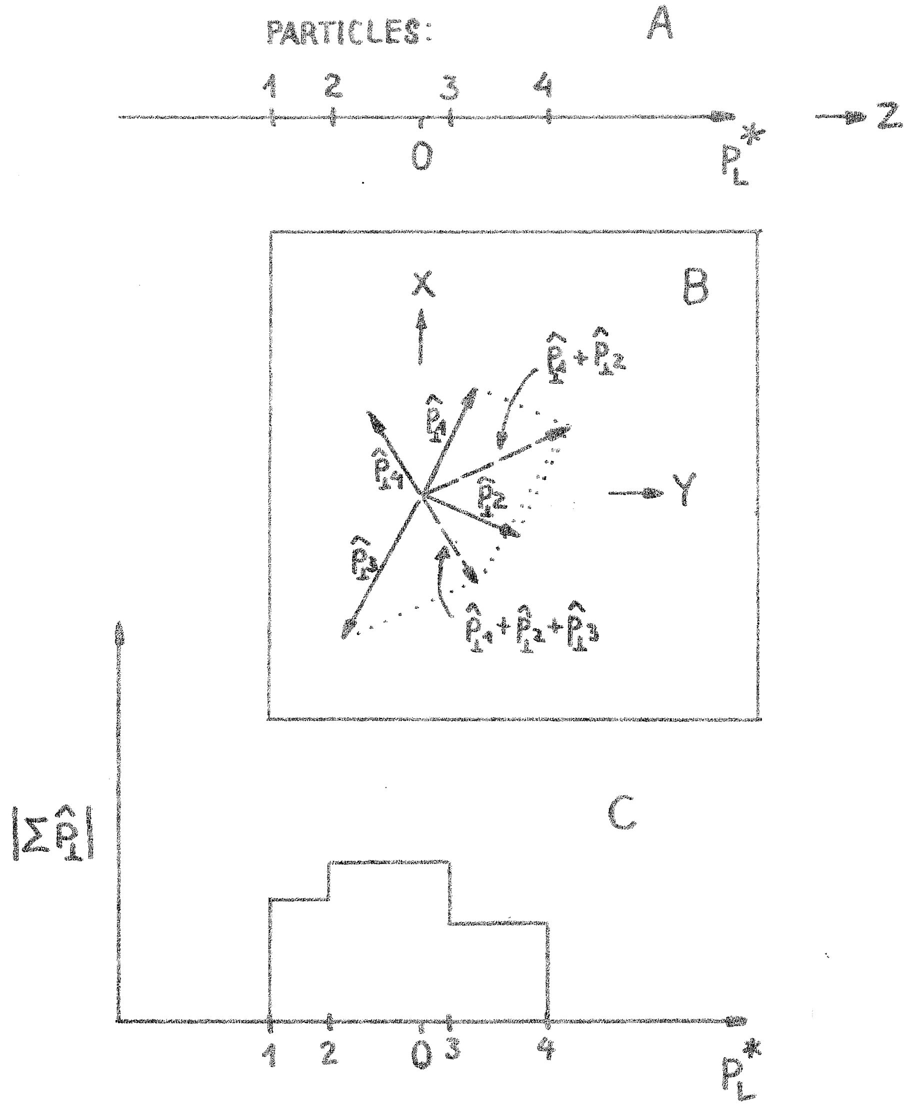

PARTICLES:
A
1
2
3
4
0
P_L^*
Z
B
X
Y
$\hat{p}_{\perp 1}$
$\hat{p}_{\perp 2}$
$\hat{p}_{\perp 3}$
$\hat{p}_{\perp 4}$
$\hat{p}_{\perp 1}+\hat{p}_{\perp 2}$
$\hat{p}_{\perp 1}+\hat{p}_{\perp 2}+\hat{p}_{\perp 3}$
C
$|\Sigma \hat{p}_{\perp}|$
1
2
0
3
4
P_L^*

Fig. 14

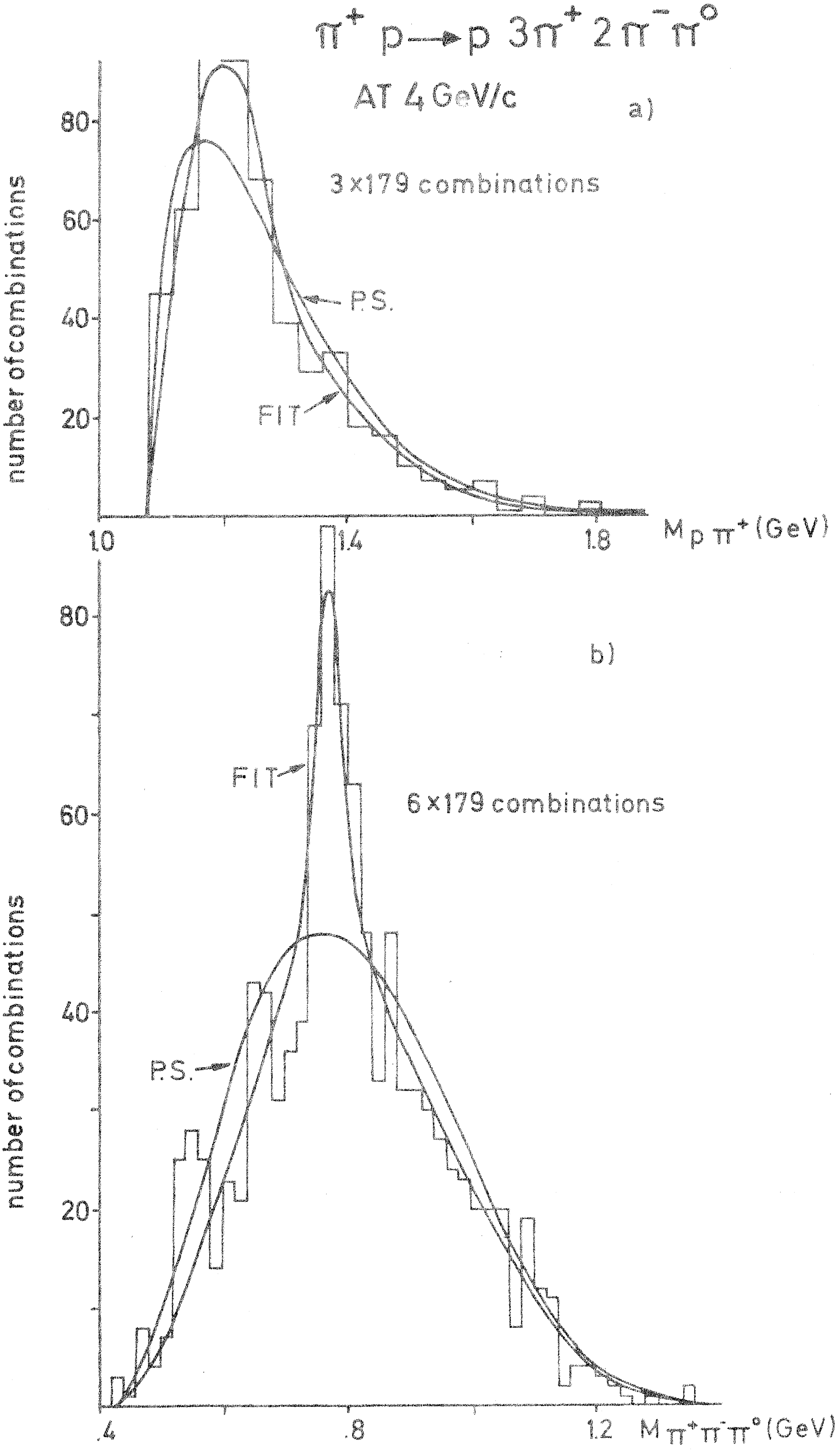

π+ p → p 3π+ 2π− π0
AT 4 GeV/c
a)
3×179 combinations
P.S.
FIT
number of combinations
80
60
40
20
1.0
1.4
1.8
M p π+ (GeV)
b)
FIT
6×179 combinations
P.S.
number of combinations
80
60
40
20
.4
.8
1.2
M π+ π− π0 (GeV)

Fig. 15

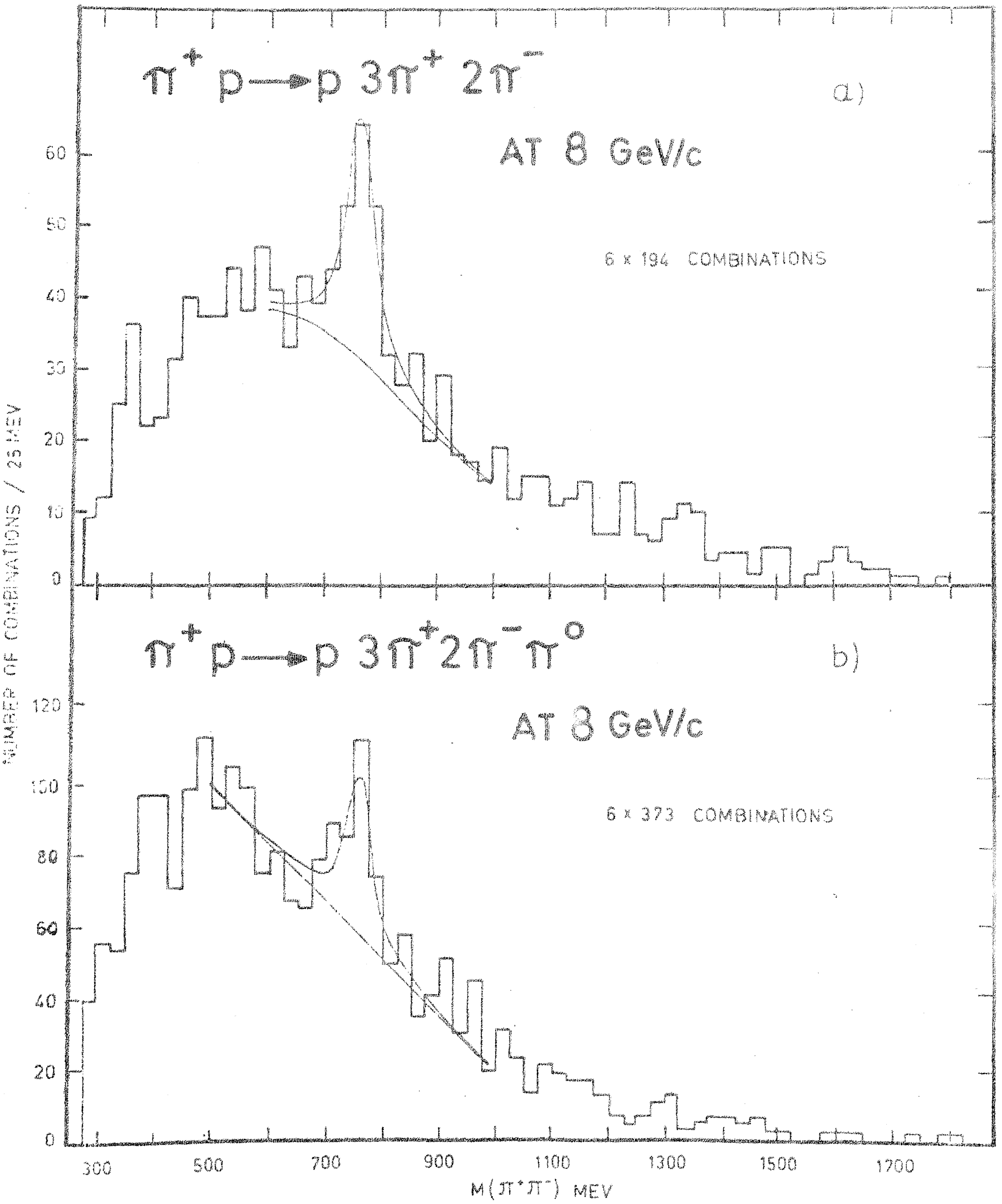
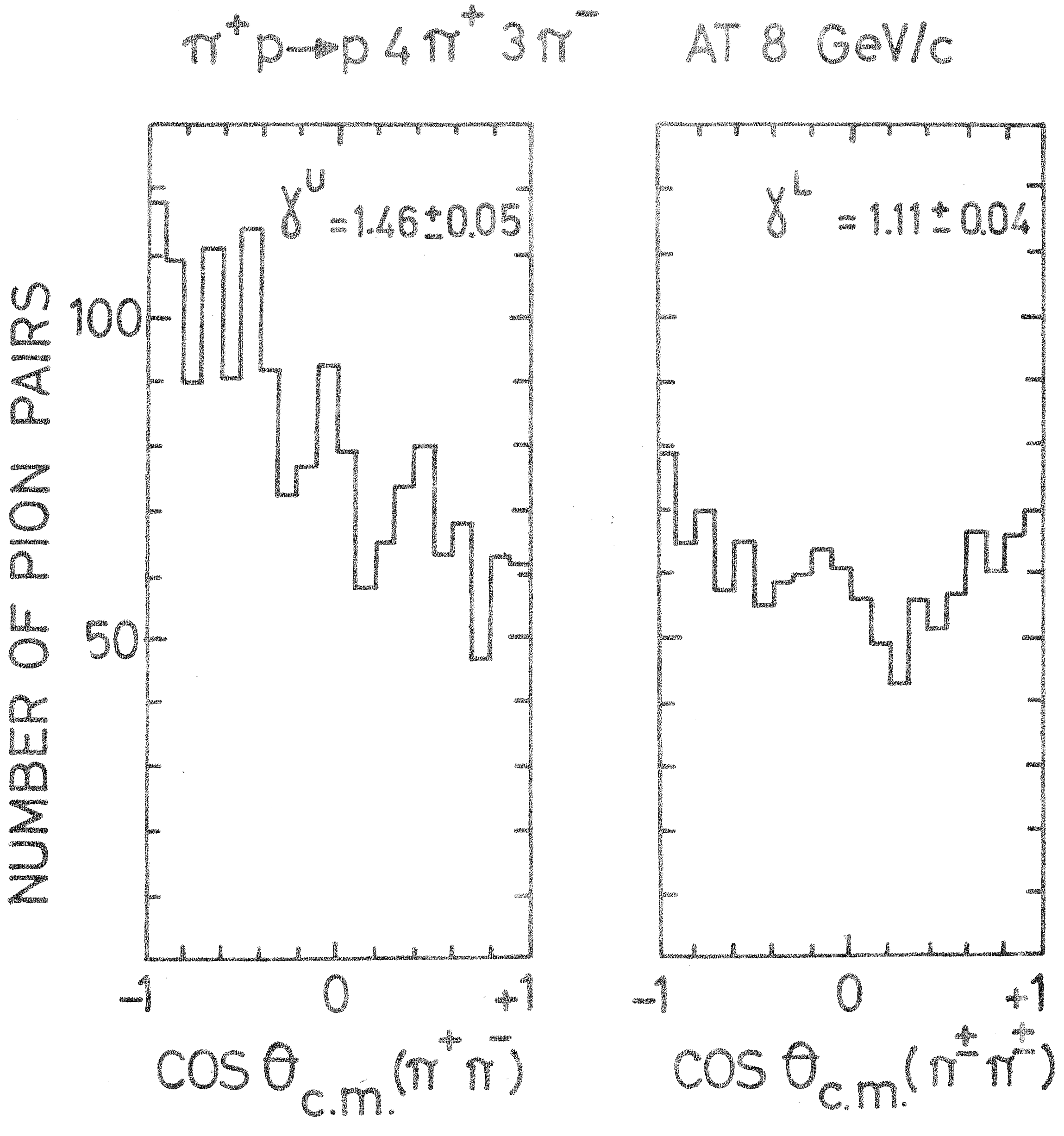

π⁺p ⟶ p 3π⁺ 2π⁻
a)
AT 8 GeV/c
6 x 194 COMBINATIONS
π⁺p ⟶ p 3π⁺2π⁻π⁰
b)
AT 8 GeV/c
6 x 373 COMBINATIONS
NUMBER OF COMBINATIONS / 25 MEV
0
10
20
30
40
50
60
0
20
40
60
80
100
120
300
500
700
900
1100
1300
1500
1700
M(π⁺π⁻) MEV

FIG.16



$\pi^+ p \rightarrow p\, 4\pi^+ 3\pi^-$ AT 8 GeV/c
$\gamma^U = 1.46 \pm 0.05$
$\gamma^L = 1.11 \pm 0.04$
NUMBER OF PION PAIRS
100
50
−1
0
+1
−1
0
+1
$\cos\theta_{c.m.}(\pi^+\pi^-)$
$\cos\theta_{c.m.}(\pi^\pm\pi^\pm)$

Fig. 17