

## Lepton-flavor violating $B$ decays in generic $Z'$ models

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LHCb has reported deviations from the Standard Model in  $b \rightarrow s\mu^+\mu^-$  transitions for which a new neutral gauge boson is a prime candidate for an explanation. As this gauge boson has to couple in a flavor nonuniversal way to muons and electrons in order to explain  $R_K$ , it is interesting to examine the possibility that also lepton flavor is violated, especially in the light of the CMS excess in  $h \rightarrow \tau^\pm\mu^\mp$ . In this article, we investigate the perspectives to discover the lepton-flavor violating modes  $B \rightarrow K^{(*)}\tau^\pm\mu^\mp$ ,  $B_s \rightarrow \tau^\pm\mu^\mp$  and  $B \rightarrow K^{(*)}\mu^\pm e^\mp$ ,  $B_s \rightarrow \mu^\pm e^\mp$ . For this purpose we consider a simplified model in which new-physics effects originate from an additional neutral gauge boson ( $Z'$ ) with generic couplings to quarks and leptons. The constraints from  $\tau \rightarrow 3\mu$ ,  $\tau \rightarrow \mu\nu\bar{\nu}$ ,  $\mu \rightarrow e\gamma$ ,  $g_\mu - 2$ , semileptonic  $b \rightarrow s\mu^+\mu^-$  decays,  $B \rightarrow K^{(*)}\nu\bar{\nu}$  and  $B_s$ - $\bar{B}_s$  mixing are examined. From these decays, we determine upper bounds on the decay rates of lepton-flavor violating  $B$  decays.  $\text{Br}(B \rightarrow K\nu\bar{\nu})$  limits the branching ratios of lepton-flavor violating  $B$  decays to be smaller than  $8 \times 10^{-5}$  ( $2 \times 10^{-5}$ ) for vectorial (left-handed) lepton couplings. However, much stronger bounds can be obtained by a combined analysis of  $B_s$ - $\bar{B}_s$ ,  $\tau \rightarrow 3\mu$ ,  $\tau \rightarrow \mu\nu\bar{\nu}$  and other rare decays. The bounds depend on the amount of fine-tuning among the contributions to  $B_s$ - $\bar{B}_s$  mixing. Allowing for a fine-tuning at the percent level we find upper bounds of the order of  $10^{-6}$  for branching ratios into  $\tau\mu$  final states, while  $B_s \rightarrow \mu^\pm e^\mp$  is strongly suppressed and only  $B \rightarrow K^{(*)}\mu^\pm e^\mp$  can be experimentally accessible (with a branching ratio of order  $10^{-7}$ ).

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### I. INTRODUCTION

While most flavor observables agree very well with their Standard Model (SM) predictions, there are some exceptions in semileptonic  $B$  decays (see for example [1] for a recent review). LHCb [2] recently found indications for the violation of lepton-flavor universality in the ratio

$$R_K = \frac{\text{Br}[B \rightarrow K\mu^+\mu^-]}{\text{Br}[B \rightarrow Ke^+e^-]} = 0.745_{-0.074}^{+0.090} \pm 0.036, \quad (1)$$

which deviates from the theoretically clean SM prediction  $R_K^{\text{SM}} = 1.0003 \pm 0.0001$  [3] by  $2.6\sigma$ . In addition, LHCb has reported deviations from the SM predictions [4–7] in the decay  $B \rightarrow K^*\mu^+\mu^-$  (mainly in an angular observable called  $P'_5$  [8]) with a significance of about  $3\sigma$  [9,10]. Furthermore, also the measurement of  $\text{Br}[B_s \rightarrow \phi\mu^+\mu^-]$  disagrees with the SM prediction [11,12] by about  $3\sigma$  [6].

Interestingly, these discrepancies can be explained in a model-independent approach by a rather large new-physics (NP) contribution  $C_9^{\mu\mu}$  to the Wilson coefficient of the

operator  $O_9^{\mu\mu}$  [the component of the usual SM operator  $O_9$  that couples to muons, see Eq. (5)] [13–19]. It is encouraging that the value for  $C_9^{\mu\mu}$  required to explain  $R_K$  (with  $C_9^{ee} = 0$ ) is of the same order as the one needed for  $B \rightarrow K^*\mu^+\mu^-$  and  $B_s \rightarrow \phi\mu^+\mu^-$  [6,20]. Taking into account the  $3\text{fb}^{-1}$  data for  $B \rightarrow K^*\mu^+\mu^-$  recently released by the LHCb Collaboration [10], the global significance is found to be  $4.3\sigma$  for NP contributing to  $C_9^{\mu\mu}$  only, and  $3.13\sigma$  in a scenario with  $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$  [18].

Many models proposed to explain the  $b \rightarrow s\mu^+\mu^-$  data contain a heavy neutral gauge boson ( $Z'$ ) which generates a tree-level contribution to  $C_9^{\mu\mu}$  [13,21–25]. If the  $Z'$  couples differently to muons and electrons,  $R_K$  can be explained simultaneously [25–29]. Since in this case lepton-flavor universality would be violated, it has been proposed to search for lepton-flavor violating (LFV)  $B$  decay modes as well [30]. This is also motivated by the CMS excess in  $\text{Br}[h \rightarrow \mu\tau]$  [31] which can be explained simultaneously together with  $R_K$ ,  $\text{Br}[B_s \rightarrow \phi\mu^+\mu^-]$  and  $\text{Br}[B \rightarrow K^*\mu^+\mu^-]$  within a single model [26,27].

While the specific model of Refs. [26,27] predicts only small effects in LFV  $B$  decays, the situation could be different in a generic model. In this article we examine the LFV decays  $B \rightarrow K^{(*)}\tau^\pm\mu^\mp$ ,  $B_s \rightarrow \tau^\pm\mu^\mp$  (and the corresponding  $\mu^\pm e^\mp$  channels) studying a simplified model

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in which the NP effects originate from a heavy new gauge boson  $Z'$  of mass  $M_{Z'}$  with generic couplings to quarks and leptons [32]. We introduce the relevant  $Z'$  couplings to  $\bar{s}b$  and charged lepton pairs  $\ell, \ell' = \tau, \mu, e$  via

$$\mathcal{L}_{Z'} \supset \Gamma_{\ell\ell'}^L \bar{\ell} \gamma^\mu P_L \ell' + \Gamma_{sb}^L \bar{s} \gamma^\mu P_L b + L \leftrightarrow R. \quad (2)$$

As the  $Z'$  is assumed to be much heavier than the scale of electroweak symmetry breaking, its couplings must respect  $SU(2)_L$  gauge invariance. This implies that the couplings to neutrinos and to left-handed charged leptons are equal:  $\Gamma_{\ell_i \ell_j}^L = \Gamma_{\nu_i \nu_j}^L$  [33]. To study bounds on the LFV  $B$  decay modes, we perform the following steps:

- (1) Motivated by the model-independent fits to  $B \rightarrow K^* \mu^+ \mu^-$ ,  $B_s \rightarrow \phi \mu^+ \mu^-$  and  $R_K$  we consider two scenarios for the  $Z'$  couplings to leptons: scenario 1 assumes vectorial couplings, i.e.  $\Gamma_{\ell\ell'}^L = \Gamma_{\ell\ell'}^R \equiv \Gamma_{\ell\ell'}^V$ , corresponding to  $C_{10}^{\ell\ell'} = C_{10}^{\ell\ell' \prime} = 0$ . Scenario 2 considers left-handed couplings, i.e.  $\Gamma_{\ell\ell'}^R = 0$ , corresponding to  $C_9^{\ell\ell'} = -C_{10}^{\ell\ell'}$ .
- (2) We use the experimental upper bound on  $B \rightarrow K^{(*)} \nu \bar{\nu}$  decays to set upper bounds on LFV  $B$  decays, independently of the values of  $\Gamma_{sb}^{L(R)}$ .
- (3) From  $B_s - \bar{B}_s$  mixing we obtain upper limits on  $\Gamma_{sb}^L$  as a function of a fine-tuning measure (to be defined later).
- (4) In the lepton sector the  $Z'$  couplings can be constrained by  $\tau \rightarrow 3\mu$  and  $\tau \rightarrow \mu \nu \bar{\nu}$ .
- (5) Taking into account the constraints (3) and (4) we derive upper limits on the branching ratios of  $B_s \rightarrow \tau^\pm \mu^\mp$ ,  $B \rightarrow K^{(*)} \tau^\pm \mu^\mp$  which are stronger than the ones obtained in (2), but depend on the amount of fine-tuning in  $B_s - \bar{B}_s$  mixing.

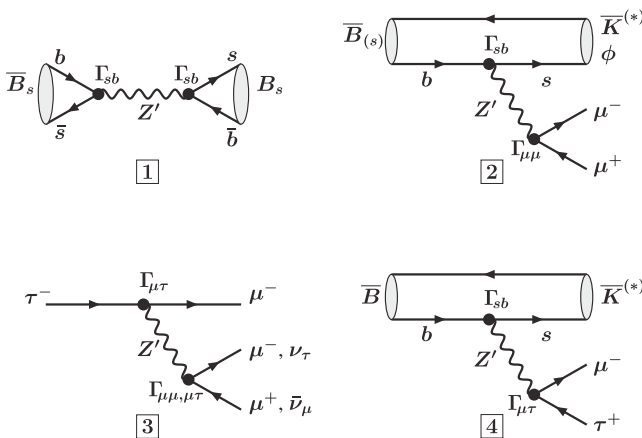


FIG. 1. Feynman diagrams illustrating the steps (1)–(4) of our analysis (see text). The diagrams display the dominant  $Z'$  contribution to  $\bar{B}_s - B_s$  mixing,  $\bar{B} \rightarrow \bar{K}^{(*)} \mu^+ \mu^-$ ,  $\bar{B}_s \rightarrow \phi \mu^+ \mu^-$ ,  $\tau \rightarrow 3\mu$ ,  $\tau \rightarrow \mu \nu \bar{\nu}$  and  $\bar{B} \rightarrow \bar{K}^{(*)} \tau^+ \mu^-$ .

In Fig. 1 we show the Feynman diagrams for the dominant  $Z'$  contribution corresponding to the steps (1)–(5) of our analysis. We apply a similar procedure to  $\mu^\pm e^\mp$  final states. In this case the best bounds on the lepton couplings are coming from  $\mu \rightarrow e \gamma$  and  $\mu \rightarrow e \nu \bar{\nu}$ .

## II. PROCESSES AND OBSERVABLES

In the Secs. II A–II E we collect the formulas for the steps (1)–(5) of our analysis outlined in the Introduction.

### A. $B_s - \bar{B}_s$ mixing

Using the notation of Refs. [34,35] for the operators describing  $B_s - \bar{B}_s$  mixing, the first diagram in Fig. 1 feeds the Wilson coefficients of

$$\begin{aligned} O_1 &= [\bar{s}_\alpha \gamma^\mu P_L b_\alpha] [\bar{s}_\beta \gamma^\mu P_L b_\beta], \\ O_5 &= [\bar{s}_\alpha P_L b_\beta] [\bar{s}_\beta P_R b_\alpha], \end{aligned} \quad (3)$$

as well as  $O'_1$  obtained from  $O_1$  by interchanging  $P_L \leftrightarrow P_R$ . The coefficients are

$$C_1^{(i)} = (\Gamma_{sb}^{L(R)})^2 / (2M_{Z'}^2), \quad C_5 = -2\Gamma_{sb}^L \Gamma_{sb}^R / (M_{Z'}^2). \quad (4)$$

For QCD renormalization group effects we use the next-to-leading order equations calculated in Refs. [34,35].

### B. $b \rightarrow s \ell^+ \ell'^-$ transitions

For  $b \rightarrow s \ell^+ \ell'^-$  transitions we need the operators

$$O_{9(10)}^{\ell\ell'} = \frac{\alpha}{4\pi} [\bar{s} \gamma^\mu P_L b] [\bar{\ell} \gamma_\mu (\gamma^5) \ell'], \quad (5)$$

and their primed counterparts found by  $P_L \leftrightarrow P_R$ .  $Z'$  contributions to other operators (such as the magnetic operator  $O_7$ ) are negligible. The diagrams of Fig. 1 give

$$C_{9,10}^{\ell\ell'} = -\frac{\pi}{\sqrt{2} M_{Z'}^2} \frac{1}{\alpha G_F V_{tb} V_{ts}^*} \Gamma_{sb}^{L(R)} (\Gamma_{\ell\ell'}^R \pm \Gamma_{\ell\ell'}^L), \quad (6)$$

which have to be multiplied by  $-4G_F V_{tb} V_{ts}^* / \sqrt{2}$  in the effective Hamiltonian.

As first noted in Refs. [13,36] a good fit to  $B \rightarrow K^* \mu^+ \mu^-$  data, leaving  $\text{Br}[B_s \rightarrow \mu^+ \mu^-]$  unchanged, is obtained with  $C_9^{\mu\mu} < 0$  and  $C_9^{\mu\mu\mu}, C_{10}^{(\prime)\mu\mu} \sim 0$ . Another interesting solution is given by  $C_9^{\mu\mu} = -C_{10}^{\mu\mu}$  [6,18].

In our analysis we use the global fit of Refs. [6,18], resulting for the two scenarios under consideration in

$$-0.53(-0.81) \geq C_9^{\mu\mu} \geq (-1.32) - 1.54, \quad (7)$$

$$-0.18(-0.35) \geq C_9^{\mu\mu} = -C_{10}^{\mu\mu} \geq (-0.71) - 0.91, \quad (8)$$

at the  $(1\sigma)$   $2\sigma$  level, respectively. The quoted ranges are in good agreement with preliminary results of Ref. [19]. Note that  $\text{Br}[B_s \rightarrow \mu^+\mu^-]$  is suppressed in scenario 2 compared to the SM. This effect is taken into account via the global fit used in our analysis.

### C. $B \rightarrow K^{(*)}\nu\bar{\nu}$

Following [37] we write the relevant effective Hamiltonian as

$$H_{\text{eff}}^{\nu\nu'} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* (C_L^{\nu\nu'} O_L^{\nu\nu'} + C_R^{\nu\nu'} O_R^{\nu\nu'}) \quad (9)$$

$$O_{L,R}^{\nu\nu'} = \frac{\alpha}{4\pi} [\bar{s}\gamma^\mu P_{L,R} b] [\bar{\nu}\gamma_\mu (1 - \gamma^5)\nu'], \quad (10)$$

$$C_{L(R)}^{\nu\nu'} = -\frac{\pi}{\sqrt{2}M_{Z'}^2} \frac{1}{\alpha G_F V_{tb} V_{ts}^*} \Gamma_{sb}^{L(R)} \Gamma_{\nu\nu'}^L. \quad (11)$$

In the approximation  $\Gamma_{sb}^R = 0$ , the branching ratio (normalized to the SM prediction) reads

$$R_{K^{(*)}}^{\nu\bar{\nu}} = \frac{1}{3} \sum_{i,j=1}^3 |C_L^{ij}|^2 / |C_L^{\text{SM}}|^2, \quad (12)$$

with  $C_L^{\text{SM}} \approx -1.47/s_W^2 \approx -6.4$ . The complete expressions can be found in Ref. [37]. The current experimental limits are  $R_K^{\nu\bar{\nu}} < 4.3$  [38] and  $R_{K^*}^{\nu\bar{\nu}} < 4.4$  [39].

Due to  $SU(2)$  invariance, we have  $C_L^{ij} = (C_9^{ij} - C_{10}^{ij})/2$ , so that  $C_L^{ij} = C_9^{ij}/2$  in scenario 1 and  $C_L^{ij} = C_9^{ij}$  in scenario 2.

### D. $\tau \rightarrow \mu\nu\bar{\nu}$ , $\mu \rightarrow e\nu\bar{\nu}$ and $\tau \rightarrow 3\mu$

The  $Z'$  boson contributes to  $\tau \rightarrow \mu\nu\bar{\nu}$  in two ways: it generates loop corrections to the  $W$  exchange diagram (as in the lepton-flavor conserving case [25]) and it mediates  $\tau \rightarrow \mu\nu\bar{\nu}$  at tree level via LFV couplings. The latter contribution decouples as  $1/m_{Z'}^2$  from the branching ratio  $\text{Br}[\tau \rightarrow \mu\nu\bar{\nu}]$  for  $\nu_i\bar{\nu}_\mu$  final states where it interferes with the SM tree-level amplitude, and as  $1/m_{Z'}^4$  for other final-state flavors  $\nu_i\bar{\nu}_j$ . We find

$$\begin{aligned} \text{Br}[\tau \rightarrow \mu\nu\bar{\nu}] &= \text{Br}[\tau \rightarrow \mu\nu\bar{\nu}]_{\text{SM}} \\ &\times \left( 1 + \frac{3\Gamma_{\mu\mu}^L \Gamma_{\tau\tau}^L \log m_W^2/m_{Z'}^2}{4\pi^2} \frac{1}{1 - m_{Z'}^2/m_W^2} \right) \\ &- \frac{8G_F m_\tau^5}{1536\sqrt{2}\pi^3 \Gamma_\tau m_{Z'}^2} \text{Re}[\Gamma_{\mu\tau}^L \Gamma_{\nu\nu}^L] + \mathcal{O}\left(\frac{1}{m_{Z'}^4}\right). \end{aligned} \quad (13)$$

The HFAG value [40] for the branching ratio reads

$$\text{BR}(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)_{\text{exp}} = (17.39 \pm 0.04)\%. \quad (14)$$

This should be compared to

$$\text{BR}(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)_{\text{SM}} = (17.29 \pm 0.03)\%, \quad (15)$$

obtained from the SM prediction in Ref. [41] and a combination of the  $\tau$  lifetime measurements in Refs. [42–47]. The difference is given by

$$\begin{aligned} \Delta_{\tau \rightarrow \mu\nu\bar{\nu}} &\equiv \text{Br}(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)_{\text{SM}} - \text{Br}(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)_{\text{exp}} \\ &= (-1.0 \pm 1.1) \times 10^{-3} \end{aligned} \quad (16)$$

at the  $2\sigma$  level, adding the error originating from the SM theory predictions linear to the experimental one. In the analogous case of  $\Gamma_{\mu e}$  we demand

$$|\Delta_{\mu \rightarrow e\nu\bar{\nu}}| \leq 4 \times 10^{-5}. \quad (17)$$

This choice restricts corrections to the Fermi constant, defined through the decay  $\mu \rightarrow e\nu\bar{\nu}$ , to the sub-per-mille level and thereby avoids conflicts with electroweak precision data.

The  $Z'$  boson further mediates the LFV three body decay  $\tau \rightarrow 3\mu$  at tree level, with the branching ratio given by (cf. e.g. [48,49])

$$\begin{aligned} \text{Br}[\tau \rightarrow 3\mu] &= \frac{m_\tau^5}{1536\pi^3 \Gamma_\tau M_{Z'}^4} [2(|\Gamma_{\mu\tau}^L \Gamma_{\mu\mu}^L|^2 \\ &+ |\Gamma_{\mu\tau}^R \Gamma_{\mu\mu}^R|^2) + |\Gamma_{\mu\tau}^L \Gamma_{\mu\mu}^R|^2 + |\Gamma_{\mu\tau}^R \Gamma_{\mu\mu}^L|^2]. \end{aligned} \quad (18)$$

Combining Belle [50] and BABAR [51] data gives  $\text{Br}[\tau \rightarrow 3\mu] \leq 1.2 \times 10^{-8}$  at 90% C.L. [40]. The corresponding decay  $\mu \rightarrow 3e$  does not affect our phenomenology, because it involves  $\Gamma_{ee}$  which we set to zero to comply with  $R_K$ .

### E. Lepton-flavor violating $B$ decays

Here we give formulas for the branching ratios of LFV  $B$  decays, taking into account the contributions from the operators  $O_9^{(\ell)\ell'\ell'}$  and  $O_{10}^{(\ell)\ell'\ell'}$  relevant for our model. For  $B_s \rightarrow \ell^+\ell'^-$  (with  $\ell \neq \ell'$ ) we use the results of Ref. [52] neglecting the mass of the lighter lepton. The branching ratios for  $B \rightarrow K^{(*)}\tau^\pm\mu^\mp$ ,  $B \rightarrow K^{(*)}\mu^\pm e^\mp$  are computed using form factors from Ref. [53] (see also Refs. [12,54]). The results read

$$\begin{aligned} \text{Br}[B_s \rightarrow \ell^+\ell'^-] &= \frac{\tau_{B_s} m_\ell^2 M_{B_s} f_{B_s}^2}{32\pi^3} \alpha^2 G_F^2 |V_{tb} V_{ts}^*|^2 \\ &\times \left( 1 - \frac{\text{Max}[m_\ell^2, m_{\ell'}^2]}{M_{B_s}^2} \right)^2 (|C_9^{\ell\ell'} - C_9^{\ell'\ell}|^2 \\ &+ |C_{10}^{\ell\ell'} - C_{10}^{\ell'\ell}|^2), \end{aligned} \quad (19)$$

$$\begin{aligned} \text{Br}[B \rightarrow K^{(*)}\ell^+\ell'^-] &= 10^{-9}(a_{K^{(*)}\ell\ell'}|C_9^{\ell\ell'} + C_9^{\ell'\ell'}|^2 + b_{K^{(*)}\ell\ell'}|C_{10}^{\ell\ell'} + C_{10}^{\ell'\ell'}|^2 \\ &+ c_{K^{(*)}\ell\ell'}|C_9^{\ell\ell'} - C_9^{\ell'\ell'}|^2 + d_{K^{(*)}\ell\ell'}|C_{10}^{\ell\ell'} - C_{10}^{\ell'\ell'}|^2), \end{aligned} \quad (20)$$

with

$\ell\ell'$	$a_{K\ell\ell'}$	$b_{K\ell\ell'}$	$c_{K\ell\ell'}$	$d_{K\ell\ell'}$	$a_{K^*\ell\ell'}$	$b_{K^*\ell\ell'}$	$c_{K^*\ell\ell'}$	$d_{K^*\ell\ell'}$
$\tau\mu$	$9.6 \pm 1.0$	$10.0 \pm 1.3$	0	0	$3.0 \pm 0.8$	$2.7 \pm 0.7$	$16.4 \pm 2.1$	$15.4 \pm 1.9$
$\mu e$	$15.4 \pm 3.1$	$15.7 \pm 3.1$	0	0	$5.6 \pm 1.9$	$5.6 \pm 1.9$	$29.1 \pm 4.9$	$29.1 \pm 4.9$

Note that the results [55] in Eqs. (19) and (20) are for  $\ell^-\ell'^+$  final states and not for the sums  $\ell^\pm\ell'^\mp = \ell^-\ell'^+ + \ell^+\ell'^-$  constrained experimentally [40]:

$$\begin{aligned} \text{Br}[B^+ \rightarrow K^+\tau^\pm\mu^\mp]_{\text{exp}} &\leq 4.8 \times 10^{-5}, \\ \text{Br}[B^+ \rightarrow K^+\mu^\pm e^\mp]_{\text{exp}} &\leq 9.1 \times 10^{-8}, \\ \text{Br}[B \rightarrow K^*\mu^\pm e^\mp]_{\text{exp}} &\leq 1.4 \times 10^{-6}, \\ \text{Br}[B_s \rightarrow \mu^\pm e^\mp]_{\text{exp}} &\leq 1.2 \times 10^{-8}. \end{aligned} \quad (21)$$

### III. PHENOMENOLOGICAL ANALYSIS

First of all, one can already derive an upper limit on LFV  $B$  decays from  $B \rightarrow K\nu\bar{\nu}$  alone, simply by employing gauge invariance [56]. As one can see from Eq. (12) the contribution for LFV couplings can only be positive. Therefore we can give a strict upper limit on  $|C_9^{\mu\tau}|$  assuming that all other contributions vanish [57]. We obtain  $|C_9^{\mu\tau}| \leq 46$  for our scenario 1 and  $|C_9^{\mu\tau}| = |C_{10}^{\mu\tau}| \leq 23$  for scenario 2. This results in upper limits on the branching ratios of  $b \rightarrow s\tau\mu$  decays:

$$\begin{aligned} \text{Br}[B \rightarrow K^*\tau\mu] &\approx \text{Br}[B_s \rightarrow \tau\mu] \approx 2\text{Br}[B \rightarrow K\tau\mu] \\ &< \begin{cases} 8 \times 10^{-5} & \text{in scenario 1,} \\ 2 \times 10^{-5} & \text{in scenario 2.} \end{cases} \end{aligned} \quad (22)$$

However, as we will show now, even stronger constraints can be obtained by employing the combined constraints from the other observables. Let us first examine the numerical impact of the leptonic constraints. As seen from Fig. 2, for our scenario 1 (vectorial couplings),  $\tau \rightarrow \mu\nu\bar{\nu}$  rules out an explanation of  $a_\mu$  via a nonvanishing  $\Gamma_{\mu\tau}^V$  (contrary to claims in Ref. [58] where  $\tau \rightarrow \mu\nu\bar{\nu}$  was not considered). The constraints from  $Z \rightarrow \mu^+\mu^-$  and  $Z \rightarrow \tau^\pm\mu^\mp$  as well as from neutrino-trident production (see Ref. [59]) are irrelevant in the displayed  $\Gamma_{\mu\mu}^V - \Gamma_{\mu\tau}^V$  region for the considered  $Z'$  masses (around 1 TeV and above). The situation is similar in scenario 2 (left-handed couplings). In this case the interference with the SM terms in  $a_\mu$  is always destructive, albeit small.

The most stringent constraints on the couplings  $\Gamma_{bs}^{L,R}$  stem from  $B_s - \bar{B}_s$  mixing. Using the 95% C.L. results on  $\Delta m_{B_s}$  by the UTfit Collaboration [60–63] one obtains

$$-0.10 < \Delta R_{B_s} \equiv \Delta m_{B_s} / \Delta m_{B_s}^{\text{SM}} - 1 < 0.23. \quad (23)$$

One can now derive limits on  $\Gamma_{sb}^L$  and  $\Gamma_{sb}^R$  via the relation

$$\Delta R_{B_s} = \frac{a_{B_s}}{M_{Z'}^2} [(\Gamma_{sb}^L)^2 + (\Gamma_{sb}^R)^2 - b_{B_s} \Gamma_{sb}^L \Gamma_{sb}^R]. \quad (24)$$

The coefficients  $a_{B_s}, b_{B_s}$  only exhibit a weak logarithmic dependence on  $M_{Z'}$  (about 3% when varying  $M_{Z'}$  from 1 to 3 TeV) and we use the values at  $M_{Z'} = 1$  TeV:

$$a_{B_s}/M_{Z'}^2 \approx 5700 \text{ TeV}^{-2}, \quad b_{B_s} \approx 8.8. \quad (25)$$

The bounds resulting from Eqs. (23) and (24) (shown by the blue contour of Fig. 3) are weakened if  $\Gamma_{sb}^L$  and  $\Gamma_{sb}^R$  have

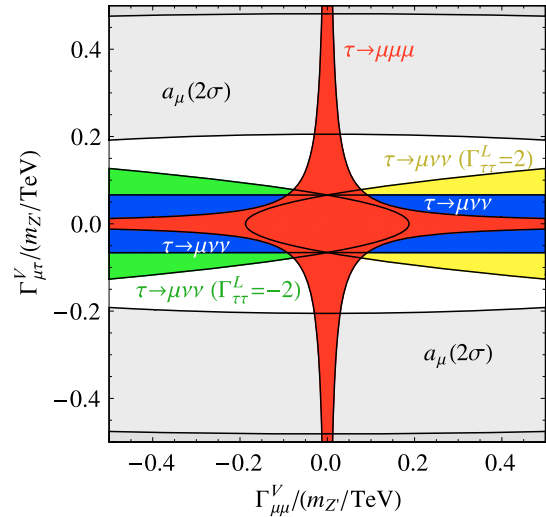


FIG. 2 (color online). Allowed  $2\sigma$  regions in the  $\Gamma_{\mu\mu}^V - \Gamma_{\mu\tau}^V$  plane from  $\tau \rightarrow \mu\nu\bar{\nu}$  for  $\Gamma_{\tau\tau}^V = 0$  (blue),  $\Gamma_{\tau\tau}^V = -2$  (yellow),  $\Gamma_{\tau\tau}^V = 2$  (green),  $\tau \rightarrow 3\mu$  (red) and  $a_\mu$  (light grey) for  $m_{Z'} = 1$  TeV. The dependence of the bounds on the  $Z'$  mass is only logarithmic. Although NP effects move  $a_\mu$  to the right direction, it cannot be explained within our model and we do not impose it as a constraint later on in our analysis.

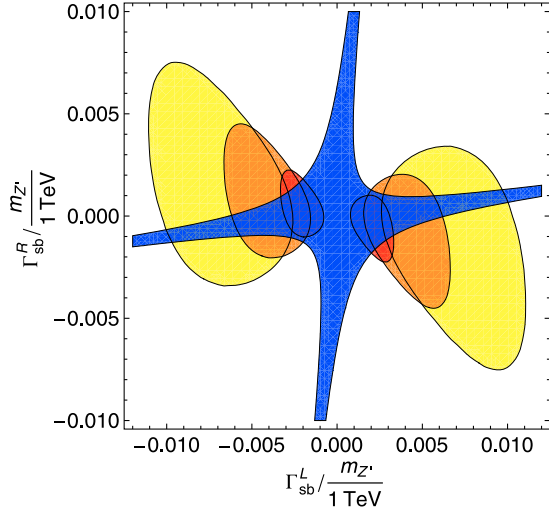


FIG. 3 (color online). Allowed regions in the  $\Gamma_{sb}^L/M_{Z'} - \Gamma_{sb}^R/M_{Z'}$  plane from  $B_s - \bar{B}_s$  mixing (blue), and from the  $C_9^{\mu\mu} - C_9^{(\prime)\mu\mu}$  fit of Ref. [6] to  $B \rightarrow K^* \mu^+ \mu^-$ ,  $B_s \rightarrow \phi \mu^+ \mu^-$  and  $R_K$ , with  $\Gamma_{\mu\mu}^V = \pm 1$  (red),  $\Gamma_{\mu\mu}^V = \pm 0.5$  (orange) and  $\Gamma_{\mu\mu}^V = \pm 0.3$  (yellow). Note that the allowed regions with positive (negative)  $\Gamma_{sb}^L$  correspond to positive (negative)  $\Gamma_{\mu\mu}^V$ . The bounds are shown for  $m_{Z'} = 1$  TeV but their dependence on the  $Z'$  mass is only logarithmic.

the same sign with  $|\Gamma_{sb}^R| \ll |\Gamma_{sb}^L|$  or  $|\Gamma_{sb}^R| \gg |\Gamma_{sb}^L|$ , as a consequence of cancellations in Eq. (24). At the  $2\sigma$  level, current  $b \rightarrow s \mu^+ \mu^-$  data requires a substantial nonzero contribution to  $C_9^{\mu\mu}$ , eliminating the option  $|\Gamma_{sb}^R| \gg |\Gamma_{sb}^L|$ . Figure 3 illustrates the combined constraints from  $b \rightarrow s \mu^+ \mu^-$  data [6,18] for different values of  $\Gamma_{\mu\mu}^V$  (scenario 1). In principle there is no upper limit on  $|\Gamma_{sb}^L|$  as long as  $b \rightarrow s \mu^+ \mu^-$  data permits small but nonvanishing contributions to the primed operators  $C_9^{\prime\mu}$  and/or  $C_{10}^{\prime\mu}$  [64]. Therefore we quantify the degree of cancellation in Eq. (24) by the following fine-tuning measure:

$$X_{B_s} = \frac{(\Gamma_{sb}^L)^2 + (\Gamma_{sb}^R)^2 + b_{B_s} \Gamma_{sb}^L \Gamma_{sb}^R}{(\Gamma_{sb}^L)^2 + (\Gamma_{sb}^R)^2 - b_{B_s} \Gamma_{sb}^L \Gamma_{sb}^R} = \frac{2a_{B_s}}{M_{Z'}^2 \Delta R_{B_s}} [(\Gamma_{sb}^L)^2 + (\Gamma_{sb}^R)^2] - 1. \quad (26)$$

Restricting  $X_{B_s}$  to an acceptable value limits the maximal size  $|\Gamma_{sb}^L|$ . As we are exclusively interested in scenarios with  $C_{9,10}^{\mu\mu} \gg C_{9,10}^{\prime\mu}$ , we neglect  $(\Gamma_{sb}^R)^2$  in Eq. (26) and express  $\Gamma_{sb}^L$  in terms of  $X_{B_s}$  and  $\Delta R_{B_s}$  as

$$|\Gamma_{sb}^L|/M_{Z'} = \sqrt{\Delta R_{B_s}(1 + X_{B_s})/(2a_{B_s})} \leq c_{B_s} \sqrt{1 + X_{B_s}}. \quad (27)$$

Note that we take all couplings  $\Gamma_{\ell\ell'}^{L,R}$  real to comply with  $CP$  data in  $B_s - \bar{B}_s$  mixing. Using the maximal  $|\Delta R_{B_s}|$  allowed by Eq. (23), we find

$$c_{B_s} = \max \left[ \sqrt{\Delta R_{B_s}/2a_{B_s}} \right] \approx 0.0045 \text{ TeV}^{-1}. \quad (28)$$

Combining the bound on  $\Gamma_{b_s}^L$  and Eqs. (13),(18) we derive upper limits for the coefficient  $C_9^{\mu\tau}$ :

$$|C_9^{\mu\tau}|^2 \leq A_{3\mu} \frac{64\pi^7 \Gamma_\tau c_{B_s}^4}{m_\tau^5 \alpha^4 G_F^4 |V_{tb} V_{ts}^*|^4} \times \max\{\text{Br}[\tau \rightarrow 3\mu]_{\text{exp}}\} \times \frac{(1 + X_{B_s})^2}{|C_9^{\mu\mu}|^2}, \quad (29)$$

$$|C_9^{\mu\tau}|^2 \leq A_{\mu\nu\bar{\nu}} \frac{96\sqrt{2}\pi^5 \Gamma_\tau c_{B_s}^2}{\alpha^2 G_F^3 m_\tau^5 |V_{tb} V_{ts}^*|^2} \times \max\{\Delta_{\tau \rightarrow \mu\nu\bar{\nu}}\} \times (1 + X_{B_s}). \quad (30)$$

For scenario 1 we obtain  $A_{3\mu}^{(1)} = 16$  and  $A_{\mu\nu\bar{\nu}}^{(1)} = 4$ , while for scenario 2 we get  $A_{3\mu}^{(2)} = 3$  and  $A_{\mu\nu\bar{\nu}}^{(2)} = 1$ .

The bounds from  $\tau \rightarrow \mu\nu\bar{\nu}$  only depend on the fine-tuning measure  $X_{B_s}$ , while those from  $\tau \rightarrow 3\mu$  also depend on the value of  $C_9^{\mu\mu}$  (and  $C_{10}^{\mu\mu}$  in scenario 2) determined from the fit to  $b \rightarrow s \mu^+ \mu^-$  data. The latter bounds disappear in the limit  $C_9^{\mu\mu} \rightarrow 0$ , as in this case the  $Z' \mu\mu$  couplings may vanish so that the  $\tau \rightarrow 3\mu$  decay does not receive contributions from  $Z'$  exchange.

From the upper bounds on  $C_{9,10}^{\tau\mu}$ , we can finally determine the maximal branching ratios for the LFV  $B$  decays with  $\tau\mu$  final states. They are shown in Fig. 4 for scenario 1 with  $X_{B_s} = 20$  and  $X_{B_s} = 100$  (in scenario 2 they are a

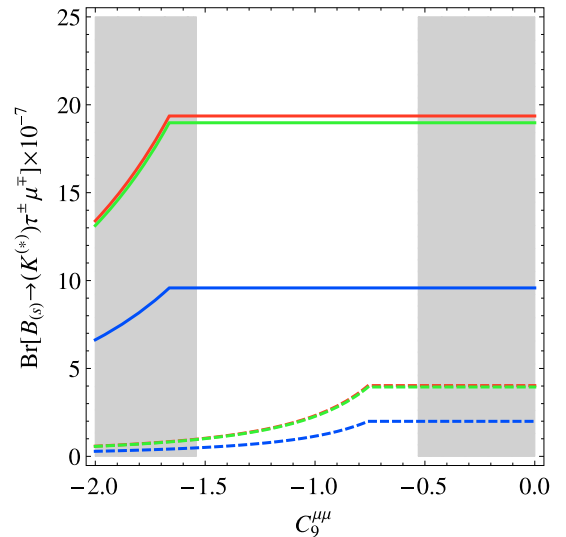


FIG. 4 (color online). Maximal value of  $\text{Br}[B \rightarrow K^* \tau^\pm \mu^\mp]$  (red),  $\text{Br}[B \rightarrow K \tau^\pm \mu^\mp]$  (blue) and  $\text{Br}[B_s \rightarrow \tau^\pm \mu^\mp]$  (green) in scenario 1 as a function of  $C_9^{\mu\mu}$  for a fine-tuning of  $X_{B_s} = 100$  (solid lines) and  $X_{B_s} = 20$  (dashed lines). The bounds are shown for  $m_{Z'} = 1$  TeV but their dependence on the  $Z'$  mass is only logarithmic.

factor of 1/2 smaller). The kink in the curves occurs at the point where the  $C_{9,10}^{\mu\mu}$ -independent constraint from  $\tau \rightarrow \mu\nu\bar{\nu}$  becomes stronger than the constraint from  $\tau \rightarrow 3\mu$ . One should note that the bounds presented in Fig. 4, which are given for  $m_{Z'} = 1$  TeV, have only a weak logarithmic dependence on the  $Z'$  mass.

Comparing these results to the experimental upper limits in Eq. (21), we see that the current experimental sensitivity is still 2 orders of magnitude weaker. However, LHCb will be able to achieve significant improvements in these channels.

In the case of  $\mu e$  final states, the stringent bound from  $\text{Br}[\mu \rightarrow e\gamma]$  renders LFV  $B$  decays unobservable in the  $C_9^{\mu\mu}$  region favored by current  $b \rightarrow s\mu^+\mu^-$  data. For  $C_9^{\mu\mu} \rightarrow 0$ ,  $\text{Br}[B \rightarrow K^{(*)}\mu^\pm e^\mp]$  can become relevant with its maximal size being constrained to  $\mathcal{O}(10^{-7})$  from  $\mu \rightarrow e\nu\bar{\nu}$ .

#### IV. CONCLUSIONS

In this article we have investigated the possible size of the branching ratios of lepton-flavor violating  $B$  decays  $B_s \rightarrow \tau^\pm\mu^\mp$ ,  $B_s \rightarrow \mu^\pm e^\mp$ ,  $B \rightarrow K^{(*)}\tau^\pm\mu^\mp$  and  $B \rightarrow K^{(*)}e^\pm\mu^\mp$  in generic  $Z'$  models. Motivated by the model-independent fit to  $b \rightarrow s$  transitions, we have considered two scenarios, one with vectorial (scenario 1) and another one with purely left-handed couplings (scenario 2) of the  $Z'$  to leptons.

From  $\text{Br}(B \rightarrow K\nu\bar{\nu})$  one obtains limits on the branching ratios of LFV  $B$  decays of  $8(2) \times 10^{-5}$  for scenario 1(2) simply by using gauge invariance. However, even stronger

bounds can be obtained by combining the leptonic constraints with a limit on the amount of fine-tuning in the  $B_s-\bar{B}_s$  system. For a fine-tuning of  $X_{B_s} \lesssim 100$ , we have found that still sizeable branching ratios of  $\mathcal{O}(10^{-6})$  are possible in both scenarios for  $\tau\mu$  final states, while for  $\mu e$  final states they can only reach  $\mathcal{O}(10^{-7})$  in a region of parameter space disfavored by the current data on  $B \rightarrow K^*\mu^+\mu^-$ ,  $B_s \rightarrow \phi\mu^+\mu^-$  and  $R_K$ .

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*Note added.*—Recently new LHCb results on  $B_s \rightarrow \phi\mu^+\mu^-$  were released, increasing the discrepancy compared to the SM to  $3.5\sigma$  [65].

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