G. DÔME SL / RFS

SL/AP/EK/ek/bt930622.tex 24 June 1993

Summary Notes of the Tenth Meeting of the

Bunch Train Study Group

held on Tuesday 22nd June 1993

Present: R. Bailey, G. Dôme, G. Geschonke, K. Hanke, G. von Holtey, E. Keil, K.H. Kissler, E. Peschardt, E. Rossa, G. Schröder

1 Comments on the Minutes

E. Peschardt made the comment that a bunch spacing of 32 RF wavelengths was convenient for the preparation of the MD in week 24. In the long run, other bunch spacings are quite possible. G. von Holtey objected to the views experessed by E. Keil on background. Below follows a text concerning background, agreed upon between G. von Holtey and E. Keil.

- Problems:
  - Synchrotron radiation power and photon flux enhanced by horizontal offsets in QS0 and QS1 by about a factor of 3 each
  - Larger opening angle of synchrotron radiation fan strongly enhances back scattering from region at about 55 m from IP
  - Present collimators at large horizontal separation between beams
  - Measured background higher than simulated background → LEP experiments tempted to multiply simulated background with crossing angle by the same factor
  - Discrepancy attributed to incomplete simulation of detection efficiencies for low-energy photons in the experiment and/or additional photon sources not in the synchrotron radiation model unknown particle density in tails of distribution, assumed to be Gaussian
- Cures:
  - Stop back scattered photons close to IP with local masks which are being studied for LEP 2 and exist in some other machines
  - Background with crossing angle more sensitive to core of beam and less uncertain

2 Background vs. Horizontal Slope of Incoming Beam

G. von Holtey[1] presented the results of an experiment in which the background rates were measured as a function of the horizontal slope of the incoming beam, excited by asymmetric

[1]See attached copies of transparencies.

horizontal bumps in all even pits. The background rates increased by about a large factor, but by less than expected from earlier simulations. Some experiments had to switch off parts of their detector because the background rate was too high.

3 Accumulation of Bunch Trains

E. Keil[2] showed transparencies summarizing the accumulation of four positron bunch trains with three bunches each.

- Accumulating bunch trains was surprisingly simple and efficient.
- A maximum current of more than 0.3 mA per bunch, and more than 3.6 mA per beam was reached.
- On the streak camera the second and third bunch are vertically blown up and show head-tail motion. E. Rossa[3] showed several streak camera pictures.
- On a wide-band position monitor in LSS7, one sees horizontal and vertical oscillations on the second and third bunch, and a lower lifetime of the third bunch.
- Logitudinally, the first and second bunch are locked to the same Q_s. The third bunch has a different Q_s and shows bursts of longitudinal oscillations.
- The bunch trains were accelerated to 45.6 GeV and squeezed.

4 Higher-Order Modes and Bunch Trains

G. Geschonke[4] showed the results of a comparison between the higher-mode losses with four positron bunches and 0.5 mA in each bunch, and four bunch trains with three bunches each and 0.3 mA per bunch.

G. Dôme[5] showed the results of calculations concerning the response of a single higher-order mode to the excitation by two bunch trains, one of electrons and one of positrons. With n bunches in a train, the power loss is modulated as a function of the bunch spacing τ, compared to the pessimistic assumption that the higher-order mode fields add in phase:

$$P \propto \left(\frac{\sin n\theta}{n \sin \theta} \right)^2 \tag{1}$$

Here $\theta = \omega\tau/2$, and ω is the circular frequency of the higher-order mode.

E. Keil[6] showed transparencies with graph of the expression for P, using the bunch spacing in units of RF wanvelengths as abscissa, and assuming that the frequency of the higher mode is 639 MHz. The transparencies demonstrate that it ought to be possible to

[2]See attached copies of transparencies.
[3]See attached copies of transparencies.
[4]See attached copies of transparencies.
[5]See attached copies of transparencies.
[6]See attached copies of transparencies.

minimize the higher-order mode power, at least for the dominant mode at 639 MHz, by choosing the bunch spacing judiciously, even taking into account that there is a spread in the frequencies of this mode among the cavities.

5 Discussion

E. Keil mentioned that he had asked for two parasitic experiments before the next MD period in week 28:

1. Fill bunch trains of just two bunches to 0.5 mA per bunch, and try to reach a beam current of 4 mA.

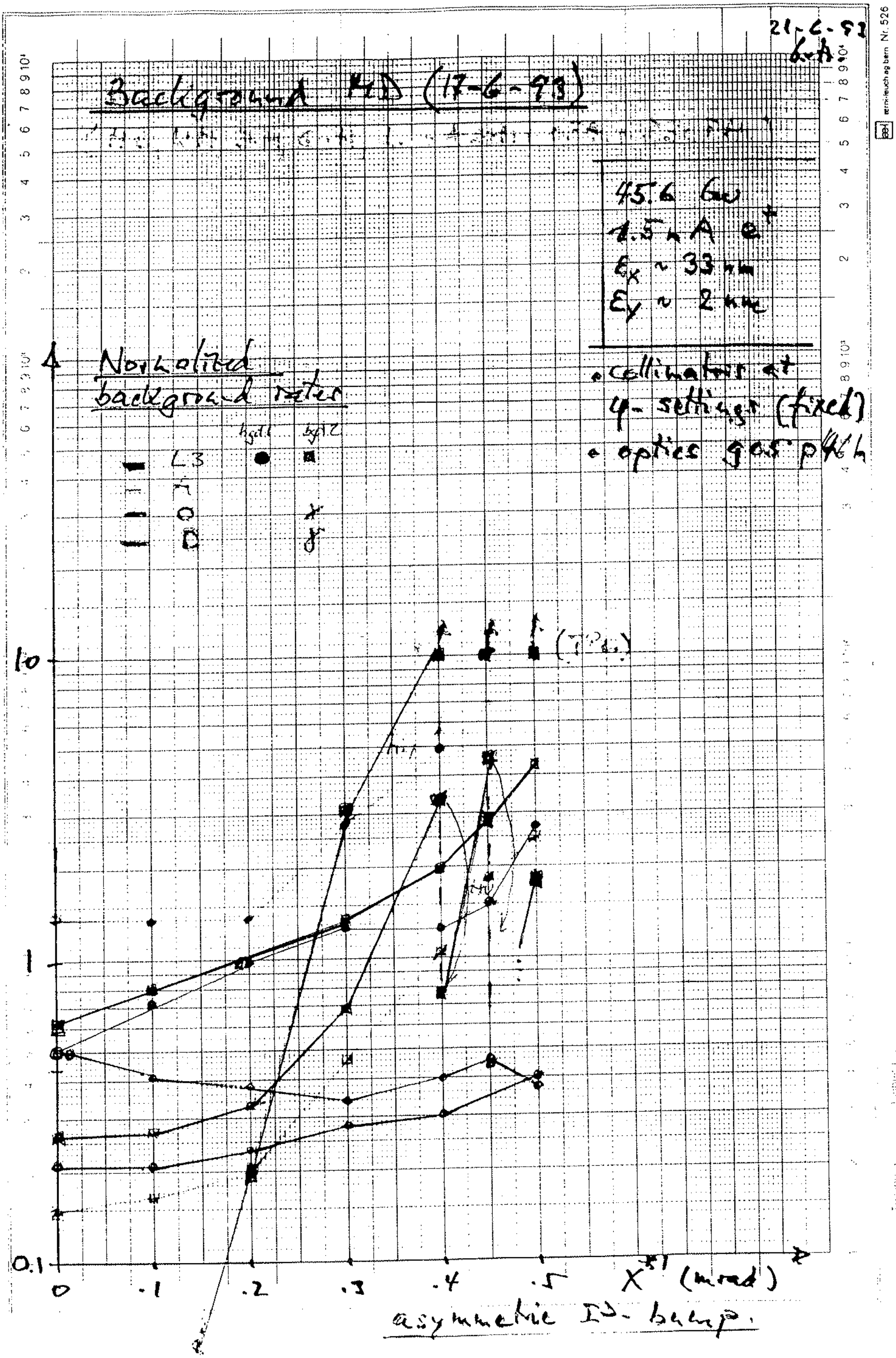
2. At the end of a physics fill, excite asymmetric horizontal bumps in the even pits, and check that two beam remain in collision.

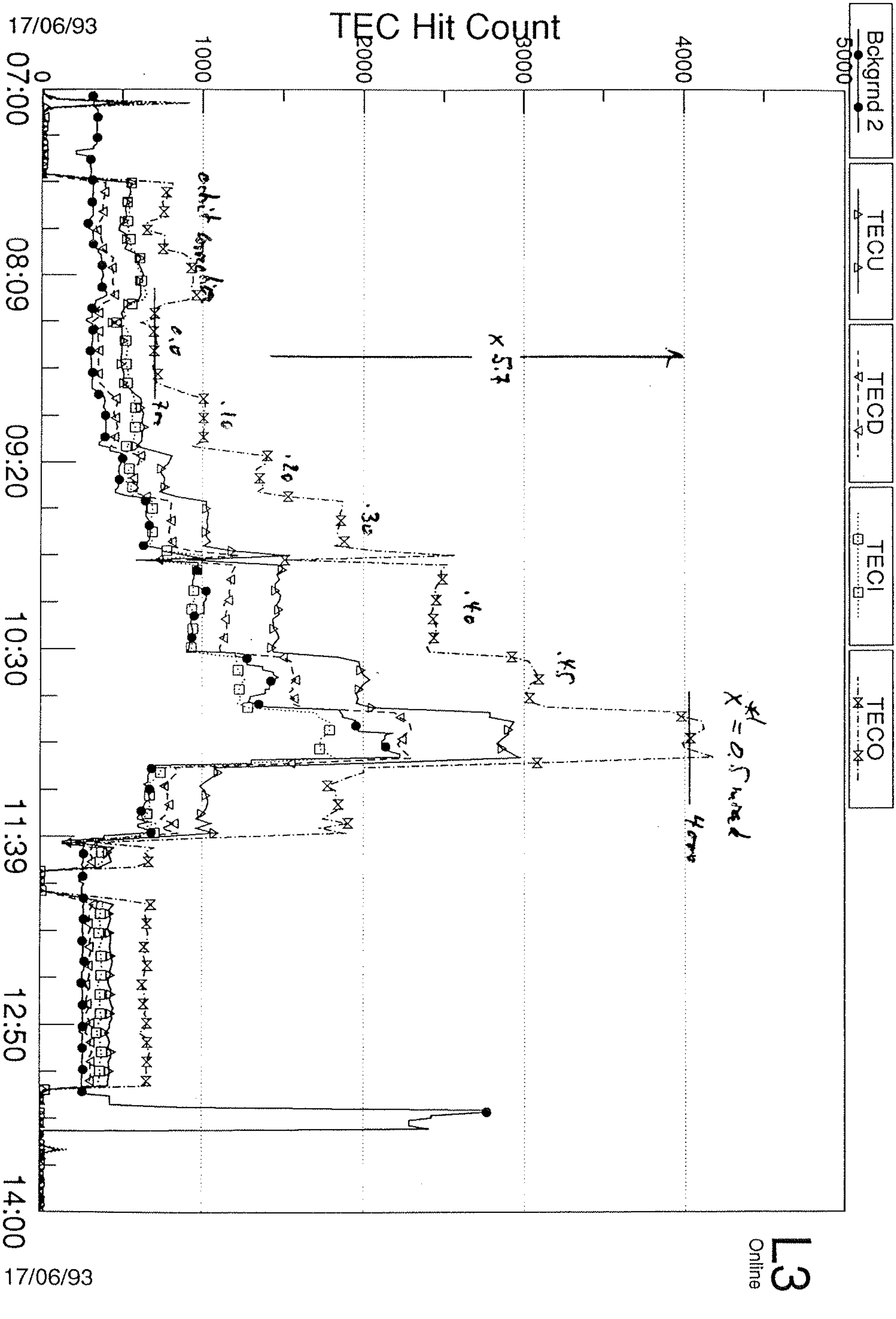
After a discussion it was agreed to submit two MD proposals for week 28, one related to background studies with incoming beams at a horizontal angle which will be looked after by G. von Holtey, and a second one devoted to the collision of bunch trains which will be looked after by E. Keil.

E. Keil reminded the working group that a report is expected by this summer, and asked the member to start thinking about their contribution.

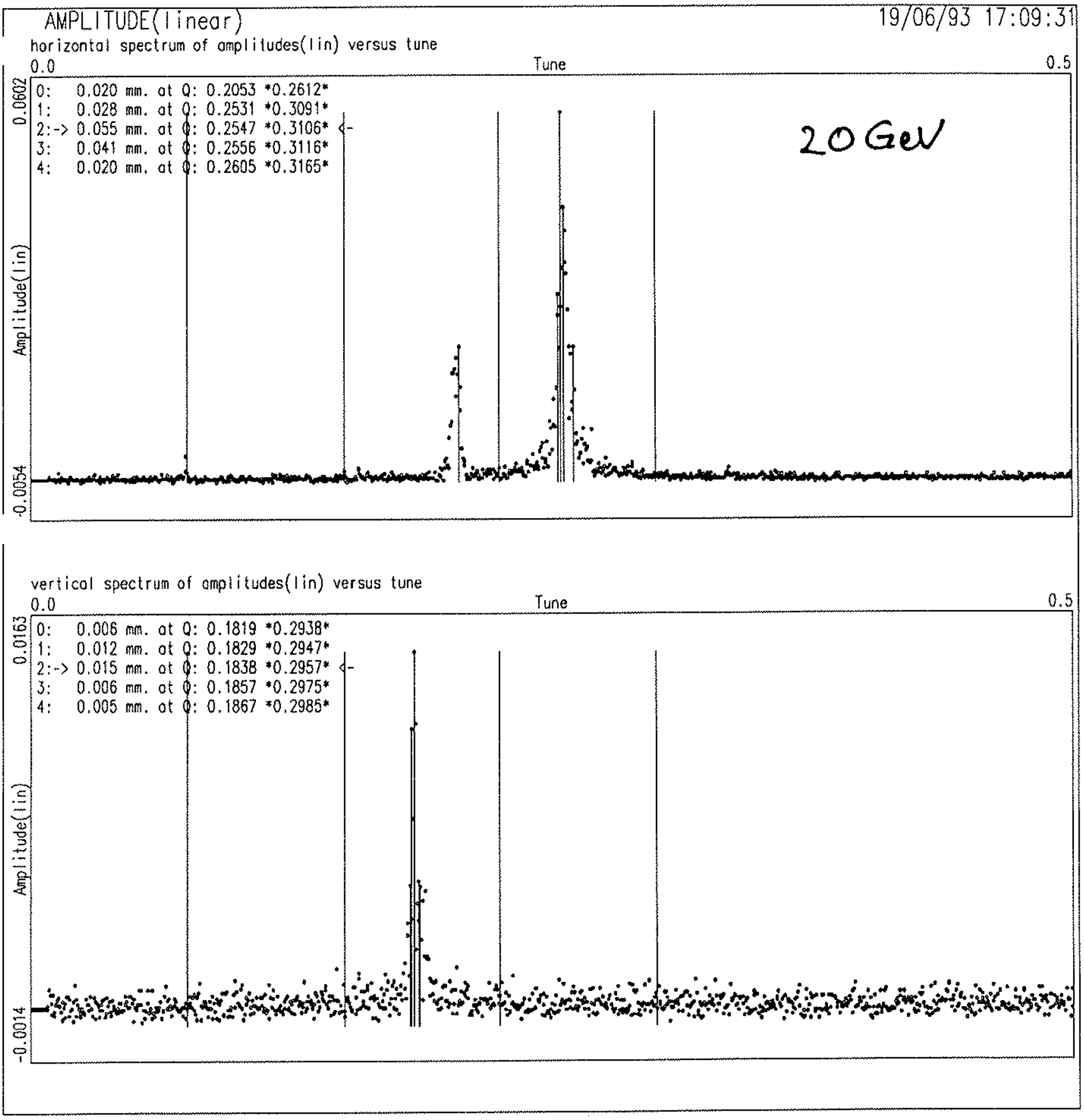
6 PostScript Output

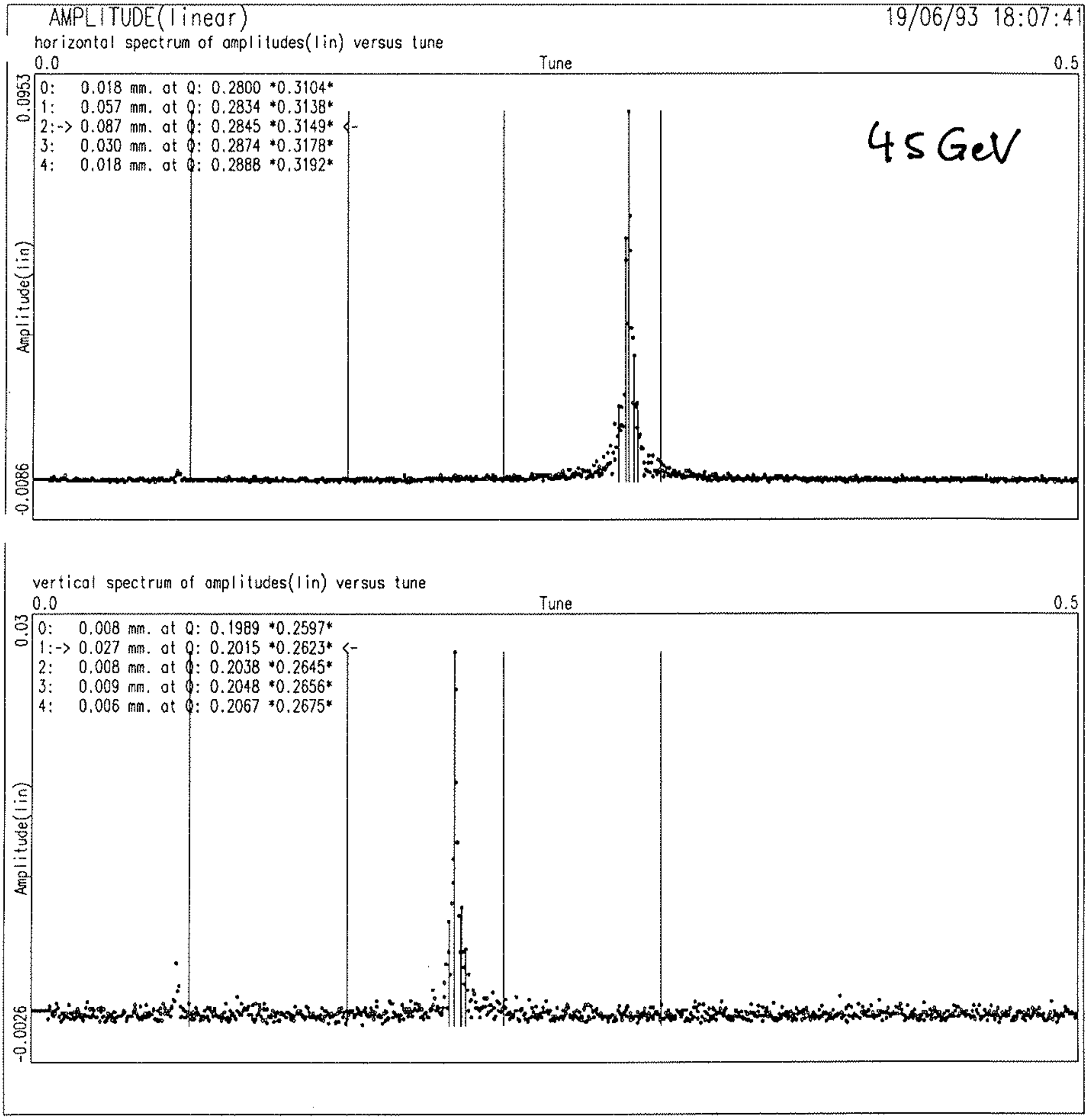
These minutes are also available as a PostScript file BT930622 PS on the A disk of KEIL.

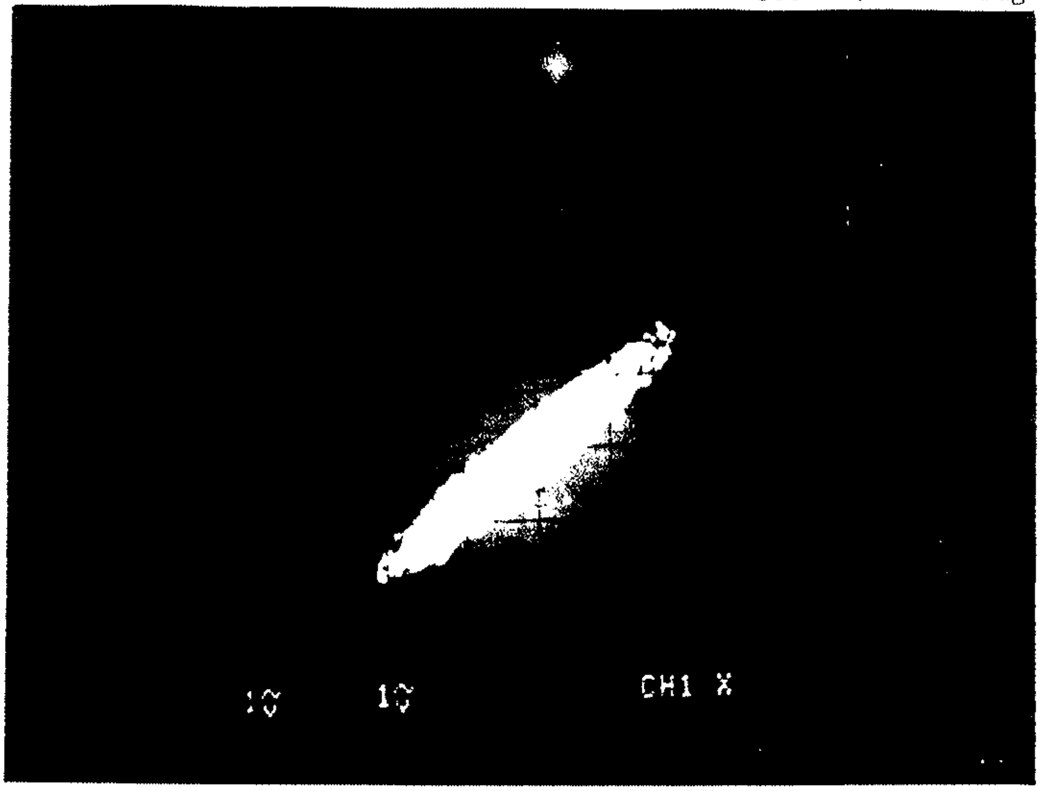

21-6-93
Background MD (17-6-93)
45.6 GeV
1.5 mA e+
εx ~ 33 nm
εy ~ 2 nm
Normalized background rates
L3
O
D
• collimators at 4 - settings (fixed)
• optics 90° p46h
10
1
0.1
0
.1
.2
.3
.4
.5
X^31 (mrad)
asymmetric ID - bump.

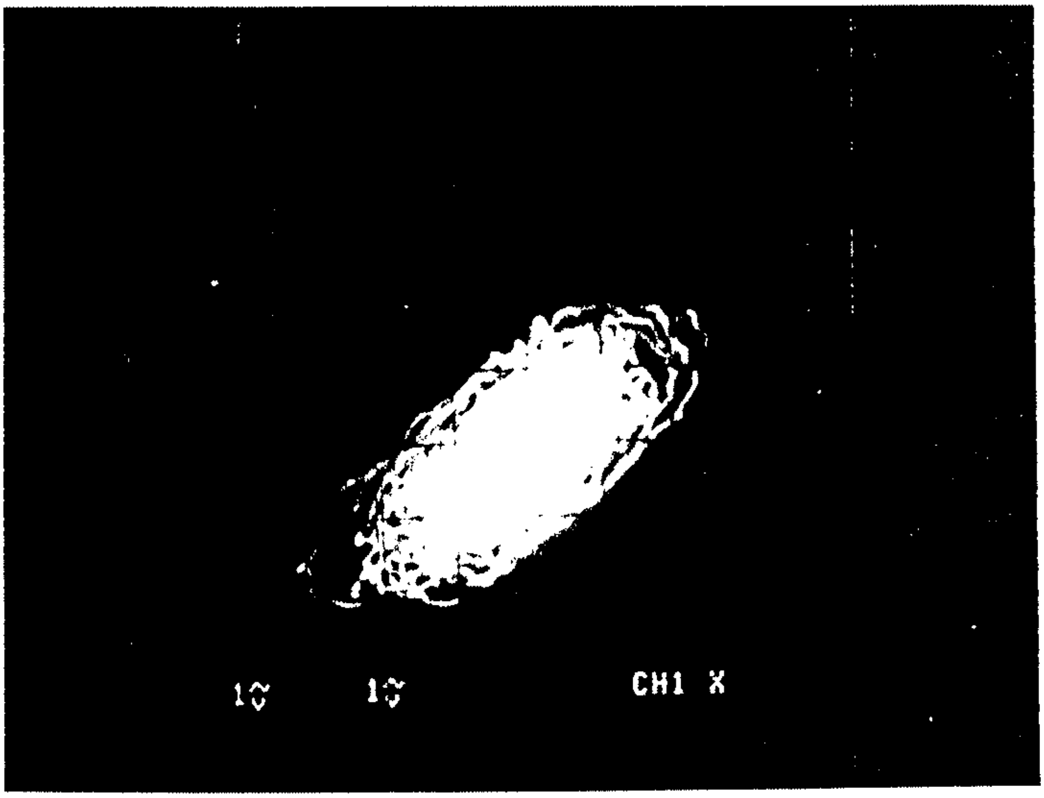

17/06/93
TEC Hit Count
0
1000
2000
3000
4000
5000
07:00
08:09
09:20
10:30
11:39
12:50
14:00
Bckgrnd 2
TECU
TECD
TECI
TECO
L3
Online
17/06/93

```
LE ->(- P                  Sat 19 Jun 93 17:21:42
Fill # 1632                E = 20.000 Gev/Beam

    NO BEAM         FILLING         COLLIDING

Intensities extracted from SPS (E9)
e1+ 24.3  e2+ 39.3     e1- 45.0   e2- 26.3
Intensities in LEP       Experimental
   Stack       Stack     Solenoids
   (uA)        (uA)                 (A)
1p 922.0  1e    0.0      L3          127
2p 900.9  2e    0.0      Aleph      4983
3p 914.8  3e    0.0      Opal       6997
4p 907.4  4e    0.1      Delphi     5002
COMMENTS Sat 19 Jun 16:50

 FILLING LEP

MD on bunch trains in progress
```


AMPLITUDE(linear)
19/06/93 17:09:31
horizontal spectrum of amplitudes(lin) versus tune
0.0
Tune
0.5
0.0602
-0.0054
Amplitude(lin)
0: 0.020 mm. at Q: 0.2053 *0.2612*
1: 0.028 mm. at Q: 0.2531 *0.3091*
2:-> 0.055 mm. at Q: 0.2547 *0.3106* <-
3: 0.041 mm. at Q: 0.2556 *0.3116*
4: 0.020 mm. at Q: 0.2605 *0.3165*
20 GeV
vertical spectrum of amplitudes(lin) versus tune
0.0
Tune
0.5
0.0163
-0.0014
Amplitude(lin)
0: 0.006 mm. at Q: 0.1819 *0.2938*
1: 0.012 mm. at Q: 0.1829 *0.2947*
2:-> 0.015 mm. at Q: 0.1838 *0.2957* <-
3: 0.006 mm. at Q: 0.1857 *0.2975*
4: 0.005 mm. at Q: 0.1867 *0.2985*


AMPLITUDE(linear)
19/06/93 18:07:41
horizontal spectrum of amplitudes(lin) versus tune
0.0
Tune
0.5
0.0953
-0.0086
Amplitude(lin)
0: 0.018 mm. at Q: 0.2800 *0.3104*
1: 0.057 mm. at Q: 0.2834 *0.3138*
2:-> 0.087 mm. at Q: 0.2845 *0.3149* <-
3: 0.030 mm. at Q: 0.2874 *0.3178*
4: 0.018 mm. at Q: 0.2888 *0.3192*
45 GeV
vertical spectrum of amplitudes(lin) versus tune
0.0
Tune
0.5
0.03
-0.0026
Amplitude(lin)
0: 0.008 mm. at Q: 0.1989 *0.2597*
1:-> 0.027 mm. at Q: 0.2015 *0.2623* <-
2: 0.008 mm. at Q: 0.2038 *0.2645*
3: 0.009 mm. at Q: 0.2048 *0.2656*
4: 0.006 mm. at Q: 0.2067 *0.2675*

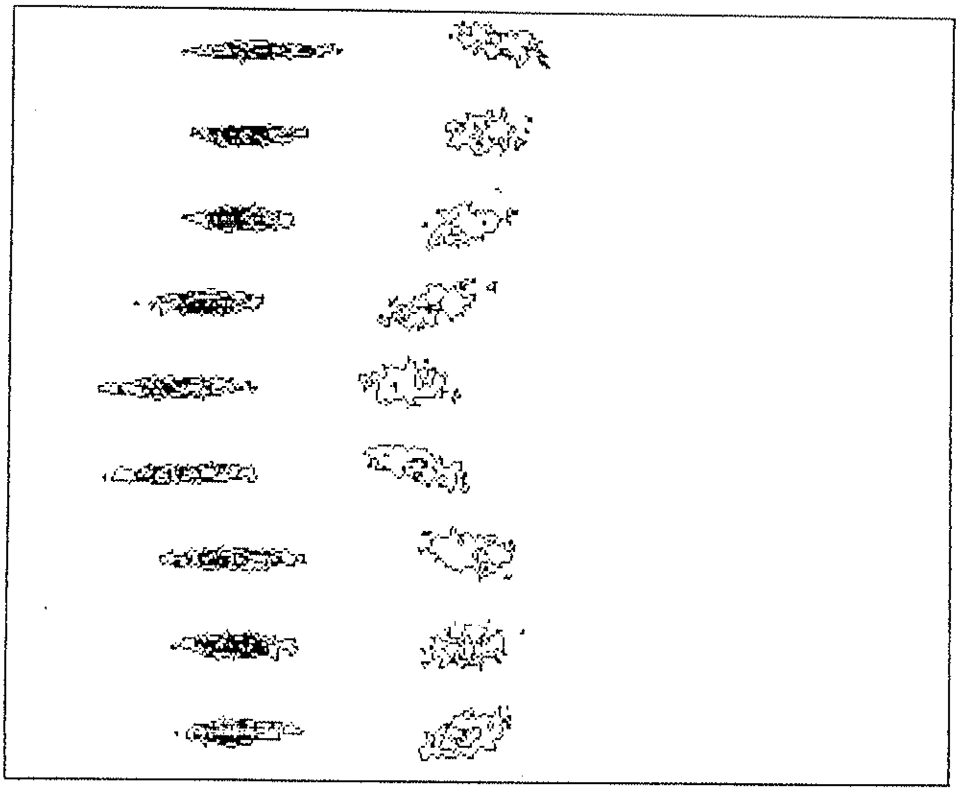

CH1 X


CH1 X

Technical observations

- On the first attempt of injecting into the second bunch, we accumulated 34 RF wavelengths behind the first bunch, although we had dialled in 32. We understand that before our MD the bunches were injected 2 buckets away from the correct buckets. This raises doubts about the resetting of the bunch address counter.
- The BOM system is supposed to read the position of the first bunch in a train. We measured the difference between orbits with one and more bunches in a train and found that it was small for a bunch spacing of 30 RF wavelengths, recorded as orbit.p_15-30-16.
- Orbit differences higher for WB than for NB monitors.
- The bunch equalizer software does not expect bunch train currents beyond 1 mA, and prevents such currents from being accumulated. This should be changed.
- Software by G. Morpurgo automatically changes the gain of the WB system with the bunch current. It should be adapted to bunch trains.
- The WB system does not work with five bunches in a train.
- Beam current transformer does not see the fifth bunch in a train.
- The software which computes the incoherent tunes from the coherent ones does not know about bunch trains.

σ_s



16.56

13.28

11.71

11.98

15.77

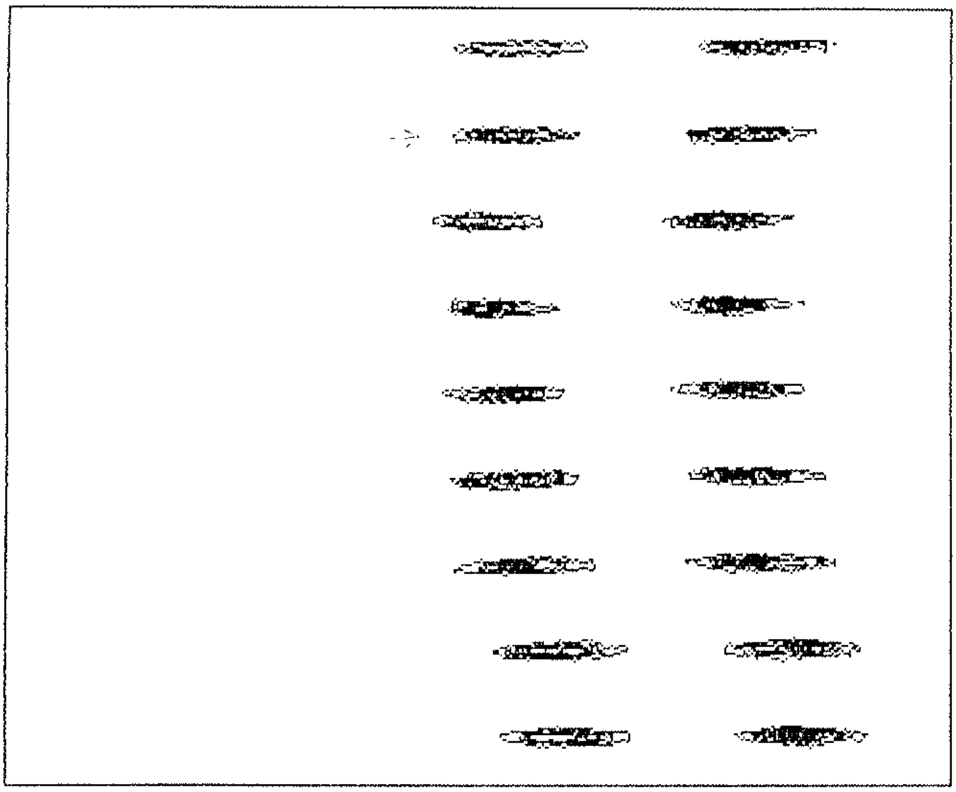
17.02

14.87

10.77

10.80

bunch no. 3 at 20 GeV



[illegible]

20:46

[illegible]

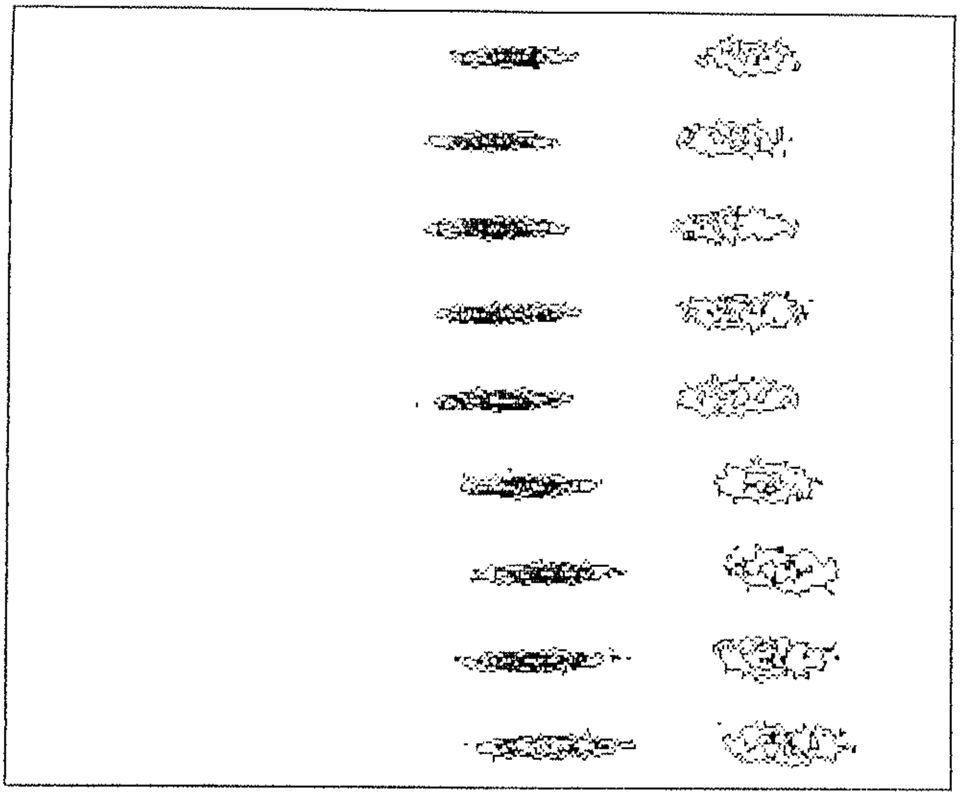
Bunch no. 1 at 20 GeV

[illegible]

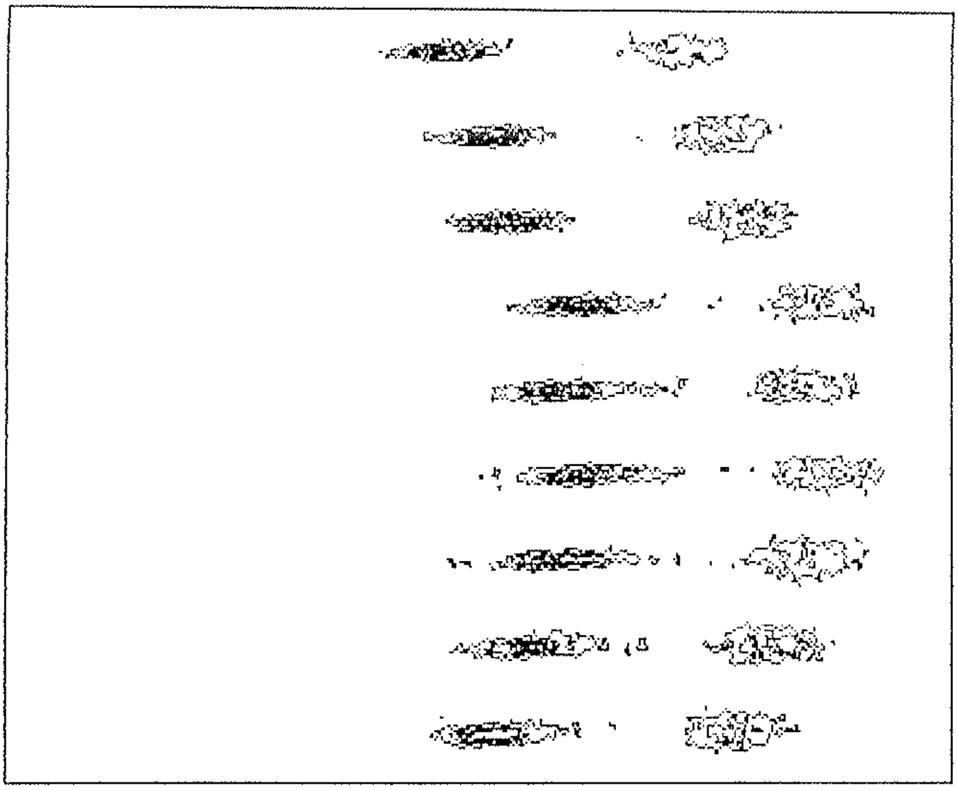
2nd of [illegible]

σ = 17 58 mm [illegible]

[illegible]



Bunch no. 2 at 20 GeV



Bunch no. 3 at 20GeV

Higher order mode measurements were done on the 8 superconducting cavities presently installed in LEP, numbered 1-4 and 13-16. A spectrum analyser and 3 wideband rf power meters were available.

Spectra

Higher order mode spectra and powers were measured in the rf units with about 50 to 100 m of coaxial cable between the cavities and the instrumentation. The spectra are corrected by the frequency dependent attenuation of the cables and the power meter readings can then be corrected by the ratio of the integrated corrected and uncorrected spectra.

An (uncorrected) spectrum of cavity 3 between 300 and 1500 MHz is shown in fig 1. The dominant contribution comes from the TM 011 mode passband at around 640 MHz.

Figs 2 to 5 show the spectra of cavity 2 around this mode with 4 single bunches and with 4 bunch trains of 3 bunchlets each. Note the "beat" pattern with peaks at 11 kHz distance and small peaks in between them. Fig 6 shows a narrow sweep (4 bunch trains with 3 bunchlets each) with lines spaced 11 kHz and higher ones spaced 45 kHz corresponding to 4 bunches.

In fig 7 the frequencies of the 4 members of the 640 MHz TM011 mode passband excited by the beam in the 8 cavities is shown. Table 1 gives the values.

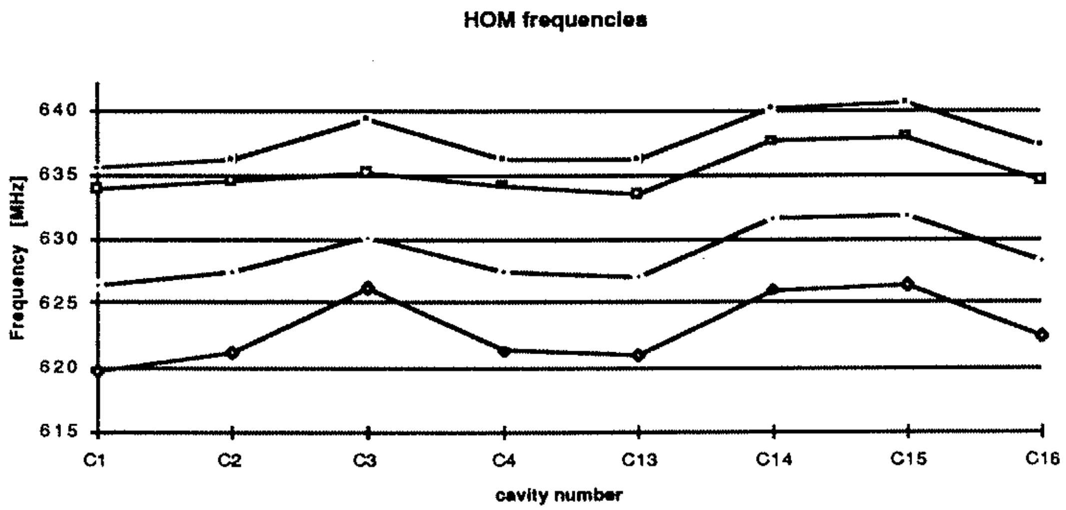


Fig 7 : frequencies of TM011 passband excited by beam.

HOM frequencies [MHZ] of TM 011 mode

C1	C2	C3	C4	C13	C14	C15	C16
635.5	636.3	639.3	636.2	636.3	640.1	640.5	637.3
633.9	634.4	635.1	634.3	633.6	637.6	637.9	634.5
626.4	627.5	630.1	627.4	627.1	631.6	631.9	628.3
619.8	621.2	626.2	621.4	621.0	626.1	626.4	622.5

Table 1 :frequencies of TM011 passband

The fundamental frequency of the cavities was not tuned to it's correct value, however the maximum tuning range for the TM010 mode is only 50 kHz. The influence on the TM011 mode is not yet measured.

HOM power

Table 2 and fig 8 give the corrected measured HOM powers for several bunch currents for 3 bunchlets per train. For comparison a measurement with single bunches is also included.

	no trains	4 trains of 3	4 trains of 3	4 trains of 3	4 trains of 3
	Power [W]	Power [W]	Power [W]	Power [W]	Power [W]
cavity nr	I_{total}=2 mA	I_{total}=3.6 mA	I_{total}=1.2mA	I_{total}=2.4 mA	I_{total}=3.6 mA
1	6.1	2.1	0.3	1.0	2.0
2	5.6	4.4	0.6	2.0	4.1
3	7.4	12.8	1.8	5.2	10.9
4	4.3	3.0	0.4	1.4	2.8
13	6.7	5.1	0.7	2.5	4.9
14	5.9	7.7	1.0	3.9	7.7
15	4.1	5.0	0.6	2.4	4.8
16	6.2	8.1	1.1	4.2	8.1
P coh	4.9	15.8	1.8	7.0	15.8
P incoh		5.3	0.6	2.3	5.3

Table 2 : Corrected HOM powers. The total power per cavity is 4 times the indicated value, because there are 4 couplers per cavity and each coupler has 2 rf absorbers. The last 2 rows give the power per cable expected for the two extreme cases of coherent addition of fields and of incoherent addition (with loss factor of 0.22 V/pC).

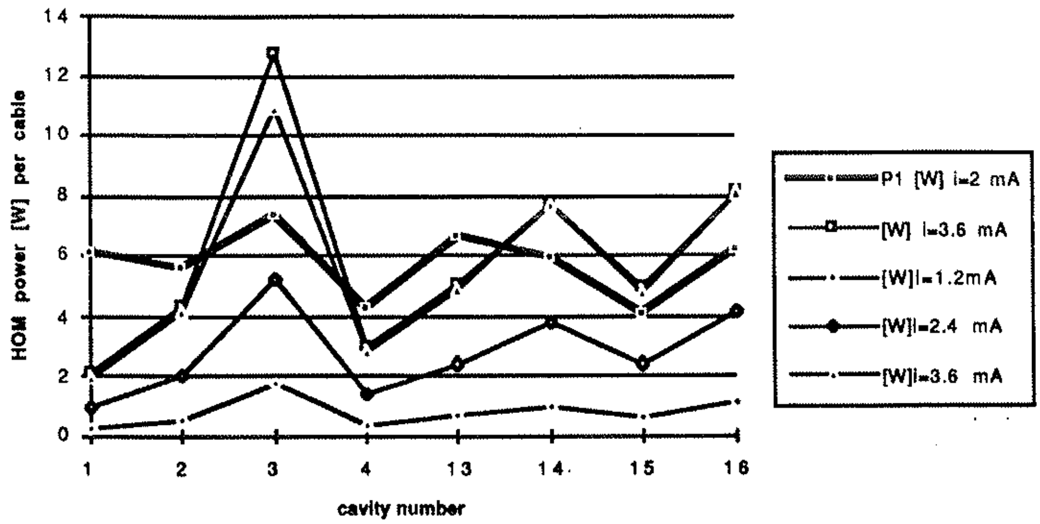


Fig 8: Corrected HOM powers per cable. The thick line is the measurement with four bunches, without bunchtrains.

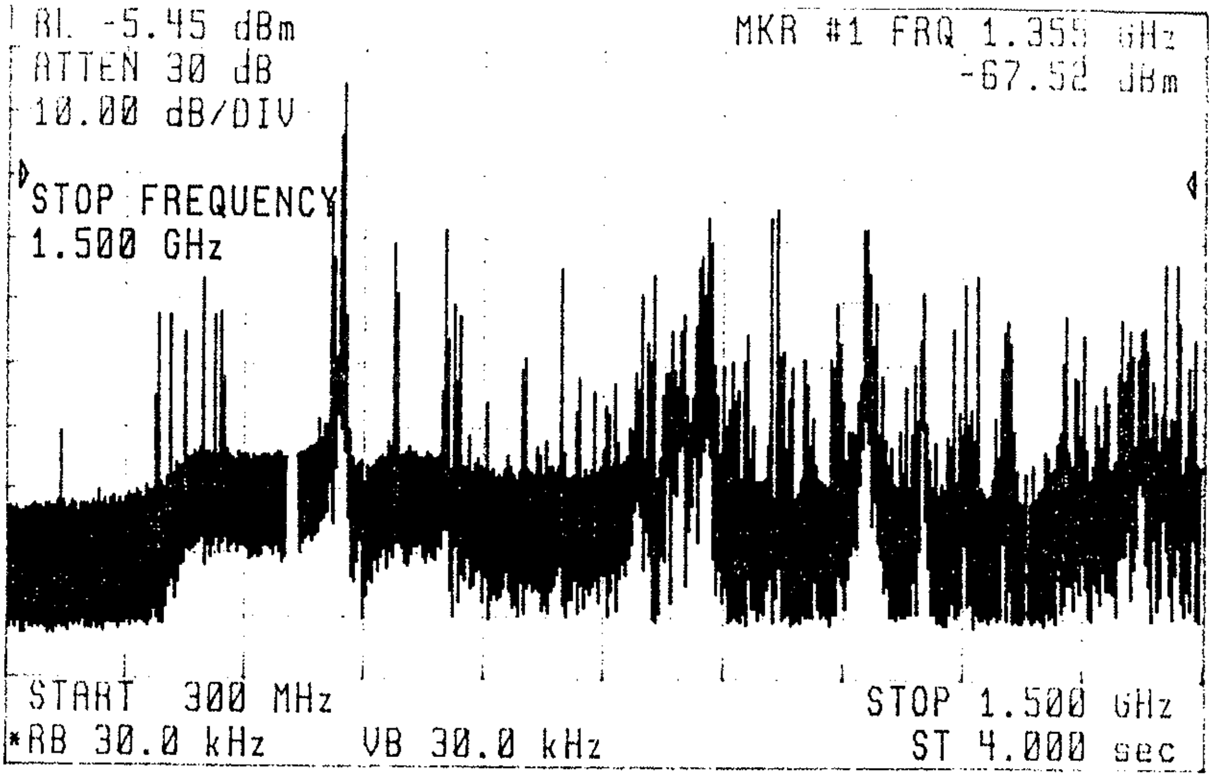


Fig. 1 HOM spectrum from 300 - 1500 MHz

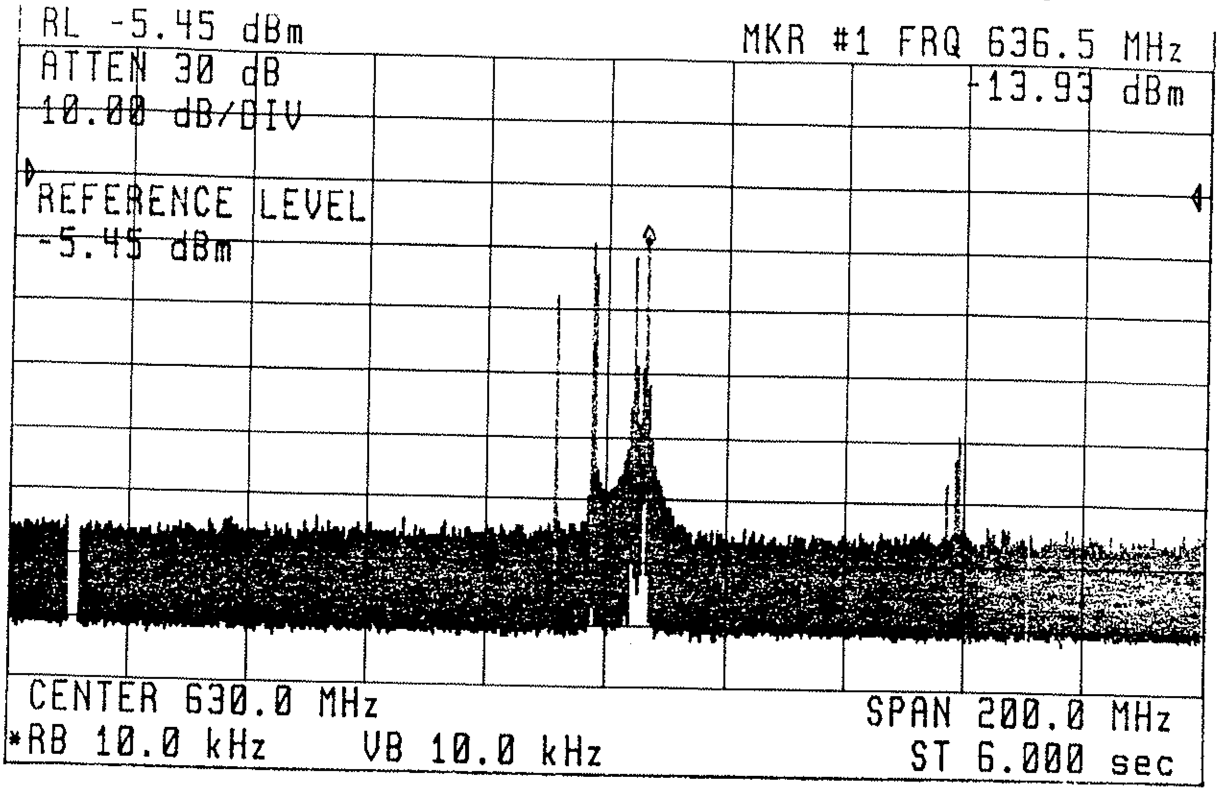


Fig 2 Hom spectrum without bunchtrains (4 bunches, one beam)

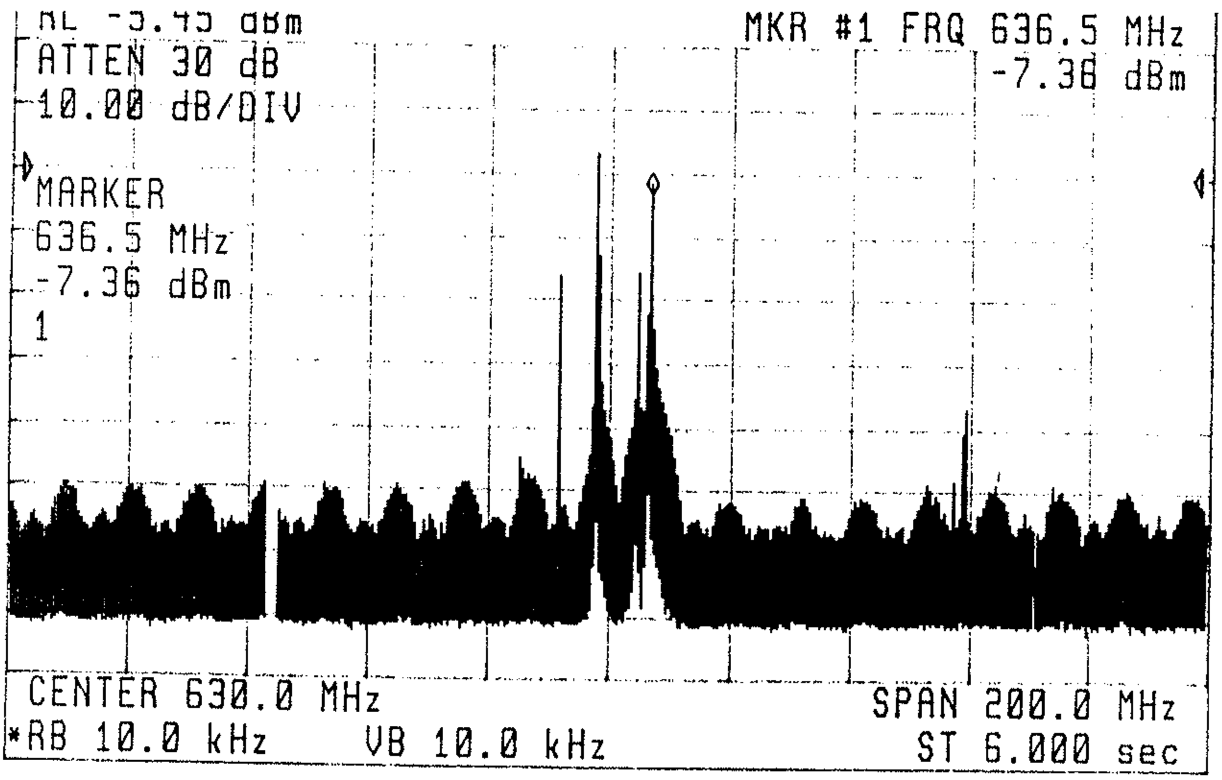


Fig 3: Hom spectrum with 4 bunchtrains, 3 bunchlets / train.

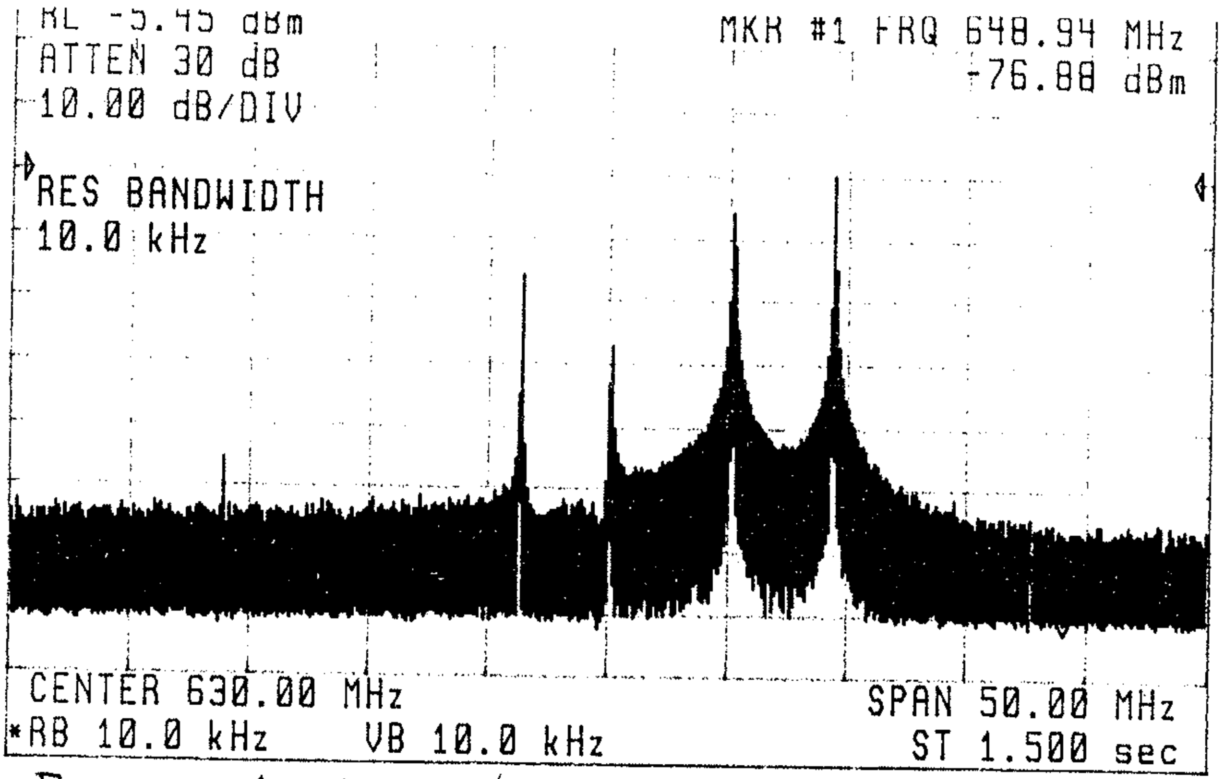


Fig. 4. Mode spectrum around TM 011 mode
no bunchtrains

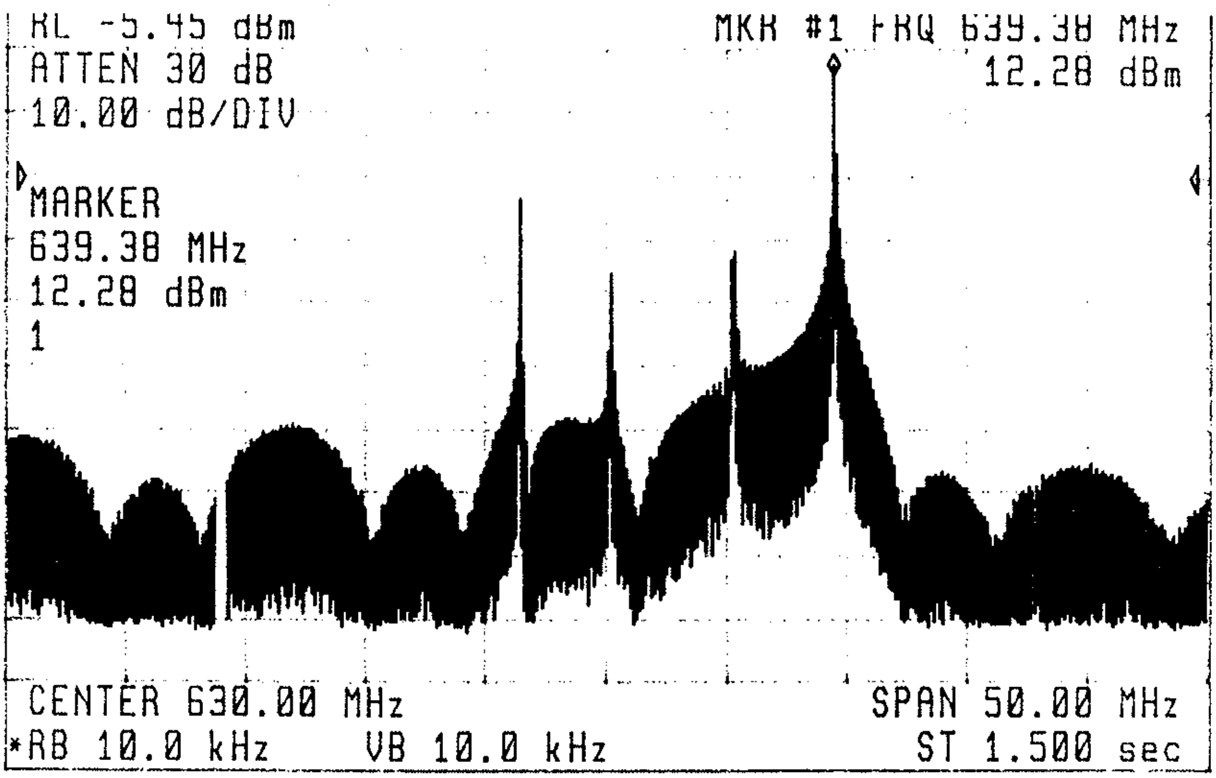


Fig 5. Mode spectrum around TM 011 mode,
4 bunchtrains (3 bunchlets)

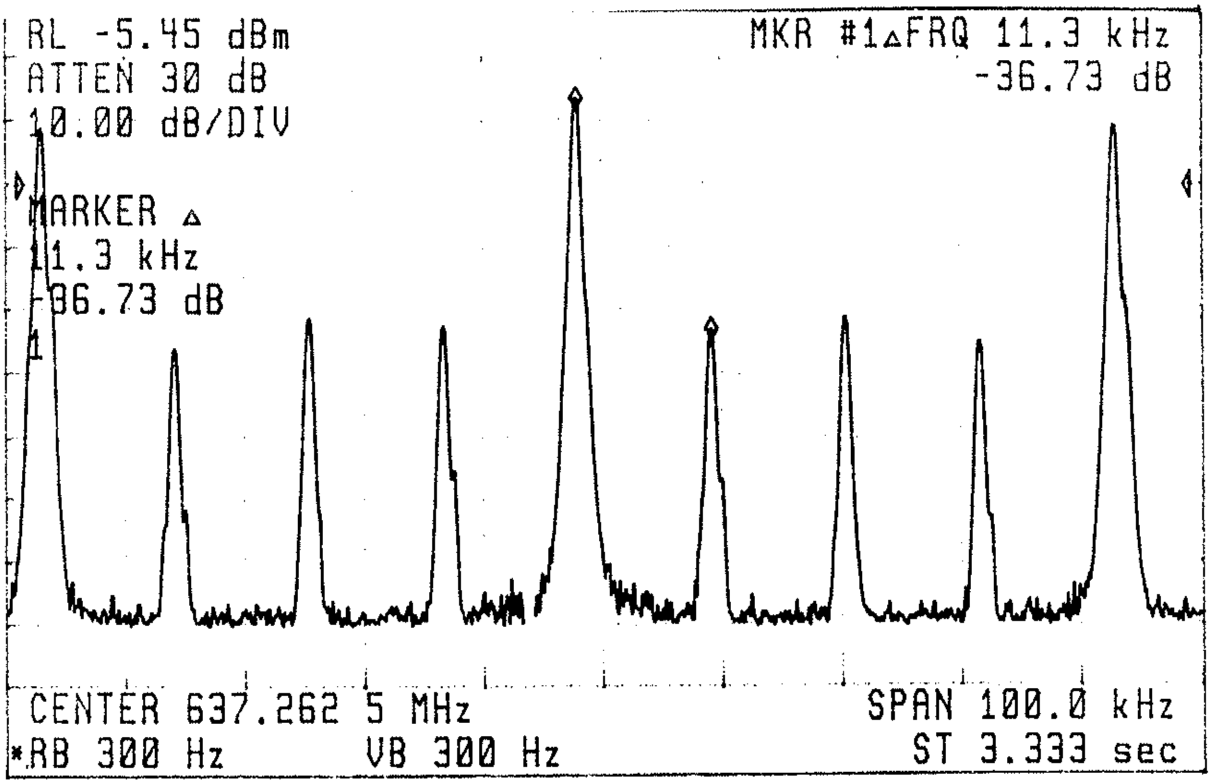
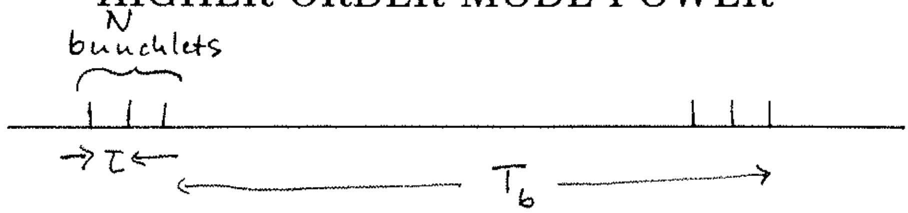


Fig 6 High resolution spectrum, 4 bunchtrains

G. Dôme, 22 June 1993

HIGHER-ORDER MODE POWER



N bunchlets of charge q and spectrum $q\ I(\omega)$ where $I(0) = 1$

$$\tau = mT_{rf} \qquad \frac{2\pi}{\tau} = \frac{1}{m}\omega_{rf}$$

For a bunchlet

$$i(t) = \frac{q}{2\pi}\int_{-\infty}^{+\infty} I(\omega)\ \mathrm{e}^{j\omega\left(t-\frac{z}{v}\right)}\mathrm{d}\omega \qquad \vec{v} = v\vec{1}_z$$

with the origin of z taken in the middle of a cavity.

Gaussian bunchlet:

$$I(\omega) = \mathrm{e}^{-\frac{1}{2}\omega^2\frac{\sigma^2}{c^2}}$$

$$\mathrm{e}^- \text{ current : } \frac{q^-}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\omega\ I(\omega)\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{N-1}\mathrm{e}^{j\omega\left(t-n\tau-pT_b-\frac{z}{v}\right)} \qquad q^- < 0$$

If the first positron bunchlet crosses the middle of the cavity a time T after the first electron bunchlet, the corresponding current is

$$\mathrm{e}^+ \text{current : } -\frac{q^+}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\omega\ I(\omega)\sum_{p=-\infty}^{+\infty}\sum_{n=0}^{N-1}\mathrm{e}^{j\omega\left(t-T-n\tau-pT_b+\frac{z}{v}\right)} \qquad q^+ > 0$$

<u>Total current</u>

$$\begin{aligned} i(t) &= \frac{q^-}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\omega\ I(\omega)\sum_{p=-\infty}^{+\infty}\mathrm{e}^{j\omega(t-pT_b)}\sum_{n=0}^{N-1}\mathrm{e}^{-j\omega n\tau}\cdot\mathrm{e}^{-j\omega\frac{z}{v}} \\ &- \frac{q^+}{2\pi}\int_{-\infty}^{+\infty}\mathrm{d}\omega\ I(\omega)\sum_{p=-\infty}^{+\infty}\mathrm{e}^{j\omega(t-T-pT_b)}\sum_{n=0}^{N-1}\mathrm{e}^{-j\omega n\tau}\cdot\mathrm{e}^{j\omega\frac{z}{v}} \end{aligned}$$

where

$$\sum_{n=0}^{N-1}\mathrm{e}^{-j\omega n\tau} = \frac{1-\mathrm{e}^{-jN\omega\tau}}{1-\mathrm{e}^{-j\omega\tau}} = \mathrm{e}^{-j(N-1)\omega\frac{\tau}{2}}\frac{\sin\left(N\omega\frac{\tau}{2}\right)}{\sin\left(\omega\frac{\tau}{2}\right)}$$

$$\sum_{p=-\infty}^{+\infty}\mathrm{e}^{-j\omega pT_b} = \sum_{p=-\infty}^{+\infty}\delta\left(\frac{\omega\ T_b}{2\pi}-p\right) = \omega_b\sum_{p=-\infty}^{+\infty}\delta(\omega-p\omega_b) \qquad \omega_b = \frac{2\pi}{T_b}$$

$$\begin{aligned} i(t) &= \frac{q^-}{2\pi}\omega_b\sum_{p=-\infty}^{+\infty} I(\omega)\mathrm{e}^{j\omega t}\mathrm{e}^{-j(N-1)\frac{\omega\tau}{2}}\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\cdot\mathrm{e}^{-j\omega\frac{z}{v}} \\ &- \frac{q^+}{2\pi}\omega_b\sum_{p=-\infty}^{+\infty} I(\omega)\mathrm{e}^{j\omega(t-T)}\mathrm{e}^{-j(N-1)\frac{\omega\tau}{2}}\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\cdot\mathrm{e}^{j\omega\frac{z}{v}} \quad \text{where} \quad \omega = p\omega_b \end{aligned}$$

Expressed in terms of phasors:

$$i(t) = \text{Re}\left[\begin{array}{c} \frac{q^-}{T_b}\sum_{p=0}^{\infty}\epsilon_p\,e^{j\omega t}I(\omega)e^{-j(N-1)\frac{\omega\tau}{2}}\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\cdot e^{-j\omega\frac{z}{v}} \\ \\ -\frac{q^+}{T_b}\sum_{p=0}^{\infty}\epsilon_p\,e^{j\omega t}e^{-j\omega T}I(\omega)e^{-j(N-1)\frac{\omega\tau}{2}}\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\cdot e^{j\omega\frac{z}{v}} \end{array}\right] \qquad \omega = p\omega_b$$

where ϵ_p is Neumann's symbol ($\epsilon_p = 1$ when $p = 0$, $\epsilon_p = 2$ when $p \neq 0$).

Driving terms

$$\begin{aligned}\int \vec{J}\cdot\vec{E}_n^*\,dV &= \int \vec{I}\cdot\vec{E}_n^*\,dz = \frac{1}{T_b}\sum_{p=0}^{\infty}\epsilon_p e^{j\omega t}I(\omega)e^{-j(N-1)\frac{\omega\tau}{2}}\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)} \\ &\cdot\left[q^-\int \vec{E}_n^* e^{-j\frac{\omega}{v}z}\vec{1}_z\,dz - q^+e^{-j\omega T}\int \vec{E}_n^* e^{j\frac{\omega}{v}z}\vec{1}_z\,dz\right] \qquad \omega = p\omega_b\end{aligned}$$

In a standing wave cavity, $\vec{E}_n$ may be taken as real. The field induced in mode $\vec{E}_n$ is

$$\begin{aligned}\vec{E}(\omega) &= -\vec{E}_n\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\,\frac{\int \vec{J}(\omega)\vec{E}_n^*\,dV}{\int |\vec{E}_n|^2\,dV} && (1)\\ k &= \frac{\omega}{c}, \qquad \text{Stored energy } W_n = \frac{\epsilon}{2}\int |\vec{E}_n|^2\,dV\end{aligned}$$

Power P_n going into the load: $P_n = \sum_{p=0}^{\infty} K(\omega)|\vec{E}(\omega) \text{ at coupling loop}|^2$

$$P_n = \sum_{p=0}^{\infty} K(\omega)\frac{|\vec{E}_n \text{ at coupling loop}|^2}{W_n}\frac{\epsilon^2}{4}\left|\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\right|^2 \underbrace{\frac{|\int \vec{J}(\omega)\vec{E}_n^*\,dV|^2}{W_n}} \qquad (2)$$

$$\frac{1}{T_b^2}\left|\epsilon_p I(\omega)\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\right|^2 \qquad \underbrace{\left|e^{j\omega\frac{T}{2}}q^-V_{\|n}^* - e^{-j\omega\frac{T}{2}}q^+V_{\|n}\right|^2}_{|V_{\|n}|^2\left|q^-e^{j\left(\omega\frac{T}{2}-\phi\right)} - q^+e^{-j\left(\omega\frac{T}{2}-\phi\right)}\right|^2}\frac{1}{W_n}$$

where

$$V_{\|n} = \int E_{zn}e^{j\frac{\omega}{v}z}\,dz = |V_{\|n}|e^{j\phi}$$

with the origin of z taken in the middle of the cavity;

$$\phi = \begin{array}{ll} 0 \text{ or } \pi & \text{if } E_{zn} \text{ is an even function of } z \\ \pm\frac{\pi}{2} & \text{if } E_{zn} \text{ is an odd function of } z\end{array}$$

At resonance, $k = k_n$ where $k_n^2 = \omega_n^2\epsilon\mu$,

$$\left|\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\right|^2 = \left|\frac{Q_n}{\omega_n\epsilon}\right|^2 \qquad\qquad \frac{|V_{\|n}|^2}{W_n} = 4k_{\|n}$$

$K(\omega)$ is a slowly varying function of ω; its value at ω_n may be obtained as follows.

At frequency ω_n, the power going into the load is

$$P = \frac{\omega_n}{Q_{n\,\mathrm{ext}}} W \tag{3}$$

where the stored energy is obtained from (1) as

$$\begin{aligned} W = \frac{\epsilon}{2}\int |\vec{E}(\omega)|^2\,\mathrm{d}V &= \frac{\epsilon}{2}\left|\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\right|^2 \frac{\epsilon}{2}\frac{\left|\int \vec{J}(\omega)\vec{E}_n^*\,\mathrm{d}V\right|^2}{W_n} \\ &= \frac{\epsilon^2}{4}\left|\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\right|^2 \frac{\left|\int \vec{J}(\omega)\vec{E}_n^*\,\mathrm{d}V\right|^2}{W_n}\end{aligned}$$

Comparing with (2) and (3) shows that

$$K(\omega_n)\frac{\left|\vec{E}_n \text{ at coupling loop}\right|^2}{W_n} = \frac{\omega_n}{Q_{n\,\mathrm{ext}}}$$

If we neglect the variation of $K(\omega)$ with ω, (2) may be rewritten as

$$P_n = \frac{\omega_n}{Q_{n\,\mathrm{ext}}}\sum_{p=0}^{\infty}\frac{\epsilon^2}{4}\left|\epsilon_p\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\right|^2 \underbrace{\frac{\left|\int \vec{J}(\omega)\vec{E}_n^*\,\mathrm{d}V\right|^2}{W_n}} \qquad \omega = p\omega_b \tag{4}$$

$$\left|I(\omega)\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\right|^2 \frac{1}{T_b^2}\left|q^- e^{j\left(\omega\frac{T}{2}-\phi\right)} - q^+ e^{-j\left(\omega\frac{T}{2}-\phi\right)}\right|^2 \frac{\left|V_{\parallel n}\right|^2}{W_n}$$

Reduction factor for $(Nq)^2$

$$\left|\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{N\sin\left(\frac{\omega\tau}{2}\right)}\right|^2$$

The peaks appear at all harmonics of $\frac{2\pi}{\tau} = \frac{1}{m}\omega_{rf}$.

In order to compute $\sum_{p=0}^{\infty}$, we consider that the last factor in (4) varies slowly with $\omega = p\omega_b$ because $\frac{\omega\tau}{2} = p\frac{\omega_b\tau}{2} = p\pi\frac{\omega_b}{\omega_{rf}}m$ where $\frac{\omega_b}{\omega_{rf}} \ll 1$, and we sum only the first factor.

$$\sum_{p=0}^{\infty}\epsilon^2\left|\epsilon_p\frac{j\omega\mu}{k_n^2\left(1+\frac{j}{Q_n}\right)-k^2}\right|^2 = \sum_{p=0}^{\infty}\left|\epsilon_p\frac{\omega}{\omega_n^2\left(1+\frac{j}{Q_n}\right)-\omega^2}\right|^2 = \frac{1}{\omega_b^2}\sum_{p=0}^{\infty}\left|\epsilon_p\frac{p}{\frac{\omega_n^2}{\omega_b^2}\left(1+\frac{j}{Q_n}\right)-p^2}\right|^2$$

It can be shown that

$$\sum_{p=0}^{+\infty}\left|\epsilon_p\frac{p}{(\alpha+j\beta)^2-p^2}\right|^2 = \frac{\pi^2}{\sinh^2(\pi\beta)+\sin^2(\pi\alpha)}\left[\frac{\sinh(2\pi\beta)}{2\pi\beta}-\frac{\sin(2\pi\alpha)}{2\pi\alpha}\right]$$

Here

$$(\alpha+j\beta)^2 = \frac{\omega_n^2}{\omega_b^2}\left(1+\frac{j}{Q_n}\right) \quad \text{hence} \quad \alpha = \frac{\omega_n}{\omega_b} \quad , \quad \beta = \frac{1}{2Q_n}\frac{\omega_n}{\omega_b} = \frac{1}{\omega_b\,\tau_n}$$

τ_n is the decay time constant of mode n. Finally (4) becomes

$$P_n = \underbrace{\frac{\omega_n}{Q_{n\,\mathrm{ext}}}\frac{1}{\omega_b^2}}_{\frac{Q_n}{Q_{n\,\mathrm{ext}}}\cdot 2\omega_b\beta}\quad\underbrace{\frac{\pi^2}{\sinh^2(\pi\beta)+\sin^2(\pi\alpha)}\cdot\left[\frac{\sinh(2\pi\beta)}{2\pi\beta}-\frac{\sin(2\pi\alpha)}{2\pi\alpha}\right]}_{\approx\frac{\pi}{\beta}\coth(\pi\beta)}\cdot\left|I(\omega)\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\right|^2 k_{\parallel n}$$

$$\cdot\frac{1}{T_b^2}\left|q^-e^{j\left(\omega\frac{T}{2}-\phi\right)}-q^+e^{-j\left(\omega\frac{T}{2}-\phi\right)}\right|^2$$

where now $\omega \approx \omega_n$. Finally

$$P_n \approx \frac{Q_n}{Q_{n\,\mathrm{ext}}}\frac{2\pi}{\omega_b}\coth(\pi\beta)\,k_{\parallel n}\quad\cdot\left|I(\omega)\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)}\right|^2 \cdot\frac{1}{T_b^2}\left|q^-e^{j\left(\omega\frac{T}{2}-\phi\right)}-q^+e^{-j\left(\omega\frac{T}{2}-\phi\right)}\right|^2 \qquad (5)$$

<u>Case of two equal beams:</u> $q^+ = -q^-$

$$\left|q^-e^{j\left(\omega\frac{T}{2}-\phi\right)}-q^+e^{-j\left(\omega\frac{T}{2}-\phi\right)}\right|^2 = \left|2q^-\cos\left(\omega\frac{T}{2}-\phi\right)\right|^2$$

<u>Case of a single beam:</u> $q^+ = 0$

$$P_n \approx \frac{Q_n}{Q_{n\,\mathrm{ext}}}\frac{1}{T_b}\coth(\pi\beta)\cdot\quad\underbrace{k_{\parallel n}\left|Nq^-I(\omega)\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{N\sin\left(\frac{\omega\tau}{2}\right)}\right|^2}_{\text{energy loss for a single passage of the bunch train in the cavity}}$$

Power lost by the beam **in the cavity**

$$= \frac{1}{T_b}\coth(\pi\beta)k_{\parallel n}\left|Nq^-I(\omega)\frac{\sin\left(N\frac{\omega\tau}{2}\right)}{N\sin\left(\frac{\omega\tau}{2}\right)}\right|^2$$

If $\beta > 1$, the interaction between bunches is negligible.
Power lost by the beam in the cavity

$$= \frac{1}{T_b} k_{\| n} \left| N q^- I(\omega) \frac{\sin\left(N\frac{\omega\tau}{2}\right)}{N \sin\left(\frac{\omega\tau}{2}\right)} \right|^2$$

$$= \frac{1}{T_b} \text{ Energy loss for a single passage of a bunch train in the cavity}$$

If $\beta \ll 1$,

$$\frac{1}{T_b} \coth(\pi\beta) \approx \frac{1}{T_b\, \pi\beta} = \frac{2}{T_b^2} \left(\frac{\omega_n}{2Q_n}\right)^{-1} = \frac{4}{T_b^2} \left(\frac{\omega_n}{Q_n}\right)^{-1}$$

Power lost by the beam in the cavity

$$= \underbrace{\left(\frac{\omega_n}{Q_n}\right)^{-1} k_{\| n}} \left| \frac{1}{T_b} N q^- 2 I(\omega) \frac{\sin\left(N\frac{\omega\tau}{2}\right)}{N \sin\left(\frac{\omega\tau}{2}\right)} \right|^2$$

$$= \frac{1}{2} R_n \qquad |I_b|^2 \qquad \text{as expected .}$$

<u>Unequal bunchlets</u>
If the N bunchlets have unequal charges q_n,

$$q \sum_{n=0}^{N-1} \mathrm{e}^{-j\omega n\tau} = q \mathrm{e}^{-j(N-1)\frac{\omega\tau}{2}} \frac{\sin\left(N\frac{\omega\tau}{2}\right)}{\sin\left(\frac{\omega\tau}{2}\right)} \Longrightarrow \sum_{n=0}^{N-1} q_n \mathrm{e}^{-j\omega n\tau} .$$

In[1]:=

```
resp[s_,k_]=(Sin[Pi h k s]/(k Sin[Pi h s]))^2
```

Out[1]=

$$\frac{Csc[h\ Pi\ s]^2\ Sin[h\ k\ Pi\ s]^2}{k^2}$$

In[2]:=

```
k=3
```

Out[2]=

3

In[3]:=

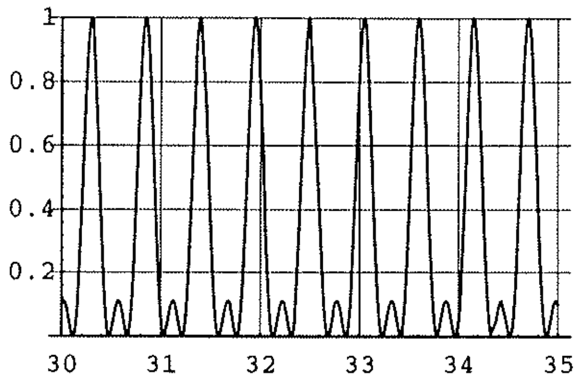
```
h=639./352.
```

Out[3]=

1.81534

In[4]:=

```
Plot[resp[s,k],{s,30,35},PlotRange->{0,1},
GridLines->Automatic]
```



Out[4]=

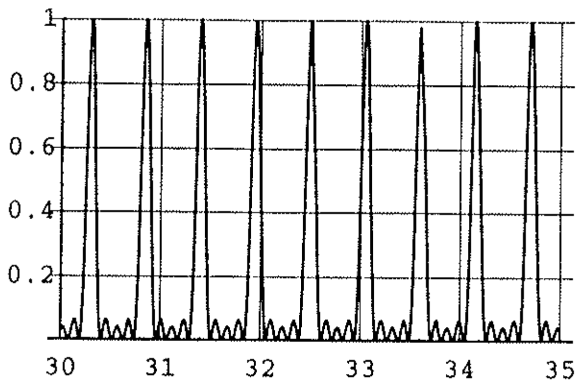
-Graphics-

In[5]:=

```
k=5
```

In[6]:=

```
In[4]
```

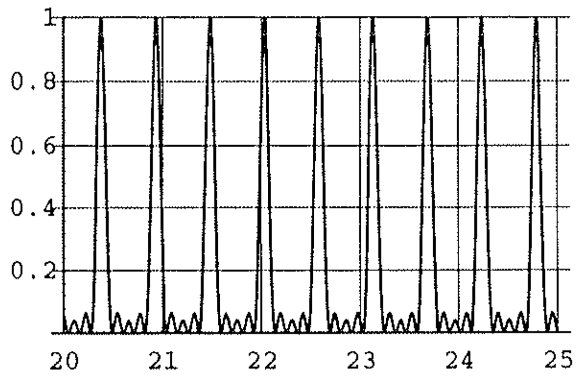


Out[6]=

-Graphics-

In[7]:=

```
Plot[resp[s,k],{s,20,25},PlotRange->{0,1},
GridLines->Automatic]
```



Out[7]=

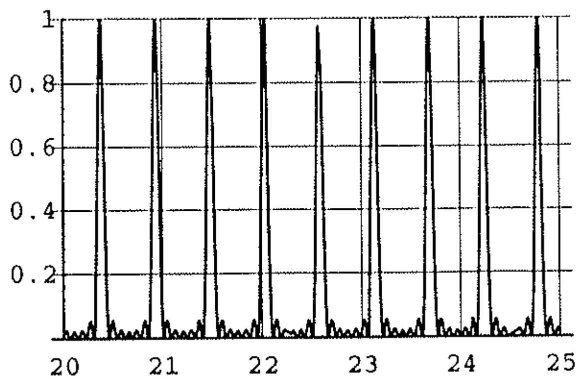
```
-Graphics-
```

In[8]:=

```
k=7
```

In[9]:=

```
In[7]
```



Out[9]=

```
-Graphics-
```