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Calibrating the sensing-coil radius by feed-down from a harmonic reference

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***Abstract* – A method for calibrating the radius of rotating, harmonic sensing coils is proposed that allows relaxing constraints on alignment and field errors of the reference quadrupole magnet. Radius calibration considering roll angle misalignment between the measurement bench, the magnet, and the motor-drive unit is studied first. We then study the calibration error when a harmonic field error of higher order is present in the calibration magnet. This also yields a calibration when a sextupole magnet is used, for example, when an in-situ calibration [1] is required. The proposed calibration method has been validated by simulations with the CERN numerical field computation program ROXIE.**

I. INTRODUCTION

In the domain of accelerator magnets, magnetic measurements are necessary for different aspects. During the prototyping phase, experimental results are used to verify design calculations, material properties, and fabrication methods [2]. Different techniques are used, which are based on sensing elements, such as harmonic coils, oscillating wires, and Hall probes, among others [3], [4]. The harmonic coil is based on Faraday's law of induction, where a radial or tangential coil [5] is turned inside the magnet's aperture. Several physical parameters, such as the coil radius, coil surface, phase angle and tilt, the number of turns, and opening angle have to be known with high precision in order to reach the required accuracy and precision in the magnetic measurements. Rotating coils can be manufactured with traditional winding methods or using printed circuit board (PCB) [6] technology. The latter is especially suited for small-aperture magnets [7]. Manufacturing errors leading to deviations from the ideal design exist in both technologies. For the PCBs there may be a misalignment between layers of different radii. Therefore a calibration is needed after the coil production.

The calibration [1], based on a reference quadrupole magnet, is currently used at CERN to calibrate rotating coil systems. In particular, the coil radius is computed by measurements performed inside the aperture of the reference quadrupole, before and after a translation in the horizontal plane. The calibration technique assumes that all the contributions of higher order harmonics are negligible in the mid-plane of the magnet. This is justified because the dominant field component is high compared to the field errors of higher order harmonics [8], [1]. However, if one of the higher harmonics is not negligible, or a sextupole magnet is used as a calibration device, this calibration technique would give rise to significant errors. Therefore a reference magnet with stringent metrological constraints of harmonic field quality and alignment is required. This is difficult to archive for magnets with small apertures or when rare-earth permanent magnet are employed.

In this paper the effect of calibrating the radius of coils using a reference quadrupole with one higher-order harmonic error is studied and a roll misalignment between the bench and the reference magnet is considered. The calibration in a sextupole is verified by simulations using the field computation program ROXIE.

II. PROPOSAL

In a measurement system employing rotating sensing coils, the radius and the equivalent surface of the coils must be known precisely in order to enable high-precision field measurements. The radius and surface are used to calculate the coil-sensitivity factors. These factors allow the computation of field harmonics from a set of Fourier coefficients of the flux, that is intercepted by the sensing coil. The coil radius is computed from two measurements performed inside the aperture of a reference quadrupole, before and after a translation in the horizontal (xz) and vertical (yz) planes. If we consider that there is no tilt and swing misalignment between the coil and magnet axes, the mathematical treatment can be limited to two dimensions. In complex notation, this displacement can therefore be written as $\triangle z = \triangle x + i\triangle y$. The formula used to calculate the radius, are based on the feed-down effect [1], under the condition that all the contributions of higher order terms can be neglected. This is justified because the dominant field component in the reference magnet is much larger compared to the field errors of higher order multipoles[8].

If one of the higher field harmonics in the reference magnet was not negligible, or if a sextupole reference magnet was used, the calibration method in [1] would results in a significant calibration error. We must also study the case of a roll-angle misalignment between the magnetic axis and the measurement bench on which the coil is displaced. It can be shown, without a loss of generality, that the displacement can be confined to the xz-plane. It is then possible to study the presence of a higher order field harmonic, which can be attributed to design constraints as well as manufacturing errors. The method proposed for the case study also holds for calibrating the sensing-coil radius in a sextupole reference magnet.

III. RADIUS CALIBRATION CONSIDERING ROLL ANGLE MISALIGNMENTS

The rotating coil radius can be calibrated in a quadrupole based on two measurements using the same equipment. It is assumed that any misalignment between the reference magnet, the coil support and the displacement table/stages remain constant between these two measurements. This is reasonable, due to the solid structure of the support posts and tables. Moreover, the coil displacement (d in Fig. 1) is known with a very high precision (< 0.01 mm).

It will be assumed that the higher multipole field errors in the reference magnet are small enough so that their influence on the measurement can be neglected; an assumption that will be challenged in the next chapter. Consider the measurement setup and the reference frames shown in Fig. 1. The quadrupole is centered at z_0 with respect to the global reference frame, and may be misaligned by the angle φ_m. The two measurements are taken at positions z_a and z_b. Only the metric distance between these two positions is known to high precision.

In principle, when the sensing coil is rotated in the field of the reference magnet, the 2π periodic voltage signal could be developed into a Fourier series. This would, however, put stringent conditions on the rotation speed of the drive system. A re-parametrization to the angular position is therefore done, by reading the trigger signals from an angular encoder, mounted between the drive system and the sensing coil. However, an angular error between the encoder and the global reference frame must be considered. This angular (encoder) error will be denoted φ_e. Since no higher multipole field errors in the calibration magnet are taken into account, only dipole $C_1 = B_1 + iA_1$ and quadrupole components $C_2 = B_2 + iA_2$ are measured at the positions z_a and z_b and expressed in the local coordinate system of the coils. The connection between the two sets of measured harmonics is made via the feed-down formula, which reads in its general form:

$$C_n(z_b) = \sum_{k=n}^{\infty} C_k(z_a) \binom{k-1}{n-1} \left(\frac{\triangle z}{R} \right)^{k-n} , \quad (1)$$

where R is the (unknown) sensing-coil radius and $\Delta z = z_b - z_a$ the displacement in the global coordinate system ($|\Delta z| = d$). This displacement can therefore be expressed

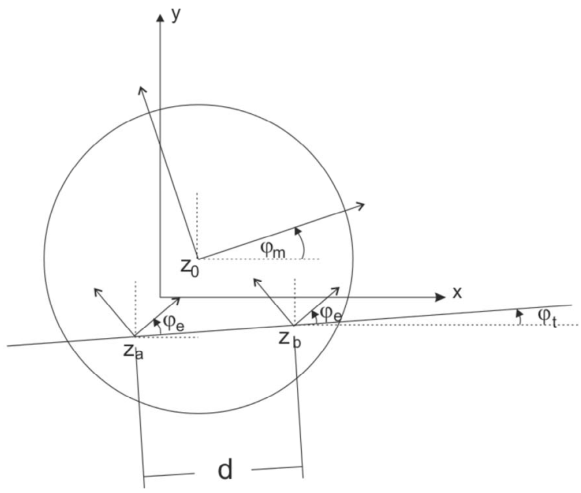

Figure 1: Setup of the measurement and the reference systems for the magnet and the measurement bench.

in the global frame as

$$\Delta z = d e^{i\varphi_t} . \quad (2)$$

To derive a formula to transform the measured multipole field errors from one local coordinate system into the other, Δz must be rotated by the angle $-\varphi_e$ into the global frame. The feed-down equation can then be written as:

$$C_2(z_b) = C_2(z_a) , \quad (3)$$

$$C_1(z_b) = C_1(z_a) + C_2(z_a) \frac{d e^{i(\varphi_t - \varphi_e)}}{R} . \quad (4)$$

Since there is no higher-order multipoles field error, the quadrupole component is not effected by feed-down. Taking the difference between the two measurements, the displacement of the reference magnet z_0 can be eliminated. In this case it is also not necessary to know the multipole harmonics in the global system, because the feed-down is related to the measurement at z_a.

$$R\left(C_1(z_b) - C_1(z_a)\right) = R\Delta C_1 = C_2(z_a) d e^{i(\varphi_t - \varphi_e)} . \quad (5)$$

The complex numbers can be transformed in a modulus and phase notation with auxiliary angles α_1 and α_2:

$$R|\Delta C_1| e^{i\alpha_1} = |C_2(z_a)| e^{i\alpha_2} d e^{i(\varphi_t - \varphi_e)} . \quad (6)$$

This equation is fulfilled, if the modulus and the phase are identical. This yields $\alpha_1 - \alpha_2 = \varphi_t - \varphi_e$ and

$$R = \frac{d|C_2(z_a)|}{|\Delta C_1|} , \quad (7)$$

which gives the expression for the equivalent measurement radius of the sensing coil. Notice, that the calibration, so obtained, is independent from the alignment errors between the reference magnet and the displacement stage. The only assumption remains that the error from the angular encoder reading φ_e does not change between the two consecutive measurements.

IV. COIL-RADIUS CALIBRATION USING HIGHER ORDER HARMONICS

The feed-down correction considering all harmonics is given in Eq. (1), where $C_n(z_b)$ are the field coefficients measured in z_b (after the coil translation) and $C_k(z_a)$ are the field coefficients in z_a. As shown in the previous section, we can assume, without a loss of generality, that the displacement of the harmonic coil is confined to the horizontal plane of the reference magnet, hence Δz reduces to Δx.

The harmonic coils can be of the radial or tangential type, intercepting the azimuthal and the radial flux components, respectively. Subsequently a radial coil is assumed, but all principles can be applied also to tangential coils.

Furthermore, let us first consider a quadrupole containing an unwanted sextupole harmonic field error due to manufacturing errors or variations in the remanence of the rare-earth permanent magnet material. As example, magnets with small apertures for linear accelerators show higher-order multipole field errors in access of 10^{-3}, while remaining acceptable for machine installation. The feed-down formula results in

$$C_1(z_b) = C_1(z_a) + C_2(z_a)\left(\frac{\triangle z}{R}\right) + C_3(z_a)\left(\frac{\triangle z}{R}\right)^2 ,$$
$$C_2(z_b) = C_2(z_a) + 2C_3(z_a)\left(\frac{\triangle z}{R}\right) ,$$
$$C_3(z_b) = C_3(z_a) ,$$

where $C_n = B_n + iA_n$ and $\triangle z$ is a movement in complex plane. In the special case of a translation $\triangle x$ in the horizontal plane, there will be no skew field harmonics excited so that $C_n = B_n$. From the above equation system we get

$$B_3(z_a)\left(\frac{\triangle x}{R}\right) = \frac{1}{2}\left(B_2(z_b) - B_2(z_a)\right) \tag{8}$$

and therefore

$$B_1(z_b) = B_1(z_a) + \frac{1}{2}B_2(z_a)\left(\frac{\triangle x}{R}\right) + \frac{1}{2}B_2(z_b)\left(\frac{\triangle x}{R}\right) . \tag{9}$$

The multipole field errors, which have been used so far, correspond to the Fourier series coefficients of the radial component of the magnetic flux density on the reference/measurement radius. As said earlier, the measurement raw data are the integrated voltage signals that correspond to the flux linkage in the sensing coil. The relation between the coefficients obtained by Fourier analysis of the vector potential (corresponding to the flux increment per trigger signal) are related to the multipole field errors by means of the coil-sensitivity factors k_n, which obviously depend on the number of coil turns N_t, and the coil length L. The calculation of the coil-sensitivity factors are given for the radial and tangential coils in document [8]. The relation between the Fourier coefficients of the vector potential $\widetilde{B}_n$ and the multipole field errors B_n is given by

$$B_n = R^{n-1}\frac{\widetilde{B}_{n+1}}{k_n} , \tag{10}$$

where

$$k_n = \frac{N_t L}{n}(R_2^n - R_1^n) . \tag{11}$$

k_n are the coil sensitivity coefficient of n^{th}-order harmonic, R_1 is the internal coil radius, and R_2 is the external coil radius. In particular

$$k_1 = N_t L W = A_c \tag{12}$$
$$k_2 = N_t L W\frac{(R_1 + R_2)}{2} = A_c R \tag{13}$$

where A_c is the effective coil area, R is the coil radius, and W is the effective width of the coil. While the harmonic field coefficients are computed using (10):

$$B_1(z_a, z_b) = \frac{\widetilde{B}_2(z_a, z_b)}{A_c} , \tag{14}$$
$$B_2(z_a, z_b) = \frac{\widetilde{B}_3(z_a, z_b)}{A_c} , \tag{15}$$

Substituting (14) and (15) into (9) yields:

$$\widetilde{B}_2(z_b) = \widetilde{B}_2(z_a) + \frac{1}{2}\triangle x\frac{\widetilde{B}_3(z_a)}{R} + \frac{1}{2}\triangle x\frac{\widetilde{B}_3(z_b)}{R} , \tag{16}$$

and therefore

$$R = \frac{\triangle x}{2}\frac{\widetilde{B}_3(z_b) + \widetilde{B}_3(z_a)}{\widetilde{B}_2(z_b) - \widetilde{B}_2(z_a)} . \tag{17}$$

Using a similar reasoning for an octupole component within a quadrupole magnet yields the equation system:

$$B_1(z_b) = B_1(z_a) + B_2(z_a)\left(\frac{\triangle x}{R}\right) + B_4(z_a)\left(\frac{\triangle x}{R}\right)^3 ,$$
$$B_2(z_b) = B_2(z_a) + 3B_4(z_a)\left(\frac{\triangle x}{R}\right)^2 ,$$
$$B_3(z_b) = 3\,B_4(z_a)\left(\frac{\triangle x}{R}\right) ,$$
$$B_4(z_b) = B_4(z_a) .$$

The coil radius can then be computed from the harmonics $\widetilde{B}_{n+1}$ of the magnetic vector potential by

$$R = \frac{\triangle x}{3} \frac{2\widetilde{B}_3(z_a) + \widetilde{B}_3(z_b)}{\widetilde{B}_2(z_b) - \widetilde{B}_2(z_a)}, \qquad (18)$$

which can be generalized for any **single** error harmonic within a quadrupole:

$$R = \frac{\triangle x}{(n-1)} \frac{(n-2)\widetilde{B}_3(z_a) + \widetilde{B}_3(z_b)}{\widetilde{B}_2(z_b) - \widetilde{B}_2(z_a)}, \qquad (19)$$

where n is the highest harmonic order; 2 for the quadrupole, 3 for the sextupole etc.

V. NUMERICAL RESULTS

The equations given above are derived for a searching coil with a very small coil cross-section compared to the surface spanned. For probes to be used in very small magnets this assumption is no more valid. We therefore study, by means of numerical simulation, the calibration error resulting from the concept of a mean surface and mean radius. For the numerical simulations the CERN field computation program ROXIE [9] , [10] was applied.

A. *Simulation of a calibration quadrupole with a sextupole field component present*

The magnet is modeled by means of current shells of an ideal ($\cos n\Theta$) current distribution that generates a pure multipole field of order n. A sextupole current shell is nested within the quadrupole. The radii of the quadrupole and the sextupole are 70 and 50 mm, respectively. A 2D simulation is sufficient because of the assumption of a longitudinal homogeneity both in the magnet and the searching coil. The tangential coil section used to test the proposed method is shown in Fig. 2. The coil is rotated by one turn. 180 samples are computed, from which the coefficients are determined by a discrete Fourier transform of the radial magnetic flux density.

The flux linkage has been computed at two coil positions within the magnet aperture. In Fig. 2 (left), the coil rotation axis and the magnetic axis of the calibration magnet are identical, while in Fig. 2, (right) the coil rotation center is displaced by 10 mm along the x-axis.

A number of flux measurements, using different rotating coil radii and displacements $\triangle x$ have been simulated in order to check the validity of Eq. (17). The proposed calibration method gives more accurate and precise radius values, while the classical method (not accounting for the higher order multipole field errors) yields errors of up to 4.5 percent. Fig.3 shows the magnetic flux density $|B|$ of the simulated magnet given by the overlapping of an ideal quadrupole shell magnet and an ideal sextupole shell magnet. The magnetic flux distribution is not symmetric and its modulus is stronger on the right hand side. Fig.4 shows the

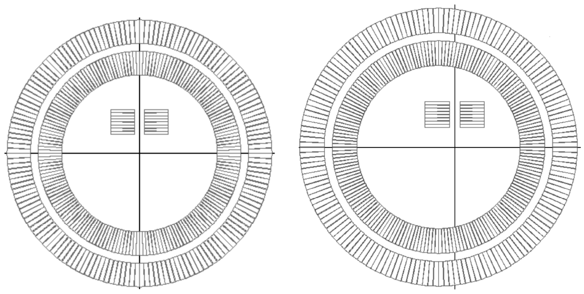

Figure 2: Two current shells and rotating coil section as modeled in ROXIE. Left: The rotating coil center coincides with the magnetic axis of the magnet. Right: The rotating coil center is shifted by $\triangle x$ = 10 mm from the magnet axis of the magnet.

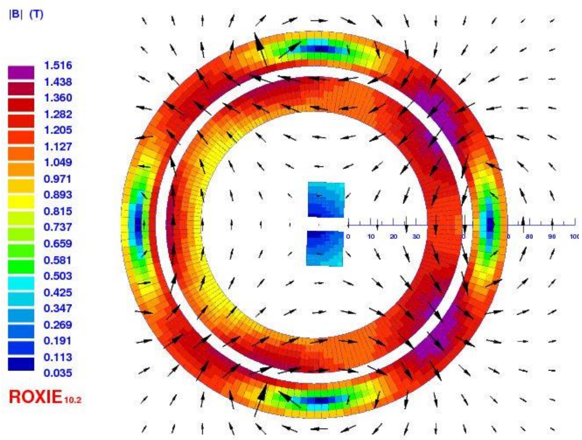

Figure 3: Magnetic flux density in the quadrupole and sextupole shell.

magnetic flux density in the cross-section of the searching coil.

The coil radius has been calculated through the classical method (Method 1, without considering the higher order multipole) in an ideal quadrupole shell magnet, for different $\triangle x$ inside the magnet aperture. These results can then be compared with the case of higher unwanted harmonics. The results obtained by Method 1 (see reference [1]) for an ideal quadrupole are shown in Table 1. These results are referred to the known coil radius of 20 mm, as modeled in ROXIE. Results of Method 1 differ by approximately 0.12 mm mainly due to the effect of the insulation between coil turns.

Table 2 gives the results obtained using the classical (Method 1) and the proposed method (Method 2) for a coil radius of 20 mm in a quadrupole magnet containing an additional sextupole harmonic. These results show that the calibration error for the classical method [1] depends on

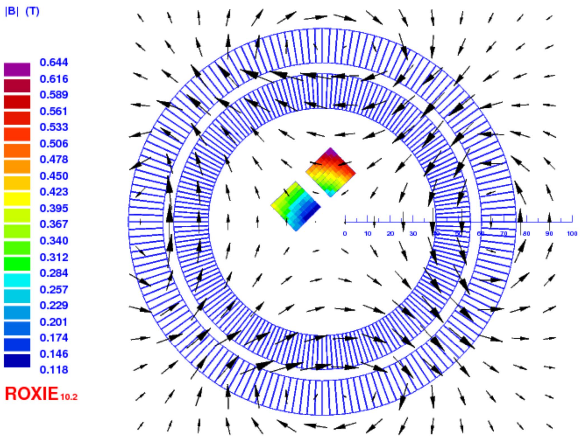

Figure 4: Magnetic flux density in a tangential rotating coil inside the magnet aperture.

Table 1: Radius calibrations resulting for movements Δx from position x_1 to x_2 using Method 1 (the exact radius is 20 mm).

x_1 mm	x_2 mm	Method1 mm
5	10	19.88
10	15	19.89
5	15	19.88

the displacements $\triangle x$, while the proposed method is more stable.

Table 2: Case study of quadrupole with one additional sextupole harmonic field error. Radius calibrations for movements Δx from position x_1 to x_2, using the traditional (Method 1) and the proposed methods (Method 2) (true coil radius of 20 mm).

x_1 mm	x_2 mm	Method1 mm	Method2 mm
10	20	19.10	19.89
15	20	19.51	19.90
10	15	19.49	19.89

B. Simulation of a quadrupole with an octupole harmonic field error

Table 3 gives the results obtained for a quadrupole magnet with an octupole field harmonic present. The proposed method gives more accurate results then the classical one because it takes into consideration the nonlinear radius dependence of the magnetic flux density. Obviously, the most accurate results are obtained when the measurements are performed close to the magnet center.

Table 3: Case study of quadrupole with one octupole additional harmonic. Radius calibrations resulting from movements Δx from position x_1 to x_2, with the traditional (Method 1) and proposed method (Method 2) (true coil radius of 20 mm).

x_1 mm	x_2 mm	Method1 mm	Method2 mm
10	15	19.67	19.85
10	20	19.32	19.76
15	20	19.52	19.78

VI. CONCLUSION

The proposed method allows to calibrate the radius and the area of a rotating coil sensor in a magnet with a harmonic field error or in a reference sextupole magnet of larger apertures. This allows a wider choice of magnets to calibrate the rotating coils.

The calibration error in the classical method must be minimized by small displacements $\triangle x$ when higher order multipole errors are present. Simulation results show that the worst case can yield a calibration error of 6% for a displacement of 10 mm. The results from the proposed method are accurate up to 0.1% for an additional sextupole harmonic, and up to 0.7% for the additional octupole harmonic.

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