The Leibbrandt–Mandelstam prescription for general axial gauge one-loop integrals

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It is well known that, in doing light-cone gauge calculations, it is mandatory to regularize the unphysical $(q\,n)^{-\beta}$ poles by use of the Leibbrandt–Mandelstam prescription. This technique is also applied to general axial gauges and it is proved that it is a suitable regularization procedure for these gauges as well. In order to find the relation between the Leibbrandt prescription and the more familiar principal value prescription with its simpler Lorentz structure the temporal gauge limit $\mathbf{n}\to 0$ is performed (within dimensional regularization). Although this limit is found to be singular for multiple poles, the analytically regularized one-loop integrals agree with the results obtained within the principal value technique for the temporal gauge.

I. INTRODUCTION

Since Mandelstam proved the UV finiteness of $N=4$ Super Yang–Mills theories by means of the light-cone gauge,[1] this (very singular) gauge has become increasingly popular. Like the axial gauges the light-cone gauge is characterized by an arbitrary but constant vector n_μ. For the axial gauges n_μ need only satisfy $n^2\neq 0$, whereas for the light-cone gauge $n^2=0$. As a consequence of such gauges additional factors $(qn)^{-1}$ appear in the momentum-space propagator of the gauge field, and loop integrals become more intricate than in covariant gauges. A major problem is the consistent treatment of the unphysical singularity $(qn)^{-\beta}$. However, for axial gauges the principal value (PV) prescription has proved to be a well-suited (but not unique) way to implement power counting and unitarity.[2] It amounts to setting[3]

$$\frac{1}{(qn)^\beta}=\lim_{\varepsilon\to 0^+}\frac{1}{2}\left(\frac{1}{(qn+i\varepsilon)^\beta}+\frac{1}{(qn-i\varepsilon)^\beta}\right). \tag{1.1}$$

But for the light-cone gauge the PV prescription is afflicted with serious peculiarities, namely[4,5]: (a) some of the divergences created by one-loop corrections manifest themselves as double poles $(\omega-2)^{-2}$ (space-time dimension 2ω); and (b) the PV prescription gives rise to poles situated in the second and third quadrant of the complex q^0 plane which effectively prohibits Wick rotation and hence the application of standard power counting.

Because of these defects of the PV technique it had to be abandoned for the light-cone gauge. Instead of it Mandelstam and Leibbrandt independently introduced the so-called light-cone (LC) prescription[1,5]

$$\frac{1}{(qn)^\beta}=\lim_{\varepsilon\to 0^+}\left(\frac{(qn^*)}{(qn)(qn^*)+i\varepsilon}\right)^\beta,\quad \varepsilon>0, \tag{1.2}$$

where $n_\mu=(n_0,\mathbf{n})$ and $n^*_\mu=(n_0,-\mathbf{n})$, and proved that this LC prescription exhibits all the necessary items of a viable regularization of the $(qn)^{-\beta}$ poles. The vital point with the LC prescription is that two space-time directions n_μ and n^*_μ are singled out to regularize the $(qn)^{-\beta}$ pole *à la* Eq. (1.2), yielding well-behaved integrals at the price of a richer tensor structure of the integrals (terms proportional to n^*p, n^*n, ..., occur) and the appearance of nonlocalities in the divergent parts.

On the other hand, we find it desirable to investigate whether the LC prescription is applicable to axial gauges as well and, in doing so, to put the regularization of axial gauge poles and light-cone poles on equal footing. For arbitrary axial gauges (and therefore arbitrary n_μ and n^*_μ) this should be a straightforward procedure. However, in the temporal gauge $\mathbf{n}=0$ $(n_\mu=n^*_\mu)$ we have to expect difficulties as can be understood from

$$\lim_{\varepsilon\to 0^+}\frac{qn^*}{(qn)(qn^*)+i\varepsilon}=\mathrm{PV}\left(\frac{1}{qn}\right)-i\pi\,\mathrm{sgn}(qn^*)\delta(qn)\,, \tag{1.3}$$

which is obviously meaningless for the temporal gauge. Indeed, the limit $\mathbf{n}\to 0$ of Eq. (1.2) is singular for $\beta>1$ and some additional regularization is necessary. By analytic continuation of the exponent of the axial pole we obtain well-defined momentum integrals, which are identical to the PV results, as we will prove.

In the following only integrals with a factor $(qn^*)^\beta((qn)(qn^*)+i\varepsilon)^{-\beta}$ are analyzed in Minkowski space with Feynman parameters and dimensional regularization. More complicated expressions can be reduced by repeated use of the identity

$$\frac{qn^*}{(qn)(qn^*)+i\varepsilon}\,\frac{(q+p)n^*}{(q+p)n(q+p)n^*+i\varepsilon}=\frac{1}{pn+i\varepsilon pn^*}\left(\frac{qn^*}{(qn)(qn^*)+i\varepsilon}-\frac{(q+p)n^*}{(q+p)n^*(q+p)n+i\varepsilon}\right). \tag{1.4}$$

Remarkably enough one does not pick up additional contributions from δ functions, as it is the case for the corresponding formula holding in the PV technique:

$$\frac{1}{qn(q+p)n}=\frac{1}{pn}\left(\frac{1}{qn}-\frac{1}{(q+p)n}\right)+\pi^2\delta(nq)\delta(np)\,. \tag{1.5}$$

The paper is organized as follows: in Sec. II we derive the general formulas for LC-regularized one-loop integrals. Section III contains some new results on the light-cone gauge, whereas Sec. IV is devoted to the trickier business of the temporal gauge.

II. AXIAL ONE-LOOP INTEGRALS AND THE LC PRESCRIPTION

Due to Eq. (1.4) integrals to be computed in one-loop calculations can be reduced to [$g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, space-time dimension 2ω],

$$I(\alpha,\beta):=\int d^{2\omega}q\,(q^2+2pq-L+i\varepsilon)^{-\alpha}(qn^*)^\beta \times((qn^*)(qn)+i\eta)^{-\beta}\,,\quad \alpha\geqslant 1,\ \ \beta\geqslant 1,\ \ \varepsilon>0,\ \ \eta>0. \tag{2.1}$$

For further covenience we define

$$\bar{I}(\alpha,\beta;f(q)):=\int d^{2\omega}q\,(q^2+2pq\text{-}L+i\varepsilon)^{-\alpha}\times((qn^*)(qn)+i\eta)^{-\beta}f(q)\,. \tag{2.2}$$

Hence

$$I(\alpha,\beta)=[\Gamma(\alpha-\beta)/\Gamma(\alpha)](-D)^\beta\bar{I}(\alpha-\beta,\beta;1)\,, \tag{2.3}$$

where $D\equiv\frac{1}{2}n^*(\partial/\partial p)$. Note that regarding Wick rotation the LC prescription in a sense is much more natural than the PV prescription, because the denominator is positive semidefinite. Therefore in case of absolute convergence the above integral is well defined by analytic continuation to the Euclidean region if $L+p^2\geqslant 0$. To evaluate $\bar{I}(\alpha-\beta,\beta;1)$ we employ the conventional Feynman trick and the Euclidean identity

$$\int d^{2\omega}q\,(aq^2+2b(nq)+g(nq)^2+f)^{-\alpha}=\left(\frac{\pi}{a}\right)^\omega\sqrt{\frac{a}{C}}\,\frac{\Gamma(\alpha-\omega)}{F^{\alpha-\omega}}\,, \tag{2.4}$$

where

$$C=a+gn^2,\quad F=f-b^2n^2/C. \tag{2.4'}$$

We obtain for $\bar{I}(\alpha-\beta,\beta;1)$,

$$\bar{I}(\alpha-\beta,\beta;1)=i\pi^\omega(-1)^\alpha\frac{\Gamma(\alpha-\omega)}{\Gamma(\alpha-\beta)\Gamma(\beta)}\int_0^1 dx\times(A\bar{A})^{-1/2}x^{-\beta}(1-x)^{\beta-1}\mathscr{L}^{\omega-\alpha}\,, \tag{2.5}$$

with the choice $n_\mu=(n_0,0,0,n_3)$ and the definitions

$$A=x+n_0^2(1-x)\,,\quad \bar{A}=x+n_3^2(1-x)\,,\quad \mathscr{L}=L+p^2+(1-x)\left(\frac{(p_3n_3)^2}{\bar{A}}-\frac{(p_0n_0)^2}{A}\right). \tag{2.6}$$

Now we apply the differential operator $(-D)^\beta$ to the integral (2.5), utilizing the general chain rule[6]

$$I(\alpha,\beta)\equiv\bar{I}(\alpha,\beta;(qn*)^\beta)=i\pi^\omega\sum_{j=0}^{[\beta/2]}(-1)^{\alpha+j}\times\frac{\Gamma(\alpha-\omega+\beta-j)}{\Gamma(\alpha)\Gamma(\beta)}\,\frac{\beta!}{(\beta-2j)!j!2^j}\times\int_0^1 dx\,x^{-\beta}(1-x)^{\beta-1}\times(A\bar{A})^{-1/2}\mathscr{L}^{\omega-\alpha-\beta+j}(D\mathscr{L})^{\beta-2j}(D^2\mathscr{L})^j\,, \tag{2.7}$$

where

$$D\mathscr{L}=x\left(\frac{n_0p_0}{A}+\frac{n_3p_3}{\bar{A}}\right),\quad D^2\mathscr{L}=\frac{1}{2}\frac{x^2n^2}{A\bar{A}}\,. \tag{2.7'}$$

For arbitrary n_μ Eq. (2.7) leads to complicated generalized hypergeometric functions.[6] However, the divergent parts proportional to $(\omega-2)^{-1}$ can be integrated elementarily and are polynomials in p^2, pn, and pn^*. But—like in the light-cone gauge—the complete graphs may contain nonpolynomial parts due to the decomposition Eq. (1.4). Note that naive power counting is fulfilled and that $I(\alpha,\beta)$ is a regular function of n_μ for $n_0\neq 0$ and $n_3\neq 0$. Fortunately, for the most interesting limiting cases, namely the light-cone gauge ($n_3=n_0$) and the temporal gauge ($n_3=0$), $I(\alpha,\beta)$ can be evaluated in terms of hypergeometric functions of one variable: Due to its homogeneity of degree $-\beta$ in n we can simplify the integral (2.7) by setting $n_0=1$. The results for general n are recovered by substituting $n_3\to n_3/n_0$ and multiplication with $1/n_0^\beta$.

Because the rest of the paper will be dealing with these two gauges we now provide the appropriate values of Eq. (2.6):

$$\begin{aligned}
&n_0^2=1,\quad p_t^2=p_1^2+p_2^2;\\
&A:=x+n_0^2(1-x)=1;\\
&\bar{A}:=x+\mathbf{n}^2(1-x)\to\begin{cases}1, & \text{light-cone gauge,}\\ x, & \text{temporal gauge;}\end{cases}\\
&\mathscr{L}=L-p_t^2+xp_0^2-\frac{x}{\bar{A}}p_3^2\\
&\quad\to\begin{cases}L-p_t^2+x(n^*p)(np), & \text{light-cone gauge,}\\ L-\mathbf{p}^2+xp_0^2 & \text{temporal gauge.}\end{cases}
\end{aligned} \tag{2.8}$$

III. THE LIGHT-CONE GAUGE

For the light-cone gauge the integral $I(\alpha,\beta)$ [Eq. (2.5)] can easily be calculated in terms of hypergeometric functions $F(a,b,c;z)$ (Ref. 6):

$$I(\alpha,\beta)=i(-1)^{\alpha}\pi^{\omega}\frac{\Gamma(\alpha+\beta-\omega)}{\Gamma(\beta+1)\Gamma(\alpha)}\times\left(\frac{2}{n^*n}\right)^{\beta}\frac{(n^*p)^{\beta}}{(L-p_t^2)^{\alpha+\beta-\omega}}\times F\left(1,\alpha+\beta-\omega,\beta+1;\frac{2}{n^*n}\frac{(n^*p)(np)}{p_t^2-L}\right). \tag{3.1}$$

Due to $D^2L:=\frac{1}{2}(x^2n^2/A\bar{A})=0$ on the light cone this integral is rendered more convergent than naive power counting would demand. Equation (3.1) is valid for arbitrary two-point functions in spontaneously broken gauge theories or QCD. Considering massless theories, i.e., $L+p^2=0$, yields the result

$$I(\alpha,\beta)=i(-1)^{\alpha}\pi^{\omega}\frac{\Gamma(\alpha+\beta-\omega)}{\Gamma(\alpha)\Gamma(\beta)(\omega-\alpha)}\times\frac{(n^*p)^{\beta}(n^*n/2)^{\alpha-\omega}}{[-(n^*p)(np)]^{\alpha+\beta-\omega}}. \tag{3.2}$$

This result is in agreement with special cases of this formula which have already been derived in the literature, e.g., see Ref. 5.

IV. THE TEMPORAL GAUGE

The central point of this paper is the investigation of the temporal gauge limit $n_\mu\to(1,\mathbf{0})$ within the LC prescription for the $q_0^{-\beta}$ poles. As already mentioned in the Introduction, switching over to the temporal gauge one encounters serious difficulties, due to the fact that the $q_0^{-\beta}$ pole is not completely regularized by the LC prescription. This feature is made explicit in the singular behavior of the momentum integrals at $n_3=0$, seen in the x integration at $x=0$. For the complete regularization of the $q_0^{-\beta}$ poles we will use analytic regularization.

As a first step we assume the exponents to be continuous; the asymptotic behavior of the momentum integrals for $n_3\to 0$ is then contained in the parameter integral

$$\int_0^1 dx\,x^{-\beta'}(x+n_3^2)^{-\alpha}=B(1-\beta',\alpha+\beta'-1)(n_3^2)^{1-\alpha-\beta}+\frac{1}{1-\alpha-\beta'}F(\alpha,\alpha+\beta'-1,\alpha+\beta';-n_3^2) \tag{4.1}$$

(note that the poles for $\alpha+\beta'=1$ cancel!). In order to find out for which β the integral Eq. (2.5) becomes singular we have to study $\bar{I}(\alpha,\beta';f(q))$ [Eq. (2.2)] for $f(q)=(qn^*)^{\beta}$. Evaluating this integral for $\beta=0,1$, and 2 we obtain

$$\bar{I}(\alpha,\beta';1)\sim\int_0^1 dx\,x^{-\beta'}(x+n_3^2)^{-1/2}=B\left(1-\beta',\beta'-\frac{1}{2}\right)n_3^{1-2\beta'}+\frac{2}{1-2\beta'}F\left(\frac{1}{2},\beta'-\frac{1}{2},\beta'+\frac{1}{2};-n_3^2\right), \tag{4.2}$$

$$\bar{I}(\alpha,\beta',qn^*)\sim n_3\int_0^1 dx\,x^{1-\beta'}(x+n_3^2)^{-3/2}+\int_0^1 dx\,x^{1-\beta'}(x+n_3^2)^{-1/2}$$
$$=B\left(2-\beta',\beta'-\frac{1}{2}\right)(n_3^2)^{1-\beta'}+\frac{2n_3}{1-2\beta'}F\left(\frac{3}{2},\beta'-\frac{1}{2},\beta'+\frac{1}{2};-n_3^2\right)+B\left(2-\beta',\beta'-\frac{3}{2}\right)n_3^{3-2\beta'}+\frac{2}{3-2\beta'}F\left(\frac{1}{2},\beta'-\frac{3}{2},\beta'-\frac{1}{2};-n_3^2\right), \tag{4.3}$$

$$\bar{I}(\alpha,\beta',(qn^*)^2)\sim n_3^2\int_0^1 dx\,x^{2-\beta'}(x+n_3^2)^{-5/2}+\int_0^1 dx\,x^{2-\beta'}(x+n_3^2)^{-3/2}$$
$$=B\left(3-\beta',\beta'-\frac{1}{2}\right)n_3^{3-2\beta'}+\frac{2n_3^2}{1-2\beta'}F\left(\frac{5}{2},\beta'-\frac{1}{2},\beta'+\frac{1}{2};-n_3^2\right)+B\left(3-\beta',\beta'-\frac{3}{2}\right)n_3^{3-2\beta'}+\frac{2}{3-2\beta'}F\left(\frac{3}{2},\beta'-\frac{3}{2},\beta'-\frac{1}{2};-n_3^2\right). \tag{4.4}$$

Thus $\bar{I}(\alpha,\beta';1)$, $\bar{I}(\alpha,\beta;qn^*)$, and $\bar{I}(\alpha,\beta';(qn^*)^2)$ are regular for $\beta'<\frac{1}{2}$, $\beta'\leqslant 1$, and $\beta'<\frac{3}{2}$, respectively, and

$$I(\alpha,\beta)\sim n_3^{1-\beta}, \tag{4.5}$$

so that $\lim_{n_3\to 0} I(\alpha,\beta)$ exists only for $\beta=1$.

For $\beta\geqslant 2$, $I(\alpha,\beta)$ can be defined by analytic continuation of $\bar{I}(\alpha,\beta',(qn^*)^{\beta})$ in β' at $\beta'=\beta$. As Hadamard's principal value is characterized by consistency with differentiation, which is guaranteed by analytic continuation, this amounts to taking the PV of $q_0^{\beta}(q_0^2+i\varepsilon)^{-\beta}$, which, in turn, is equivalent to the PV of $q_0^{-\beta}$ (this is easily "tested" with the basis $\{q_0^n|n\in\mathbb{N}\}$ of $L_2[-1,1]$). Thus we have proved that the LC prescription is not a complete regularization of multiple axial poles in the temporal gauge limit; further regularization by means of analytic continuation eventually is equivalent to the PV prescription.

In order to confirm this general argument we now turn to the evaluation of $I(\alpha,\beta)$ in the temporal gauge. We first define

$$c := \alpha - \omega + \beta - j, \qquad z := \frac{p_0^2}{p^2 + L}, \quad \frac{z}{1-z} = \frac{p_0^2}{L - \mathbf{p}^2} \tag{4.6}$$

which we insert into Eq. (2.7),

$$I(\alpha,\beta) = \frac{i\pi^\omega(-1)^\alpha}{\Gamma(\alpha)\Gamma(\beta)} \sum_{j=0}^{[\beta/2]} \frac{\beta!(-1)^j}{(\beta-2j)!j!4^j} \times \int_0^1 dx\, x^{-1/2-j}(1-x)^{\beta-1} \times \frac{\Gamma(c)p_0^{\beta-2j}}{(L-\mathbf{p}^2)^c(1-xz/(1-z))^c}. \tag{4.7}$$

Using the integral representation of the hypergeometric function $F(a,b,c;z)$ (Ref. 7),

$$F(a,b,c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 dt\, t^{b-1} \times (1-t)^{c-b-1}(1-tz)^{-a} \tag{4.8}$$

and formula (9.132) of Ref. 6 we obtain

$$I(\alpha,\beta) = \frac{i\pi^\omega(-1)^\alpha}{\Gamma(\alpha)} \sum_{j=0}^{[\beta/2]} \frac{\beta!}{(\beta-2j)!} \times \frac{\pi}{j!4^j} \frac{\Gamma(c)}{\Gamma(\frac{1}{2}+j)\Gamma(\beta-j+\frac{1}{2})} \times \frac{p_0^{\beta-2j}}{(L+p^2)^c} F\left(\beta,c,\beta-j+\frac{1}{2};z\right). \tag{4.9}$$

The sum is calculated in the Appendix, yielding the final result

$$I(\alpha,\beta) = \frac{i\pi^\omega(-1)^\alpha}{n_0^\beta\Gamma(\alpha)} \begin{cases} \dfrac{\Gamma(\alpha+\beta/2-\omega)\Gamma(\frac{1}{2})}{\Gamma((\beta+1)/2)} \dfrac{F(\beta/2,\alpha+\beta/2-\omega,\frac{1}{2};z)}{(L+p^2)^{\alpha+\beta/2-\omega}}, & \beta \text{ even}, \\ \dfrac{\Gamma(\alpha+(\beta+1)/2-\omega)\Gamma(\frac{1}{2})}{\Gamma(\beta/2)} 2p_0 \dfrac{F((\beta+1)/2,\alpha+(\beta+1)/2-\omega,\frac{3}{2};z)}{(L+p^2)^{\alpha+(\beta+1)/2-\omega}}, & \beta \text{ odd}, \end{cases} \tag{4.10}$$

$n_\mu = (n_0, \mathbf{0})$, which is in complete agreement with the PV result of Konetschny.[3]

V. SUMMARY

In this paper we have proved that the LC prescription is a well-defined regularization of the *axial* gauge poles as well and derived the general formula for axial one-loop integrals within the LC prescription. We discussed the limiting cases of the light-cone gauge, where we found some new formulas, and the temporal gauge. For the latter the LC prescription does not regularize the $q_0^{-\beta}$ poles sufficiently: the limit $\mathbf{n}\to 0$ is singular. However, the analytically regularized LC results turn out to be identical to the results obtained within the PV technique. Hence, in a way, we put the regularization of the axial gauges and of the light-cone gauge on the same basis.

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APPENDIX: PROOF OF EQ. (4.10)

In order to prove the relation under scrutiny [Eq. (4.10)] we proceed from the identity

$$zF(a,b+1,c+1;z) = [c(c-1)/b(c-a)][F(a,b,c;z) - F(a-1,b,c-1;z)]. \tag{A1}$$

Accordingly we define a new function

$$\bar{F}(a,b,c;z) = [\Gamma(b)z^{c-a}/\Gamma(c)\Gamma(a-c+1)] \times F(a,b,c;z). \tag{A2}$$

Then the functions $\mathscr{F}(\beta,j)$ and the coefficients $c(\beta,j)$,

$$\mathscr{F}(\beta,j) = \bar{F}(\beta,a-j,\beta-j+\tfrac{1}{2};z),$$

$$c(\beta,j) = \begin{cases} \dfrac{\Gamma(\beta+1)}{\Gamma(\beta-2j+1)\Gamma(j+1)4^j}, & 0\leqslant j\leqslant\beta/2, \\ 0, & j\in\mathbb{Z}\setminus[0,\beta/2], \end{cases} \tag{A3}$$

fulfill the recursions

$$\begin{aligned} \mathscr{F}(\beta,j) &= \mathscr{F}(\beta-1,j+1) - (\beta-j-\tfrac{3}{2})\mathscr{F}(\beta,j+1), \\ c(\beta,j) &= c(\beta-2,j-1)(\beta-j-\tfrac{1}{2}) + c(\beta-2,j). \end{aligned} \tag{A4}$$

Now we rewrite Eq. (4.9) in terms of $c(\beta,j)$ and $\mathscr{F}(\beta,j)$ and perform the sum using the recursion given above ($0\leqslant k\leqslant[\beta/2]$):

$$\sum_{j=0}^{[\beta/2]} c(\beta,j)\mathscr{F}(\beta,j) = \sum_{j=0}^{\infty} c(\beta-2k,j)\mathscr{F}(\beta-k,j+k)$$
$$= \sum_{j=0}^{\infty} c(\beta-2k-2,j-1)(\beta-j-\tfrac{1}{2}-2k)\mathscr{F}(\beta-k,j+k) + c(\beta-2k-2,j)$$
$$\times[\mathscr{F}(\beta-k-1,j+k+1)-(\beta-2k-j-\tfrac{3}{2})\mathscr{F}(\beta-k,j+k+1)]$$
$$= \sum_{j=0}^{\infty} c(\beta-2k-2,j)\mathscr{F}(\beta-k-1,j+k+1)$$
$$= c\left(\beta-2\left[\frac{\beta}{2}\right],0\right)\mathscr{F}\left(\beta-\left[\frac{\beta}{2}\right],\left[\frac{\beta}{2}\right]\right) = \mathscr{F}\left(\beta-\left[\frac{\beta}{2}\right],\left[\frac{\beta}{2}\right]\right). \quad \text{(A5)}$$

Inserting $\mathscr{F}(\beta,j)$ and $c(\beta,j)$ with $a=\alpha+\beta-\omega$ then yields Eq. (4.10)

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