P. N. LEBEDEV PHYSICAL INSTITUTE
USSR ACADEMY OF SCIENCES

Физический институт имени П.Н.Лебедева

FIAN

PREPRINT 114

A.A. BYKOV, A.V. LEONIDOV,
A.D. MIRONOV

QUANTUM PROPERTIES OF QCD STRING AND HEAVY QUARKONIA SPECTROSCOPY

Moscow – 1988

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High Energy Physics

Preprint N° 114

QUANTUM PROPERTIES OF QCD STRING AND HEAVY QUARKONIA SPECTROSCOPY

A.A.Bykov, A.V.Leonidov, A.D.Mironov

Moscow 1988

The situation with quantum corrections to large-R behaviour of potential induced by the string is considered theoretically as well as experimentally. The sensitivity of the heavy quarkonia spectroscopy to this corrections is discussed. We obtain the restrictions on possible value of the central charge of the Virasoro algebra, which is conjectured to correspond to QCD string.

1. It is well known, what extreme difficulties arise in QCD, when one attempts to describe long-distances physics, i.e. the confinement phase. To develop a quantitative description of confinement one faces a problem of natural theoretical objects adequate to this regime - such as quarks and gluons for the QCD description of the physics of small distances, where perturbation theory is applicable. It seems highly probable that such natural objects are strings, for which confinement is realized already at classical level. For example, stretching of a string between quark and antiquark naturally explaines the impossibility of extracting an isolated quark (antiquark) from a meson. To confirm the string picture of long-distance QCD one certainly must take into account the quantum properties of strings and, which is very important, to compare the results of corresponding theoretical computations to experimental data making as small "ad hoc" assumptions as possible. In particular, different string models, even being equivalent classically, differ at the quantum level, so one of the questions is whether it is possible to distinguish them experimentally - for example, to compare the predictions of usual Nambu string [1] with that of rigid (Polyakov, smooth) string [2].

The most interesting string model quantity to test is its central charge. All string models have connection with two-dimensional conformal field theories, which are characterized by their Virasoro algebras which, in turn, are defined by the value of the central charge (see, e.g. [3]). The aim of this paper is to show, that the value of the central charge, corresponding to the QCD-string conformal algebra can be determined by comparison with the data on heavy-quarkonium spectroscopy. Let us mention, that ref. [4] contains a comparison of QCD and rigid string calculation of

Wilson loop in the framework of QCD sum rules. Comparison of string theory and QCD is also discussed in [5].

2. One of the basic objects in the physics of open string - the energy of its ground state (rectangular world surface) can be naturally connected with the static potential of quark interactions.

For ordinary Nambu string the asymptotics of the potential at large distances R between quark and antiquark in quarkonium takes form [6]:

$$V(R)\Big|_{R\to\infty} = \mu R - \frac{\pi}{12}\cdot\frac{1}{R} + \dots \qquad (1)$$

where μ is a string tension, and the second term in the r.h.s. is a leading quantum correction to a classical linear potential, having its origin in the quantum fluctuations of the string. It was shown [7] that for a general two-dimensional conformal theory this term has a universal form $-\frac{\pi c}{24}k$, where k is a coefficient specifying the boundary conditions (string type) — 1 for open and 4 for closed, and c is a corresponding central charge.

Let us now discuss the possible value of the central charge, corresponding to a rigid string, which is a possible natural object for QCD [2]. First of all, let us mention different values for the Coulomb term for rigid strings, existing in the literature: c=2 [8], and c=6h(h/8-1)/32 [9], where h is an undetermined constant. It seems to us that ref. [10] confirmed the answer of ref. [8].

3. In order to compute the characteristics of heavy quarkonia, having the static potential with the asymptotics of the form[1)]

1) As the rigid string theory is not conformally invariant at all distances, the usual relation between 1/R term in (1) and tachyon mass is, generally speaking, broken.

$$V(R)\Big|_{R\to\infty} = \mu R - \frac{\pi \hat{C}}{12} \cdot \frac{1}{R} + \ldots \qquad (2)$$

where $2\hat{C}$ is a central charge[1], it is necessary to reconstruct a potential for all R. We'll use a QCD-potential model (see, e.g. [11]), in which one doesn't use additional parameters and the parameterless interpolation of the QCD β-function is used [11]. The static potential is reconstructed in the effective one-gluon approximation with a coupling constant, generated by RG equation. The long-distance asymptotics of the potential constructed is exactly (2), and the short-distance one is given by ordinary perturbation theory formula.

4. The simplest characteristics of heavy quarkonia, calculable in the framework of potential models, are energy spectrum and leptonic widths of s-levels. We calculated these characteristics and some of the widths of radiative transitions using the potential constructed as in [11] for various values of $\hat{C}$ (i.e. if conformal situation is realized, for various possible central charges) for the Υ -family of heavy quarkonia.[2] Our results are listed in Tables 1,2.

From tables 1,2 we see, that having $\hat{C} > 1$ we rapidly fall into sharp disagreement with experiment already for the mass of 2S level. As for the values $\hat{C} < 1$, $\hat{C}=0$ is excluded (2 MeV error in M(1P) is twice tigger than are the possible errors of nonrelativistic potential model [11] and the experimental is of order of 1 MeV [12]). The value $\hat{C}=1$ was long ago found to be in good agreement with experiment [11]. We also tried $\hat{C}=1/2$ and $\hat{C}=5/8$[3] and

[1] This $\hat{C}$ is a central charge for degree of freedom. We are taking into account the fact that we have two independent degrees of freedom in physical gauge (d=4).

[2] Corresponding results for toponium family will be published elsewhere.

[3] The minimal value, possible in ref. [9].

also found, that in our scheme we can't distinguish between $\hat{C}=1/2$, 5/8, 1. Let us mention, that $\hat{C}=1/2$ is the smallest value, permitted by unitarity requirement[1)] [13]. Finally, let us mention the possibility of adding a constant term to potential. As for Nambu-Goto string, it is absent [6], and in rigid string case corresponding term is negative [8,10]. In the latter case we would reduce the central charge, and experimental restrictions on possible value of the central charge[2)] don't change. We can conclude, therefore, that the possible values of $\hat{C}$ are

$$0 < \hat{C} \leqslant 1 . \qquad (3)$$

5. Let us discuss the possible theoretical outcome of this conclusion. The value $\hat{C}=1$ corresponds to usual bosonic string (trivial critical point). The case $\hat{C}<1$ deserves more attention. Let us mention, firstly, that if, as is suggested by Polyakov [2], the theory must be modified by additional topological Θ-term, and if this Θ-term provides additional nontrivial zero of β-function, then the corresponding central charge $\hat{C}$ must be, due to Zamolodchikov theorem [14] less than trivial, which, as we discussed, is most probably $\hat{C}=1$. Secondly, as for $\hat{C}<1$ for unitary two-dimensional conformal theories only discrete values of central charge and of anomalous dimensions are permitted [13], and as in the case of rigid string anomalous dimensions smoothly depend on rigidity coefficient [15], we can conclude, that the conformal theory, corresponding to QCD-string can be non-unitary(and, therefore, modular non-invariant [16]). For the case of rigid string it was pointed out in [15]. This important issue certainly needs a more deep understanding.

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1) See sect.5 below.

2) We thank I.V.Andreev, who drew our attention to this point.

6. We are grateful to the participants of the High energy physics sector of Theoretical Physics Department for useful discussions.

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Table 1. Energy spectrum of Υ -family (GeV)

lev. \ C	0	1/2	5/8	1	3/2	exp.
1 S	9.46	9.46	9.46	9.46	9.46	9.46
2 S	10.02	10.02	10.02	10.02	10.37	10.02
3 S	10.36	10.35	10.35	10.35	10.41	10.35
4 S	10.62	10.61	10.61	10.61	10.59	10.58
1 P	9.92	9.91	9.91	9.91	10.26	9.90
2 P	10.26	10.26	10.26	10.26	10.62	10.25

Table 2. Leptonic and photonic widths (KeV)

	lev. \ C	0	1/2	5/8	1	3/2	exp
lept.	(1 S)	1.22	-	1.12	1.13	-	1.17
	(2S)/ (1S)	0.42	-	1.41	0.42	-	0.45
	(3S)/ (1S)	0.31	-	0.30	0.30	-	0.32
	(4S)/ (1S)	0.24	-	0.23	0.25	-	0.24
phot.	(2S 1P)	-	-	4.3	4.1	-	4.2
	(3S 2P)	-	-	6.44	6.5	-	6.5
	(2P 2S)	-	-	44.1	46.4	-	-

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Our address is: USSR, 117924, Moscow B–333, Leninsky prospect, 53

Т – 04461. Подписано в печать 19. 01. 1988 г.
Заказ № 344. Тираж 100 экз. П.л. 0,5.

Отпечатано в Отделе научно-технической информации ФИАН СССР
Москва, В-333, Ленинский проспект, 53