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Theoretical considerations on the effect of polishing in vacuum breakdown connected with the growth of protrusions.

This report has been presented for the working party on vacuum breakdown and vacuum discharges held at the University College of Swansea on the 12th - 13th April, 1965.

INTRODUCTION

A few years ago our group at CERN undertook the investigation of electrical breakdown across large gaps (1 to 10 cm) in vacuum (10^{-6} to 10^{-3} Torr) as related to the improvement of electrostatic separators. This application requires a spark free vacuum gap of a few centimetres spacing between plane electrodes of large area (more than one square metre) with the highest possible field. In this type of research we have been led to study the behaviour of different kinds of electrodes by a series of tests, each one to investigate the effects of a different parameter on breakdown. The principal results of these studies (such as alumina coating of cathodes) has been published in various papers, (Ref. 1, 2 and 3).

Among the fundamental hypotheses introduced in the paper presented in Boston, (Ref. 2,3) for the explanation of the pressure effect, the presence on cathodic surfaces of sharp protrusions was assumed. It is in fact very well known from electron microscope studies that such protrusions appear and grow even on smooth surfaces after an electric field of sufficient magnitude has been produced (Ref. 4,5,6,7). The theoretical analysis of this phenomenon could give a better understanding of conflicting results about surface polishing and of the effect observed that higher breakdown strengths are obtained with harder electrodes.

GROWTH OF PROTRUSIONS

In a recent paper, (Ref. 5,8,9) Charbonnier proposed an explanation of the growing of protrusions on metal surfaces which is based on the isotropic approximation of Herring's theory of the build up phenomenon, and shows that a balance is achieved between surface tension and field forces when the tip radius and the field of the emitter surface are related by the approximate expression (for a given field emission tip geometry)

$$E^2 = 2 \cdot \frac{\gamma}{\varepsilon_o r}$$

r = radius of the emitter tip (m)

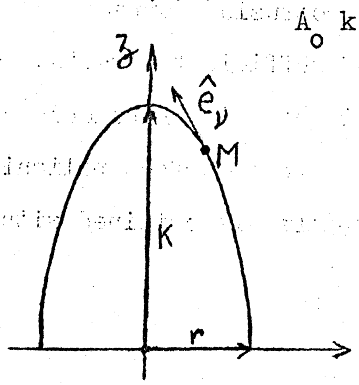
γ = surface tension $\frac{N}{m}$

E = field at the emitter tip $\frac{V}{m}$

$\varepsilon_o = \frac{10^{-9}}{36\,\pi} \quad \frac{As}{Vm}$

It was then thought worthwhile to apply to our geometry the same Herring's postulate which consists essentially in a computation of the chemical potential gradient on the protrusion surface. The chosen protrusion geometry being a semi-ellipsoid of revolution standing on a flat surface, an interesting expression is found for the flux of material $\vec{J}$ which flows per unit time across a line of unit length, perpendicular to the direction of the migration at any point of the surface for any ratio λ of the major and minor axis of the ellipsoid. ($\lambda = 0$ plane, $\lambda = 1$ sphere, $\lambda = \infty$ perfect needle).

$$\vec{J}(\lambda,\nu) = -\frac{\gamma \Omega_o^2 D_o e^{-\frac{Q}{RT}}}{A_o\, k\, T\, r^2}\, A(\lambda,\nu) \left[1 - \frac{\varepsilon_o E_o^2 r}{\gamma C(\lambda,\nu)}\right] \hat{e}_\nu \quad \left(\frac{m^3}{ms}\right)$$



$\lambda = \frac{K}{r} \qquad \nu = \frac{z}{K}$

D_o = diffusivity constant for surface migration $\left(\frac{m^2}{s}\right)$

A_o = surface area per atom $\left(\frac{m^2}{atom}\right)$

Ω_o = volume area per atom $\left(\frac{m^3}{atom}\right)$

Q = activation energy $\left(\frac{J}{mole}\right)$
T = temperature of the protrusion (^{o}K)
k = Boltzmann constant = 1.38 10^{-23} $\left(\frac{J}{^{o}K\ atom}\right)$
R = 8.31 $\left(\frac{J}{^{o}K\ mole}\right)$
E_o = mean gap field $\left(\frac{V}{m}\right)$
r = radius of the shank of the protrusion (m)

$A(\lambda,\nu)$ and $C(\lambda,\nu)$ represent dimensionless coefficients which depend only on the point M and on the ratio λ.

$$A(\lambda,\nu) = \lambda(\lambda^2 - 1)\nu\sqrt{1-\nu^2}\ \frac{4 + (1-\nu^2)(\lambda^2-1)}{\left[1 + (1-\nu^2)(\lambda^2-1)\right]^3}$$

$$C(\lambda,\nu) = \frac{\lambda^2 - 1}{\lambda\beta^2(\lambda)}\ \frac{4 + (1-\nu^2)(\lambda^2-1)}{\left[1 + (1-\nu^2)(\lambda^2-1)\right]^{1/2}} \qquad \beta(\lambda) = \frac{(\lambda^2-1)^{3/2}}{\lambda\ \mathrm{Log}(\lambda + \sqrt{\lambda^2-1}) - \sqrt{\lambda^2-1}}$$

The sign $|\vec{J}(\nu,\lambda)|$ determines the dynamical behaviour of the protrusion. If $|\vec{J}|$ is negative, atoms migrate from the apex toward the shank: on the contrary, if $|\vec{J}|$ is positive, the atoms migrate from the shank to the apex and the protrusion grows.

$|\vec{J}|$ can be written : $|\vec{J}| = -J_o\ A(\lambda,\nu)\left(1 - \frac{\varkappa E_o^2}{C(\lambda,\nu)}\right)$

with : $J_o = \frac{\gamma\Omega_o^2 D_o e^{-\frac{Q}{RT}}}{A_o\ k\ T\ r^2}$

$$\varkappa = \frac{\varepsilon_o r}{\gamma}$$

We can distinguish between different values of the parameters λ, ν and E_o.

1) If $\lambda < 1$ $A(\lambda,\nu)$ and $C(\lambda,\nu) < 0$ for all values of ν and E_o then $|\vec{J}|$ is positive (or zero for $\lambda = 0$) and the protrusion grows.

2) If $\lambda = 1$ $A(\lambda,\nu) = 0$ and $|\vec{J}|$ becomes :

$$|\vec{J}| = 9\ J_o \varkappa E_o^2\ \nu\sqrt{1-\nu^2}$$

Thus, in the absence of electric field, $|\vec{J}| = 0$ everywhere on the surface of the sphere. This is an equilibrium shape because, when $E_o = 0$,

$$|\vec{J}| \text{ is } > 0 \quad \text{for} \quad \lambda < 1$$
$$\text{"} \quad = 0 \quad \text{for} \quad \lambda = 1$$
$$\text{"} \quad < 0 \quad \text{for} \quad \lambda > 1$$

In this sense the plane is not an equilibrium shape because we have seen that $|\vec{J}| > 0$ for $\lambda < 1$ and 0 for $\lambda = 0$.

When $E_o > 0$, $|\vec{J}|$ is always positive and zero for $\nu = 0$ or 1 (the apex and shank of the protrusion). Then, when a field is applied to an equilibrium sphere shape, the protrusion grows even for the smallest field.

3) If $\lambda > 1$ $A(\lambda,\nu)$ is always positive or zero (for $\nu = 0,1$).

In this case the sign of $|\vec{J}|$ depends only on the sign of the bracket function :

$$\Gamma(\lambda,\nu,E_o) = 1 - \frac{\varkappa E_o^2}{C(\lambda,\nu)}$$

$C(\lambda,\nu)$ is always positive for all values of ν. Thus the sign of $\Gamma(\lambda,\nu,E_o)$ depends only on the intensity of the applied field.

If $\Gamma > 0 \rightarrow E_o^2 < \dfrac{C(\lambda,\nu)}{\varkappa}$ the protrusion always decreases but at a smaller rate than in the absence of the electric field.

If $\Gamma = 0 \rightarrow E_o^2 = \dfrac{C(\lambda,\nu)}{\varkappa}$ the protrusion is stable whatever its temperature may be.

If $\Gamma < 0 \rightarrow E_o^2 > \dfrac{C(\lambda,\nu)}{\varkappa}$ the protrusion grows.

The condition of full growth $\Gamma < 0$ is equivalent to :

$$u^2 - u\,\frac{2\lambda^2 + 6 - K_o^2}{\lambda^2 - 1} + \frac{\lambda^4 + \lambda^2(6 - K_o^2) + 9}{(\lambda^2 - 1)^2} < 0$$

with $u = \nu^2$ $\qquad K_o = \dfrac{\varkappa E_o^2\,\lambda\,\beta^2}{\lambda^2 - 1}$.

The discussion of this condition with respect to the possible values for λ and K_o^2 gives the following result (Fig. 1).

For $1 < \lambda < \sqrt{3}$ the protrusion decreases for any value of ν

$$\text{if} \quad K_o < \frac{\lambda^2 + 3}{\lambda}$$

For $\sqrt{3} < \lambda < \infty$ the protrusion decreases for any value of ν

$$\text{if} \quad K_o < 2\sqrt{3}$$

For $1 < \lambda < 3$ the protrusion grows for any value of ν

$$\text{if} \quad K_o > 4$$

For $3 < \lambda < \infty$ the protrusion grows for any value of ν

$$\text{if} \quad K_o > \frac{\lambda^2 + 3}{\lambda}$$

For the intermediate cases, parts of the protrusion surface migrate from the shank to the apex and others from the apex to the shank according to the values of the roots u_1, u_2 of the above condition. In particular, if $K_o > 4$ the tip of the protrusion grows for any value of λ between 1 and the infinity.

Now, let us see what will happen to a protrusion of a given value of λ, according to the value of the applied field E_o.

For this purpose, let us consider the condition of full growth:

$$E_o^2 > \frac{\gamma}{k\varepsilon_o} \frac{4(\lambda^2 - 1)}{\lambda\beta^2} \qquad \text{for } 1 < \lambda < 3$$

$$E_o^2 > \frac{\gamma}{k\varepsilon_o} \frac{(\lambda^2 + 3)(\lambda^2 - 1)}{\lambda^2\beta^2} \qquad \text{for } 3 < \lambda < \infty$$

putting $\varphi(\lambda) = \dfrac{4(\lambda^2 - 1)}{\lambda\beta^2}$ $\qquad \Theta(\lambda) = \dfrac{(\lambda^2 + 3)(\lambda^2 - 1)}{\lambda^2\beta^2} = \dfrac{(\lambda^2 + 3)}{4\lambda} \varphi(\lambda)$

we see immediately that $\varphi(1) = \varphi(\infty) = \Theta(1) = \Theta(\infty) = 0$

$$\varphi(3) = \Theta(3)$$

The maximum of $\varphi(\lambda)$ and of $\theta(\lambda)$ is reached for :

$$\lambda \cong \sqrt{3} \qquad \beta(\lambda) \cong 5 \text{ and } \varphi_{max} = 0.1888$$
$$\theta_{max} = 0.1633$$

which gives for the maximum field :

$$E_{omax} = 1.461 \sqrt{\frac{\gamma}{k}} \ \frac{kV}{cm} \qquad \left(\text{Tungsten at } 2700^{\circ}K \quad E_{omax} \cong 2.5 \ \frac{MV}{cm}\right)$$

Thus, for a certain kind of material given by γ and α polishing state described by k, if the macroscopic field E_o is less than E_{omax}, the protrusions with $\lambda > \lambda_4$ (intercept of $\theta(\lambda)$ with the straight line $\varphi_o = \frac{k\varepsilon_o}{\gamma} \cdot E_o^2$) will grow indefinitely and could produce consequently sparks, the protrusions with $\lambda \not< \lambda_1$ will stabilize their shape to $\lambda = \lambda_1$, intercept of $\varphi(\lambda)$ with φ_o line. For λ between λ_1 and λ_4 we must consider different cases :

For $\lambda > \sqrt{3}$ the condition of full decrease is : $E_o^2 < \frac{\gamma}{k\varepsilon_o} \ \omega(\lambda)$
with $\omega(\lambda) = \frac{\sqrt{3}}{2} \ \varphi(\lambda)$

For $\lambda < \sqrt{3}$ the condition of full decrease is : $E_o^2 < \frac{\gamma}{k\varepsilon_o} \ \theta(\lambda)$

Then, for $\lambda < \lambda_2$, intercept of φ_o with $\omega(\lambda)$ and for $\lambda > \lambda_1'$, intercept of φ_o with $\theta(\lambda)$, $(\lambda_1' \cong \lambda_1)$, the protrusions will decrease and stabilize to $\lambda = \lambda_1' \cong \lambda_1$.

It can be shown that for $\lambda_2 < \lambda < \lambda_4$, two intervals can be distinguished :

For $\lambda < \lambda_4$ but $> \lambda_3$, intercept of φ_o with $\varphi(\lambda)$, the tip of the protrusion grows but the shank decreases, $|\vec{J}|$ being zero for

$$\nu_2 = \sqrt{\frac{2\lambda^2 + 6 - K_o^2 - K_o\sqrt{K_o^2 - 12}}{2(\lambda^2 - 1)}}$$

For $\lambda < \lambda_3$ but $> \lambda_2$ the tip and the shank decrease but the middle part of the protrusion grows between

$$\nu = \nu_2 \text{ and } \quad \nu = \nu_1 = \sqrt{\frac{2\lambda^2 + 6 - K_o^2 + K_o\sqrt{K_o^2 - 12}}{2(\lambda^2 - 1)}}$$

If all the protrusions have $\lambda < \lambda_2$, it is theoretically impossible to get a spark. This is a theoretical explanation of the effect of the polishing. Moreover, we can see that if $E_o > E_{omax}$, all the protrusions will grow indefinitely, and no matter what the polishing state before applying the field may be, it will be possible to get breakdowns. The theory also gives an explanation of the conditioning process; by increasing the voltage, the protrusions with the highest value of λ will be eliminated by growth and sparking. The rate of increasing depends strongly on the protrusion temperature and on the activation energy; from this, we understand why when the protrusion emits electrons and heats up, a sharp point is quickly formed and we conceive that the process must be divergent.

CONCLUSION

These preliminary results seem to give us a better understanding of the protrusion growing phenomena and especially a new limitation of the maximum field that a material could support; this field is directly proportional to the square root of the "hardness". The results obtained show that the best polishing state would be a plane surface made of a kind of hemisphere with the smallest possible value of r, standing side by side with a "λ" ratio smaller than $\sqrt{3}$. An experimental proof of the fact that the smaller the value of r the higher the field is given by F. McCoy, C. Coenraads and M. Thayer (Ref. 10). In that respect a crystalline structure is evidently worse than an amorphous one because of its anisotropy and its shape of facets (Ref. 8). This remark could be an element in the explanation of the better breakdown voltage obtained either with glass or alumina coated cathodes, the surface shape of which is very similar to the ideal one, as shown by recent electron shadow microscope pictures.

F. Rohrbach.

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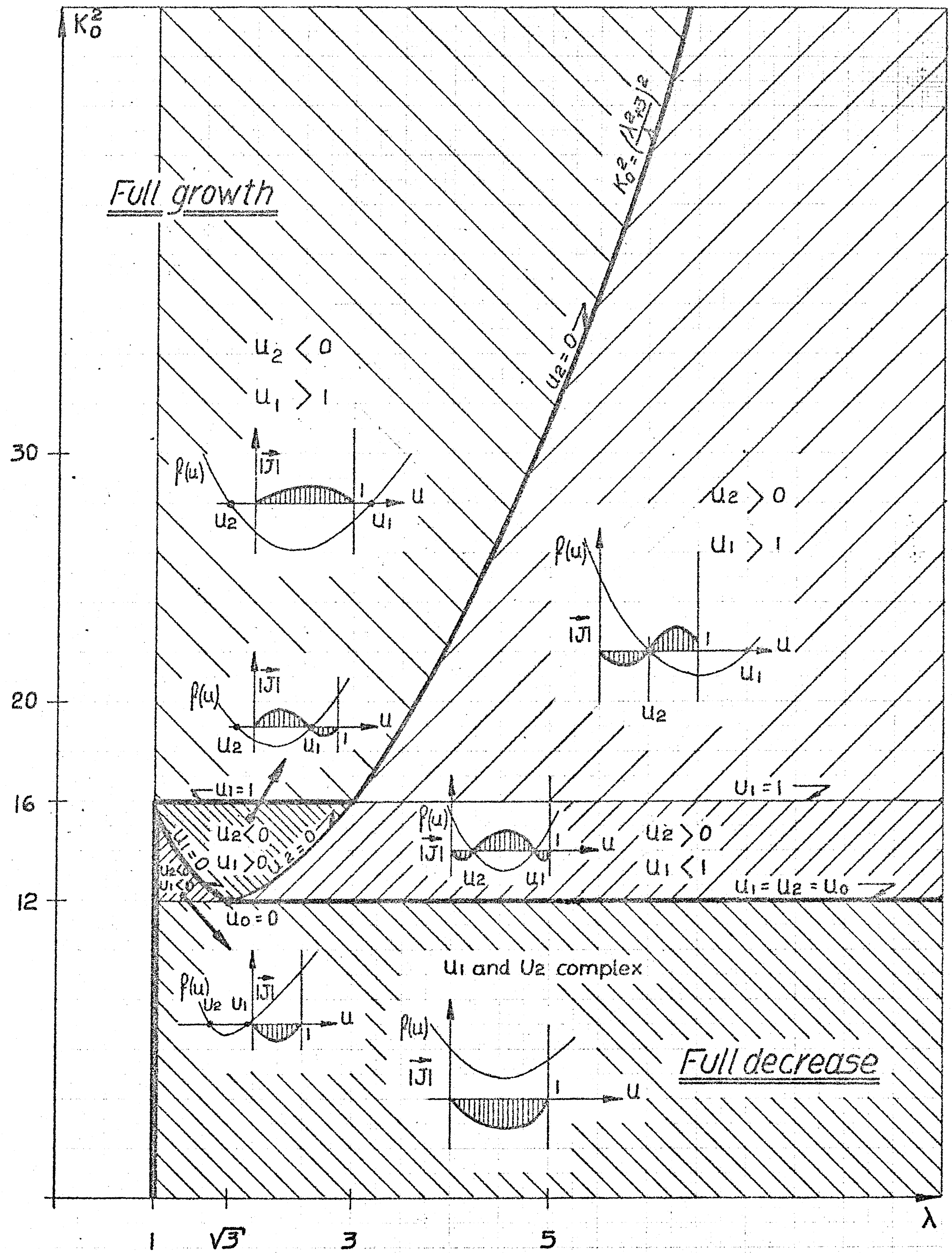

Discussion of the equation $f(u) = u^2 - u\,\dfrac{2\lambda^2 + 6 - K_0^2}{\lambda^2 - 1} + \dfrac{\lambda^4 + \lambda^2(6 - K_0^2) + 9}{(\lambda^2 - 1)^2}$

Fig. 1

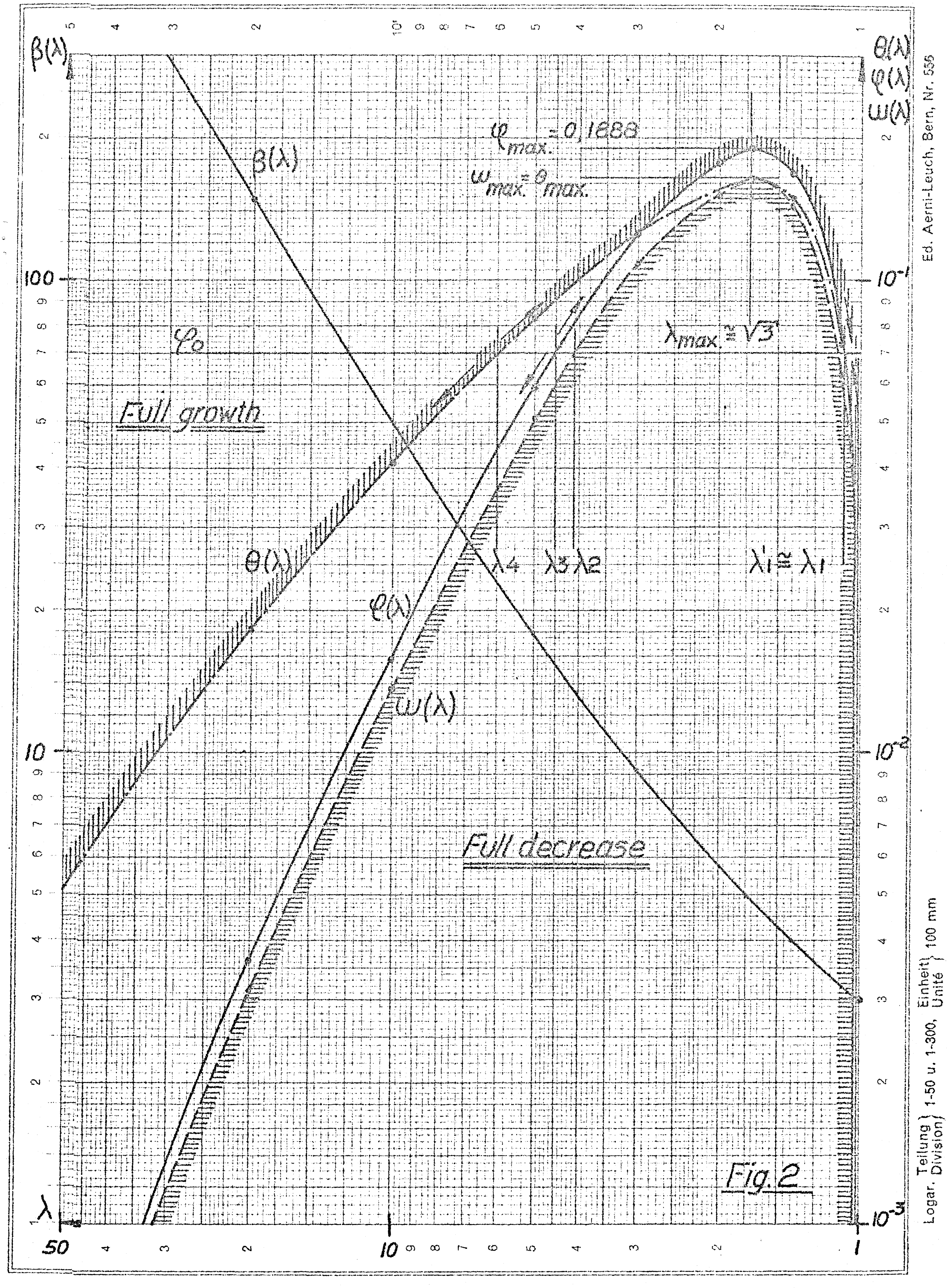
β(λ)
θ(λ)
φ(λ)
ω(λ)
φ_max = 0,1888
ω_max = θ_max
φ₀
λ_max ≅ √3
Full growth
Full decrease
λ4
λ3 λ2
λ'1 ≅ λ1
100
10
1
10⁻¹
10⁻²
10⁻³
λ
50
10
1
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Fig. 2