

Observational Aspects of Symmetries of the Neutral B Meson System

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Abstract

We revisit various results, which have been obtained by the BABAR and Belle Collaborations over the last twelve years, concerning symmetry properties of the Hamiltonian, which governs the time evolution and the decay of neutral B mesons. We find that those measurements, which established CP violation in B meson decay, 12 years ago, had as well established T (time-reversal) symmetry violation. They also confirmed CPT symmetry in the decay ($T_{CPT} = 0$) and symmetry with respect to time-reversal ($\epsilon = 0$) and to CPT ($\delta = 0$) in the $B^0\bar{B}^0$ oscillation.

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1 Introduction

A system of neutral mesons such as B^0, \bar{B}^0 or K^0, \bar{K}^0 is a privileged laboratory for the study of weak-interaction's symmetries. Even though the phenomenological framework is well understood since long time [1–4], recent discussions in the physics community [5] show that it may be useful to revisit a few points, in order to fully (and correctly) exploit the experimental results. This process is then at the origin of the present note.

We focus on the $B^0 \bar{B}^0$ system, and refer to experimental results [6–9] that have been achieved by measurements of the decay products of $B^0 \bar{B}^0$ pairs created in the entangled antisymmetric state

$$|\Psi\rangle = (|B^0\rangle |\bar{B}^0\rangle - |\bar{B}^0\rangle |B^0\rangle) / \sqrt{2} \quad (1)$$

where the first B in this notation moves in direction \vec{p} and the second in direction $-\vec{p}$.

Within the Weisskopf-Wigner approximation [1] the time evolution of a neutral B -meson, and its decay into a state f is described by the amplitude A_{Bf} ,

$$A_{Bf} = \langle f | T e^{-i\Lambda t} | B \rangle \quad (2)$$

where T and Λ are represented by constant, complex 2×2 matrices $T = (T^{ij}) = \langle f^i | T | B^j \rangle$ and $\Lambda = (\Lambda^{ij}) = \langle B^i | \Lambda | B^j \rangle$, $i, j = 1(2)$. We consider experiments with final states $f^i = J/\Psi K^i$ or $f^i = \mu^i \nu_\mu (\bar{\nu}_\mu) X$. Here $B^{1(2)}, K^{1(2)}$ and $\mu^{1(2)}$ stand for the flavour eigenstates $B^0 (\bar{B}^0), K^0 (\bar{K}^0)$ and $\mu^+ (\mu^-)$ or $e^+ (e^-)$, respectively.

We recall that a symmetry is a property of the hermitian Hamiltonian ($H = H_0 + H_{weak}$) of the Schrödinger equation which is defined in a space sufficiently complete to include all the particle states under consideration, also the decay products [1]. Thus the aim of the experiments is to establish properties of the weak interaction Hamiltonian H_{weak} by measuring observable combinations of the elements of Λ and of T , which represent these properties.

2 Observables of Symmetries

Together with a parametrization of the matrices Λ and T , the equations (1) and (2) are a sufficient basis for the description of the symmetry properties of the experimental results [6–9]. Symmetry properties of the Hamiltonian often manifest themselves in an especially simple and direct way in relations between measured quantities. Here, *Table 1* gives a summary, with definitions and derivations as found in [1–4], and the phase conventions of [2]. Our approach is analogous to [10].

Table 1: A symmetry of H_{weak} implies vanishing values among the observables $\Lambda_T, \Lambda_{CPT}, T_T, T_{CPT}$. Channels are assumed to have one single amplitude.

| Symmetry of H_{weak} | requires for the matrix Λ | requires for the matrix T |
|------------------------|--|--|
| T | $\Lambda_T \equiv \Lambda^{21} ^2 - \Lambda^{12} ^2 = 0$ | $T_T \equiv \text{Im}(T^{11*} T^{22}) = 0$ |
| CPT | $\Lambda_{CPT} \equiv \Lambda^{22} - \Lambda^{11} = 0$ | $T_{CPT} \equiv T^{11} ^2 - T^{22} ^2 = 0$ |
| CP | $\Lambda_T = 0$ and $\Lambda_{CPT} = 0$ | $T_T = 0$ and $T_{CPT} = 0$ |

Let us pose

$$\Lambda^{11} = m - i\gamma/2 - \delta \Delta m, \quad \Lambda^{22} = m - i\gamma/2 + \delta \Delta m, \quad (3)$$

$$\Lambda^{12} = (1 - 2\epsilon) \Delta m/2, \quad \Lambda^{21} = (1 + 2\epsilon) \Delta m/2 \quad (4)$$

with real $m, \gamma, \Delta m, \epsilon$, and complex δ . For the observables of the symmetry violations in the matrix Λ , i.e. in the $B^0 \bar{B}^0$ oscillation, we deduce from eqs. (3), (4), and *Table 1*

$$\Lambda_T = 2\epsilon (\Delta m)^2 + \mathcal{O}(\epsilon^2), \quad (5)$$

$$\Lambda_{CPT} = 2\delta \Delta m. \quad (6)$$

We note, that with eqs. (3), (4) and (5) the difference of the widths of the eigenstates of Λ becomes $\Delta\Gamma = 2\Delta m \cdot \text{Im}(\sqrt{1 - 4\epsilon^2 + 4\delta^2})$. We recognize that, if $\Delta\Gamma = 0$, our matrix Λ still allows for a finite ϵ ($|\epsilon| < 1/2$), in accordance with [11]. This is in contrast to widely repeated affirmations [12], that $\Delta\Gamma = 0$ would imply time-reversal symmetry of Λ , i.e. $\epsilon = \Lambda_T = 0$.

In terms of $\Lambda = M - \frac{i}{2}\Gamma$ ($M = M^\dagger, \Gamma = \Gamma^\dagger$), $\Lambda^{12} = |M^{12}| e^{i\phi_M} - \frac{i}{2} | \Gamma^{12} | e^{i\phi_\Gamma}$, the relation to eqs. (3) to (5) is given by $\Delta m = 2 |M^{12}|$, $\epsilon = -\frac{1}{4} | \Gamma^{12} | / |M^{12}| \times \sin(\phi_\Gamma)$, $\phi_M = 0$ and $\Delta\Gamma \approx -2 | \Gamma^{12} | \cos(\phi_\Gamma)$. We admit $| \Gamma^{12} | \ll |M^{12}|$.

In order to calculate the amplitude A_{Bf} in eq.(2), we need to evaluate the exponential in terms of Λ . We do this by summing up the power series (as explained in [10]). Let $U = (U^{ij}) = e^{-i\Lambda t}$ and find

$$U^{11} = U_0(\cos(\omega t) + i 2\delta \sin(\omega t)), \quad U^{22} = U_0(\cos(\omega t) - i 2\delta \sin(\omega t)), \quad (7)$$

$$U^{12} = U_0(-i(1-2\epsilon)\sin(\omega t)), \quad U^{21} = U_0(-i(1+2\epsilon)\sin(\omega t)), \quad (8)$$

$$|U_0|^2 = e^{-\gamma t}, \quad \omega = \Delta m/2 + \mathcal{O}(|\delta|^2, |\epsilon|^2). \quad (9)$$

We assume

$$T^{12} = T^{21} = 0, \quad (10)$$

with complex T^{11}, T^{22} , corresponding to the " $\Delta b = \Delta S$ rule".

From Table 1 and with the normalization $|T^{11}|^2 + |T^{22}|^2 = 2$ we deduce the useful identity among the (diagonal) elements of T ,

$$T_T^2 + T_{CPT}^2/4 + (\text{Re}(T^{11*}T^{22}))^2 \equiv (|T^{11}|^2 + |T^{22}|^2)^2/4 = 1. \quad (11)$$

Results based on eqs. (1) to (11) will turn out to be sensitive to all the four symmetry parameters in Table 1.

3 Experiments

3.1 General description

Call $\mathcal{A}_{f_1, f_2}(t)$ the amplitude for the decay of an entangled, antisymmetric $B^0\bar{B}^0$ pair into a final state with the two observed particles f_1 (at time $t = 0$) and f_2 (at time $t > 0$). Intermediate, unobserved states are not considered, although these might be of interpretational interest.

With specific choices of the two final states f_1, f_2 , we can represent the complete set of results of the CP , T and CPT symmetry violation studies listed in Table 2 and performed by [6,7] through [9], by making use of eq. (12) below [2, 13], which is a combination of (1) and (2),

$$\begin{aligned} \mathcal{A}_{f_1, f_2}(t) &= \langle f_1, f_2 | \Psi \rangle \\ &= (\langle f_1 | T | B^0 \rangle \langle f_2 | T U | \bar{B}^0 \rangle - \langle f_1 | T | \bar{B}^0 \rangle \langle f_2 | T U | B^0 \rangle) / \sqrt{2}. \end{aligned} \quad (12)$$

The variety of expected frequency distributions $|\mathcal{A}_{f_1, f_2}(t)|^2$ is displayed in Table 2. We find that the parameters of the data analysis are the T and CPT violation parameters of the T matrix, T_T and T_{CPT} , concerning the decay, and those, $p_i, q_i, i=1, 2, 5, 6$, concerning mainly the $B^0\bar{B}^0$ oscillation matrix Λ . In the limit of CP symmetry of Λ the p_i, q_i all vanish. Then, T_T and T_{CPT} , are exactly associated each with its own proper time dependence: T_T with $\pm \sin(\Delta m t)$, T_{CPT} with $\pm \cos(\Delta m t)$. Table 2 also allows to read off the relations of the measured distributions to the symmetry violating parameters of Λ and T , as demonstrated below, and also to construct combinations of data which are signatures for specific violations.

3.2 The earlier results

The experiments [6–8] have measured in 2001/2 all the data sets listed in Table 2, and thereby discovered CP violation in the matrix T . We show now that the data furthermore establish time-reversal symmetry violation in H_{weak} , and are compatible as well with CPT symmetry of the T matrix as with $\epsilon = 0, \delta = 0$, i.e. CP symmetry of Λ .

To this purpose we consult Table 2 and calculate

$$\{1\} - \{2\} = (p_1 - p_2) + (T_{CPT} - (p_1 - p_2)) \cos(\Delta m t) + (2T_T + (q_1 - q_2)) \sin(\Delta m t).$$

Similarly, we calculate $\{5\} - \{6\}$ and summarize the results as follows.

$$\begin{aligned} CP_{S(L)} &\equiv |\mathcal{A}_{\mu^-, J/\psi K_{S(L)}^0}(t)|^2 - |\mathcal{A}_{\mu^+, J/\psi K_{S(L)}^0}(t)|^2 \\ &\propto 4\epsilon \mp 4\text{Re}(\delta) \cdot \text{Re}(T^{11*}T^{22}) \\ &+ \{T_{CPT} - 4\epsilon \pm 4\text{Re}(\delta) \cdot \text{Re}(T^{11*}T^{22})\} \cos(\Delta m t) \\ &+ \{\pm 2 T_T - 4\text{Im}(\delta)\} \sin(\Delta m t). \end{aligned} \quad (13)$$

The experimental results for CP_S and CP_L show no time independent terms, $4\epsilon \mp 4\text{Re}(\delta) \cdot \text{Re}(T^{11*}T^{22}) \approx 0$, and no $\cos(\Delta m t)$ signals, $\{T_{CPT} - 4\epsilon \pm 4\text{Re}(\delta) \cdot \text{Re}(T^{11*}T^{22})\} \approx 0$. From this we conclude $\epsilon \approx 0$, $4\text{Re}(\delta) \cdot \text{Re}(T^{11*}T^{22}) \approx 0$, and $T_{CPT} \approx 0$. The $\sin(\Delta m t)$ amplitudes are equal but with opposite signs, and, in absolute value, < 2 , implying $\text{Im}(\delta) \approx 0$ and $|T_T|^2 < 1$. From (11) now follows $\text{Re}(T^{11*}T^{22}) \neq 0$ and thus $\text{Re}(\delta) \approx 0$. The p_i and q_i defined in Table 2 are thus all compatible with zero.

Table 2: *The measurements, classified according to eq. (12).* General expressions for the expected frequency distributions in terms of $\mathbb{T}_{CPT}, \mathbb{T}_T, \epsilon, \delta$. In the limit $\epsilon = \delta = 0$, they are all of the form $(1 \pm \frac{1}{2}\mathbb{T}_{CPT} \cos(\Delta m t) \pm \mathbb{T}_T \sin(\Delta m t)) e^{-\gamma t}$. μ^- is a shorthand for $\mu^- \bar{\nu}_\mu X$ or $e^- \bar{\nu}_e X$, μ^+ for $\mu^+ \nu_\mu X$, etc. By the "Δb = ΔQ rule", a $B^0(\bar{B}^0)$ decays semileptonically always into $\mu^+ + \dots$ ($\mu^- + \dots$). $|K_{S(L)}\rangle = (|K^0\rangle \pm |\bar{K}^0\rangle)/\sqrt{2}$ has been used. All 10 measurements have been performed.

| Name of measurement | 1^{st} decay | 2^{nd} decay | $ \mathcal{A}_{f_1, f_2}(t) ^2 \propto a + b \cos(\Delta m t) + c \sin(\Delta m t)$ | | | |
|-------------------------------|----------------|----------------|---|------------------------------|--------------------------------------|-----------------------|
| | f_1 | f_2 | a | b | c | |
| $B^0 \rightarrow K_S^0$ | {1} | μ^- | $J/\Psi K_S^0$ | $1 + p_1$ | $+\frac{1}{2}\mathbb{T}_{CPT} - p_1$ | $+\mathbb{T}_T + q_1$ |
| $\bar{B}^0 \rightarrow K_S^0$ | {2} | μ^+ | $J/\Psi K_S^0$ | $1 + p_2$ | $-\frac{1}{2}\mathbb{T}_{CPT} - p_2$ | $-\mathbb{T}_T + q_2$ |
| $K_L^0 \rightarrow \bar{B}^0$ | {3} | $J/\Psi K_S^0$ | μ^- | $1 + p_1$ | $+\frac{1}{2}\mathbb{T}_{CPT} - p_1$ | $-\mathbb{T}_T - q_1$ |
| $K_L^0 \rightarrow B^0$ | {4} | $J/\Psi K_S^0$ | μ^+ | $1 + p_2$ | $-\frac{1}{2}\mathbb{T}_{CPT} - p_2$ | $+\mathbb{T}_T - q_2$ |
| $B^0 \rightarrow K_L^0$ | {5} | μ^- | $J/\Psi K_L^0$ | $1 + p_5$ | $+\frac{1}{2}\mathbb{T}_{CPT} - p_5$ | $-\mathbb{T}_T + q_5$ |
| $\bar{B}^0 \rightarrow K_L^0$ | {6} | μ^+ | $J/\Psi K_L^0$ | $1 + p_6$ | $-\frac{1}{2}\mathbb{T}_{CPT} - p_6$ | $+\mathbb{T}_T + q_6$ |
| $K_S^0 \rightarrow \bar{B}^0$ | {7} | $J/\Psi K_L^0$ | μ^- | $1 + p_5$ | $+\frac{1}{2}\mathbb{T}_{CPT} - p_5$ | $+\mathbb{T}_T - q_5$ |
| $K_S^0 \rightarrow B^0$ | {8} | $J/\Psi K_L^0$ | μ^+ | $1 + p_6$ | $-\frac{1}{2}\mathbb{T}_{CPT} - p_6$ | $-\mathbb{T}_T - q_6$ |
| $\bar{B}^0 \rightarrow B^0$ | {9} | μ^+ | μ^+ | $\frac{1}{2}(1 - 4\epsilon)$ | $-\frac{1}{2}(1 - 4\epsilon)$ | 0 |
| $B^0 \rightarrow \bar{B}^0$ | {10} | μ^- | μ^- | $\frac{1}{2}(1 + 4\epsilon)$ | $-\frac{1}{2}(1 + 4\epsilon)$ | 0 |

The terms with ϵ and δ (upper signs for p_1, p_5, q_1, q_5).

$$\begin{aligned}
p_1(p_2) &= \epsilon (\pm 2 - \mathbb{T}_{CPT}) \mp 2\text{Re}(\delta) \cdot \text{Re}(\mathbb{T}^{11*}\mathbb{T}^{22}) - 2\text{Im}(\delta)\mathbb{T}_T \\
p_5(p_6) &= \epsilon (\pm 2 - \mathbb{T}_{CPT}) \pm 2\text{Re}(\delta) \cdot \text{Re}(\mathbb{T}^{11*}\mathbb{T}^{22}) + 2\text{Im}(\delta)\mathbb{T}_T \\
q_1(q_2) &= \epsilon \cdot 2\mathbb{T}_T - \text{Im}(\delta)(\pm 2 + \mathbb{T}_{CPT}) \\
q_5(q_6) &= -\epsilon \cdot 2\mathbb{T}_T - \text{Im}(\delta)(\pm 2 + \mathbb{T}_{CPT})
\end{aligned}$$

Identity:

$$q_1 + q_6 - (q_2 + q_5) = 0$$

The experiment [8] has set a stringent limit on T -symmetry violation in the Λ matrix of the $B^0\bar{B}^0$ system with a direct measurement of ϵ . See *Tables 2 and 3*. The method is analogous to the one of the CPLEAR, experiment [14, 15] for the $K^0\bar{K}^0$ system, where also a signature for T -violation ("Kabir asymmetry") has been directly measured. The experiments make use of the general identity, valid in two dimensions (see [10]), $\Lambda^{21}/\Lambda^{12} \equiv (e^{-i\Lambda t})^{21}/(e^{-i\Lambda t})^{12} = \mathbb{U}^{21}/\mathbb{U}^{12}$ from which

$$\epsilon \approx \frac{1}{4} \frac{|\Lambda^{21}|^2 - |\Lambda^{12}|^2}{|\Lambda^{21}|^2 + |\Lambda^{12}|^2} \equiv \frac{1}{4} \frac{|\mathbb{U}^{21}|^2 - |\mathbb{U}^{12}|^2}{|\mathbb{U}^{21}|^2 + |\mathbb{U}^{12}|^2} = \frac{1}{4} \frac{|\mathcal{A}_{\mu^-\mu^-}|^2 - |\mathcal{A}_{\mu^+\mu^+}|^2}{|\mathcal{A}_{\mu^-\mu^-}|^2 + |\mathcal{A}_{\mu^+\mu^+}|^2}, \quad (14)$$

the connection from the data to the T -symmetry violation signal ϵ , follows - without any assumptions on CPT symmetry or on the value of $\Delta\Gamma$ of the Λ matrix.

In summary, the discovered CP violation in the $B^0\bar{B}^0$ system is T -symmetry violation in the decay-amplitude matrix \mathbb{T} , $\mathbb{T}_T \neq 0$ with $\mathbb{T}_{CPT} \approx 0$. In the $K^0\bar{K}^0$ system, however, the CP -violation is T -symmetry violation in oscillations, $\Lambda_T \neq 0$ with $\Lambda_{CPT} \approx 0$.

3.3 Recent results

The analysis by [9] compares rates, whose asymmetries are displayed in *their* Fig. 2 and whose differences are listed in *Table 3*, below. Our results contradict their affirmation: "Any difference in these two rates is evidence for T -symmetry violation" [9], since a T -symmetric, CPT -violating Hamiltonian $H_{weak}(\mathbb{T}_T = 0, \mathbb{T}_{CPT} \neq 0)$ would just also create such rate differences.

Nevertheless, the measured frequency distributions show a dominant $\sin(\Delta m t)$ dependence, meaning that $\mathbb{T}_{CPT} \approx 0$, and with the previous knowledge about the vanishing of the q_i , that $\mathbb{T}_T \neq 0$, i. e. T symmetry violation is confirmed. (More combinations are discussed in [5]). In the lower part of *Table 3*, we indicate rate combinations which are signatures of T - or CPT -symmetry violations.

Table 3: A selection of expectations for the experiment of Ref. [9]. Due to the presence of T_{CPT} and the q_i , our results contradict the attempt [16, 17] to define the differences $\{2a\}$ to $\{2d\}$, each as a signature for T violation. In the lower part, signatures for T - and CPT - symmetry violations are indicated.

| Display in [9] | Rates compared | Expected \propto $\times \cos(\Delta m t)$ | $\times \sin(\Delta m t)$ |
|------------------|-------------------------------|---|---|
| <i>Figure</i> 2a | $\{2\} - \{7\} \equiv \{2a\}$ | $-\Gamma_{CPT}$ | $-2 \Gamma_T + q_2 + q_5$ |
| 2b | $\{4\} - \{5\} \equiv \{2b\}$ | $-\Gamma_{CPT}$ | $+2 \Gamma_T - q_2 - q_5$ |
| 2c | $\{6\} - \{3\} \equiv \{2c\}$ | $-\Gamma_{CPT}$ | $+2 \Gamma_T + q_1 + q_6$ |
| 2d | $\{8\} - \{1\} \equiv \{2d\}$ | $-\Gamma_{CPT}$ | $-2 \Gamma_T - q_1 - q_6$ |
| Signatures are | for Γ_T | $-8 \Gamma_T \sin(\Delta m t)$ | $\propto \{2a\} - \{2b\} - \{2c\} + \{2d\}$ |
| | Γ_{CPT} | $-4 \Gamma_{CPT} \cos(\Delta m t)$ | $\propto \{2a\} + \{2b\} + \{2c\} + \{2d\}$ |
| | Λ_T | 4ϵ | $\approx (\{10\} - \{9\}) / (\{10\} + \{9\})$ |

4 Conclusion

The experiments [6] and [7] have discovered CP violation in the $B^0 \bar{B}^0$ system. Our analysis shows that this CP violation is dominantly T violation, with the same statistical significance. Furthermore, their data sets contain the information which allows for the estimation of all symmetry-violating parameters indicated in *Table 1*. CP symmetry of the matrix Λ , which governs the $B^0 \bar{B}^0$ oscillation, is confirmed.

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