CERN/TC/NBC 65-4
17.6.1965
G. KELLNER

Geometrical Reconstruction for 150 cm BNHBC

(results valid for experiments after Jan.1965 only)

The aim of this note is to describe the necessary modifications in the geometry program for the reconstruction of events in the British Hydrogen Bubble Chamber. Since lens distortions and tilt of the film plane with respect to the reference plane of the chamber can not be neglected, an additional treatment of IEP measurements has to be included.

All calculations were done with the program PYTHON, written by J. Zoll (see T.C.Program Library, Section : UTILITY-PYTHON). Copious reference is given to this manual to avoid a too long description.

The following description is grouped into several sections :

A) Calculation of Chamber Constants
B) Results
C) Application in Geometry Program
D) Discussion
E) Appendix I : Grid Plate
F) Appendix II : Grid calibration
G) Appendix III : Fiducial marks on chamber windows

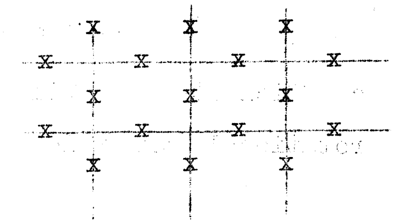
It should be stressed that these results are only valid for experiments after January 1965 since the camera plate has been modified at this time.

A) Calculation of Chamber Constants

Two methods have been used to determine these parameters : a) a grid plate, and b) actual chamber fiducials.

a) A grid plate (glass plate with lines engraved on the face towards the cameras, see Appendix I) has been photographed in 3 positions, corresponding approximately to hydrogen surface of front glass, centre of chamber and hydrogen surface of back glass. Only 8 horizontal and 23 vertical lines (184 intersection points) could be seen on all photographs.

Measurements were done such that points on a line were measured in between two intersecting lines. **Any** two neighbouring points were used to calculate the equation of the line. Intersection of the two lines through each point give then the best possible



PS/4944/dmh

values for the coordinates of these intersection points on the film (see PYTHON, INMESH).

Various combinations of two positions were used to calculate camera positions, film tilt and lens distortions.

The influence of film stretch was removed by measuring the 4 camera based fiducials (fiducials in the film gate) which are printed onto every photograph.

The transformation - from measured camera based fiducials on the film to fiducials in space - is found by a least-squares fit (see PYTHON, SHAPE).

To get the position of these marks with high accuracy, they were photographed separately with orthochromatic plates (Gevaert 23D56) and measured on a precision microscope.

After alignment of two grid positions in space (telescope readings, see Appendix II) an iteration cycle was entered for the calculation of camera positions. Parameters which were fixed during the iteration cycle were : camera coordinates (the chamber was shifted relative to the cameras instead), inter-lens spacing of cameras, distance between two grid positions and distances between fiducial marks. After 10 - 20 iterations the restriction of fixed inter-lens spacing was removed. Usually the results are slightly improved.

Film tilt and lens distortions were calculated by a least-squares fit to the following expression (see PYTHON, CAMPOS, DISTIP) :

$$\sum_{i=1}^{N} w \left((x' - x^T)^2 + (y' - y^T)^2 \right) = \text{Min.}$$

$$\binom{x}{y}' = \binom{x}{y} \left(1 + \alpha_1 \frac{x}{f} + \alpha_2 \frac{y}{f} + \alpha_3 \frac{xy}{f^2} + \alpha_4 \frac{x^2}{f^2} + \alpha_5 \frac{y^2}{f^2} + \alpha_6 \frac{(x^2 + y^2)^2}{f^4} \right)$$

x, y measured coordinates on film (transformed into the optic axis system)

x^T, y^T true coordinates, obtained by projection of the chamber fiducials (grid intersection points) onto an ideal film plane

w weight

f distance film-lens (then the α_i are dimensionless and independent of the arrangement)

Note: The correct formula for fitting would be

$$\binom{x}{y}' = \binom{x}{y} \left(1 + \beta_1 x + \beta_2 y + \beta_3 r^2 + \beta_4 r^4 \right) \qquad r^2 = x^2 + y^2$$

β_1, β_2 coefficients for film tilt

β_3, β_4 rotational symmetric lens distortions

α_3, α_4, α_5 were introduced to study effects of non-rotational arrangement and other, not obvious, contributions.

b) Chamber fiducials : The distortion coefficients resulting from the previous calculations with the grid plate are now used as fixed parameters in the iteration cycle for the camera positions. (Other fixed parameters are : camera coordinates, fiducial mark distance, depth of chamber, hydrogen refractive index, glass thickness and refractive index). Again, linear distortions of the film are removed by measuring the camera based fiducials before the iteration cycle starts. Since calculation of camera position is a standard, well-known, procedure no further description is necessary.

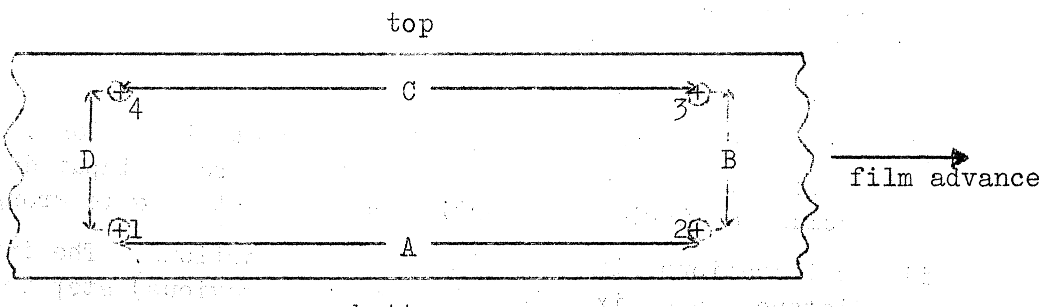
The fiducial mark position in space as well as depth of the chamber, were taken from a report of P. Williams (CERN/TC/NBC 65-1). All 34 fiducials have been used though not all of them are visible on all 3 views (see Appendix III). Those fiducials have been given a small weight and did not affect the least squares fitting.

The chamber working conditions are 70 P.S.I. at 27°K. This corresponds to a hydrogen density of 0.0605 g/cm^3, and a refractive index of 1.0945 (calculated from Progress in Cryogenics, vol.2, p.112, 1960)

B) Results

1) Camera based fiducial marks

Relative position of fiducials on film gate.



Film gate (face to chamber)

Camera	A	B	C	D
Top (1)	83.083	28.022	83.068	27.874
Centre (2)	81.940	28.191	81.712	28.247
Bottom (3)	84.101	28.141	83.902	28.009

Distances are given in mm. with an error of 0.002 mm.
(Note: A black spot in fiducial 2 of camera 3 might be useful for orientation of measurements).

PS/4944/dmh

B) 2) Analysis of film stretch

A typical example is given for the transformation

$$(x') = (A)(x - x_0)$$

found between measured camera based fiducials and the positions in space mentioned above :

$$\begin{pmatrix} x \\ y \end{pmatrix}' = 2.545333 \begin{pmatrix} 0.999674 & 0.008175 \\ -0.008274 & 1.000257 \end{pmatrix} \begin{pmatrix} x - 16699.05 \\ y - 6672.72 \end{pmatrix}$$

This transformation (see PYTHON, B 505, TRANAL) can be decomposed into :

i) a rotation of the coordinate system into the axis of the stretch

$$\alpha = -1.650817$$

ii) a diagonal matrix which removes the stretch

$$2.546086 \begin{pmatrix} 1.000000 & 0.000000 \\ 0.000000 & 0.999408 \end{pmatrix}$$

iii) a rotation and translation of the coordinate system to coincide with the (arbitrarily chosen) system, in which the camera based fiducials are given in space

$$\beta = 1.659042, \quad x_0 = 16699.05, \quad y_0 = 6672.72$$

The differences between true and transformed (measured) pattern are small (few microns). Other views and measurements give similar results for the film stretch.

3) Grid measurements

The results for the near-centre and the near-far positions of the grid will be discussed.

The calculations were done in 3 steps :

i) 2 iterations without taking into account distortions. This results in more accurate values for the rough input data and avoids meaningless distortion coefficients due to wrong setup.

ii) 20 iterations with calculation of distortions. The inter-lens distances are fixed in this (and the previous) step to ensure that no unphysical deformation of the camera base is introduced.

iii) 10 iterations with calculation of distortions without fixing inter-lens distances. Since the actual position of the optical axis of the camera cannot be given precisely the above restriction would not be meaningful.

The easiest way to check the results is a comparison of χ^2- values after the various steps. (Table 1). Corrections are calculated for plane 1 and plane 2 separately (both planes are used for calculation of camera positions, but only one plane is used for calculation of distortions at a time)

The distortion coefficients, together with errors and χ^2 of the fit, are shown separately for the plane 1 and plane 2 after step iii) (Table 2).

The errors (see PYTHON, DISTIP) are determined by :

$$\Delta\alpha_k = \left[\frac{1}{N - 7 - 1} \cdot \chi^2 \cdot A^{-1}_{k,k}\right]^{1/2} \quad \text{with}$$

$$\chi^2 = \sum_{i=1}^{N} w_i \left\{ (x_i^N - x_i')^2 + (y_i^N - y_i')^2 \right\}$$

N = number of points

w = weight = $1/((v_1\delta_1)^2 + (v_2\delta_2)^2)$

v = relative error of point

δ = global error on points

$A^{-1}_{k,k}$ = diagonal elements of the inverted matrix.

The measurement errors, which are important for the value of χ^2, were taken as

δ_1 = 0.0013 cm. for the error on the grid intersection points (quoted in Appendix I)

δ_2 = 0.0005 cm. (2 fringes) for the error on the calculated intersection points on the film

Correction for distortions gives a drastical improvement of the film measurements. In Fig. 1 the differences between true and measured (transformed) intersection points are plotted, without and with correction for distortions.

After correction the mean error (Δx, Δy) on a measured point is about 100 μ in space.

4) Chamber fiducial marks

A similar procedure as mentioned under 3) above has been applied for these calculations, except that the distortion coefficients remained fixed during the iteration cycle. Various sets of coefficients were used.

Parameters which remained unchanged for all calculations are shown in Table 3.

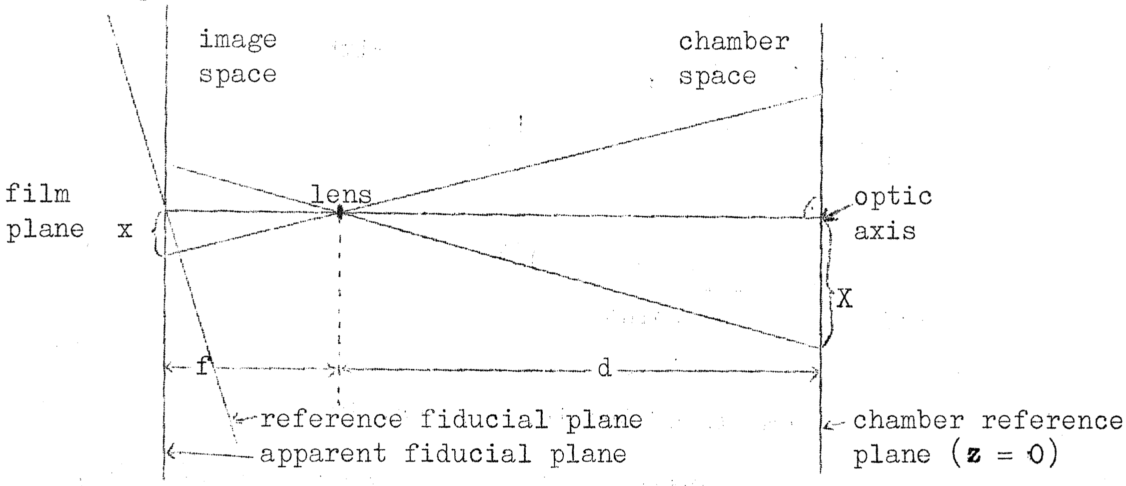
The results for the camera positions, calculated with the distortion coefficients found in the near-centre and near-far positions of the grid, are shown in Table 4 and Table 5. Also given are the χ^2 - values for the front and back fiducial plane.

The fiducial marks on the chamber windows as well as on the reference fiducial plane (see section C for definition) are given (again for all 4 combinations, as above) in Table 6 and Table 7.

The mean error on reconstructed fiducial marks is about 100 μ for Δx, Δy and 300 μ for Δz (see PYTHON, NEWFID, RECON)

C) <u>Application in Geometry Program</u>

The definitions used in the description are shown in the following sketch :



The reference fiducial plane, shown above, carries the image of the fiducial marks on the chamber windows, projected along the optic axis through the distorting lens onto the tilted film plane and scaled to chamber units (usually cm.). If no lens distortions are present and the film plane is parallel to the chamber reference plane (z = 0) the reference fiducial plane is identical to the apparent fiducial plane.

The following steps have to be executed in the reconstruction for each view :

1) Transfer reference fiducials into optic axis system for each camera. This is necessary only if the fiducials are given in a unique system attached to the chamber

2) Find transformation between fiducials on the measured film and the reference fiducial plane (least-squares fit).

3) Apply this transformation to any measured point (fiducial mark, point, point of track). This results in :

   a) transformation into optic axis system on the film,

   b) elimination of linear distortions due to film stretch, obliquity of IEP axes, etc.

   c) scaling of film measurements to chamber units.

4) Correct for film tilt and lens distortion (points and points of tracks only). The same formula which was used for the determination of the coefficients (see section A) is used. Only d (z-coordinate of camera) has to be taken instead of f (distance film lens) in this formula.

Correspondence between chamber and film system (to first order optics for simplicity) is given by :

$$x = X.\frac{f}{d} \quad \text{in film units}$$

Application of the transformation in 3) scales to chamber units

$$x' = \frac{d}{f} . x$$

Division by d gives now :

$$\frac{x'}{d} = \frac{x}{f}$$

(For chambers where no lens distortions and film tilt are present step 4) will just be skipped)

After last view : Reconstruction in space using available views.

The necessary modifications have been included in the new CERN Fortran/Fortran IV versions of THRESH

D) Discussion

The results quoted above show clearly that reconstruction of events without correcting for distortions will not give reliable geometry output. It is therefore advisable to use one of the given 4 sets of data (near-centre with corrections for plane near or centre, near-far with corrections for plane near or far). Any of these sets can be used but, at least for high energy tracks which lie usually close together in the middle of the chamber, it is probably best to use the correction coefficients found for the centre position of the grid (near-centre, corrections for plane centre).

About 1000 events of the 10 GeV/c K^- experiment have been passed through THRESH and GRIND and the results seem to be correct. The expected percentage of fits is observed and probability distributions for secondary and primary vertex look reasonable. Further analysis of the data is still to be done.

One interesting fact is the significant difference between the distortion coefficients $\alpha_4(x^2)$ and $\alpha_5(y^2)$, which should be about equal, since rotational lens distortions are proportional to $r^2 = x^2 + y^2$. There exist several explanations, but no one of them is really convincing. This result will be studied in more detail.

As a final remark it should be stated that points in space can be resonstructed with about the same precision as in the 80 cm chamber. The 75 μ(for Δx, Δy) quoted for this chamber should be compared with the 100 μ found above, but due to the larger demagnification (15 instead of 10 for the front glass) this gives the same precision for the reconstruction from film measurements.

Tables and Figures :

Table 1. χ^2 -values (179 degrees of freedom) for the least-squares fit of the film measurements to the grid intersection points in space. Results for the fit to plane 1 and plane 2, resp., are given.

Table 2. Distortion coefficients, their errors and χ^2 -values of the fit (separately for the fit to plane 1 and plane 2).

Table 3. Chamber parameters fixed during the iteration cycle for camera positions.

Table 4. Camera coordinates and χ^2 -values after least-squares fit for fiducial marks, with near-centre position of grid used for calculation of distortions. For comparison results are shown if no distortion corrections are introduced.

Table 5. Ditto, with near-far position of grid.

Table 6. Fiducial marks on the chamber and on the reference plane in a unique coordinate system attached to the chamber with near-centre position of grid used for calculation of distortions (corrections for near and centre).

Table 7. Ditto, with near-far position of grid. (corrections for near and far)

Figure 1. Results of least-squares fit of the film measurements to the grid intersection points in space. View 1 of the near-centre positions of the grid (fit for plane 1) has been chosen as an example. The differences Δx and Δy after step i) and step iii) are shown for the two planes.

Figure 2. Grid Plate

Figure 3. Visible and not visible fiducial marks on the 150 cm BNHBC. Numbering of fiducials according to CERN/TC/NBC 65-1.

APPENDIX I. (Fig.2)

Grid plate for testing lens distortions

The figure shows the general layout of the rulings on the plate, the lines being ruled at 2 inch intervals on a jig borer at 20°C. The rulings are in two sections A and B, the relation between these two being given below. In the A section, (left hand section), the coordinate axes are roughly as indicated, the exact position of the origin being such that the point P has coordinates in inches (32.000, 14.000) and the grid rulings are parallel to the axes of coordinates. Let the grid intersections be denoted by pairs of numbers (m,n) which are nominally equal to the (x,y) coordinates, and actually equal to them in the A section. Thus the point Q in the B section is labelled (44, 28) although these are not exactly its coordinates.

Then the actual coorodinates of grid points in the B section, referred to the axes in the A section, are given as follows :

$$x = m + 0.643 \times 10^{-4} n + 0.0003$$
$$y = n - 0.643 \times 10^{-4} m - 0.0005 \qquad \text{if } m \geqslant 38$$

(N.B. In fact this transformation implies the following:

The coordinates of the point (46, 28) are (46.0021, 27.9965) and the B system is rotated anticlockwise 0.643×10^{-4} radian with respect to the A system).

The special crosses 1, 2, 3 are in line with the nominal positions of the axes of cameras 1, 2, 3; the special crosses A, B, C are in line with the alignment telescope holes in the lens plate.

The grid plate is actually concave on its ruled surface with a sperical curvature of about 1.7×10^{-5} inch^{-1} ; the ruling plane was parallel to the tangent plane at the point symmetrically situated between the camera axes.

A. Thetford
W.T. Welford
June 1964.

PS/4944/dmh

APPENDIX II.

Camera Calibration, 14th January, 1965

Before this calibration the film gates were replaced; the new gates incorporate an improved suction system which should hold the film flat with 100% reliability. The "race-track" camera fiduciary has been replaced by a camera fiduciary system of four X crosses at the corners of a rectangle of approximate dimensions 28 by 84 mm. Also the reference surface on the lens plate has now been re-adjusted so that it is parallel to the film plane within about a milliradian.

The grid plate, described in the accompanying note, was photographed simultaneously by the three cameras; three positions of the grid plate were taken, corresponding approximately to the inside of the camera side window (near), the mid-plane of the chamber (middle) and the inside of the illumination side window (far). As usual the cameras are numbered 1, 2, 3 in order from the top, i.e. the top camera is No.1. The exact positions of the grid plate are given below.

In order to define the positions of the grid plate unambiguously we set up right-handed coordinate axes Oxyz, with positive z direction normal to the datum surface on the lens plate, along the axis of alignment telescope hole A, and pointing from the lens plate towards the grid plate; the positive y direction is downwards and parallel to the line joining the centres of the alignment telescope holes B and C in the lens plate. Thus positive x is roughly parallel to the beam direction. The grid plate is set so that, as seen from the cameras, its x and y axes are roughly parallel to these. The actual positions of the grid plate are then specified by the following data :

L, M, the x and y direction cosines of the normal to the plate in the region near the mark A.
Δx, Δy, the coordinates of mark A.
h, the distance which mark B is below the axis of the alignment telescope hole B. (This gives the rotation of the grid plate in its own plane).
Δz, the displacement of mark A along the z ayis relative to the middle position of the grid plate.
The numerical values are as follows :

Position	L (radians)	M (radians)	Δx (inches)	Δy (inches)	h (inches)	Δz (inches)
Near	-0.00003	0.00006	-0.016	0.004	0.028	-8.604
Middle	0.00017	0.00002	0.040	0.020	0.028	0
Far	0.00007	0	0.021	0.009	0.027	8.730

Mark A was 60.4"($\pm$0.1) from the datum plane, in the middle position.

The temperature was 20°C for the near and middle positions, 18°C for the far position. The thermal expansion coefficient of the grid plate is 8×10^{-6}.

W.T. Welford.

PS/4944/dmh

APPENDIX III.

Fiducial marks on chamber

On the attached Fig.3 all visible and not visible fiducial marks (outside the dashed lines) are drawn. The numbering convention is the same as indicated in CERN/TC/NBC 65-1, that means front glass fiducials are numbered 21 through 37, back glass fiducials 1 through 17.

Table 1.

a) Near-centre

Plane	step	Camera 1		Camera 2		Camera 3	
		Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2
Near	i)	18645	12148	18206	7117	22151	10608
	ii)	354	351	403	355	831	618
	iii)	352	352	400	354	840	621
Centre	i)	18645	12148	18206	7117	22151	10608
	ii)	771	824	2070	714	2023	1009
	iii)	389	351	634	306	975	578

b) Near-far

Plane	step	Camera 1		Camera 2		Camera 3	
		Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2
Near	i)	18645	4092	18206	3587	22151	6212
	ii)	356	136	405	412	829	578
	iii)	355	136	402	412	833	577
Far	i)	18645	4092	18206	3587	22151	6212
	ii)	2876	1296	1786	743	6661	1691
	iii)	398	143	467	386	994	516

Table 2

a) Near-centre

Plane	Camera	$\alpha_1 \pm\Delta\alpha_1$	$\alpha_2 \pm\Delta\alpha_2$	$\alpha_3 \pm\Delta\alpha_3$	$\alpha_4 \pm\Delta\alpha_4$	$\alpha_5 \pm\Delta\alpha_5$	$\alpha_6 \pm\Delta\alpha_6$	χ^2
near	1	-0.001294 0.000298	0.008127 0.001137	-0.002430 0.001017	0.021043 0.001055	0.046252 0.002854	-0.009808 0.002069	352
	2	-0.001062 0.000223	0.001596 0.000444	-0.001499 0.000976	0.018236 0.001119	0.037746 0.003069	-0.002115 0.002081	400
	3	-0.003670 0.000430	-0.005268 0.001611	0.004206 0.001388	0.012428 0.001376	0.030015 0.003957	-0.001263 0.002352	840
centre	1	-0.000681 0.000406	0.008879 0.001538	-0.002172 0.001542	0.021948 0.001531	0.046295 0.004421	-0.009793 0.003587	401
	2	-0.002304 0.000269	0.000178 0.000533	0.000763 0.001364	0.023501 0.001558	0.042707 0.004272	-0.013351 0.003908	312
	3	-0.004439 0.000482	-0.005198 0.001813	0.002157 0.001822	0.010430 0.001807	0.025801 0.005204	-0.000091 0.004214	582

b) Near-far

Plane	Camera	$\alpha_1 \pm\Delta\alpha_1$	$\alpha_2 \pm\Delta\alpha_2$	$\alpha_3 \pm\Delta\alpha_3$	$\alpha_4 \pm\Delta\alpha_4$	$\alpha_5 \pm\Delta\alpha_5$	$\alpha_6 \pm\Delta\alpha_6$	χ^2
near	1	-0.001200 0.000293	0.007873 0.001112	-0.002264 0.000976	0.020082 0.001011	0.044102 0.002730	-0.008909 0.001882	355
	2	-0.000987 0.000217	0.001550 0.000433	-0.001402 0.000932	0.017403 0.001071	0.036042 0.002939	-0.001938 0.001910	402
	3	-0.003518 0.000420	-0.005093 0.001562	0.004010 0.001322	0.011947 0.001309	0.028738 0.003762	-0.001235 0.002127	833
far	1	-0.000443 0.000370	0.010119 0.001403	-0.002145 0.001644	0.020381 0.001702	0.048960 0.004597	-0.007472 0.005637	181
	2	0.000288 0.000379	0.000522 0.000756	-0.000540 0.002165	0.013943 0.002478	0.032410 0.006811	0.001671 0.007813	388
	3	-0.003119 0.000583	-0.007102 0.002173	0.000200 0.002454	0.015065 0.002429	0.031064 0.006988	-0.008013 0.007033	516

Table 3

Total glass thickness (cm)	:	19.6500
Refractive index of glass	:	1.5230
Depth of chamber (cm)	:	46.4200
Refractive index of hydrogen	:	1.0945
Density of hydrogen (g/cm^3)	:	0.0605
Vacuum path (mean value, cm)	:	122.1900
Reference plane (z = 0)	:	back of front glass

Table 4

a) distortion corrections for plane 1 (near)

Camera	x	y	z	front	back
1	47.9717	24.0108	141.9272	10.0	14.3
2	0.0195	0.0276	141.8488	5.1	9.0
3	48.0270	-24.0385	141.7502	15.0	21.2

b) distortion corrections for plane 2 (centre)

Camera	x	y	z	front	back
1	47.9438	24.0003	141.8935	11.6	12.8
2	- 0.0043	0.0357	141.7864	7.7	9.8
3	47.9892	-24.0505	141.7807	16.1	20.2

c) distortions ignored

Camera	x	y	z	front	back
1	47.9530	24.0196	142.1459	319.4	233.0
2	0.0342	0.0287	141.9600	112.8	145.9
3	48.0102	-24.0480	141.9140	321.2	270.4

Table 5

a) distortion corrections for plane 1 (near)

Camera	x	y	z	front	back
1	47.9712	24.0111	141.9327	11.9	14.0
2	0.0202	0.0277	141.8550	6.0	9.5
3	48.0267	-24.0390	141.7562	17.5	23.0

b) distortion corrections for plane 2 (far)

Camera	x	y	z	front	back
1	47.9817	23.9712	141.8982	16.0	14.7
2	- 0.0078	0.0473	141.8953	6.3	11.0
3	47.9901	-24.0773	141.7480	14.5	21.3

Table 6

a) with distortion corrections for plane 1 (near) - front fiducial plane

Fiducial Mark	chamber x	chamber y	Camera 1 x	Camera 1 y	Camera 2 x	Camera 2 y	Camera 3 x	Camera 3 y
front 21	- 16.2098	20.1352	-19.4796	19.9497	-17.0041	21.1603	-19.9386	22.5520
22	3.6216	20.4961	1.3889	20.3264	3.8733	21.5009	1.2227	22.8106
23	23.7675	20.0999	22.5506	19.8996	25.0115	21.0678	22.5468	22.2973
24	43.5873	20.3778	43.3432	20.2012	45.8080	21.3654	43.3788	22.5334
25	63.5024	19.9488	64.2740	19.7422	66.7738	20.9420	64.2239	22.0709
26	-36.3974	1.6315	-41.4689	0.3733	-38.2341	1.7171	-41.5153	3.0706
27	-16.4155	1.5724	-19.7478	0.4076	-17.2326	1.6487	-19.7687	2.9211
28	3.5664	1.5132	1.2902	0.3593	3.7481	1.5885	1.2840	2.8320
29	23.5484	1.4541	22.3157	0.3028	24.7454	1.5242	22.2997	2.7514
30	43.5303	1.3950	43.3117	0.2380	45.7634	1.4724	43.3031	2.6996
31	63.5122	1.3358	64.3035	0.1732	66.8188	1.4081	64.3108	2.6560
32	83.4942	1.2767	85.3373	0.1124	87.9198	1.3112	85.3554	2.6164
33	-16.4844	-17.5237	-20.2013	-19.7876	-17.3053	-18.4201	-19.8188	-17.1833
34	3.5102	-17.4697	1.1286	-19.6060	3.6852	-18.3531	1.2426	-17.1489
35	23.4928	-17.5593	22.2662	-19.6046	24.6783	-18.4548	22.2541	-17.2501
36	43.4741	-17.5880	43.2593	-19.5765	45.6797	-18.4858	43.2494	-17.2813
37	63.4927	-17.5868	64.1996	-19.5620	66.7541	-18.5089	64.2820	-17.2632

PS/4944/dmh

Table 6 cont.

a) with distortion corrections for plane 1 (near) - back fiducial plane

Fiducial Mark	Chamber x	Chamber y	Camera 1 x	Camera 1 y	Camera 2 x	Camera 2 y	Camera 3 x	Camera 3 y
back								
1	-16.4854	19.6055	- 3.6527	20.4820	-13.1557	15.6699	- 3.7463	10.9913
2	3.7263	19.8012	12.7062	20.5884	2.9834	15.8278	12.5455	11.0481
3	23.6485	19.9778	28.5212	20.7880	18.9148	15.9772	28.5065	11.2124
4	43.6937	19.7979	44.5456	20.6559	34.9670	15.8608	44.5881	10.9376
5	63.6575	19.6783	60.5074	20.5577	51.0192	15.7860	60.5146	10.9902
6	-36.2378	0.8216	-19.6562	5.3769	-28.9736	0.6570	-19.6456	- 4.0733
7	-16.2558	0.8200	- 3.5216	5.4205	-12.9755	0.6527	- 3.5014	- 4.1131
8	3.7263	0.8183	12.5583	5.4434	2.9728	0.6608	12.5688	- 4.1484
9	23.7083	0.8167	28.5751	5.4499	18.9667	0.6524	28.5857	- 4.1546
10	43.6904	0.8150	44.5417	5.4566	34.9731	0.6564	44.5697	- 4.1606
11	63.6724	0.8134	60.5333	5.4549	51.0376	0.6689	60.5415	- 4.1378
12	83.6544	0.8117	76.5501	5.4446	67.2017	0.6690	76.5750	- 4.1235
13	-16.0962	-18.1728	- 3.4505	- 9.8463	-12.8740	-14.5266	- 3.3476	-19.3446
14	3.7248	-18.1646	12.5246	- 9.8060	2.9788	-14.5186	12.5953	-19.3587
15	23.7443	-18.2368	28.5666	- 9.8207	18.9935	-14.5893	28.6078	-19.4143
16	43.6889	-18.1679	44.5377	- 9.7468	34.9708	-14.5396	44.5716	-19.3584
17	63.7152	-17.9696	60.5679	- 9.6019	51.0808	-14.3983	60.5688	-19.1875

Table 6 - cont.

b) with distortion corrections for plane 2 (centre)

Fiducial Mark	Chamber x	Chamber y	Camera 1 x	Camera 1 y	Camera 2 x	Camera 2 y	Camera 3 x	Camera 3 y
front								
21	-16.2358	20.1559	-19.5155	19.9701	-17.0213	21.1785	-19.9666	22.5747
22	3.5958	20.5071	1.3555	20.3373	3.8526	20.5085	1.1966	22.8249
23	23.7415	20.1012	22.5193	19.9008	24.9869	21.0647	22.5223	22.3030
24	43.5614	20.3695	43.3144	20.1929	45.7798	21.3518	43.3562	22.5309
25	63.4763	19.9309	64.2473	19.7242	66.7418	20.9178	64.2029	22.0600
26	36.4324	1.6619	-41.5162	0.4016	-38.2573	1.7496	-41.5529	3.1003
27	-16.4505	1.5931	-19.7927	0.4260	-17.2596	1.6705	-19.8045	2.9421
28	3.5314	1.5243	1.2476	0.3681	3.7173	1.5997	1.2499	2.8446
29	23.5134	1.4555	22.2754	0.3018	24.7109	1.5247	22.2673	2.7556
30	43.4953	1.3867	43.2737	0.2274	45.7252	1.4622	43.2726	2.6954
31	63.4771	1.3179	64.2678	0.1530	66.7768	1.3873	64.2820	2.6435
32	83.4591	1.2491	85.3039	0.0825	87.8741	1.2797	85.3284	2.5954
33	-16.5286	-17.5029	-20.2555	-19.7713	-17.3425	-18.3947	-19.8627	-17.1640
34	3.4660	-17.4586	1.0769	-19.5994	3.6443	-18.3383	1.2006	-17.1380
35	23.4486	-17.5579	22.2168	-19.6077	24.6337	-18.4507	22.2138	-17.2476
36	43.4299	-17.5962	43.2122	-19.5893	45.6314	-18.4924	43.2109	-17.2871
37	63.4484	-17.6047	64.1548	-19.5844	66.7021	-18.5262	64.2453	-17.2775
back								
1	-16.5117	19.6263	- 3.6866	20.4952	-13.1763	15.6872	- 3.7775	11.0066
2	3.7001	19.8122	12.6742	20.5941	2.9600	15.8369	12.5157	11.0569
3	23.6224	19.9792	28.4909	20.7865	18.8887	15.9782	28.4781	11.2149
4	43.6675	19.7896	44.5171	20.6470	34.9380	15.8536	44.5609	10.9336
5	63.6313	19.6603	60.4806	20.5415	50.9873	15.7707	60.4889	10.9799
6	-36.2731	0.8520	-19.6988	5.3958	-28.9990	0.6850	-19.6842	- 4.0530
7	-16.2911	0.8407	- 3.5624	5.4320	-13.0038	0.6727	- 3.5387	- 4.0992
8	3.6910	0.8293	12.5192	5.4475	2.9417	0.6726	12.5329	- 4.1409
9	23.6730	0.8180	28.5379	5.4466	18.9328	0.6560	28.5512	- 4.1535
10	43.6551	0.8067	44.5062	5.4461	34.9363	0.6519	44.5365	- 4.1659
11	63.6371	0.7954	60.4996	5.4370	50.9980	0.6562	60.5097	- 4.1494
12	83.6191	0.7841	76.5181	5.4193	67.1593	0.6481	76.5445	- 4.1415
13	-16.1407	-18.1522	- 3.4983	- 9.8366	-12.9100	-14.5041	- 3.3909	-19.3321
14	3.6802	-18.1536	12.4786	- 9.8036	2.9400	-14.5042	12.5533	-19.3525
15	23.6998	-18.2354	28.5223	- 9.8256	18.9519	-14.5830	28.5672	-19.4145
16	43.6443	-18.1762	44.4952	- 9.7590	34.9264	-14.5414	44.5323	-19.3650
17	63.6708	-17.9876	60.5273	- 9.6214	51.0336	-14.4083	60.5310	-19.2005

Table 7

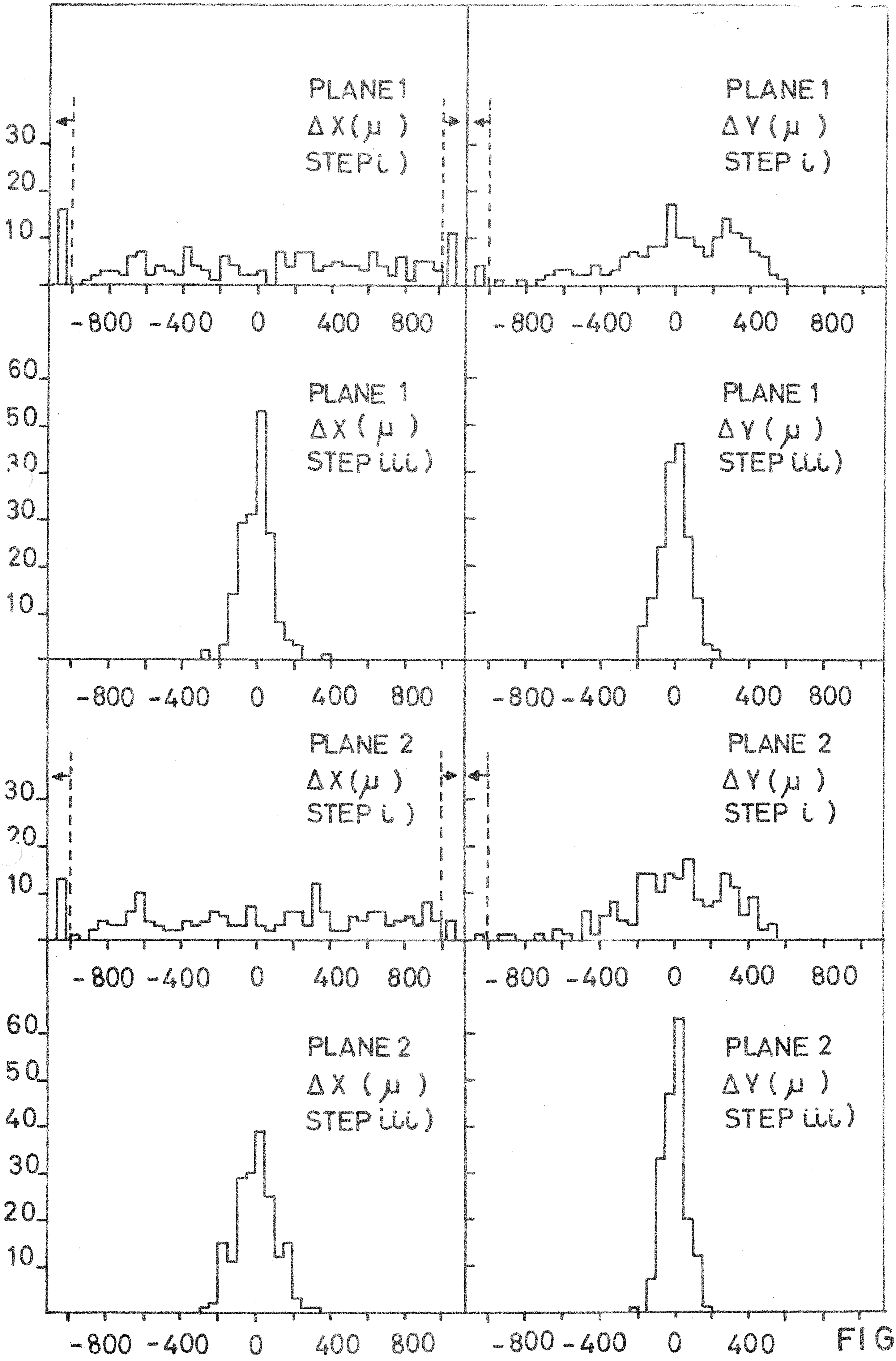
a) with distortion corrections for plane 1 (near)

Fiducial Mark	Chamber x	Chamber y	Camera 1 x	Camera 1 y	Camera 2 x	Camera 2 y	Camera 3 x	Camera 3 y
front								
21	-16.2092	20.1342	-19.4892	19.9478	-17.0050	21.1614	-19.9452	22.5563
22	3.6222	20.4954	1.3822	20.3255	3.8743	21.5024	1.2183	22.8148
23	23.7682	20.0997	22.5468	19.8996	25.0144	21.0696	22.5446	22.3013
24	43.5879	20.3779	43.3422	20.2021	45.8127	21.3676	43.3789	22.5372
25	63.5030	19.9494	64.2759	19.7439	66.7804	20.9445	64.2261	22.0745
26	-36.3964	1.6301	-41.4806	0.3678	-38.2365	1.7162	-41.5243	3.0731
27	-16.4145	1.5714	-19.7566	0.4030	-17.2332	1.6481	-19.7754	2.9234
28	3.5674	1.5126	1.2844	0.3557	3.7494	1.5882	1.2794	2.8341
29	23.5494	1.4539	22.3127	0.3000	24.7485	1.5243	22.2973	2.7533
30	43.5313	1.3951	43.3116	0.2361	45.7684	1.4728	43.3030	2.7013
31	63.5132	1.3364	64.3063	0.1722	66.8257	1.4089	64.3129	2.6576
32	83.4952	1.2776	85.3429	0.1123	87.9286	1.3123	85.3597	2.6178
33	-16.4831	-17.5247	-20.2092	-19.7950	-17.8056	-18.4224	-19.8258	-17.1831
34	3.5116	-17.4703	1.1236	-19.6124	3.6868	-18.3551	1.2379	-17.1489
35	23.4942	-17.5595	22.2641	-19.6101	24.6818	-18.4565	22.2516	-17.2503
36	43.4755	-17.5878	43.2600	-19.5811	45.6851	-18.4872	43.2491	-17.2816
37	63.4941	-17.5862	64.2032	-19.5657	66.7614	-18.5099	64.2839	-17.2637
back								
1	-16.4848	19.6045	- 3.6601	20.4809	-13.1561	15.6706	- 3.7513	10.9943
2	3.7269	19.8006	12.7010	20.5879	2.9844	15.8289	12.5422	11.0510
3	23.6491	19.9776	28.5181	20.7883	18.9172	15.9785	28.5049	11.2152
4	43.6943	19.7981	44.5448	20.6569	34.9708	15.8623	44.5882	10.9402
5	63.6581	19.6789	60.5087	20.5594	51.0244	15.7878	60.5164	10.9927
6	-36.2368	0.8203	-19.6651	5.3730	-28.9752	0.6561	-19.6524	- 4.0718
7	-16.2548	0.8190	- 3.5284	5.4173	-12.9757	0.6521	- 3.5065	- 4.1117
8	3.7274	0.8177	12.5537	5.4409	2.9740	0.6605	12.5654	- 4.1471
9	23.7093	0.8165	28.5728	5.4481	18.9694	0.6523	28.5839	- 4.1534
10	43.6914	0.8152	44.5415	5.4555	34.9771	0.6566	44.5696	- 4.1596
11	63.6734	0.8140	60.5353	5.4545	51.0431	0.6693	60.5431	- 4.1369
12	83.6555	0.8127	76.5543	5.4449	67.2087	0.6697	76.5783	- 4.1227
13	-16.0948	-18.1738	- 3.4565	- 9.8516	-12.8739	-14.5285	- 3.3528	-19.3448
14	3.7262	-18.1652	12.5207	- 9.8106	2.9803	-14.5203	12.5917	-19.3590
15	23.7457	-18.2370	28.5648	- 9.8246	18.9965	-14.5908	28.6060	-19.4147
16	43.6902	-18.1677	44.5381	- 9.7500	34.9751	-14.5408	44.5714	-19.3590
17	63.7166	-17.9690	60.5706	- 9.6043	51.0866	-14.3992	60.5703	-19.1882

Table 7

b) with distortion corrections for plane 2 (far)

Fiducial Mark	Chamber x	Chamber y	Camera 1 x	Camera 1 y	Camera 2 x	Camera 2 y	Camera 3 x	Camera 3 y
front								
21	-16.1986	20.1701	-19.5006	19.9848	-17.0007	21.2011	-19.9318	22.5954
22	3.6332	20.5084	1.3769	20.3392	3.8807	21.5182	1.2320	22.8310
23	23.7787	20.0894	22.5469	19.8896	25.0221	21.0611	22.5580	22.2943
24	43.5988	20.3448	43.3483	20.1689	45.8224	21.3352	43.3925	22.5077
25	63.5134	19.8932	64.2873	19.6873	66.7914	20.8880	64.2394	22.0224
26	-36.4072	1.6894	-41.5199	0.4239	-38.2563	1.7787	-41.5321	3.1355
27	-16.4254	1.6075	-19.7899	0.4349	-17.2513	1.6864	-19.7833	2.9622
28	3.5565	1.5257	1.2568	0.3640	3.7329	1.6025	1.2716	2.8502
29	23.5384	1.4439	22.2908	0.2849	24.7337	1.5144	22.2894	2.7466
30	43.5202	1.3620	43.2954	0.1975	45.7553	1.4388	43.2952	2.6718
31	63.5021	1.2802	64.2958	0.1102	66.8143	1.3507	64.3051	2.6053
32	83.4840	1.1983	85.3382	0.0267	87.9188	1.2299	85.3520	2.5427
33	-16.5160	-17.4885	-20.2652	-19.7681	-17.3468	-18.3857	-19.8554	-17.1443
34	3.4787	-17.4571	1.0737	-19.6093	3.6474	-18.3424	1.2084	-17.1329
35	23.4612	-17.5695	22.2199	-19.6306	24.6441	-18.4679	22.2220	-17.2571
36	43.4425	-17.6208	43.2217	-19.6251	45.6491	-18.5227	43.2196	-17.3111
37	63.4610	-17.6424	64.1706	-19.6331	66.7271	-18.5697	64.2546	-17.3160
back								
1	-16.4748	19.6407	- 3.6666	20.5003	-13.1578	15.7054	- 3.7504	11.0158
2	3.7372	19.8135	12.6992	20.5891	2.9842	15.8452	12.5433	11.0548
3	23.6595	19.9675	28.5209	20.7718	18.9185	15.9765	28.5062	11.2018
4	43.7045	19.7648	44.5518	20.6225	34.9734	15.8420	44.5892	10.9094
5	63.6682	19.6225	60.5199	20.5072	51.0282	15.7489	60.5176	10.9446
6	-36.2484	0.8793	-19.6928	5.4062	-28.9954	0.7079	-19.6679	- 4.0331
7	-16.2665	0.8550	- 3.5516	5.4324	-12.9946	0.6855	- 3.5220	- 4.0905
8	3.7156	0.8306	12.5349	5.4381	2.9564	0.6756	12.5500	- 4.1433
9	23.6976	0.8063	28.5584	5.4274	18.9531	0.6490	28.5686	- 4.1670
10	43.6797	0.7819	44.5315	5.4170	34.9622	0.6350	44.5544	- 4.1905
11	63.6617	0.7576	60.5297	5.3981	51.0295	0.6292	60.5280	- 4.1851
12	83.6437	0.7332	76.5530	5.3706	67.1964	0.6110	76.5632	- 4.1883
13	-16.1284	-18.1380	- 3.4968	- 9.8407	-12.9102	-14.4965	- 3.3848	-19.3239
14	3.6925	-18.1523	12.4849	- 9.8176	2.9453	-14.5065	12.5598	-19.3553
15	23.7120	-18.2472	28.5334	- 9.8495	18.9627	-14.5954	28.5741	-19.4284
16	43.6566	-18.2010	44.5111	- 9.7927	34.9428	-14.5637	44.5397	-19.3900
17	63.6832	-18.0254	60.5481	- 9.6649	51.0557	-14.4406	60.5388	-19.2365



PLANE 1
ΔX(μ)
STEP i)
PLANE 1
ΔY(μ)
STEP i)
PLANE 1
ΔX(μ)
STEP iii)
PLANE 1
ΔY(μ)
STEP iii)
PLANE 2
ΔX(μ)
STEP i)
PLANE 2
ΔY(μ)
STEP i)
PLANE 2
ΔX(μ)
STEP iii)
PLANE 2
ΔY(μ)
STEP iii)
-800 -400 0 400 800

FIG.1

P3/4-94-4

GRID PLATE

CROSSES AT 2.000" INTERVALS
RULINGS GOOD TO 0.0005"

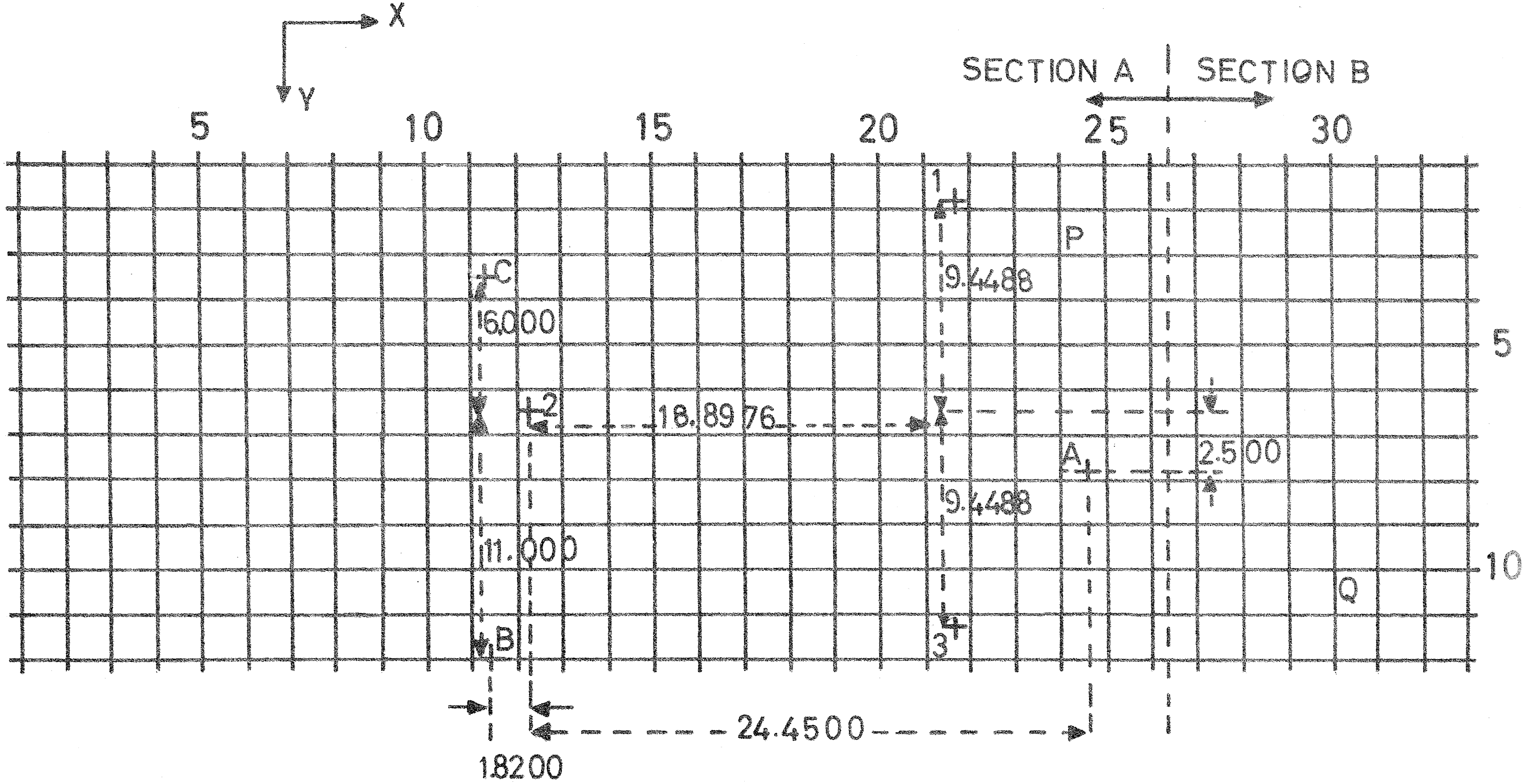
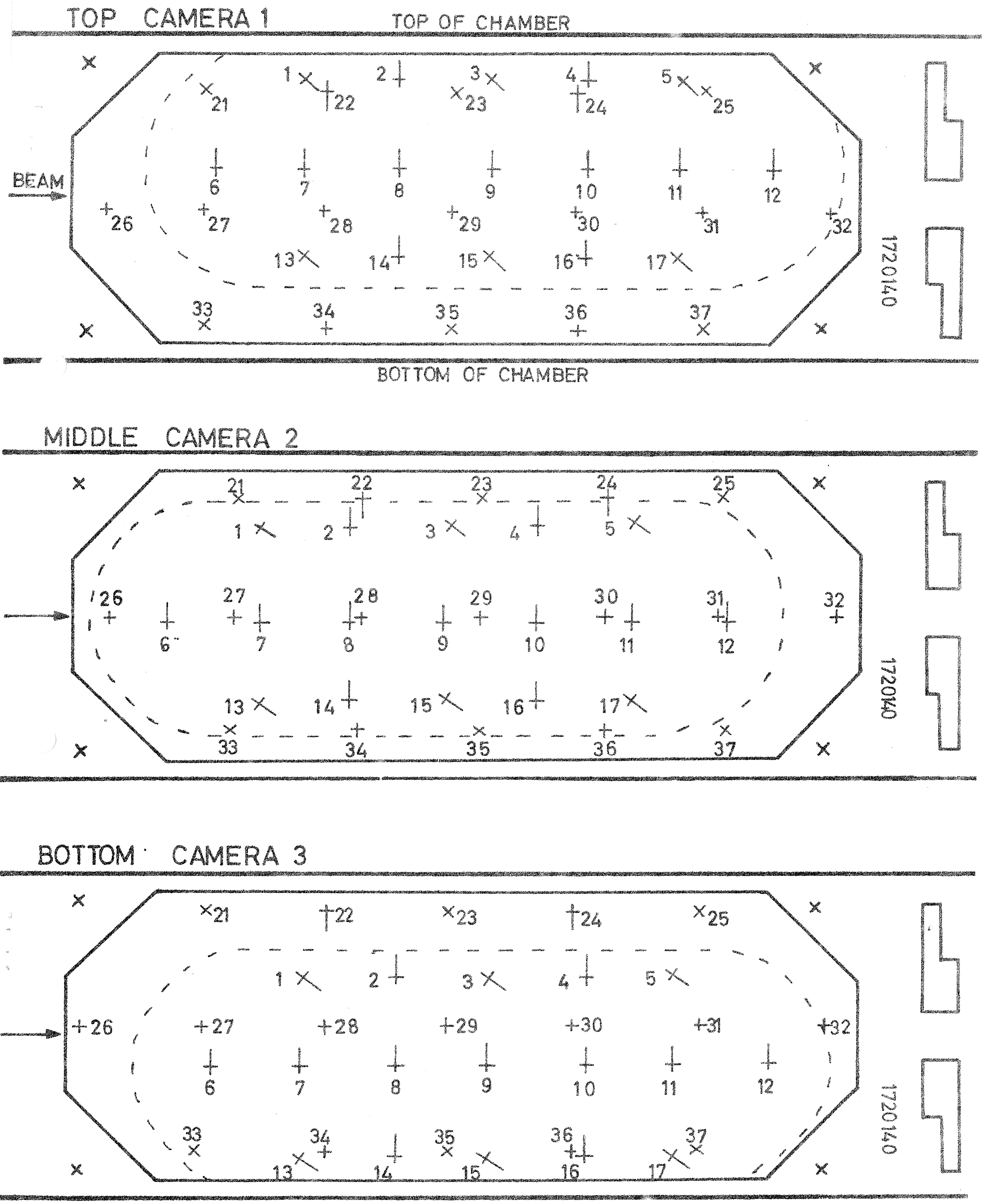


FIG. 2

150 cm BRITISH HYDROGEN BUBBLE CHAMBER

ALL FIDUCIAL MARKS VISIBLE AND NOT VISIBLE



PS/4944 JUNE 1965

FIG.3